Stages of Problem-Solving in Answering HOTS-Based Questions in Differential Calculus Courses

Eko Andy Purnomo[1], Y.L. Sukestiyarno[2*], Iwan Junaedi[2], Arief Agoestanto[2]

[1] Doctoral Students of Graduate School, Universitas Negeri Semarang, Indonesia,
[2] Department of Mathematics, Universitas Negeri Semarang, Indonesia

ekoandy@unimus.ac.id, sukestiyarno@mail.unnes.ac.id*, iwanjunmat@mail.unnes.ac.id, arief.mat@mail.unnes.ac.id

Abstract: Students are required to have the ability to implement mathematics in solving everyday life problems. A good solving process will produce an excellent solving ability. The existing problem-solving stages cannot be used in solving problems with the Higher Order Thinking Skills (HOTS) category. The aims of this research are 1). to know students' problem-solving process in solving HOTS category questions; 2). To design the stages of problem-solving that can be used to solve HOTS category questions. Twenty-four students who took the differential calculus course in mathematics education at one of Indonesia's private universities made up the study group. The solving process is divided into high, medium, and low. The descriptive and qualitative research method describes the Polya stages' problem-solving approach. Researchers used four sets of problem-solving by making nine indicators. Based on the analysis results, it can be concluded that several indicators of the problem-solving stages have not been appropriately implemented, including I4, I8, and I9. In addition, the steps at the Polya problem-solving stage of devising a plan and looking back must be improved. Based on the findings above, research recommendations design the stages of problem-solving consisting of 6 steps. Future research will develop steps of problem-solving with the characteristics of the HOTS category questions.

Keywords: HOTS, Problem-Solving, Polya

INTRODUCTION

Problem-solving skill is the highest skill in mathematics. It is one of the mathematical skills that students must possess (Akay & Boz, 2010; Dagan et al., 2018; Nurkaeti, 2018; Osman et al., 2018; Eichmann et al., 2019; Rott et al., 2021; Sulistyaningsih et al., 2021; Purnomo et al., 2022). Problem-solving requires applying several mathematical principles and knowledge to non-routine, open-ended, real-life problems, which helps solve everyday problems. Problem-solving is how to solve the non-routine problems (Temur, 2012; Güner & Erbay, 2021; Pardiansyah et al., 2021) and complex problems (Greiff, S & Fischer, 2013) to find the most appropriate solution (Greiff, S & Fischer, 2013; Yayuk & Husamah, 2020). The importance of this ability means that every student-teacher candidate must have good solving skills (Barnett & Francis, 2012; Purnomo & Mawarsari, 2014; Santoso, 2016). The problem-solving process requires the ability to understand the problem and the willingness to face the problem and solve the problem (Bingolbali, 2011; Dostál, 2015; Purnomo et al., 2020; Thienngam et al., 2020). Problem-solving skills are used in applying mathematics in everyday life. Students who have good problem-solving skills will quickly and precisely solve problems.

Many research results that the problem-solving abilities are still low (Hidayat & Irawan, 2017; Kusuma et al., 2017; Kilic & Sancar-Tokmak, 2017; Yeni et al., 2020; Aziz et al., 2021; Güner & Erbay, 2021). Factors causing low problem-solving abilities include students who are not used to solving problems (Abdullah et al., 2015), especially those involving real-life situations (Aziz et al., 2021; Puteh et al., 2017). Solving problems has many obstacles, including the problem-solving process containing errors that do not follow the legal problem-solving stages (Rott et al., 2021). Problem-solving has many problems for students, but not much research has been done on developing problem-solving models.

Improving problem-solving stages will make it easier for students to solve problems. Previous research with the problem-solving theme only described the stages of Polya's problem-solving (Pardiansyah et al., 2021), described the steps in terms of conceptualization and problem-solving abilities (Delahunty et al., 2020), knowing the impact of students' abilities in solving non-routine math problems (Saadati & Felmer, 2021), analyzed the problem-solving process in terms of the language of mathematics (Strohmaier et al., 2019), investigated critical thinking and problem-solving abilities used by students (Shanta & Wells, 2020), students' cognitive barriers in the process of solving mathematics problems (Antonijević, 2016), describing mathematics teaching through issue solving (Zhang & Cai, 2021), fourth and eighth-grade students' misconceptions in solving issues (Delahunty et al., 2020). There are no recommendations to improve the problem-solving stages based on previous research.

One of the subjects that frequently applies problem-solving is the Differential Calculus course. The Differential Calculus course is a material that is rarely used to solve problems in implementing derivative materials in everyday life. Applying mathematics in this course can be in economics, engineering, chemistry, and others. Based on observations in Differential Calculus courses, many students do not have good problem-solving skills. The difficulty in solving student problems means that the questions apply derivatives in the High Order Thinking Skills category. In solving problems in the application of results, it is necessary to have stages of solving according to the characteristics of the HOTS category questions.

Problem-solving is a complex activity that includes higher-order thinking skills (Simamora et al., 2018; Gursan & Yazgan, 2020). Problem-solving is an activity that encourages students to use HOTS. It is referred to as the level of thinking needed to shape the 21st-century generation that has the potential to compete globally with intelligence, creativity, and innovation (Hamzah et al., 2022). The achievement of the HOTS thinking process includes high knowledge, which provides for analytical, evaluative, and synthetic thinking levels. According to Bloom, education should focus on competence (subject mastery) and higher-order thinking outcomes. HOTS focuses on developing students' ability to analyze and evaluate by inferring existing information and creating (synthesizing) something new (Anderson et al., 2001; Wilson, 2016). A student can't differentiate and HOTS if he succeeds in solving the top four Bloom taxonomic indicators (Aziz et al., 2021).

Based on research, many students have low HOTS abilities (Misrom et al., 2020; Kim How et al., 2022; Andin & Aziz, 2019; Abdullah et al., 2015). Based on the research results, High Order Thinking Skill needs to be developed in problem-solving (Aziz et al., 2021; Osman et al., 2018; Mohd Rusdin et al., 2019; Ismail et al., 2022). Low problem-solving ability causes students to have still difficulty solving questions in the HOTS category (Amir, 2015; Karimah et al., 2018; Santoso, 2016; Misrom et al., 2020). Improving problem-solving abilities can be done by being trained (Abosalem, 2015), getting used to interacting with problem-solving problems (Barnett & Francis, 2012; Andin & Aziz, 2019), and getting used to solving complex problems (Purnomo et al., 2014; Yeni et al., 2020; Purnomo et al., 2022), and integrating digital literacy (Kim How et al., 2022). Future research explores possible patterns of teachers dealing with problem-solving and identifies the most effective discourse patterns when teaching mathematics through problem-solving (Zhang & Cai, 2021), focusing on the integration of digital literacy in improving HOTS (Kim How et al., 2022)

In this study, the stages of solving Polya's problem is elaborated. There are four stages of Polya problem-solving recently, however it less relevant to solve HOTS-based questions. For that reason, researchers modify and add several stages to make it more relevant. Many researchers can understand the profile of issue-solving abilities through these indicators. Based on the analysis of problem-solving steps, the students' errors in solving problems will be seen. Through the mistakes made, the root of the problem is students' inability to solve problems. The situation suggests a new completion stage to improve problem-solving, especially HOTS category questions.

LITERATURE REVIEW

Polya Problem Solving

Problem-solving arises from psychological and pedagogical confusion in mathematical problems (Schoenfeld, 1987). Problem-solving is based on various cognitive processes, such as metacognitive processes (Schoenfeld, 1992), attention, memory, and language (Jitendra et al., 2015). Numerous theories propose the phases of problem-solving in the problem-solving model.

The first experts to introduce problem-solving were Dewey, Wilson, et al., Bransford & Stein, Schoenfeld, Burton & Stacey, and Polya. Each theory of solving stages has its characteristics. Based on the analysis, each approach has its advantages and disadvantages. This study will look at students' problem-solving abilities profiles based on the theory of Polya's problem-solving stages..

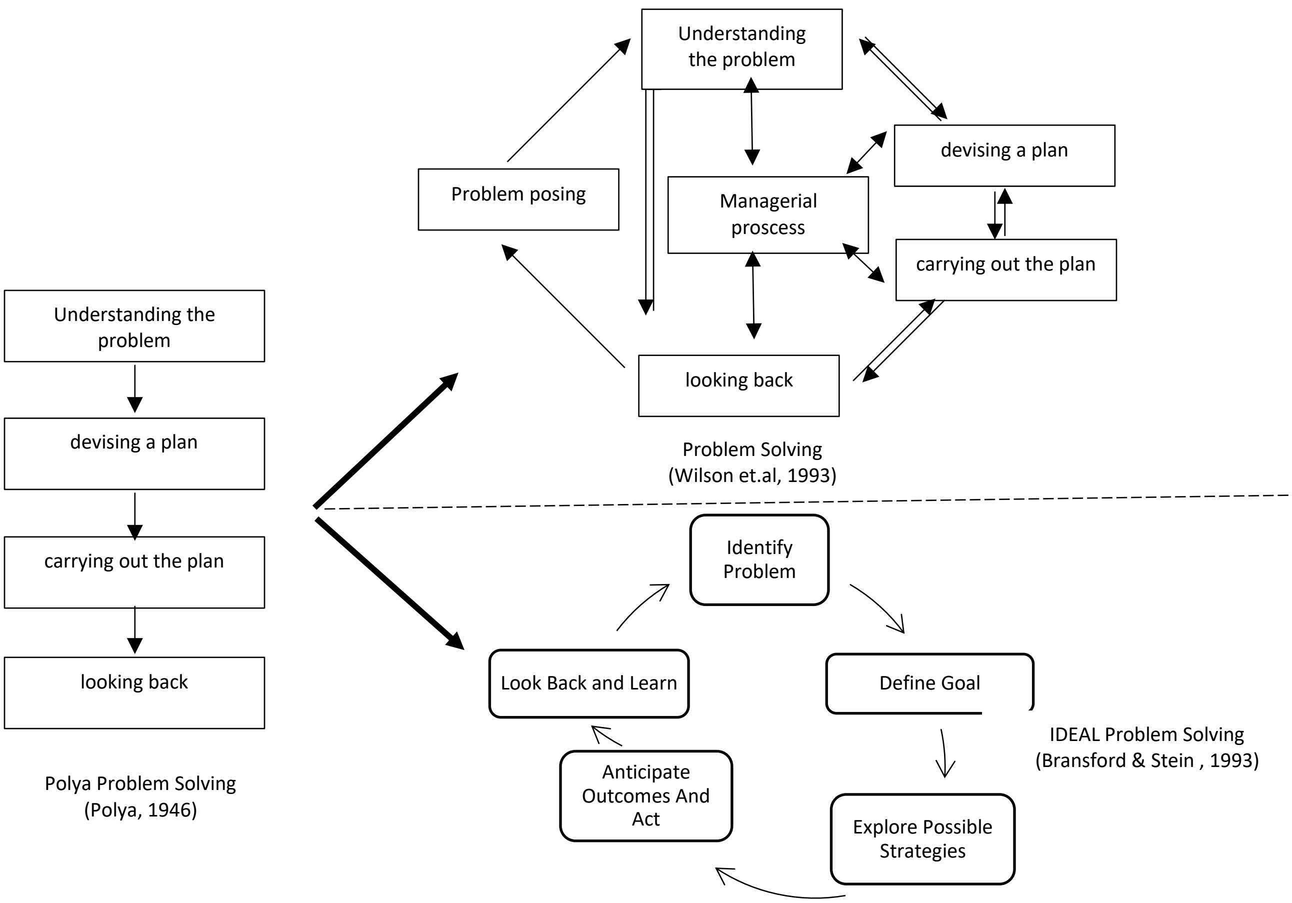


Figure 1. Modify Polya problem-solving stages

There are four stages of problem-solving skills: understanding the problem, devising a plan, carrying out the project, and looking back (Robson & Polya, 1946). Polya problem solving has been modified by several researchers, including (Wilson et.al, 1993) and (Bransford & Stein, 1993). An overview of Polya's troubleshooting modification can be seen in Figure 1 above. Figure 1 shows that the modification of problem-solving carried out by Wilson et al. (1993) is still in the same stages as Polya. The looking backstage in Polya is divided into two phases: anticipate outcomes & act, and look back & learn. Wilson et al. added four steps plus the managerial part of the process and problem posing. Bransford & Stein (1993) added five stages. Using the findings from examining the Polya solving phases, it can still be modified and adapted to problem-solving characteristics.

Higher Order Thinking Skills (HOTS)

Students must think higher in receiving new information, organizing and storing information in long-term memory, connecting details and existing knowledge, and processing data to solve a problem. This ability is known as HOTS. It is a method of creative thinking, problem-solving process, and critical situations in making good decisions (Behar-horenstein, 2011) in complex cases (Andin & Aziz, 2019). HOTS in Bloom's revised Taxonomy, namely application skills, analyzing, evaluating, and creating (Anderson et al., 2001; Wilson, 2016).

Mathematical Modelling

Mathematical modeling is a tool for deciphering a system's dynamics and forecasting future outcomes (Varaki & Earl, 2006). Mathematical modeling has been positioned at the forefront of many levels of education globally as modeling strengthens purposeful problem-solving skills, connects mathematics to the real world, and makes mathematics more meaningful and relevant. Mathematical modeling is identifying a situation in the real world, confirming assumptions and choices, and then using a mathematical model to derive a solution that can be translated back into the real world. Using mathematical modeling in learning mathematics allows students to understand mathematics more meaningfully, learn mathematics by relating it to real life, and eliminate the inadequacy of available problems (Yasa & Karatas, 2018). The Figure below illustrates that mathematical modeling involves a multistep and iterative process.

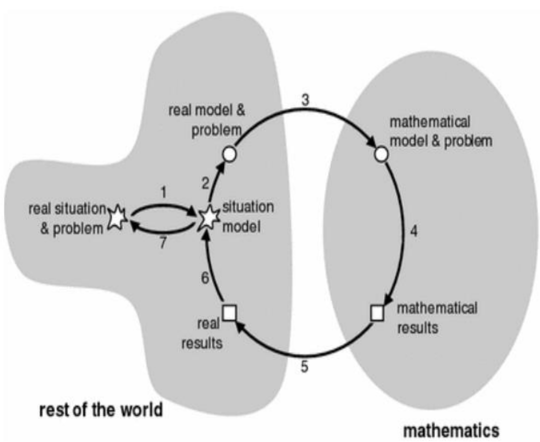


Figure 2. Mathematical modeling process

RESEARCH METHODOLOGY

This research employed a qualitative descriptive study that described students' problem-solving abilities. Students from the mathematics education study program in Central Java, Indonesia, made up the research group. The sample of this study was 24 students who had done educational internships. The research sample was selected using the cluster sampling technique to obtain three categories, namely high ability (A1), moderate ability (A2), and low ability (A3). There were three categories of questions such as easy, medium, and complex types. Problem-solving problems in this study were chosen as non-routine problems that could not be solved directly (Saadati & Felmer, 2021). Problem-solving questions consist of the HOTS category with the ability to analyze, evaluate, and create. The instrument employed in this study was a problem-solving test consisting of three items tested by expert validation with an assessment of content, context, and language aspects. The results of the validation showed that the average overall score was 4.5 (out of a total score of 5), which include very valid category. The following are indicators of problem-solving abilities in this study.

HOTS Ability Category	Question	The Steps in Answering the Questions
Analyzing (C4) Easy	An open box is made of a zinc sheet in a square measuring 12 cm inside. By cutting at each end of the congruent squares, determine the size of the maximum volume of the box!	Problem Solving Stages: 1. Describing the requested conditions 2. Determining the mathematical model 3. Solving with derivatives
Evaluating (C5) Medium	The operating cost of a truck is estimated to be (30+v/2) cents per mile when driven at v miles/hour. Drivers are paid 14 dollars per hour. With a speed limit of 40≤v≤60. At what rate would it be cheapest to ship to a city k miles away?	Problem Solving Stages: 1. Describing the requested conditions 2. Determining the mathematical model 3. Connecting variables with each other 4. Solving with derivatives
. Creating (C6) Difficult	Finding the size of a rectangular, cylindrical cylinder with as large as possible can be placed inside a rectangular cone!	Problem Solving Stages: 1. Describing the requested conditions 2. Setting the variables 3. Finding a mathematical model, then determine the maximum quantity 4. Connecting variables with each other 5. Solving using derivatives

Table 1. Problem Solving Questions using HOTS Category

The researchers collected the data by triangulation, namely evaluation tests, observations, and in-depth interviews (Creswell, 2014; Sukestiyarno, 2020). The evaluation test has three levels: easy, medium, and high. Determination of problem-solving indicators employs Polya indicators (Robson & Polya, 1946; Argarini, 2018; Puspa et al., 2019; Nurkaeti, 2018). Based on the indicators of these studies, the indicators are shown in Table 2.

Problem Solving Steps	Indicator	Code
Understanding the problem (2 indicators)	1. Analyzing question	I1
	2. Focusing question	I2
Devising a plan (2 indicators)	3. Associating the relationship between known data to find things that are not known.	I3
	4. Determining the method used to solve the problem	I4
Carrying out the plan (2 indicators)	5. Executing the solution plan and checking every step.	I5
	6. Determining the solution to the problem and writing down the solution or answer to the problem	
	1. Determining the solution to the problem and writing down the solution or answer to the problem	I6
Looking back (3 indicator)	2. Determining conclusion	I7
	3. Is there an alternative to get a different result?	I8
	4. Checking the accuracy of answers to questions	I9

Table 2. Polya Problem Solving Indicator

Data from tests, interviews, and observations were analyzed by content analysis consisting of three activities: data reduction, data presentation, and conclusion (Creswell, 2012; Miles et al., 2014). By coding the interview files, data was reduced. Coding is used to make monitoring crucial information about how exposed existing data is more accessible. The next step is to show the data after it has been reduced. The following stage is to use in-depth interviews to confirm the data. Whereas, concluding the field data is the final stage. To create a comprehensive picture, data analysis employs an inductive method where conclusions are made after a thorough investigation of small cases (Sukestiyarno, 2020).

FINDINGS

In this study, to see the profile of problem-solving abilities by giving three questions with easy, medium, and high difficulty levels. The results of student work were investigated and compared according to their level of knowledge. The work results were analyzed using the problem-solving ability stages whose indicators have been determined. The analysis results had been a reference in looking at the profile of students' problem-solving abilities.

The first analysis used the easy category problem with the question "*An open box is made of a square sheet of zinc measuring 12 cm sides. By folding at each the end of the congruent squares, please, determine the maximum volume of the squares*!" The results of students' answers in easy category questions can be seen below.

Stage 1: *understanding the problem*

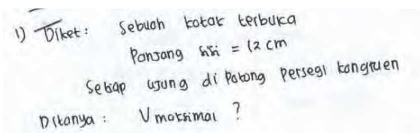
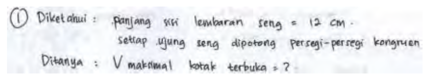
<table>
<tr><td>High Skill (A1)</td><td>Medium Skill (A2)</td><td>Low Skill (A3)</td></tr>
<tr><td>
</td><td>
</td><td>Do not write</td></tr>
<tr><td>Translation :
Given :
• An open box with a side length of 12 cm
• Each end of the square is cut with a congruent square shape
Asked: Maximum Volume?</td><td>Translation :
Given :
• Zinc sheet side length = 12 cm
• Each zinc end is cut into a congruent square
Asked: Max volume of the open box?</td><td></td></tr>
</table>

Figure 3 : The results of student work on the analyzing (C4) questions in the problem solving stage 1

In stage 1, indicator 1 (I1), students A1 and A2 completely write everything known in the questions. For A3, students do not write what is known in the questions. Based on the interview results, A3 did not write down what was known in the questions because A3 felt that this was not necessary to answer the questions. What is known is that there is a box measuring 12 cm in length, while the question is the maximum volume. In stage 1, indicator 2 (I2), students do not write down that are not known. Students should be able to write down something they don't know because later, it will become a basis for planning in answering questions.

An in-depth interview was conducted to find out why A3 students ignored to write what information was known. The results of interviews showed that students actually were able to read the questions well. However, their absence of writing the information because they consider it as unnecessary and unimportant. The students considered that solving problems could be directly solved as long as they understand the concepts and do the calculations well.

Stage 2 : *devising a plan*

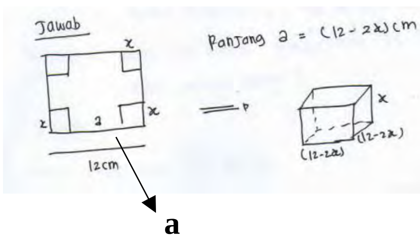
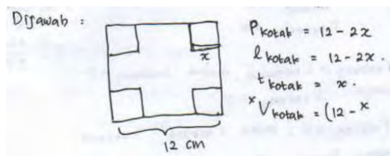
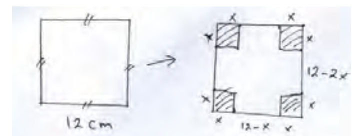
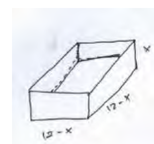
High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
		 
Translation : Length a = (12-2x) cm	**Translation :** • box length = (12-2x) cm • box width = (12-2x) cm • box height = x cm • box volume = 12-x cm	

Figure 4: The results of student work on the analyzing (C4) category questions in the problem-solving stage2

In stage 2, indicator 1 (I3), students A1, A2, and A3 have found a relationship between the data and the unknown, namely the shape's size and its pictures. A1, A3 are written in full from the net and its size and the shape of the space formed. Even A3 students wrote down from the initial process that it was known that there was a box with a height of 12 cm, then each corner would be cut along the x length and the space that would be formed. Student A2 wrote down only the picture and its measurements. The interview results with student A3 showed that students were able to correlate the unknown information from the questions, so they could find new information. However, they could not visualize it in pictures. Indicator 2 stage 2 (I4) all students do not write explicitly in the answers, but only in the students' thoughts. The solving steps for students A1, A2, and A3 are the same: finding the mathematical model of the volume of space, finding the stationary point, and substituting the critical point in the volume equation.

From the results of in-depth interviews with students A1, A2, and A3, it was found that all students successfully identified the stages of solving the problem well; however, solution plans they had in mind were not written down. They ignored the plan and tended to write the solution directly to save the time.

Stage 3: Carrying out the plan

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
$V_{kotak} = p \times l \times t$ $= (12-2x)(12-2x)(x)$ $= (144-24x-24x+4x^2)x$ $= (4x^2-48x+144)x$ $= 4x^3-48x^2+144x$ $V_{maksimal} \rightarrow V' = 0$ $12x^2-96x+144=0 \quad :12$ $x^2-8x+12=0$ $(x-6)(x-2)=0$ $x-6=0 \vee x-2=0$ $x=6 \quad x=2$ untuk $x=2$ $V(x) = 4x^3-48x^2+144x$ $V(2) = 4(8)-48(4)+144(2)$ $= 32-192+288$ $= 32+96$ $= 128 \text{ cm}^3$ untuk $x=6$ ~~$V(6) = 4)$~~ $V(x) = 4x^3-48x^2+144x$ $V(6) = 4(6)^3-48(6)^2+144(6)$ $= 4(216)-48(36)+864$ $= 864-1728+864$ $= 0$	$V_{kotak} = p \times l \times t.$ $= (12-2x)(12-2x)x.$ $= (144-24x-24x+4x^2)x$ $= 144x+48x^2+4x^3$ $V = 144x-48x^2+4x^3$ Syarat $V'=0.$ $144-96x+12x^2=0. \quad :12$ $12-8x+x^2=0.$ $(x-6)(x-2)=0.$ $x=6 \vee x=2.$ $x=2 \longrightarrow V$ $V = 144x-48x^2+4x^3$ $= 144(2)-48(2)^2+4(2)^3$ $= 288-192+32$ $= 128 \text{ cm}^3.$ $x=6 \longrightarrow V$ $V = 144x-48x^2+4x^3.$ $= 144(6)-48(6)^2+4(6)^3$ $= 864-1728+864$ $= 1728-1728.$ $= 0 \text{ cm}^3$ Jadi, volume maksimal kotaknya adalah 128 cm^3.	$V = plt$ $= (12-2x)(12-2x)x$ $= (144-48x+4x^2)x$ $= 4x^3-48x^2+144x$ $V' = 12x^2-96x+144$ $0 = x^2-8x+12$

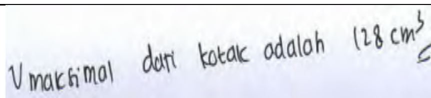
		
Translation : The maximum volume of the box is 128 cm^3	**Translation :** The maximum volume of the box is 128 cm^3	

Figure 5: The results of student work on the analyzing (C4) category questions in the problem-solving stage 3

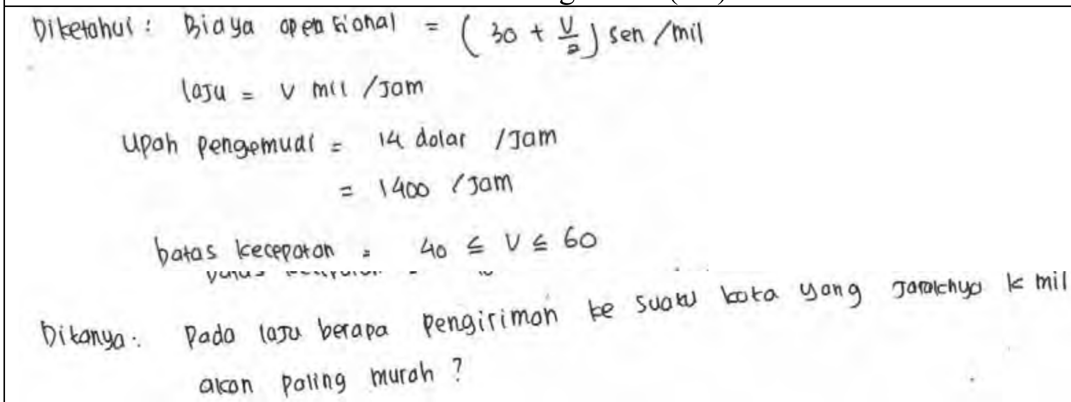
In stage 3, indicator 1 (I5), All students have found the correct volume of space shapes, and all are correct. The next step is to see the conditions for getting to the critical point with the condition $V' = 0$. In this step, all students have implemented it, but A3 does not explicitly write it down. A1 and A2 get the critical point, namely $x = 6$ *or* $x = 2$; A3 doesn't get the crucial point. To get the maximum volume, A1 and A2 plug the two critical points into the volume equation. The two students answered the questions correctly, but A3 was only up to determining the requirements for getting the maximum volume. The interviews result with A3 students showed that students needed more time to complete them. In stage 3 indicator 2 (I6), students A1 and A2 have written the correct answer, such as the maximum volume of 128 cm^3. The results of interviews with A3 students showed that students had not accomplished the question because they did not have enough time.

Stage 4: looking back

In stage 4, there are three indicators I7, I8, and I9. In indicator I7, students have done it, but it is not written explicitly in the answer. At I8, only A1 students tried to find other alternative solutions. The result of interview with student A1 showed that A1 was able to solve the problem well, however she/he was little hesitated when writing the solution. At that time, A1 substituted the x value in the length, width, and height equations to figure out the volume. Whereas, the interview result with student A2 stated that students were pretty sure to write the answer and o not even think about other alternative ones. Therefore, in stage I9 both A1 and A2 have successfully carried out this step and answer the question accurately. On the other hand, student A3 only answered the questions up to the maximum volume requirement. According to the result of interviews with students A1 and A2, the answers they provided were in accordance with the questions such as determining the volume and the value of the volume. The medium category problem is given the following questions: "The operating costs of a truck are estimated to be $(30 + \frac{v}{2})$ cents per mile while driving by mile/hour. Drivers are paid 14 dollars per hour. With speed limitation at $40 \leq v \leq 60$. At what rate would shipping costs to a city mile away be cheapest? " The results of student answers can be seen below.

Stage 1: ***understanding the problem***

High Skill (A1)


Diketahui : Biaya operasional = $\left(30 + \frac{v}{2}\right)$ sen /mil
laju = v mil /Jam
Upah pengemudi = 14 dolar /Jam
= 1400 /Jam
batas kecepatan = $40 \le v \le 60$
Ditanya: Pada laju berapa pengiriman ke suatu kota yang Jaraknya k mil akan paling murah ?

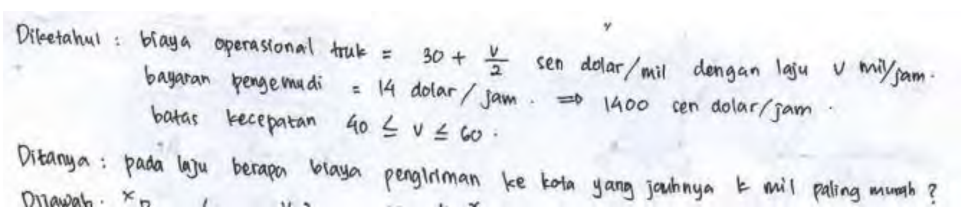
Translation :

Given :

- operating costs = (30+v/2)cents/mile
- Speed = v miles/hour
- Driver fare = 14 dollars/hour = 1400/hour
- Speed limit = 40≤v≤60

Asked : At what rate will shipping to a city k miles away be cheapest?

Medium Skill (A2)


Diketahui : biaya operasional truk = $30 + \frac{v}{2}$ sen dolar/mil dengan laju v mil/jam.
bayaran pengemudi = 14 dolar/jam. ⇒ 1400 sen dolar/jam.
batas kecepatan $40 \le v \le 60$.
Ditanya : pada laju berapa biaya pengiriman ke kota yang jauhnya k mil paling murah ?

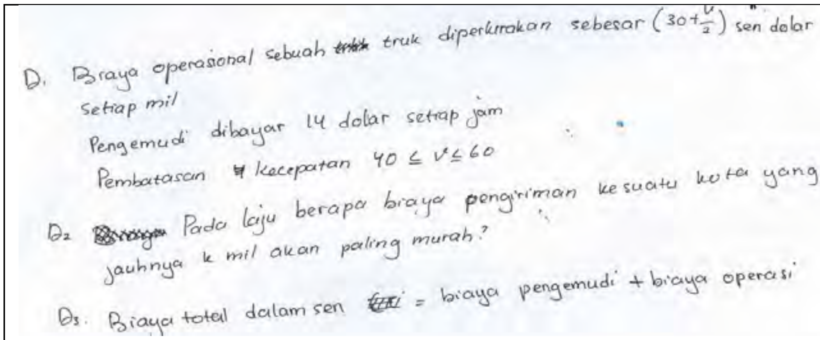
Translation :

Given :

- Truck operating costs = (30+v/2) cents/mile at the rate of v miles/hour
- Driver fare = 14 dollars/hour = 1400/hour
- Speed limit = 40≤v≤60

Asked : At what rate is the cheapest delivery to the city k miles away?

Low Skill (A3)



Translation :

D_1 (given)

- The operating cost of a truck is estimated at (30+v/2) cents/mile
- Drivers are paid 14 dollars every hour
- Speed limiting $40 \leq v \leq 60$

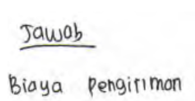
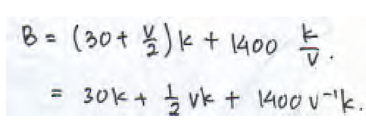
D_2 (Asked): at what rate will it be cheapest to a city k miles away?

D_3 (answer): Total costs in cents = driver fees + operational costs

Figure 6: The results of student work on the evaluating (C5) category questions in the problem-solving stage 1

In stage 1 indicator 1 (I1), students A1, A2, and A3 have written everything that is known, namely operational costs $(30 + \frac{v}{2})$ cents, driver's wages 14 dollars per hour, and speed limitation at $40 \leq v \leq 60$. In stage 1, indicator 2 (I2) of students A1, A2, and A3 has found a relationship between operational costs and wages and results. Through this relationship, a mathematical model will be found. The result of the interview showed that all students had successfully analyzed the information and focused on the questions, so they could properly write down the known information and questions.

Step 2: *devising a plan*

High Skill(A1)	Medium Skill (A2)	Low Skill (A3)
		

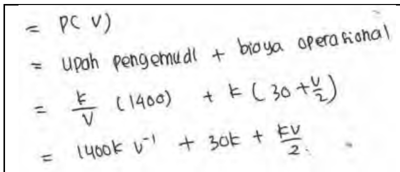
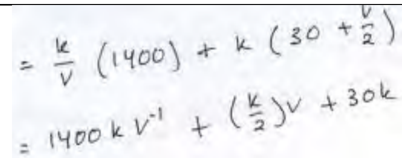
		
Translation : Total costs = $P(v)$ = driver fare + operational costs $=\frac{k}{v}(1400) + \mathrm{k}\left(30 + \frac{v}{2}\right)$ $= 1400kv^{-1} + 30k + \frac{kv}{2}$	**Translation :** $\mathrm{B} = \mathrm{k}\left(30 + \frac{v}{2}\right) + \frac{k}{v}(1400)$ $= 30k + \frac{1}{2}vk + 1400\ v^{-1}k$	**Translation :** $=\frac{k}{v}(1400) + \mathrm{k}\left(30 + \frac{v}{2}\right)$ $= = 1400kv^{-1} + \frac{kv}{2} + 30k$

Figure 7: The results of student work on the evaluating (C5) category questions in the problem-solving stage 2

In stage 2, indicator 1 (I3), All students have found a relationship between known and unknown data by looking for shipping costs. Shipping costs $p(v) = 1400\ v^{-1} + 30k + \frac{kv}{2}$.. In stage 2, indicator 2 (I4) is not written explicitly in the answer. Students already know the flow of solving existing problems. The steps are to find the critical point in the obtained mathematical model. The results of the interviews showed that students had already known the flow of solving the existing problems. This stage focused on looking for critical points in the mathematical model. However, they failed to write it down because they thought that this activity was unnecessary to be accomplished. