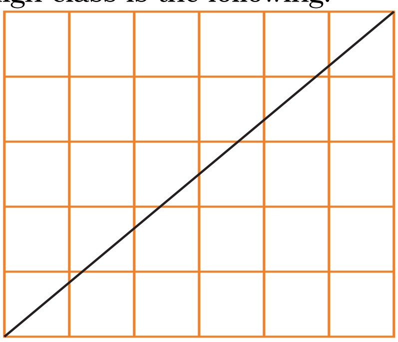
Diversions

with Barry Squire

Prediction is a great skill to have in any walk of life: it can, in fact, save your life at times. While the two investigations I pose for you in this edition may not be that dramatic, they might just increase your appreciation of some important connections between grids and rectangles and the divisors of numbers that appear in the dimensions of those rectangles (now that's a huge hint!).

Walk across my swimming pool?

Quite often the simplest ideas lead to some great results. A teacher was once challenged to look for ways to get his students thinking, rather than just doing sums. What resulted was a book called *Starting Points* (Banwell, Saunders & Tahta, OUP, 1972) and one of the questions posed to encourage some mental doodling, lateral thinking and discussion in a junior high class is the following.



- On squared paper draw a rectangle five squares by six squares.
- Draw a diagonal across this rectangle.
- How many squares does the diagonal pass through?
- Do this for other rectangles.
- Can you forecast the number of squares passed through for any sized rectangle? That is if you know the length and breadth can you predict the number of squares the diagonal cuts?

Now that seems an innocent enough question. Your task is to get some sheets of 1 cm grid paper and start drawing. I suggest some systematic routine be established or you may well miss the main idea. Working with a friend helps greatly.

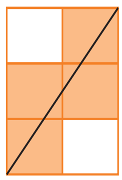
Play with sets of rectangles of a particular length and varying widths always keeping a table of results, that is, how many squares the diagonal passes through as compared to the length and breadth of the rectangle.

You will notice cases where the diagonal passes through an intersection of squares. When does that happen? What does that do to the square count?

A table like the following might help keep track of your findings:

Dimension	Squares cut
2×3	4
6×5	10

Example: a way to show the cut squares using a 2×3 rectangle

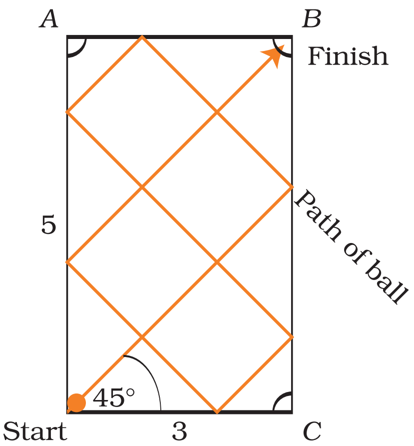


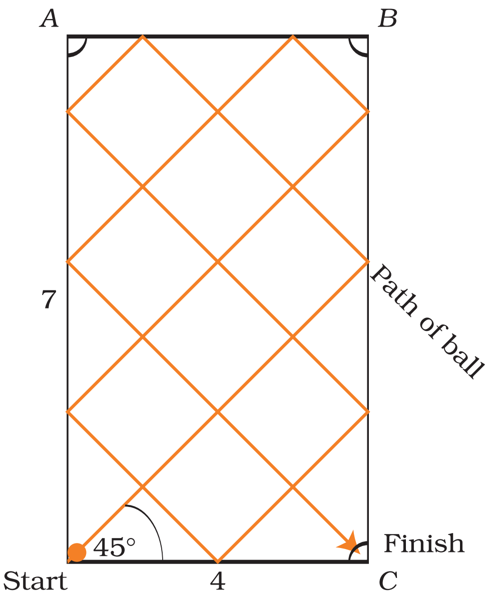
Try to write up your conclusion in as concise a way as you are able, so that, given the dimensions l and b of any size rectangle, you will be able to predict in terms of l and b how many squares the diagonal will cut. Keep a track of your predicted "formulae" as they occur to you as well as your test cases and the success or failure of those tests until you get to a prediction that seems to cover all cases!

Another "pool"? Tricky billiards

Next is a great activity from Randall Souviney's book of problems (*Solving Problems Kids Care About*, Goodyear, 1981, p. 132) with an appropriate title pun. Here is the scenario for our hero Bill Yud

> *Bill Yud pool*
> Bill Yud was a avid pool player. He enjoyed impressing his friends with his feats of skill. He invented a new pool table that could be adjusted to almost any size and had pockets in only three of the four corners. Someone could call out any size table and old Bill would think for a minute then point to one of the pockets. Next he would place the ball in the corner with no pocket and shoot out at a 45° angle. That ball would scoot all over the table and sure enough would fall into the chosen pocket. He never missed! Can you figure out what Bill was up to? Here are a few examples.





Use some 1 cm grid paper and "play" several games on different sized "Bill Yud" tables.

Organise the data in a table that indicates table dimensions and in which of the 3 pockets A, B, or C the ball finishes.

Look for patterns and particularly look for tables with similar "track" patterns. How are they related? This will simplify your task.

Try to predict how many rebounds the ball will have in its track around the table (include start and finish each as a "bounce").

Erratum

An error appeared in the Diversions solutions in *AMT 16*(4), page 17. The second paragraph should have read:

> Hopefully you will have seen that $n = 4$ and $n = 6$ do not yield prime numbers for $2^n - 1$ and hence there is no perfect number in those cases. The other 4 values for n in the table result in the four perfect numbers you verified earlier by factor sum.

and instead of showing $2n - 1$.