Accessing results from research on technology in mathematics education

Kaye Stacey
University of Melbourne
<k.stacey@unimelb.edu.au>

In his recent article in this journal, John Gough (2004) concludes that the practical results of the considerable academic research undertaken at the University of Melbourne into the uses and impacts of graphics calculators and CAS (computer algebra systems) should be made more widely available in journals for teachers. This short article is intended to provide teachers and others with a brief guide to our research and how the findings and products can be obtained most easily. In this way, we hope that teachers might be able to access what they need from our research. The publications referenced here are the more accessible articles, mainly in Australian teachers' conferences and journals, rather than those internationally published. Our websites (HREF1 and HREF2) have complete listings and many of the referenced articles can be downloaded from there (go to HREF1 and then 'Selected recent publications').

A quick guide to our research

Since the early 1990s, a team at The University of Melbourne has been working on various education issues related to the use in secondary mathematics teaching of graphics calculators, including those with a computer algebra facility (CAS), and related computer software. We have done this because digital technology affects mathematics and mathematics teaching very fundamentally. The development of tools to make computation easier, more accurate and faster, has always been important to progress in mathematics; from counting aids, to the abacus, to logarithms, the slide rule and now the historic change to digital processing in the computer and calculator. In the world of work, these improved methods of mathematical computation are unproblematic. There are issues of access and affordability and training for sensible use, but beyond what can easily be done mentally, computation by machine is preferred at work to computation by humans.

Schools, however, need a multifaceted response. They need to teach students how to use modern methods of computation, and consequently we

are now well into a time of substantial curriculum adjustment. However, schools also have to ensure that students are masters of technology, not its slaves. We aim to give students a deep understanding of mathematical processes, both to equip them to operate appropriately in everyday situations and to guide their use of machines. This needs to be at a simple level ('Have I received approximately the right change?') and at a complex level ('Why can't my CAS solve this quintic equation in exact mode?'). Deep understanding will involve learning both with and without technology. Finding the best mix needs research. In addition to these aspects of student learning, there are important practical issues of equity, teacher training, assessment, access etc. In all, this makes a very comprehensive mix of interesting and important research questions.

Our early University of Melbourne work, especially that of Gary Asp, Barry McCrae, John Dowsey, David Tynan and myself, was with spreadsheets and computer graphing. The latter has had its greatest impact in schools with the advent of the graphics calculator. Some selected resources and articles from this work are described below. This experience demonstrated the substantial impact that a personal technology (i.e., portable and relatively inexpensive) could have on daily classroom practices; an impact that was most clearly demonstrated when Year 12 examinations in our state permitted graphics calculator use. This experience led our team, now greatly enriched by Lynda Ball, Peter Flynn, Robyn Pierce, Margaret Kendal and others, to research the more controversial issue of access to full CAS in examinations, and this has been the main focus in recent years. One of these research projects, the CAS-CAT project (HREF1), accompanied the development of the new Year 12 subject Mathematical Methods (CAS) for the Victorian Certificate of Education. Stacey (2003) summarises many results of this project. The rest of this paper provides a guide to selected curriculum materials and research papers, written by our team members, our postgraduate students and also by some of the teachers with whom we conduct our research.

Teaching resources

Graphic Algebra —
Teaching units introducing functions in context with a graphics calculator

Our early research projects resulted in the creation of a textbook of replacement units for students in Years 8–11. *Graphic Algebra* (Asp, Dowsey, Stacey & Tynan, 1995; 1998) contains units covering linear, quadratic, exponential and reciprocal functions studied in real contexts with a functions approach using graphics calculators. These texts are useful for anyone introducing ideas about functions, as well as making first use of graphics calculators or computer packages such as *Graphmatica* in middle secondary years. The teachers' guide includes comments on the material and 'quick start' hints for using the most popular types of graphics calculators. Our research and evaluations of students' learning showed that students could learn to use graphing technology at the same time as they learned the new mathematical material, and

that they can learn new material in the context of real problems, although some lessons need to focus on the abstract mathematical objects. It also showed that developing a strong concept of the viewing window (which involves ideas of domain and range, scale and zooming) was critical to good use of graphing functionality.

Materials for teaching with CAS

The CAS-CAT website contains all of the teaching materials which were prepared for the first pilot of the subject Mathematical Methods (CAS) in Victoria. There are versions for CAS calculators from Casio, Hewlett-Packard and Texas Instruments. These materials are not polished, so you need to check them before use. There is also a project task, designed to be completed over several hours, related to the incidence of HIV-AIDS (see also Stillman, 2002; Stillman & Stacey, 2002). There are schematic maps showing the architecture of each brand of calculator, to help your students orient themselves as they explore the menus of these powerful machines. Teachers can access these resources from the CAS-CAT website (HREF1) by clicking on 'Resources for Teachers'. Many of our published papers also contain teaching suggestions and classroom examples, mainly for senior years; see, for example, Ball, Leigh-Lancaster and Stacey (2001) and Asp, Ball, Flynn and Stacey (2002).

Algebraic expectation quiz

Robyn Pierce developed this novel quiz to monitor students' algebraic insight. Just as students need good number sense for operating in everyday life and for using a four-function calculator well, older students need 'algebraic insight' to supplement their use of a computer or calculator with a CAS. Students need strong basic algebraic skills and a good intuition about symbols (i.e., algebraic insight) so that they do not reach for CAS for simple tasks.

The quiz can be administered to your class in just a few minutes, to pinpoint areas where their algebraic insight needs strengthening and re-administered to track progress. Download the quick quiz from the CAS-CAT website and administer it with a computer slide show. Two articles (Ball, Stacey & Pierce, 2001; Ball, Pierce & Stacey, 2003) provide results from senior mathematics students, so that you can see how your students compare. You can also quickly make up your own version for younger or more advanced students. Access the quiz from the CAS-CAT website 'Resources for Teachers' page (HREF1).

An orientation to teaching for algebraic insight can add new dimensions to teaching algebra. The components of this algebraic insight are set out by Pierce and Stacey (2002b). Algebraic insight can become a focus of classroom conversations which help students get an overall feeling of algebraic objects. For example, a teacher might routinely start discussion of a question by looking at the overall structure of an expression or equation, pointing out what are the main structural groups, what is varying and what is staying the same. Observations can also include the identification of key features and structures of algebraic expressions and how these key features and structures

are evident in graphical, symbolic and numerical representations. Pierce and Stacey (2002b) has more ideas. Teachers may be interested to read in Flynn, Berenson and Stacey (2002) about a survey asking teachers what by-hand skills should be mastered and which can be allocated to CAS. These questions will stimulate staffroom debate, and help teachers make explicit their beliefs and values about teaching mathematics. Whatever one's position on exactly what skills are needed, this exercise is thought-provoking and a worthwhile group exercise.

New classroom practices

There are many ways in which teaching practices might change when new technology is introduced. Several of the teachers with whom we have worked have written about their experiences of discovering how to teach with technology as well as the experiences of students in their classes. The articles by Garner (2002; 2004) and Tynan (2002) in the list below are examples. The experiences of other teachers, and the changes that they made, are reported by Stacey (2001). One interesting finding is that even in our small sample of teachers, markedly different teaching practices were evident. One teacher was always careful to develop 'understanding' without technology, stressing first of all the by-hand steps and then moving to CAS for harder questions. In contrast, another developed understanding by using CAS as a magic carpet to take students on a whirlwind tour of all the central ideas of the topic, and returned only later to look in more depth at the by-hand skills. From our research, we have no grounds for recommending the first 'white box, then black box' approach over the second 'black box then white box' approach.

