

# MATHEMATICS FROM STILL AND MOVING IMAGES

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Digital photos and digital movies offer an excellent way of bringing real world situations into the mathematics classroom. The technologies surveyed here are feasible for everyday classroom use and inexpensive. Examples are drawn from the teaching of Cartesian coordinates, linear functions, ratio and Pythagoras' theorem using still images, and quadratic functions using moving images. Resources and tips for creating suitable images for analysis are given.

The aim of this article is to demonstrate the potential for teaching mathematics using some of the technologies available to analyse still and moving images. These technologies have been awkward to use in the past, but are now becoming feasible for regular classroom use. In particular, working with still images is now very easy, with both equipment and software readily available. The curriculum ideas described here have been developed as part of the RITEMATHS project (Real world problems and Information Technology Enhancing Mathematics), which is a collaboration between university researchers, six Victorian secondary schools and Texas Instruments. Further curriculum ideas and information about software can be obtained from our project website ([HREF1](#)), which will be regularly updated.

Educationally, some of the benefits of using still and moving images can be:

- to bring real world problems alive in the classroom,
- to personalise mathematics, thereby

increasing engagement of students,

- to integrate mathematics with many other subjects,
- to make good use of information technology and students' interest in it.

These benefits will be illustrated below by outlining some lesson ideas for junior secondary school using different software. Details of how to obtain the software are given at the end of the article. These lesson ideas are focussed on mathematical topics, but there is very rich potential to use this approach in integrating work with most other subjects (e.g. science, technology, media studies or art, physical education).

Both students or teachers can take appropriate photos and videos using home digital still and video cameras, and so there are real possibilities for capturing the immediacy of an event at school, and using images of personal interest to students. However, taking good images for analysis requires thought, to overcome problems such as perspective distorting measurements and angles. For this reason, developing a school collection of shared images is recommended.

For many years, it has been recognised that good teaching of secondary mathematics links the three representations of algebraic ideas: the symbolic, the numerical and the graphical. The use of digital images lets us add a visual fourth representation that is much closer to the real physical situation, and to do it in a way where the analysis can be fully integrated with the other representations.

## Mathematics from still images

### GridPic and the equations of straight lines

GridPic was created at Luther College by a talented former student, Gerard Visser, to the specifications of the mathematics teaching staff led by Ian Edwards. An image is easily imported into GridPic and Cartesian coordinates of points can be obtained automatically. For example, Figure 1 shows a photo of a scissor lift at the Ballarat Air Show, taken by Maxine Andrews of Mt Clear College. A window on the right hand side shows the coordinates of the marked points, in this case (3.3, -6.3) and (-7.6, -8.9). Straight lines, quadratics in different algebraic forms, and other common curves can be plotted in the same window by inputting coefficients in the designated entry-boxes. Four straight lines are shown on the photo. When they find the equations of the three lines with positive gradient, students will see that parallel lines have the same coefficient of  $x$  in their equations (here +0.5), whilst the “reflected” line has slope  $-0.5$ . This provides a new dimension to fitting functions to data. GridPic can be used with and without the photo, which provides a transition into abstract mathematics. GridPic was designed by teachers who wanted a product that was focussed on their curriculum, did not require familiarity with spreadsheets or allow access to automatic curve fitting functions and which controlled distracting variation between students such as placement of axes and scale. Availability is described on the RITEMATHS website (HREF1).

### DigitiseImage and a Water Jet

DigitiseImage (HREF2) is a small, free simple-to-use software application, created by Jeff Waldock of Sheffield Hallam University. It places a grid on the image, and outputs the Cartesian co-ordinates of selected points to a spreadsheet where curve fitting or calculations can be carried out. The user can choose the origin, the scale and the size of the markers on the points. Just one mouse click exports the points to Excel, where they appear automatically in columns A ( $x$ -coordinates) and B ( $y$ -coordinates) and are also displayed on an  $X$ - $Y$  scatter plot. Figure 2 shows another image

taken by Maxine Andrews of a water jet for irrigation in a paddock near her school in Ballarat. Points along the water jet have been marked and their coordinates (shown in this case in “twips” in the window on the right) can be exported to a spreadsheet, where the quadratic of best fit can be found automatically or manually. It is important that students understand the common occur-

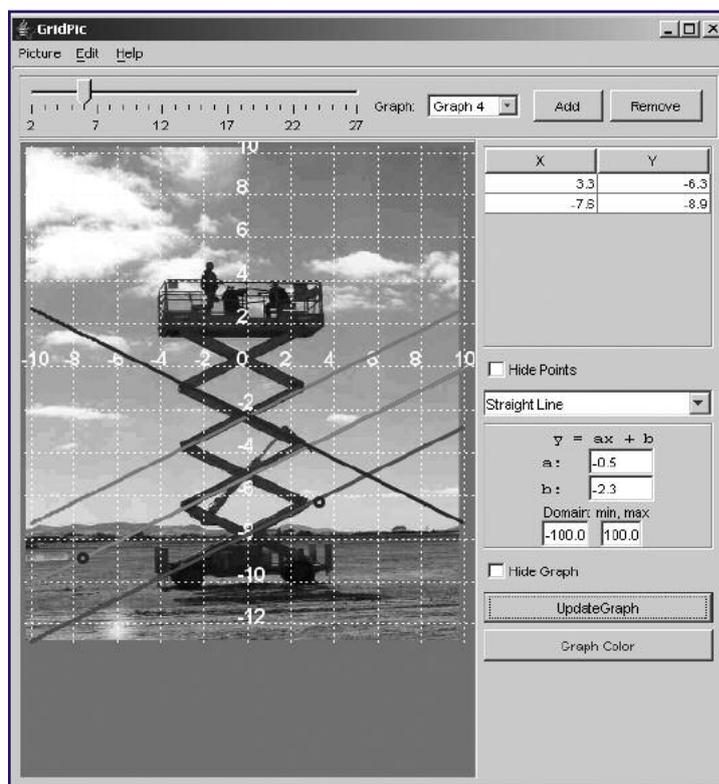


Figure 1. Screen shot from GridPic, using photo by Maxine Andrews of Mt Clear College.

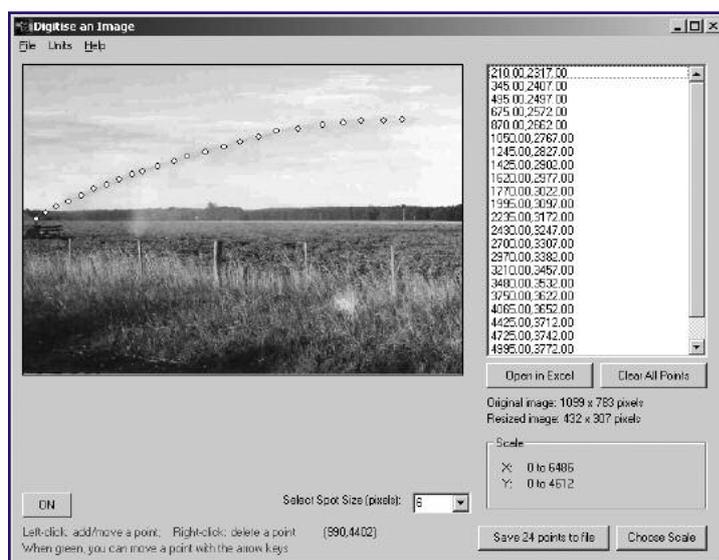
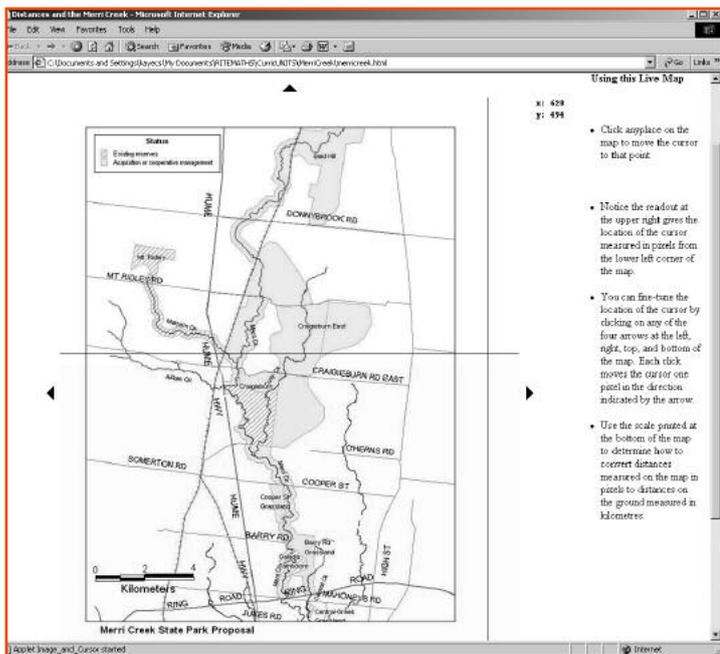


Figure 2. Screen shot from DigitiseImage, using photo by Maxine Andrews of Mt Clear College.

rences in real life of the mathematics that they learn, and quadratics should be associated with projectile motion of many sorts, including water jets. Oldknow (2003, HREF3) provides other examples and discusses how data can be taken, instead of into a spreadsheet, into *TI Interactive!* or graphics calculators to better deal with scale issues involved with this elegant, simple-to-use and powerful freeware.



## Image\_and\_Cursor and the Merri Creek

Image\_and\_Cursor is a small java application, which can be used in many ways by being embedded in web pages containing written instructions and information for students. Figure 3 shows a map of Merri Creek, in a webpage of instructions. The Image\_and\_Cursor applet outputs the coordinates of any selected points on the map. The coordinates are measured in pixels with (0,0) at the bottom left-hand corner of the image. This simple application could be used to familiarise students with Cartesian coordinates, as well as to take measurements from the map, for example to calculate the distance between Mt Ridley and the creek.

With a further “lite app” embedded in the webpage, the coordinates of marked points can be listed and then either exported to a spreadsheet or entered by hand into a graphics calculator. Lengths, distances and areas can then be calculated. For example, from a list of coordinates of points along a segment of the creek, Pythagoras’ theorem could be used to find the length of the section, by using Excel and filling down a formula such as  $(=sqrt((A2-A1)^2+(B2-B1)^2))$  and adding the lengths obtained.

Image\_and\_Cursor is one of a series of “lite apps” accessible through the *Journal of On-*

Figure 3. Screen shot of a webpage showing map courtesy of the Friends of Merri Creek, with coordinates of point showing at top right hand side. The webpage is based on material from JOMA.



Figure 4. A photo imported into The Geometer’s Sketchpad, showing two positions of a vertical line used for measurements.

*Line Mathematics* (JOMA) (HREF4). These are most easily used with a direct Internet connection, but they can be downloaded. An interesting example is “Water Squirt” which has some of the curve fitting capabilities of GridPic. The RITEMATHS website (HREF1) has links and information about how to handle the files involved. It is easier to use the images provided, but your own images can be substituted with just a little programming.

### Introduction to similarity and ratio using dynamic geometry

Digital images can be imported into major dynamic geometry applications, such as *The Geometer’s Sketchpad* (HREF5) and *Cabri Geometry* (HREF6). They are then available for curve fitting or as a backdrop to geometric constructions. For example, Figure 4 shows a motorbike leaning against a wall with a large triangular feature. The lengths and ratios of the sides of the small and large right-angled triangles can be measured and the ratios can be calculated and seen to be invariant as the left-most vertical line moves.

In Figure 5, dragging the left-hand top point (B) of the picture changes its size, and possibly its shape. Students are challenged to move point B to keep the picture looking the same; an idea that is the basis of geometric similarity. The figure shows the trace of point B as the student tried to reduce the picture keeping it the same shape, and also the diagonal line upon which it should have moved (added after the students tried this challenge). This exercise draws on students’ intuitive ideas of similarity and the growing familiarity that many of them have developed by using DVDs with aspect ratio, to introduce formal mathematical ideas and properties of similar rectangles, and hence similar triangles.

### Specialised image software

Other specialised software for analysing images may be useful for project work integrated with other subjects, especially at senior levels. For example, the free product ImageJ (HREF7) can be used to count the number of microbes on a slide, or give the percentage of a photo of a leaf that is bright green, or find the area under a curve.

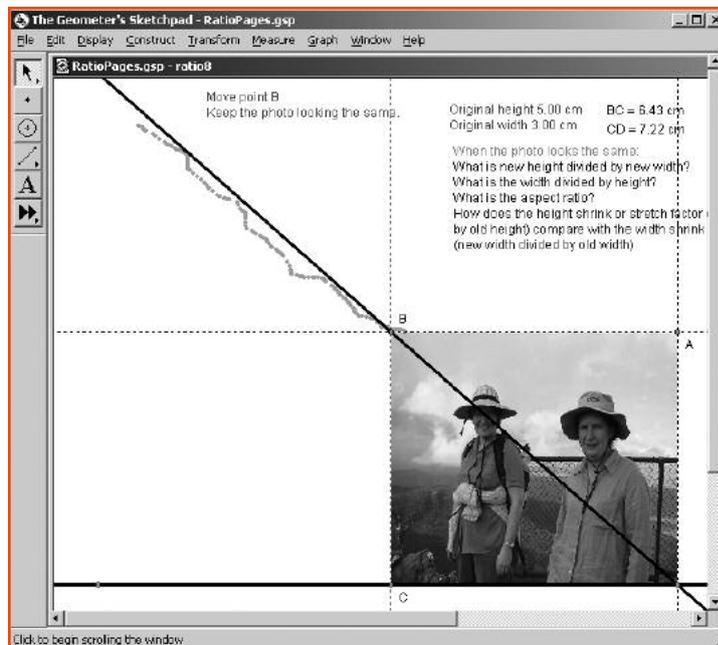


Figure 5. A photo of two of the authors imported into *The Geometer’s Sketchpad*, showing trace of point B which a student was trying to move so that the picture stayed the right shape.

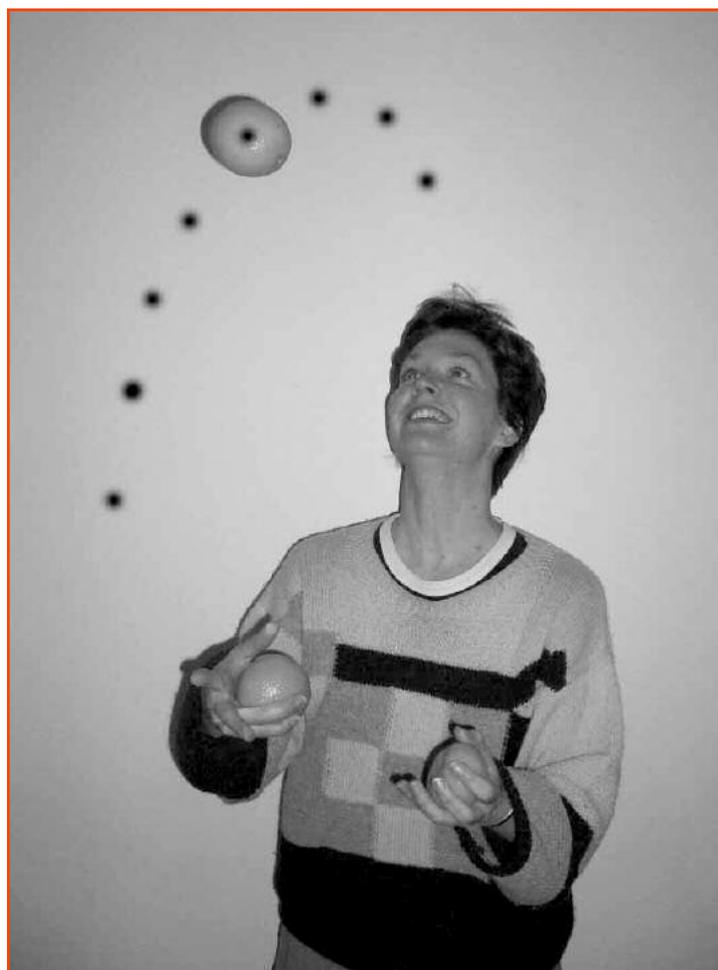


Figure 6. Data from video can be analysed with respect to time, by marking each frame.

## Mathematics from digital movies

Data highly relevant to students' lives can be captured from digital movies taken with domestic digital video cameras. There are many products available for the analysis of movies, including some free products discussed below, and although they remain a little harder to use than software for still images, their use is now becoming feasible with only a moderate time investment. The software purchased by many school science departments for use with physical data logging equipment may be suitable. For example, *Logger Pro* (HREF8) software collects a wide variety of data using Vernier's *LabPro* interface (e.g. light intensity, pH, temperature, etc), but can also be used to obtain data from video images. It supports statistical analysis and curve fitting, can synchronise movies with data collection, and can playback experiments at different speeds. Coordinating a purchase of software and hardware with your science department is highly recommended.

Movies are analysed in a similar way to still images. Typically, a point (such as the centre of the juggling ball) is marked by hand on each frame of the video. Figure 6 shows the juggling ball marked in one frame and the positions of the ball in other frames. The video analysis software exports the coordinates of each of the points. The software calculates time in seconds from the frame-number. Hence it is possible to graph  $y$  against  $x$ , or either  $x$  or  $y$  against time. For realistic modelling, it is good to have a scale in the movie (e.g., a ruler behind the juggler on the wall). In the case of an object thrown into the air, the graph of  $y$  against  $x$  will (ideally) be a parabola, with the gradient at the start showing the angle at which the object is thrown. The graph of  $x$  against  $t$  will be approximately a straight line and the graph of  $y$  (in metres) against  $t$  (in seconds) will be a parabola, with first term approximately  $-4.9t^2$  (reflecting acceleration due to gravity).

Analysing movies lends itself to investigations in physics, and especially to ideas of position, velocity and acceleration and all of the advanced software will move automatically between position-time graphs, velocity-time graphs and acceleration-time graphs. Since these are rate of change concepts, this can be

of great value in teaching calculus. There are, however, a variety of situations that can be used to support teaching other mathematics topics. For example, as well as movies of fair ground rides, demonstration movies included with our early version of *Measurement in Motion* (HREF9) included a candle burning and changes in cross-sectional area of coloured liquid in a transparent cube as the cube changes position.

We have experimented successfully with two free video analysis products. *Datapoint* (HREF10) is very simple to use. It provides a list of coordinates  $(t, x, y)$ , which can be copied into a spreadsheet or graphics calculator. *Vidshell* (HREF11), highly recommended by Adrian Oldknow (2003), provides more features and a great set of sample movies. The commercial products such as *Measurement in Motion*, *World in Motion* (HREF12) and *Logger Pro* provide more features and are certainly worth investigating with your science department. Further details, sample activities and links on all of the above are provided on the RITEMATHS website (HREF1).

## Obtaining useful images and movies

Creating a good movie or image for digital analysis requires some thought. Sometimes all that is needed is the general shape of an object. For example, you may want to see if a rainbow is part of a circle or not. You may want to see which of a water jet, a cross section of a headlight or a hanging chain are parabolic in shape. Taking such photos requires a direct view of the object, so that it is not distorted by the perspective, and a normal lens that does not distort much. The photo needs to be taken "front on", with all parts of the image approximately the same distance from the camera. Of course, it is also possible to study perspective effects themselves by comparing measured distances on an image with the measured distances on the real object. For example, our early version of *Measurement in Motion* included a movie of a truck travelling along a freeway directly away from the camera on a bridge across the freeway. The change in its apparent size could be calculated by measuring the truck in

successive frames. Landmarks along the freeway could assist in calibrating distance away from the camera.

If measurements are required, it is a good idea to place something of known size in the photo. For example, a metre ruler on the wall immediately behind the juggler in Figure 6 would have been useful. Note again that the ruler would need to be very close to the juggler to avoid problems of the apparent size of ruler and object changing with distance from the camera. Different software tools provide different ways of linking the “real” scale and the scale in the photos, which may be measured in pixels or in twips (one twentieth of a printer’s point, and equal to 1/567 of a centimetre). Sometimes conversion to the real world units can be done within the software; sometimes not. Obviously there are many opportunities here for students to use their understanding of ratio and proportion.

Movies for analysis can be very short. Twenty or fewer frames are adequate for many purposes, including to analyse motion such as a basketball shot. Because not many frames are involved, the mark-up process need not be sophisticated. Clicking on each frame to identify a point is not a hard task. Similarly, if the program does not have its own mathematical analysis tools (e.g. spreadsheet and graphing capability), it is good if data is immediately exported to another program, but retyping into a graphics calculator or spreadsheet is not necessarily an onerous task. Some experimentation with the correct digital compression formats is also required, so that images are not blurred digitally in the storage process.

In summary, using still and moving images in teaching mathematics is a technique that is just beginning to be practical. We already have many good ideas for how this can be used, and some excellent resources. However, there is still great potential for creative new ideas to grasp the opportunities to bring the real world into the classroom.

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