

Student's Reversible Thinking Processes: An Analysis Based on Adversity Quotient Type Climbers

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Abstract

Reversibility thinking carried out mentally in mathematical operations has an important role in the process of understanding concepts as it involves developing a thinking process from beginning to end and from end to beginning. This qualitative research aims to describe students' reversible thinking processes in solving algebra problems, specifically in climber-type adversity quotient. A total of 2 students who have the climber type were selected as participants from 36 potential subjects involved in the research. Data collection was carried out by providing Adversity Response Profile questionnaires, Test of Thinking Reversible, and semi-structured interviews. Our participants solved problems according to polya stages and through forward and reverse processes. The results of data analysis show that in the forward process, there are 2 aspects of reversible thinking, namely negation and reciprocity, while in the reverse process, they involve 2 aspects of reversible thinking, namely the capability to return to initial data after obtaining the result and negation. The results suggest that student's ability to perform mental reversals indicates a strong reversible thought process, leading to more precise cognitive thinking.

Keywords

Adversity quotient, climbers, algebra, reversible thinking.

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Procesos de Pensamiento Reversible de los Estudiantes: Un Análisis Basado en Escaladores de Tipo Cociente de Adversidad

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Resumen

El pensamiento de reversibilidad llevado a cabo mentalmente en operaciones matemáticas tiene un papel importante en el proceso de comprensión de conceptos ya que implica desarrollar un proceso de pensamiento de principio a fin y de fin a principio. Esta investigación cualitativa tiene como objetivo describir los procesos de pensamiento reversible de los estudiantes en la resolución de problemas de álgebra, específicamente en el cociente de adversidad tipo escalador. Un total de 2 estudiantes que tienen el tipo escalador fueron seleccionados como participantes entre 36 sujetos potenciales involucrados en la investigación. La recolección de datos se llevó a cabo mediante cuestionarios de perfil de respuesta a la adversidad, prueba de pensamiento reversible y entrevistas semiestructuradas. Nuestros participantes resolvieron problemas según las etapas de polia y mediante procesos directos e inversos. Los resultados del análisis de datos muestran que en el proceso directo, hay 2 aspectos del pensamiento reversible, a saber, la negación y la reciprocidad, mientras que en el proceso inverso, involucran 2 aspectos del pensamiento reversible, a saber, la capacidad de regresar a los datos iniciales después de obtener los datos iniciales. resultado y negación. Los resultados sugieren que la capacidad del estudiante para realizar inversiones mentales indica un fuerte proceso de pensamiento reversible, lo que lleva a un pensamiento cognitivo más preciso.

Palabras clave

Coefficiente de adversidad, escaladores, álgebra, pensamiento reversible.

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Thinking is a required process to find ideas that enable someone to solve a problem (Tomé et al., 2019). It is a constructive process that helps individuals form new representations of any object or event by changing the available information. It involves several mental activities, such as inferring, abstracting, reasoning, imagining, judging, and solving problems (Sternberg, 2013). As a mental process, thinking involves more than just memorization and understanding. It describes how people act when faced with problems and situations, thereby, aiding students to solve problems that exist in society (Sternberg, 2013; Nur et al., 2024).

Every student has a different path in solving math problems, depending on their specific traits (Tambychik & Meerah, 2010; Mihajlovic et al., 2016; Dyah & Setianingsih, 2019). Students present excellent problem solvency when they are equipped with a good understanding of the problem. When attempting to solve a problem, students engage in a thinking process to find a solution to the problem (Albay, 2019). This thinking process represents the competency for transferring the knowledge and skills developed during learning a new context (Sa'dijah et al., 2021). In specific, the ability to manage problems and obstacles in life and then turn them into challenges that must be solved is known as Adversity Quotient (AQ) (Zimmerman, 2010).

AQ can serve as a parameter of how a person reacts to existing obstacles or difficulties. This is because everyone has an instinctive need to react, which drives them to engage in adaptive behavior and productive pursuits (Espanola, 2016). AQ is equivalent to the ability to survive and overcome life's issues and challenges, including the four CO2RE dimensions, consisting of control, origin, ownership, reach, and endurance (Dewi et al., 2022). A person's AQ is classified into three categories, quitters, campers, and climbers.

When students engage in a series of thinking activities they may develop the ability to move back and forth between two perspectives, known as reversible thinking. Reversibility is understood as a process where a person controls their thoughts to return to their starting point (Fitmawati et al., 2019; Kang, Mee-Kwang & Lee, 1999; Mafulah & Juniati, 2020). Through reversible thinking, students see problems face to face, enabling them to solve complex problems and understand two reciprocal lines on the spectrum from different points of view.

Reversibility plays an important role in the formation of material concepts in students' thinking activities. Utilizing reversible thinking, students can better comprehend and avoid confusion when working on different questions, as demonstrated by the teacher. There are two virtues of reversible thinking based on Piaget's theory (Wong, 1977). The first theory emphasizes that reversible thinking is a psychological mechanism that underlies the development of the concept of child conservation. The second theory emphasizes the flexibility of hindsight and foresight as well as the acceleration of cognitive structures in reversible thinking, which takes on a central meaning in the broader context of Piaget's theory of intellectual development.

The direct and the inverse problem are two related things that can be conceptualized in two ways: a) as an inverse problem immediately after the direct, and b) as an independent inverse problem (Muir, 1988). In the basic experiment, the inverse problem is presented immediately after the direct problem, which can influence the student's approach. The process of transitioning from direct thinking to inverted thinking allows for a non-repetitive path in the reverse order. This applies to the direct theorem and its inverse. The path from A

to D can differ from the path from D to A. Students can move from one to another in various ways. In some cases, there is a direct and reverse association of $A \longleftrightarrow D$ and otherwise. Meanwhile, in some other cases, there is only the general direction of movement of thinking: $A \rightleftarrows D$ and otherwise, but it is significant that in both cases, thinking hitherto traveling from A to D, is now moving in the opposite direction.

Experimental problems related to the formation of direct and inverse associations, as well as changes in the general direction of thinking, have been identified (Muir, 1988; Kang, Mee-Kwang & Less, 1999; Ramful, 2015). When students work on routine problems frequently given in school, such as $7 + 6 = 13$, then the student also understands the direction of solving the opposite, namely $13 - 6 = 7$. Likewise, when students are able to solve the problem $5 \times 8 = 40$, then they also master $40 \div 8 = 5$. However, in broader cases that involve more than 1 mathematical operation, students sometimes need more in-depth analysis that allows them to work with various strategies.

To study this relationship, the conceptual relationship between formal and the reversal of thinking in solving algebraic forms must be established. Solving an algebraic form involves finding all the numbers that satisfy the problem, resulting in the right solution. The reversibility of thinking is implicit in solving algebraic equations, and there are various methods for achieving this reversibility. Piaget's theory of intellectual development posits that two different forms of inversion and compensation can occur at the level of concrete operations. The relationship between two forms of reversibility in the same system is defined as a feature of a formal operation, whereas reversal is defined as an action by which reversing a process cancels its effect (Börnert-Ringleb & Wilbert, 2018).

To reinforce schemata, students must be able to think in two reversible ways, making two-way linkages between concepts, principles, and methods (Pebrianti & Juandi, 2023). When students have reversible thinking skills, they present the ability to 1) think naturally in two opposite directions, 2) solve problems in a forward or backward pattern, and 3) minimize mistakes in every decision-making. Therefore, reversible thinking is essential for every student (Maf'ulah et al., 2019).

Based on the literature review, a group of students was asked to work on a routine problem at school. They were asked to find the value of x in the form $\frac{12}{x-2} + 14 = 16$, then check the correctness of their answer. Among the answers that emerged, there were aspects of reversible thinking used by students, as seen in Figure 1.

Figure 1
Student Answers and Results of Checking Answers

$\frac{12}{x-2} + 14 = 16$	<i>Translation.</i>	
$\frac{12}{x-2} + 14(x-2) = 16(x-2)$		
$12 + 14x - 28 = 16x - 32$		
$14x - 16 = 16x - 32$		
$-16 + 32 = 16x - 14x$		
$16 = 2x$		
$\frac{16}{2} = x$		
$8 = x$		
		$\frac{12}{x-2} + 14 = 16$
		$12 + 14(x-2) = 16(x-2)$
	$12 + 14x - 28 = 16x - 32$	
	$14x - 16 = 16x - 32$	
	$-16 + 32 = 16x - 14x$	
	$16 = 2x$	
	$\frac{16}{2} = x$	
	$8 = x$	

$\begin{array}{r} 12 + x = 16 \\ \underline{8-2} \\ 12 + x = 16 \\ \underline{6} \\ 2 + x = 16 \\ \underline{x = 16 - 2} \\ x = 14 \end{array}$	<p style="text-align: right;"><i>Translation.</i></p> $\begin{array}{l} 12 \\ \underline{8-2} + x = 16 \\ \frac{12}{6} + x = 16 \\ 2 + x = 16 \\ x = 16 - 2 \\ x = 14 \end{array}$
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As presented in Figure 1, the student's answer references an equation in which both sides are operated with the same number, $x - 2$. The student groups similar terms on both sides and uses inverse operations to obtain the value of x by operating $16 = 2x$. The related operation is division. Then, in checking the answers, students looked for the value 14 in the initial equation by substituting the value $x = 8$ that was obtained in the previous answer into the equation, and to obtain the value $x = 14$, students operated $2 + x = 16$ involving the inverse of the related operation, namely subtraction.

Research related to reversible thinking has been carried out by several researchers, including research focusing on the ability to think reversibly in graph concepts (Sutiarso, 2020), reversible thinking in the case of fuzzy representations (Kang, Mee-Kwang & Lee, 1999), reversible ability in compiling linear equations (Balingga et al., 2016), and the relationship between the concept of reversibility and arithmetic (Wong, 1977). However, there has not been much research that examines reversible thinking processes at the school level in students involving algebraic fractions and in terms of students' adversity quotient.

Therefore, this study evaluates students' reversible thinking processes in solving algebra problems using the Adversity Quotient Climber type (AQC) review. Of the previous studies available, none has investigated students' reversible thinking processes in AQ review, especially the climber type. AQC was chosen because it shows students' ability to persist and try to solve the problems given using their strategies and knowledge. The results of this research provide information about the importance of having reversible thinking skills in solving problems and become the basis for designing learning based on the construction of thinking processes both generally and specifically.

Methodology

This study utilizes a qualitative approach to accurately describe the characteristics of situations or phenomena that occur in the field (Busetto, 2020). The research process comprised of stages, beginning with the preparation stage, which involved compiling instruments and carrying out validation. Then, the implementation stage was performed by selecting research subjects and collecting data, and the analysis stage was completed by data processing.

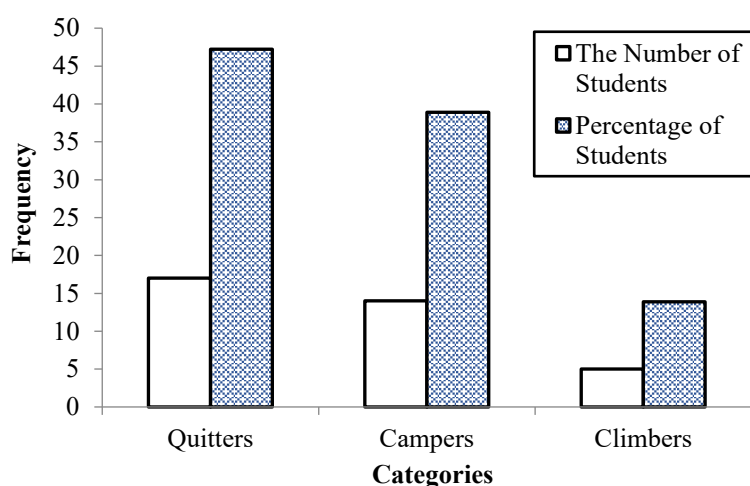
This research was conducted on 36 ninth-grade students of State Middle Schools in Blitar City, East Java, Indonesia, who had studied all algebra material competencies. The research employed the Adversity Response Profile (ARP) questionnaire adapted from Stoltz Theory to determine the student's Adversity Quotient (AQ) type. Meanwhile, the Test of Thinking

Reversible (TTR) question containing algebra contextual problems was used to find out students' thinking processes in working on questions using reversible thinking aspects. Lastly, the semi-structured interview guidelines were utilized for confirming as well as exploring information directly related to students' written answers.