An issue that concerned all of the teachers was how to help students become discriminating users of CAS. Students should not reach for CAS for simple tasks, but they do need to know how to make effective use of CAS when it can enhance their work. Pierce and Stacey (2002a, 2004) describe a framework that organises the components of 'effective use of CAS'. Teachers could use the framework to monitor students' progress towards becoming discriminating users and also use it as a checklist of important abilities to direct attention to in class. Students need skills in inputting and outputting information and in manipulating it and switching between symbolic, graphical and numerical (tabular) representations. Many students will also become aware of the potential of technology support to help them learn, not just to help them get answers.

Many teachers are concerned with the overhead of learning to use complicated machines. Our research results show that students with a positive attitude can learn to use their calculators well, both to solve mathematical problems and to help them explore concepts. There will always be new technical difficulties to overcome. When new mathematical topics are encountered, new facilities on the machine need to be mastered, and so new technical difficulties arise on a regular basis. However, this process becomes gradually easier as the fundamental architecture and logic of the calculator

or program is understood. From our research, attitude seems to be a more important determinant of success in learning to use technology than prior skills.

All of our project teachers were concerned about how students would write responses to mathematical questions when the 'working' is done inside the CAS. It was clear that copying intermediate results from the calculator screen was not a good practice to encourage. Additionally, intermediate results are often not available. To assist to develop a new practice, we developed the the Reasons, Information and Inputs, Plan, Answers (RIPA) framework (Ball & Stacey, 2003) to help students communicate the overall plan of their solutions, regardless of whether they do the calculations by hand or by CAS.

Finally, it is essential when contemplating any curriculum change to have clear goals about what might be achieved. My own priorities in incorporating CAS into senior mathematics (see Stacey, Asp & McCrae, 2001) were principally to enable students to engage more fully in solving real world problems, to increase the congruence between real mathematics and school mathematics and to encourage deeper learning. Teachers, on the other hand, very often put the goal of encouraging deeper learning much higher than the others. They also rank other goals highly, such as providing access to senior mathematics for students whose unassisted algebra skills would be a serious handicap. A survey at your own school may help clarify what it is most important to achieve with any changes.

Assessment when CAS is available

The great mathematical power of a CAS calculator or software program, especially its ability to work with algebraic symbols, means that examination processes must change if CAS is permitted. About 60% of the pre-CAS examination questions in Victoria are affected, and in many cases trivialised, by permitting CAS (McCrae & Flynn, 2001). The questions affected are often the most straightforward, and so removing all of these affected questions would make examinations much harder. A different response is required. Flynn (2001) makes some suggestions. An interesting finding is that the assessment is differently affected for different mathematical topics. Calculus tends to be handled very easily by CAS, since so many of the senior secondary questions are in fact very routine (e.g., find maximum or turning point). We see calculus questions as advanced because they incorporate a wide range of earlier developed skills, rather than because they require insightful thinking. On the other hand, there are many difficulties in using CAS for trigonometry questions (see Stacey & Ball, 2001) that are caused by the redundancy and rich relationships between the trigonometric functions. CAS use often changes the mathematical demand of a question from carrying out a procedure to matching an answer in a given algebraic form and this is very evident with trigonometric functions. So what an exam question assesses without CAS may be very different from what it assesses with CAS. In a multiple choice examination, without CAS a question may be assessing differentiation, while with

CAS the same question may be assessing knowledge of trigonometric relationships.

The VCAA website contains official information about the Victorian CAS Mathematics subject, copies of the examination papers, and some supplementary questions that extend what can be done with a CAS beyond today's familiar question types (see HREF3 and HREF4).

Conclusion

This short article has highlighted the issues that have received most of our attention in our recent research projects on technology in mathematics. We are now turning our focus to the middle secondary years, where technology may be more used as a pedagogical tool to increase the engagement of students in mathematics. For example, with Gloria Stillman and Jill Brown (University of Melbourne) and Robyn Pierce and Sandra Herbert (University of Ballarat) we are looking at a wider range of mathematical computer tools, some of which 'do the maths' (as does a CAS) and some of which are 'real world interfaces' bringing the real world into the classroom. Details and curriculum resources can be found from the RITEMATHS project website (HREF2). Anyone who needs more information on any of these topics is welcome to contact me. We believe firmly that the value of research is greatly enhanced if its many products, including research findings, curriculum units, teaching ideas and professional development courses are made easily available to others. The teacher professional journals have an important role to play in this endeavour.

References

Note: Some of these references are also available from HREF1 or HREF5.

Asp, G., Ball, L., Flynn, P. & Stacey, K. (2002). Expanding algebra by substituting CAS. In C. Vale, J. Roumeliotis & J. Horwood (Eds), *Valuing Mathematics in Society. Proceedings of the 39th Annual Conference of the MAV* (pp. 348–364). Melbourne: Mathematical Association of Victoria.

Asp, G., Dowsey, J., Stacey, K. & Tynan, D. (1995). *Graphic Algebra: Explorations With a Function Grapher (Student Edition and Teacher Edition).* Melbourne: Curriculum Corporation.

Asp, G., Dowsey, J., Stacey, K. & Tynan, D. (1998). *Graphic Algebra: Explorations with a Graphing Calculator.* Berkeley: Key Curriculum Press.

Gough, J. (2004) Algebra skills and traps and diagnostic teaching for the future. *Australian Senior Mathematics Journal, 18* (2), 43–54.

Ball, L. & Stacey, K. (2003). What should students record when solving problems with CAS? Reasons, information, the plan and some answers. In J. T. Fey, A. Cuoco, C. Kieran, L. Mullin, & R. M. Zbiek (Eds), *Computer Algebra Systems in Secondary School Mathematics Education* (pp. 289–303). Reston, VA: The National Council of Teachers of Mathematics.

Ball, L., Leigh-Lancaster, D. & Stacey, K. (2001). Teaching senior maths with CAS: Major issues for curriculum, assessment and teaching. In C. Vale, J. Horwood & J. Roumeliotis (Eds), *A Mathematical Odyssey. Proceedings of the 38th Annual Conference of the MAV* (pp. 265–275). Melbourne: Mathematical Association of Victoria.

Ball, L., Pierce, R. & Stacey, K. (2003). Recognising equivalent algebraic expressions: An important component of algebraic expectation for working with CAS. In N. A. Pateman, B. J. Dougherty & J. T. Zilliox (Eds), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 15–22). Honolulu: PME.

Ball, L., Stacey, K. & Pierce, R. (2001). Assessing algebraic expectation. In J. Bobis, B. Perry, M. Mitchelmore (Eds), *Numeracy and Beyond. Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1* (pp. 66–73). Sydney: MERGA.

Flynn, P. (2001). Examining mathematics with CAS: Issues and possibilities. In C. Vale, J. Horwood & J. Roumeliotis (Eds), *2001 A Mathematical Odyssey. Proceedings of the 38th Annual Conference of the MAV* (pp. 285–298). Melbourne: Mathematical Association of Victoria.

Flynn, P., Berenson, L. & Stacey, K. (2002). Pushing the pen or pushing the button: A catalyst for debate over future goals for mathematical proficiency in the CAS-age. *Australian Senior Mathematics Journal, 16* (2), 7–19.

Garner, S. (2002). 'I can do maths now': How does CAS affect the teaching and learning of year 12 mathematics? In C. Vale, J. Roumeliotis & J. Horwood (Eds), *Valuing Mathematics in Society. Proceedings of the 39th Annual Conference of the MAV* (pp. 389–400). Melbourne: Mathematical Association of Victoria.

Garner, S. (2004) The CAS classroom. *Australian Senior Mathematics Journal, 18* (2), 28–42.

HREF1. CAS-CAT website. http://extranet.edfac.unimelb.edu.au/DSME/CAS-CAT/ University of Melbourne. Accessed 6 October 2004.

HREF2. RITEMATHS website. University of Melbourne. Accessed 6 October 2004. http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS

HREF3. Mathematical Methods (CAS). Victorian Curriculum and Assessment Authority. Accessed 6 October 2004. http://www.vcaa.vic.edu.au/vce/studies/mathematics/caspilot/casindex.html

HREF4. VCE Examinations site (Victorian Curriculum and Assessment Authority). Accessed 11 October 2004. http://www.vcaa.vic.edu.au/vce/studies/mathematics/caspilot/casexams.html

HREF5. Kaye Stacey's Home Page. Accessed 11 October 2004. http://staff.edfac.unimelb.edu.au/~kayecs.

McCrae, B. & Flynn, P. (2001). Assessing the impact of CAS calculators on mathematics examinations. In B. Lee (Ed.), *Mathematics Shaping Australia. Proceedings of the 18th Biennial Conference of the Australian Association of Mathematics Teachers.* Adelaide: AAMT.

Pierce R. & Stacey, K. (2002a). Monitoring effective use of computer algebra systems. In B. Barton, K. Irwin, M. Pfannkuch & M. Thomas (Eds), *Mathematics Education in the South Pacific, Vol. 2* (Proceedings of the 25th Annual conference of the Mathematics Education Research Groups of Australasia, pp. 575–582). Auckland: MERGA.

Pierce. R. & Stacey, K. (2002b). Algebraic insight: The algebra needed to use computer algebra systems. *Mathematics Teacher, 95* (8), 622–627.

Pierce, R. & Stacey, K (2004). A framework for monitoring progress and planning teaching towards effective use of computer algebra systems. *International Journal for Computers and Mathematics Learning, 9*(1), 59–93.

Stacey, K. (2001). *Teaching with CAS in a Time of Transition.* Accessed 25 February 2005 from http://www.lonklab.ac.uk/came/events/freudenthal/2-Presentation-Stacey.pdf.

Stacey, K. (2003) Using computer algebra systems in secondary school mathematics: Issues of curriculum, assessment and teaching. In W. C. Yang, C. Chu, T. de Alwis, M. G. Lee (Eds), *Proceedings of Eighth Asia Technology Conference in Mathematics* (pp. 40–54). Hsin-Chu, Taiwan: Chung Hua University.

Stacey, K. & Ball, L. (2001). How might computer algebra change senior mathematics: The case of trigonometry. In B. Lee (Ed.), *Mathematics Shaping Australia* (Proceedings of the 18th Biennial Conference of the Australian Association of Mathematics Teachers). Adelaide: AAMT.

Stacey, K., Asp, G. & McCrae, B. (2000). Goals for a CAS-active senior mathematics curriculum. In M. O. J. Thomas (Ed.), *Proceedings of TIME 2000 An International Conference on Technology in Mathematics Education* (pp. 244–252). Auckland: University of Auckland & Auckland University of Technology.

Stillman, G. (2002). Students' models, methods, limitations and assumptions for an application task in a CAS calculator environment. In M. Goos & T. Spencer (Eds), *Mathematics ~ Making Waves* (Proceedings of the 19th Biennial Conference of the Australian Association of Mathematics Teachers). Adelaide: AAMT.

Stillman, G. & Stacey, K. (2002). Designing an application task for Mathematical Methods using AIDS data. In C. Vale, J. Roumeliotis & J. Horwood (Eds), *Valuing Mathematics in Society. Proceedings of the 39th Annual Conference of the MAV* (pp. 470–480) Melbourne: Mathematical Association of Victoria.

Tynan, D. (2002). Re-learning in a CAS-aided classroom — a teacher reflects. In M. Goos & T. Spencer (Eds), *Mathematics ~ Making Waves* (Proceedings of the 19th Biennial Conference of the Australian Association of Mathematics Teachers). Adelaide: AAMT.