The History of the Calculus

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Introduction

Calculus is a mathematical concept that is fundamental to how we understand the world around us. Whether it is in the world of technology, finance, astronomy, sociology, medicine, or any other field you could name, calculus in one form or another can be found.

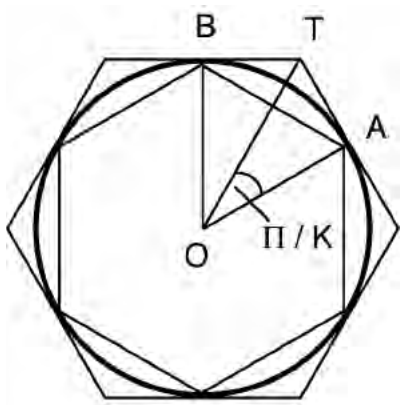
Who first found it? How did they discover the concept? Is it mathematically rigorous? How do we know it works?

Archimedes and the method of exhaustion

Calculus has its roots in the work of the mathematicians and philosophers of the golden age of Greece. Thought was most important to the early Greeks, who longed for an understanding of how the world around them was put together.

Today the most well known of these great men of the mind was a philosopher named Archimedes of Susa, who lived around 225 BC. It was Archimedes who laid the foundations for what we know today as *integral calculus*, in his development of the "method of exhaustion".

Archimedes approximated the areas of otherwise unmeasurable shapes using shapes whose area could be calculated. The most common example of this was Archimedes' calculation of the area of a circle using inscribed polygons. Archimedes found that by increasing the number of sides of an inscribed polygon, the area of the polygon became closer to that of the circle. By continually increasing



$OA = 1$
$AB = \sin(\Pi / K)$
$AT = \tan(\Pi / K)$

where $K = 3 \times 2^{n-1}$

Archimedes (225 BC)
and his area approximation calculation

the number of sides, he "exhausted" the circle by reducing the area of the circle not covered by the polygon. He established that the area of the circle was exactly proportional to the square of its radius, and defined the constant of proportionality — what we today know as π.

Another prominent example of Archimedes' work was his method for calculating the area of a parabolic section using triangles. Using this method, Archimedes calculated quite a large number of values still in use today, including the surface area of a sphere and the volumes of cylinders and cones. As with the majority of the mathematics of Ancient Greece, Archimedes' work was based upon geometric working ratios and area, rather than the algebraic definitions with which we are familiar. However the principles that Archimedes established in this method were used many centuries later in developing calculus as we know it today.

Calculus takes form

In the 16th century, the order of the day was mechanics. As such, it suddenly seemed useful to consider once again the concepts of rates of change and area. Other motivations for continuing this research lay in astronomy — planetary motion was a favourite among these topics — and a number of more "pure" concepts.

The mathematicians who made contributions to this work are numerous, with Fermat, Roberval and Cavalieri preparing the way for the entry of the greats of calculus. The two men who are acknowledged as the fathers of modern calculus are Sir Isaac Newton and his contemporary from across the channel, Gottfried Leibniz. These two mathematicians independently reached the same conclusions, but neither man was able to sufficiently establish the logical roots of their work. This was despite the fact that both men had methods that could equally well answer complicated questions, reaching the same, correct answer every time.

The fact that these two men independently reached the same results was a point of contention for many years. Newton in particular was sure than Leibniz had based his work upon Newton's, and remained bitter towards the German mathematician for the rest of his life. When Newton was appointed to a position in London's Royal Society, he used his position to influence the argument, to the point where he personally chaired the "independent" board that investigated the matter, and published the findings under another name.

The methods used by both men were frequently attacked by other, disbelieving mathematicians and even theologians, for in particular the use of "infinitesimals" was not seen as a mathematically sound method. It was not until the 19th century when Cauchy published his definition of what we today call the "limit" that any formal and rigorous basis was established for the calculus.

Isaac Newton
Born: 4 January 1643
in Lincolnshire, England
Died: 31 March 1727
in London, England

Gottfried Leibniz
Born: 1 July 1646
in Leipzig, Saxony
Died: 14 November 1716
in Hannover, Hanover

The calculus of Newton

The man who today is best known for his three Laws of Motion, used exactly those principles in establishing his method for differentiation. Called his "method of fluxions", Newton's work was based upon the idea of the motion of particles — he named these fluents — along curved paths. The horizontal and vertical velocities, x' and y', he named the fluxion of the particle. An example of his work is shown on page 4.

Much of the controversy surrounding whom of the two mathematicians had developed the calculus first arose from Newton's reluctance to publish his findings. Although many mathematicians saw his work, he was not satisfied with the rigour of his work, and considering that a publishing house went bankrupt after publishing work by his mentor Barrow, it is not really surprising.

An example from Newton's Method of Fluxions

I The Relation of Flowing Quantities (Fluents) to one another being given, to determine the Relation of their Fluxions.

If the Relation of Flowing Quantities x, y be

$$x^3 - ax^2 + axy - y^3 = 0$$

it will be found that

$$\dot{x} : \dot{y} : : 3y^2 - ax : 3x^2 - 2ax + ay.$$

In moment 'o':

$$x \to x + \dot{x}\, o\,,\ y \to y + \dot{y}\, o.$$

Hence

$$x^3 - ax^2 + axy - y^3 \to$$

$$x^3 + 3x^2\dot{x}\, o + 3\dot{x}^{\,2}oox + \dot{x}^{\,3}o^3 - ax^2 - 2a\dot{x}\, ox - a\dot{x}^{\,2}oo +$$
$$axy + a\dot{x}oy + a\dot{y}\, ox + a\dot{x}\,\dot{y}\, oo - y^3 - 3y^2\dot{y}\, o - 3\dot{y}\, ooy - \dot{y}^{\,3}o = 0.$$

Expunge $x^3 - ax^2 + axy - y^3$ (= 0) and divide through by o:

$$3x^2\dot{x} + 3\dot{x}^{\,2}\ \mathrm{o}x + \dot{x}^{\,3}\mathrm{o}^2 - 2a\dot{x}\,\mathrm{o}x - \mathrm{a}\dot{x}^{\,2}\mathrm{o} + a\dot{x}\, y + a\dot{y}\, x + a\dot{x}\,\dot{y}\,\mathrm{o} - 3y^2\dot{y}\ - 3\dot{y}\mathrm{o}y - \dot{y}^{\,3} = 0.$$

But whereas o is supposed to be infinitely little, the terms that are multiplied by it will be nothing in respect of the rest. Therefore I reject them:

$$3x^2\dot{x} - 2a\dot{x}\,\mathrm{o}x + a\dot{x}\, y + a\dot{y}\, x - 3y^2\dot{y}\ - \dot{y}^{\,3} = 0$$

$$-\dot{x}\,(3x^2 - 2ax + ay) + \dot{y}\,(ax - 3y^2) = 0,$$

whence the Result.

Newton's contributions to mathematics ended in 1699 when he was appointed as the master of the Royal Mint in his home city of London. He ended his days as an administrator, and a very rich man. However, it is unlikely that his influence on the world and the way we see it will ever be forgotten.

The calculus of Leibniz

Working independently from Newton, German Gottfried Leibniz also discovered the principles of calculus. Mainly considering the analysis of graphs, he thought of values x and y ranging over domains containing infinite numbers of values: the distance between them he denoted dx and dy — terminology still in use today.

It is from Leibniz's work that we also get the terminology of the integral as $\int$ and the derivative as $\frac{d}{dx}$.

Leibniz spent much of the later part of his career defending himself against the attacks of Newton and his supporters, who claimed that he stole his ideas from two letters that he received from Newton years earlier. Leibniz based his defence upon errors Newton had made in calculating second and higher derivatives, which were not made in his own work. However by the stage this information was released the argument had degenerated to a purely nationalistic debate.

Today it is widely accepted that Leibniz developed his theories regarding calculus independently, and his approach to the work was very different to that of Newton. Despite this he also was unable to provide satisfactory proof of his work, and in the end published without proofs. The first part of his work to be published was described by Jacob Bernoulli as an enigma, for although it produced correct answers, it could not be summarily proven to be correct.

Calculus today

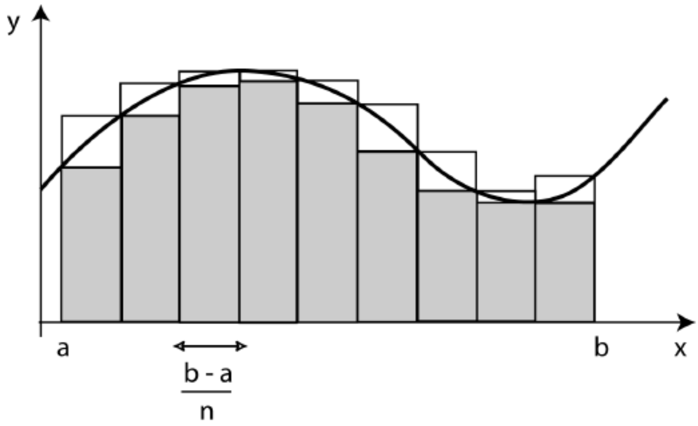
Today calculus is taught in high schools and universities across the world. With the aid of computers we can solve problems previously unsolvable, and with complicated numerical approximations, we can generate useful solutions for problems that otherwise could not be reached. But the fundamentals are still the same.

Integration

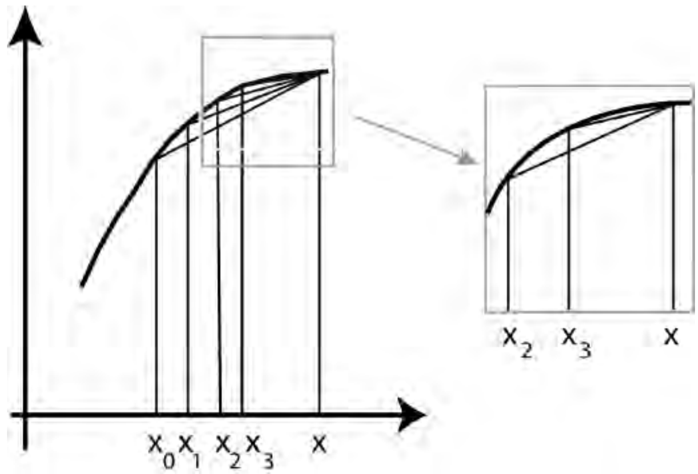
Integration is often thought of by most as calculating the area between a positive curve and the x-axis, and it is based upon a set of principles developed from Archimedes' work. Today we generally consider a set of rectangles, with heights determined by the function, and equal base lengths spanning the interval to be integrated (a … b). The area under the curve lies between the lower sum (shown in grey, never more than the actual area) and the upper sum (grey plus white rectangles, never less), and is approximated ever more closely by these sums as the number of rectangles (n) approaches infinity.

Differentiation

Differentiation is usually described as finding slopes of tangents, and is best described geometrically. Given two points on a curve, the slope of the chord that joins then is close to the slope of the curve. If we consider the effect of moving the two points closer together we see that the slope of the chord approached that of the curve.



Integration



Differentiation

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