Before being used for data collection, the instrument was first validated by an expert in the field of mathematics education, an expert in the field of mathematics, and an expert in the field of psychology for the instrument’s content/material, construction, and language. After expert validation, empirical validation or instrument testing was carried out on students. The validators concluded that the instrument was suitable for research purposes.

The results of administering the ARP questionnaire are shown in Figure 2.

Figure 2
AQ Type Grouping Results



As illustrated in Figure 2, 47.22% (17) students, are in the quitters category, 38.89% (14) students in the campers category, and 13.89% (5) students in the climbers category. After completing the ARP questionnaire, students were provided with a Test of Thinking Reversible (TTR) question sheet. In this process, students work on questions accompanied by a process of checking the answers again. The test question is presented in the following.

Mrs. Siti has some money. She spent a quarter of it on buying rice, a sixth on buying sugar, and a third of the rest on paying the fare. If now Mrs. Siti's remaining money is IDR 175,000.00. How much money did Mrs. Siti have at first?

The results of students' answers to TTR questions were examined using reversible thinking indicators in solving algebra questions, either through a forward or backward process (Mafulah et al., 2017; Sangwin & Jones, 2017). The forward process is defined as a process in which the subject creates another equation that is equivalent to the initial equation. The reverse process, on the other hand, involves returning the equation to its initial form. Table 1 presents the indicators of reversible thinking among students in this study.

Table 1
Indicators of Students' Reversible Thinking Processes in This Study

Process	Aspect	Indicator		Step
		Reversibility	Problem-Solving	
Forward	Negation	Subjects use the inverse properties of related operations in creating new equations	a. The subject understands the question by understanding the information from the question	Understanding the problem
	Reciprocity	The subject uses compensation properties or other relationships equivalent to known equations in creating new equations	b. The subject understands the question by understanding the information asked in the question	
				a. The subject can determine other unknown conditions (if any) b. Subjects can use all the information in the question c. Subjects can make plans or steps to solve the given problems
			Subjects can work on existing questions according to the steps based on the chosen solution method	Carrying out the plan
Reverse	Negation	The subject uses the inverse property of the operations related to the steps or strategies when returning the equation to its original form	Subjects can re-check the work that has been obtained using the correct method or steps	Looking back
	Reciprocity	The subject uses compensatory properties or other relationships equivalent to known equations in reversing the equations		
	Capability to return to initial data after obtaining the result	The subject changes the new equation that has been created to its initial form using the correct procedure		

The study employed a three-stage data analysis process: data reduction, data presentation, and drawing conclusions. Data reduction was carried out by selecting, categorizing, and focusing on the results of the ARP questionnaire grouping and students' answers to the TTR questions. The study focused on two subjects in the climber category, coded as WMW and

ADZ, based on the provision of the TTR questions. In specific, they were selected because (1) the problem-solving carried out comprised of a forward and reverse process, (2) there were differences in the strategies or steps taken in solving problems that affected the emergence of different aspects and indicators of reversible thinking, and (3) the subject presented the skills to express opinions or reasons based on the results of written work. After that, the reduced data was presented, and a conclusion was drawn.

Results

The data and analysis conclusions of the reversible thinking process from TTR and semi-structured interviews according to the Polya phases are described in the following.

Reversible Thinking Processes on The Subject WMW

The results of the work from the WMW subjects are shown in Figure 3.

Figure 3

Results of The Work from WMW (Forward Process)

<p> Diket: Uang bu Siti = x Beras = $\frac{1}{4}x$ Gula = $\frac{1}{6}x$ Angkutan = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$ sisa uang = Rp 175.000,00 Ditanya = uang bu Siti mula-mula. Djawab: $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175.000$ $\frac{4x}{4} - \frac{1x}{4} - \frac{2x}{6} - \frac{1x}{3} + \frac{1x}{4} + \frac{1x}{6} = 175.000$ $\frac{36x - 9x - 12x + 12x + 3x + 2x}{36} = 175.000$ $\frac{14x}{36} = 175.000$ $14x = 175.000 \cdot 36$ $14x = 6.300.000$ $x = \frac{6.300.000}{14}$ $x = 450.000$ </p>	<p><i>Translation.</i></p> <p> Known. Mrs. Siti's money = x Rice = $\frac{1}{4}x$ Sugar = $\frac{1}{6}x$ Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$ Leftover money = IDR 175,000.00 Asked. Mrs. Siti's money at first? Answered. $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$ $\frac{4}{4}x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}x + \frac{1}{12}x + \frac{1}{18}x = 175,000$ $\frac{36x - 9x - 6x - 12x + 3x + 2x}{36} = 175,000$ $\frac{14}{36}x = 175,000$ $14x = 175,000 \cdot 36$ $14x = 6,300,000$ $x = \frac{6,300,000}{14}$ $x = 450,000$ </p>
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Figure 3 shows that WMW uses the forward process by analyzing what is known from the question to analyze and answer the question. At the stage of understanding the problem, WMW has been capable of stating, identifying and formulating problems using appropriate sentences. On the answer sheet, the WMW writes in advance that Mrs. Siti's money = x , so to buy rice is $\frac{1}{4}x$, to buy sugar is $\frac{1}{6}x$, for parking fees $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$, the rest is 175,000 and he question asks for Mrs. Siti's initial money.

In addition, the excerpts from the researcher's (R) interview with WMW regarding the stages of understanding the problem are described in the following.

- R : "What do you understand from this question?"
 WMW : "Mrs. Siti has money. $\frac{1}{4}$ of it is for rice, $\frac{1}{6}$ for sugar and the remaining $\frac{1}{3}$ for paying the fare, the remaining 175,000."
 R : (points to the answer sheet).

$Onyklus = \frac{1}{3} (x - \frac{1}{4}x - \frac{1}{6}x)$	<i>Translation.</i> Parking fees = $\frac{1}{3} (x - \frac{1}{4}x - \frac{1}{6}x)$
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"For exes, you write $\frac{1}{3} (x - \frac{1}{4}x - \frac{1}{6}x)$. What do you mean by that?"

- WMW : "For expenses, it's $\frac{1}{3}$ of the rest. That means $\frac{1}{3}$ of Mrs. Siti's money minus the cost of buying rice, sugar, and the cost itself."
 R : "Good. Then what's the question?"
 WMW : "How much money does Mrs. Siti have in total, sir?"
 R : "Okay."

From the above excerpts, the subject demonstrated an ability to comprehend and accurately convey the intent of the questions. As such, they could effectively rephrase the problem using appropriate mathematical language.

Furthermore, in addition to analyzing information thoroughly, WMW can also provide guidance on developing a problem-solving plan, including a planning stage. This is evident in the researcher's interview with WMW, as shown in the following excerpts.

- R : "In your opinion, what concepts must be understood to solve the problem?"
 WMW : "In my opinion, we have to master algebraic operations first, sir. Because there are variables here."
 R : "Next, how?"
 WMW : "So we have to know what $\frac{1}{4}x$, $\frac{1}{6}x$, and $\frac{1}{3} (x - \frac{1}{4}x - \frac{1}{6}x)$ means. Then we operate them."
 R : "In your opinion, apart from the way you did, are there other ways to answer this question?"
 WMW : "Hmm. I don't think there is. But I don't know (with a chuckle)."
 R : (points to the answer sheet).

<p>Alat:</p> <p>Uang bu siti = x</p> <p>Beras = $\frac{1}{4}x$</p> <p>Gula = $\frac{1}{6}x$</p> <p>Parkir = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$</p> <p>... sisa uang = Rp 175.000,00</p> <p>Ditanya = uang bu Siti mula-mula</p>	<p><i>Translation.</i></p> <p>Known.</p> <p>Mrs. Siti's money = x</p> <p>Rice = $\frac{1}{4}x$</p> <p>Sugar = $\frac{1}{6}x$</p> <p>Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$</p> <p>Leftover money = IDR 175,000.00</p> <p>Asked. Mrs. Siti's money at first?</p>
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“After you write down what is known, then how do you calculate Mrs. Siti's initial money?”

WMW : “The original money was reduced by $\frac{1}{4}x$ then reduced by $\frac{1}{6}x$ and again reduced by $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$. Then the result should be 175,000.”

R : “Okay.”

From the interview excerpts, the subject had a good problem-solving plan and strategy, equipped with excellent mastery of the concept of algebraic operations to solve problems.

After making a problem-solving plan, WMW solved the problems following the plan (carrying out the planning stage) that had been made. WMW writes $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$ as a first step in problem-solving. At this stage, there are reversible thinking aspects and indicators in the process that appear, namely reciprocity, when the WMW subject writes the form from $\frac{14}{36}x = 175,000$ comes $14x = 175,000$.³⁶ Then, WMW uses the inverse of the related operation to obtain a new equation, namely when dividing the form $14x = 3,600,000$ by the number 14 to obtain the value x .

The results of excerpts from the researcher’s (R) interview with WMW regarding the stages of solving the problem according to the plan are presented in the following.

R : (points to the answer sheet).

<p>$x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175.000$</p> <p>$\frac{4x}{4} - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}x + \frac{1}{12}x + \frac{1}{18}x = 175.000$</p>	<p><i>Translation.</i></p> <p>$x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$</p> <p>$\frac{4}{4}x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}x + \frac{1}{12}x + \frac{1}{18}x = 175,000$</p>
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“Try to explain the shape you make, namely $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$.”

WMW : “So Mrs. Siti’s money is deducted by $\frac{1}{4}x$ to buy rice, $\frac{1}{6}x$ to buy sugar and the remaining $\frac{1}{3}$ for parking fees. For those who parked, it’s because $\frac{1}{3}$ of the rest is like this (points to the answer).”

R : “Then, how?”

WMW : “I made a new equation from the results of the previous equation operation by equating the denominators. The shape becomes $\frac{4}{4}x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}x + \frac{1}{12}x + \frac{1}{18}x = 175,000$.”

R : “Okay. Now pay attention to your answer here (points to the answer sheet).”

$$\frac{14}{36}x = 175.000$$

$$14x = 175.000 \cdot 36$$

<i>Translation.</i>
$\frac{14}{36}x = 175,000$
$14x = 175,000 \cdot 36$

“Try to explain your step.”

WMW : “Here, I removed the denominator 36. I multiplied both sides by 36, too. The result is $14x = 175,000 \cdot 36$ ”

R : “Next you write $x = \frac{6,300,000}{14}$. What do you mean?”

WMW : “Previously, the form was $14x = 175,000 \cdot 36$. So, to find the value of x , I used the opposite operation of multiplication, namely division. So 175,000 is multiplied by 36, then the result is divided by 14.”

R : “Okay.”

The interview excerpts suggest that the subject has a step-by-step problem-solving strategy. The subjects utilized a forward process, incorporating negation, to systematically derive new equations.

Additionally, during the locking backstage phase, the WMW re-examined previously obtained answers by testing them in reverse until they returned to the original information. Figure 4 displays the WMW's re-checked answers at the locking backstage.

Figure 4
The Results of The Work of WMW Subjects (Reverse Process)

	<p style="text-align: right;"><i>Translation.</i></p> <p>Known. Mrs. Siti's money = 450,000 $x = 450,000$ Rice = $\frac{1}{4}x$ leftover money = p Sugar = $\frac{1}{6}x$ Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$ Asked. Mrs. Siti's leftover money? Answered. $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = p$ $(450,000 - \frac{1}{4}(450,000) - \frac{1}{6}(450,000) - \frac{1}{3}(450,000 - \frac{1}{4}(450,000) - \frac{1}{6}(450,000))) = p$ $\frac{1}{3}(450,000) + \frac{1}{12}(450,000) + \frac{1}{18}(450,000) = p$ $450,000 - 112,500 - 75,000 - 150,000 + 37,500 + 25,000 = p$ $450,000 - 275,000 = p$ $175,000 = p$</p>
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As illustrated in Figure 4, WMW has solved the problem properly. This indicates that the WMW can correct the answers found in the previous test question. At this stage, the WMW completes the reverse process with the aspects of reversible thinking, which allow to return to

initial data after obtaining the result. The WMW changed the new equation that had been made to the initial form using the correct procedure by writing $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = p$ as a first step to get the correct result.

In addition, excerpts from the interview showing the locking backstage are shown in the following.

R : “Tell me about the steps you took to answer the question?”

WMW : (points to the answer sheet).

	<p style="text-align: right;"><i>Translation.</i></p> $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = p$ $450,000 - \frac{1}{4}(450,000) - \frac{1}{6}(450,000) - \frac{1}{3}(450,000) + \frac{1}{12}(450,000) + \frac{1}{18}(450,000) = p$ $450,000 - 112,500 - 75,000 - 150,000 + 37,500 + 25,000 = p$
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“We’re down, sir. For rice, it means $\frac{1}{4}$ of 450,000 right? Buy sugar for $\frac{1}{6}$ of 450,000 for $\frac{1}{3}$ of the remainder. See you later how much is left.”

R : “Have you encountered any difficulties?”

WMW : “I was a little confused earlier about the fee, especially about the remaining $\frac{1}{3}$ of the money, but I found it in the end. I hope I got it right.”

R : “Are you sure your answer is correct?”

WMW : “Sure, sir, because the results are the same as my earlier answer.

R : “Okay.”

From the above interview excerpts, the subject should review the reverse questions to ensure the accuracy of the previously obtained answers properly. The backstage locking mechanism demonstrates the reversibility of addition and subtraction operations, allowing for a return to the initial data after obtaining the result and negation.

Reversible Thinking Processes on The Subject of ADZ

The results of the ADZ subject’s work are shown in Figure 5.

Figure 5
The Results of The Work of ADZ Subjects (Forward Process)

<p>diket:</p> <p>uang bu siti = u</p> <p>beras : $\frac{1}{4}u$</p> <p>gula : $\frac{1}{6}u$</p> <p>ongkor = $\frac{1}{3}(u - \frac{1}{4}u - \frac{1}{6}u)$</p> <p>Sisa uang = Rp 175.000,00</p> <p>Ditanya: Uang bulif mula-mula?</p> <p>Jawab: $u - \frac{1}{4}u - \frac{1}{6}u - \frac{1}{3}(u - \frac{1}{4}u - \frac{1}{6}u) = \text{Rp } 175.000$</p> <p>$= \frac{12}{12}u - \frac{3}{12}u - \frac{2}{12}u - \frac{1}{3}(\frac{12}{12}u - \frac{3}{12}u - \frac{2}{12}u) = \text{Rp } 175.000$</p> <p>$= \frac{7}{12}u - \frac{1}{3}(\frac{7}{12}u) = \text{Rp } 175.000$</p> <p>$\frac{7}{12}u - \frac{7}{36} = 175.000$</p> <p>$= \frac{21}{36}u - \frac{7}{36} = 175.000$</p> <p>$= \frac{14}{36}u = 175.000$</p> <p>$u = 175.000 \times \frac{36}{14}$</p> <p>$= \frac{3.150.000}{7}$</p> <p>$= 450.000$</p>	<p><i>Translation.</i></p> <p>Known.</p> <p>Mrs. Siti's money = x</p> <p>Rice = $\frac{1}{4}x$</p> <p>Sugar = $\frac{1}{6}x$</p> <p>Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$</p> <p>Left over money = IDR 175,000.00</p> <p>Asked. Mrs. Siti's money at first?</p> <p>$x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$</p> <p>$\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x - \frac{1}{3}(\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x) = 175,000$</p> <p>$\frac{7}{12}x - \frac{1}{3}(\frac{7}{12}x) = 175,000$</p> <p>$\frac{12}{12}x - \frac{7}{36}x = 175,000$</p> <p>$\frac{12}{21}x - \frac{7}{36}x = 175,000$</p> <p>$\frac{36}{36}x - \frac{7}{36}x = 175,000$</p> <p>$\frac{14}{36}x = 175,000$</p> <p>$x = 175,000 \cdot \frac{36}{14}$</p> <p>$x = \frac{3,150,000}{7}$</p> <p>$x = 450,000$</p>
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Figure 5 shows that the ADZ performs a forward process, where the subject analyzes the known information and asks questions first. ADZ could identify and formulate problems using appropriate sentences at the understanding stage. Further, the subject wrote in advance on the answer sheet that Mrs. Siti's money = x , to buy rice is $\frac{1}{4}x$, to buy sugar is $\frac{1}{6}x$, for parking fees, is $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$ and the rest is 175,000. The question asked was Mrs. Siti's initial money.

Excerpts from the researcher's (R) interview with ADZ subjects regarding the stage of understanding the problem are presented in the following.

R : "Do you understand the question? If you understand, try to explain what that means?"

ADZ : "Understood, sir. Mrs. Siti has some money, but we don't know how much. Well, $\frac{1}{4}$ of it is used to buy rice, $\frac{1}{6}$ to buy sugar and $\frac{1}{3}$ of the remaining to pay for the parking fare. Then, after using all kinds of money, only 175,000 remained. The question is, how much money did she have initially, sir? "

R : (points to the answer sheet).

<p>ongkor = $\frac{1}{3}(u - \frac{1}{4}u - \frac{1}{6}u)$</p>	<p><i>Translation.</i></p> <p>Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$</p>
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"Your answer says the fee is $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$. What do you mean?"

ADZ : “The money for the parking fare is $\frac{1}{3}$ of the rest. So $\frac{1}{3}$ of Mrs. Siti’s money has been deducted for the cost of buying rice, sugar, and parking fees.”
 R : “Good. What’s the question then?”
 ADZ : “Mrs. Siti’s initial money again, sir.”
 R : “Okay.”

According to the above interview excerpts, they demonstrated an ability to comprehend and effectively respond to the questions posed. Additionally, the subject displayed proficiency in translating mathematical problems into their own words.

Aside from being able to analyze what is known and asked, ADZ also presents excellent skills in formulating a problem-solving plan (devising a planning stage) properly, as suggested by the researcher’s (R) interview with ADZ subjects in the following excerpts.

R : “In your opinion, what concepts must be understood to solve the problem?”
 ADZ : “Concepts on algebraic operations, sir.”
 R : “Sure? Why algebra?”
 ADZ : “Yes, sir. Because here we have to operate the $\frac{1}{4}x$, $\frac{1}{6}x$, $\frac{1}{3}$ of the remaining money so that we can find the original money.”
 R : “In your opinion, apart from the concept of algebraic operations, are there other concepts that can be used to answer the question?”
 ADZ : “In my opinion, sir, I have no idea of another concept.”
 R : (points to the answer sheet)

<p>diket : uang bu siti = u beras = $\frac{1}{4}u$ gula = $\frac{1}{6}u$ ongkos = $\frac{1}{3}(u - \frac{1}{4}u - \frac{1}{6}u)$ Sisa uang = Rp 175.000,00 Ditanya : Uang bu siti mula - mula ?</p>	<p><i>Translation.</i></p> <p>Known. Mrs. Siti's money = x Rice = $\frac{1}{4}x$ Sugar = $\frac{1}{6}x$ Parking fees = $\frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x)$ Left over money = IDR 175,000.00 Asked. Mrs. Siti's money at first?</p>
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“From what you wrote, to answer the question, what steps did you take?”
 ADZ : “So after we write down everything that is known, then we look for it using equations $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$.”
 R : “Okay.”

The interview excerpts indicated that the subject has an excellent problem-solving plan or strategy, which involves the mastery of the concept of algebraic operations.

Aside from the creation of a problem-solving plan, ADZ can solve problems following the plan (carrying out the planning stage). As the first step in problem-solving, the ADZ subject writes the formula $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$. There are reversible thinking aspects and indicators in the process that appear at this stage, such as negation when the ADZ writes the form from $\frac{14}{36}x = 175,000$ becomes $x = 175,000 \cdot \frac{36}{14}$. Further, to obtain a

new equation, ADZ uses the inverse of the related operation by eliminating $\frac{14}{36}$ by adding the multiplication $\frac{36}{14}$ operation or the $\frac{14}{36}$ division operation on both sides.

The results of the researcher's (R) interview with ADZ regarding the stages of solving the problem according to the plan are presented in the following excerpt.

R : (points to the answer sheet).

$u = \frac{1}{4}u - \frac{1}{6}u - \frac{1}{3}\left(u - \frac{1}{4}u - \frac{1}{6}u\right) = Rp\ 175.000$ $\frac{12}{12}u - \frac{3}{12}u - \frac{2}{12}u - \frac{1}{3}\left(\frac{12}{12}u - \frac{3}{12}u - \frac{2}{12}u\right) = Rp\ 175.000$	<p style="text-align: right;"><i>Translation.</i></p> $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = 175,000$ $\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x - \frac{1}{3}\left(\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x\right) = 175,000$
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“Try to explain the shape you wrote, namely $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = 175,000$.”

ADZ : “ x is Mrs. Siti's money that will be sought. Here x is minus one quarter, one-sixth, and one-third left.”

R : “Then, how?”

ADZ : “I made a new equation from the results of the previous equation operation by equating the denominator to $\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x - \frac{1}{3}\left(\frac{12}{12}x - \frac{3}{12}x - \frac{2}{12}x\right) = 175,000$.”

R : “Okay. Now pay attention to your answer here (points to the answer sheet).”

$\frac{7}{12}u - \frac{1}{3}\left(\frac{7}{12}u\right) = Rp\ 175.000$ $\frac{7}{12}u - \frac{7}{36} = 175.000$ $\frac{21}{36}u - \frac{7}{36} = 175.000$ $\frac{14}{36}u = 175.000$	$u = 175.000 \times \frac{36}{14}$ $= \frac{3.150.000}{14}$ $= 450.000$	<p style="text-align: right;"><i>Translation.</i></p> $\frac{14}{36}x = 175,000$ $x = 175,000 \cdot \frac{36}{14}$
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“Here you write that $\frac{14}{36}x = 175,000$ next you write $= 175,000 \cdot \frac{36}{14}$. Try to explain why is that?”

ADZ : “We are looking for the value of x . So 175,000 divided by $\frac{14}{36}$ ”

R : “Why did you write it $x = 175,000 \cdot \frac{36}{14}$ in the next step?”

ADZ : “Because it is divided by a fraction, it means the same as multiplying by the opposite number. So, I wrote $= 175,000 \cdot \frac{36}{14}$ ”

R : “Okay.”

The interview excerpt suggested that the subject was capable of solving the problem according to the planned steps or strategy. The subject systematically performed algebraic operations involving aspects and indicators of reversible thinking, namely negation.

Furthermore, during the backstage locking, the ADZ subject re-examined the answers in reverse by testing the previously obtained answers into operation until they returned to the original information. ADZ's answers during the locking backstage (re-checking answers) are shown in Figure 6.

Figure 6
The Results of The Work of ADZ (Reverse Process)

	<p style="text-align: right;"><i>Translation.</i></p> <p>Known. Mrs. Siti's money = 450,000 Rice = $\frac{1}{4}x = \frac{1}{4} \times 450,000 = 112,500$ Sugar = $\frac{1}{6}x = \frac{1}{6} \times 450,000 = 75,000$ Parking fees = $\frac{1}{3}(450,000 - 112,500 - 75,000) = \frac{1}{3}(262,500) = 87,500$ Leftover money = x $112,500 + 75,000 + 87,500 + x = 450,000$ $275,000 + x = 450,000$ $x = 450,000 - 275,000$ $x = 175,000$</p>
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Figure 6 implies that ADZ subjects demonstrate excellent problem-solving skills. This allows the subject to double-check the previously formulated answers to the test questions. At this point, the subject used reversible thinking to perform a reverse process, representing the ability to return to initial data after obtaining the result. ADZ then used the correct procedure to return the new equation to its original form, namely $112,500 + 75,000 + 87,500 + x = 450,000$. In the previous answer, ADZ carried out the forward process by writing $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}(x - \frac{1}{4}x - \frac{1}{6}x) = 175,000$ as the initial step of problem-solving. Therefore, ADZ used the inverse nature of related operations, addition and subtraction, when carrying out problem-solving steps.

Excerpts from the interview at the locking backstage are shown in the following.

R : “Could you please explain the steps or strategy you took to answer the question?”

ADZ : (points to the answer sheet).

	<p style="text-align: right;"><i>Translation.</i></p> <p>Known. Mrs. Siti's money = 450,000 Rice = $\frac{1}{4}x = \frac{1}{4} \times 450,000 = 112,500$ Sugar = $\frac{1}{6}x = \frac{1}{6} \times 450,000 = 75,000$ Parking fees = $\frac{1}{3}(450,000 - 112,500 - 75,000)$ $= \frac{1}{3}(262,500)$ $= 87,500$</p>
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”In the first step, we calculate how much rice to buy. How much sugar to buy? How much? We will continue to operate to get the rest of the money.”

R : “Are there any difficulties that you encountered?”

ADZ : “I don’t think there is, sir.”

R : “Now, look at your answer again (points to the answer sheet).”

	<p style="text-align: right;"><i>Translation.</i></p> <p>$275,000 + x = 450,000$ $x = 450,000 - 275,000$</p>
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“Explain the steps you did!”

ADZ : “We are looking for the value of x that satisfies the equation. We need to find the value of x that will make the equation true. To do so, we must determine how much

needs to be added to 275,000 to get a result of 450,000. The calculation is as follows: $x = 450,000 - 275,000$, which simplifies to $x = 175,000$.”

- R : “What kind of operation do you use this for?”
- ADZ : “I use the inverse of addition, sir.”
- R : “Are you sure your answer is correct?”
- ADZ : “Sure, sir. Because the results are to my answer earlier.”
- R : “Okay.”

The information obtained from the above interview excerpts indicated that the subject is capable of properly checking previously obtained answers by working on the reverse question. At this locking backstage, the reversibility of the operations appears, such as addition and subtraction, allowing the return to initial data after obtaining the result and negation.

Based on the results of answers and interviews with both subjects, the obtained aspects and indicators of reversible thinking that appear during problem-solving procedures are presented in Table 2.

Table 2
Aspects and Indicators of Reversible Thinking Carried Out by Subject

Initial Subject	Forward Process Subject Activity	Reverse Process Subject Activity	Aspects and Indicators that Appear
WMW	The subject operates the shape $\frac{14}{36}x = 175,000$ become shape $14x = 175,000 \cdot 36$	-	Reciprocity, multiply both sides of the equation by the same number, in this case, 36. By doing so, the denominator 36 is removed, and compensation is made by multiplying both sides by 36
	The subject operates the shape $14x = 3,600,000$ become shape $x = \frac{3,600,000}{14}$	-	The subject used negation by inverting multiplication and division operations to create new equations. To find the value of x , divide 6,300,000 by 14. The subject stated that 14 times x equals 6,300,000.
	-	The subject writes the formula $x - \frac{1}{4}x - \frac{1}{6}x$ $-\frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right)$ $= p$ in the first step until finding the correct result	Capability to return to initial data after obtaining the result, where the subject performs step-by-step with the application of the correct algebraic operations starting from the shape $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = p$ to get the right result, namely $p = 175,000$

Initial Subject	Forward Process Subject Activity	Reverse Process Subject Activity	Aspects and Indicators that Appear
ADZ	The subject operates the shape $\frac{14}{36}x = 175,000$ Become shape $x = 175,000 \cdot \frac{36}{14}$	-	Negation, where the subject uses the inversion of multiplication and division operations to create new equations. The subject multiplies x by 14 and then divides it by 36. The result is 175,000. Then, the subject looks for x by multiplying 175,000 by 36 and then dividing it by 14.
		The subject writes a new equation formula, namely $112,500 + 75,000 + 87,500 + x = 450,000$ after identifying each part until finding the correct result	Capability to return to initial data after obtaining the result, where the subject performs the step to correct application of algebraic operations starting from identifying parts: rice = $\frac{1}{4}x = \frac{1}{4} \times 450,000 = 112,500$; sugar = $\frac{1}{6}x = \frac{1}{6} \times 450,000 = 75,000$; parking fees = $\frac{1}{3}(450,000 - 112,500 - 75,000) = \frac{1}{3}(262,500) = 87,500$ to get the correct result, namely $p = 175,000$
	-	The subject operates the shape $275,000 + x = 450,000$ to form $x = 450,000 - 275,000$	Negation, where the subject uses inversion of addition and subtraction operations to create new equations. The subject says that 275,000 plus x makes 450,000. Next, find x , then 450,000 minus 275,000.

Table 2 summarizes several different problem-solving strategies or steps. In the forward process, even though both subjects make the same initial equation with different calculation algorithms, they present different aspects and indicators of reversible thinking. From the WMW subject, the aspects that appear are reciprocity and negation, while in the ADZ subject utilizes negation. In the reverse process, the WMW subject demonstrates the capability to return to initial data after obtaining the result, while the ADZ subject shows both the capability to return to initial data after obtaining the result and negation.

Discussion

Problem-solving in the Reversible Test of Thinking (TTR) was given to Adversity Quotient Climbers (AQC) subjects, coded by WMW and ADZ, through two processes, the forward and the reverse process. These two processes were carried out to identify the process of a) the formation of a two-way process and b) the reversal of mental processes in the thinking

process. According to Muir (1988), developing reversible thinking processes and thinking about mathematical processes from input to output and output to input can be associated with one's mathematical ability. This reverse process can be used to verify the accuracy of answers and assess the subject's comprehension of the material after completing the forward process. Mafulah & Juniati (2020) described that the forward process is carried out to identify aspects that arise in solving future problems, while the reverse process is used to reveal aspects in the reversal process. WMW and ADZ subjects carry out the forward process in polya steps through 3 stages, namely understanding the problem, preparing a solution plan, and solving the problem according to plan.

When understanding the problem, both subjects analyze the information about the problem by identifying the known information and asking questions using their own language mathematically. Riswang et al. (2021) explained that in identifying problems and determining goals, AQC subjects use their own words to express aspects of the question that are known and asked. Karimah et al. (2018) also concluded that students in the climber category are able to identify existing information and relationships between information relevant to problem-solving. At this stage, both subjects used x as a substitute variable for Mrs. Siti's money and were able to rewrite the meaning of the question correctly.

When preparing a problem-solving plan, WMW and ADZ subjects retrieve information by repeating the concept of algebraic operations. They perform this considering that the information contained in the problem requires analysis before being used as a reference for solving the problem. Marsitin et al. (2022) asserts that creating a problem-solving plan aids in determining the relationship between known information and unknown information to calculate a certain variable. In line with the results of research (Juwita, 2020), describing that subjects in the climber category can link or look for relationships between information and previous knowledge, providing compelling arguments regarding the chosen settlement plan. At this stage, apart from strengthening understanding of the concept of algebraic operations, both subjects also create equations based on information analysis as a reference for problem-solving. Their equation states $x - \frac{1}{4}x - \frac{1}{6}x - \frac{1}{3}\left(x - \frac{1}{4}x - \frac{1}{6}x\right) = 175,000$.

At the problem-solving stage, WMW and ADZ subjects connect the analyzed information with a problem-solving plan to determine goals. They process information by associating all relevant knowledge and asking questions based on problem-solving strategies. The method used is an advanced procedure that emphasizes reversible aspects of thinking, such as negation. Both subjects carry out the aspect of negation when changing forms $\frac{14}{36}x = 175,000$ to $x = 175,000 \cdot \frac{36}{14}$.

In the next stage, WWM and ADZ subjects carry out the reverse process on the final answer from the forward process that has been completed by linking existing information back to their problem-solving. WMW and ADZ also check the problem-solving answers in reverse through the last polya stage called the locking back stage. Both subjects work on questions using the opposite process but through different strategies or steps. Similarly, Mafulah & Juniati (2020) reported that the forward and backward processes do not necessarily have to proceed in the same way. The subject's process of creating and returning a new equation to the initial equation can be done through different processes. At this stage, the

WMW subject carries out similar steps to the forward process. In contrast to MWM, ADZ carries out the stages of understanding the problem and making a solution plan simultaneously. ADZ has been observed to identify the problem by directly multiplying each part by the result obtained in the forward process, namely 450,000 resulting in a new equation of $112,500 + 75,000 + 87,500 + x = 450,000$.

At first, the WMW and ADZ subjects face difficulty creating new equations, so they return to their original form. However, after reviewing the information repeatedly, both subjects are convinced of the possible strategy or steps. Linearly, a study by Juwita (2020) defining that climbing students sometimes experience difficulty in returning answers to initial questions at the re-examination stage. This can occur due to limited skills in processing the information to achieve goals.

The individual with the Climber's Adversity Quotient type tends to keep trying to find solutions to the problems they face using their own methods or strategies. When AQC subjects have difficulty finding an equation formula to solve a problem, they carefully repeat the analysis of existing information to understand the ideas to solve the problem. This is in line with research on Marsitin et al. (2022) that the higher a person's AQ level reflects their higher efforts in achieving mathematical understanding abilities and easily face the difficulties they experience.

Conclusion

The subjects in this study employ a reversible thinking process with different problem-solving strategies. Differences in strategy may arise when identifying problems and responding to information in questions. This process is closely linked to the mastery and application of concepts, as well as reviewing the results obtained. Mathematical symmetrical (reciprocal) relationships, direct and reverse operations, as well as simple skills in formula reading and problem-solving are distinct types. AQC students' internal psychological basis involves reconstructing the direction of mental processes, transitioning from directing to reverse thinking and forming two-way (reversible) associations.

To solve problems, AQC students take two solution paths, namely the forward process and the backward process. In the advanced process, the subject identifies problems that involve x as a substitute variable, creates new equations that link the relationship between variables and carries out calculation operations that involve reversible thinking aspects. In this process, two aspects of reversible thinking emerge, namely negation and reciprocity. In the reverse process, the AQC subject formulates the answer in the forward process as information and makes one of the information in the forward process a question. Thus, in the problem-solving process, the opposite process occurs with the reversible aspect of thinking emerging, namely the ability to return to the initial data after obtaining results and negations.

Reversible thinking is an important technique in problem-solving. By working on a problem in two ways, students can test their results and ensure accuracy. It involves approaching a problem from multiple perspectives, which can increase cognitive abilities and confidence in the correctness of answers. By working on a problem in two ways, students can test their results and ensure accuracy. By working on a problem in two ways, students can test

their results and ensure accuracy. This technique is particularly useful in mathematics. Furthermore, reversible thinking promotes active and relevant thinking in identifying networks or solving complex mathematical problems.

It is important to note that this research is limited to routine school questions and only analyzes the responses of the participants. The analysis of the reversible thinking processes of students with the Adversity Quotient Type of Climbers (AQC) provides preliminary information for further study of the same problem from a different perspective. This research can serve as study material or a reference for mathematics teachers developing learning that accommodates students' AQ differences.

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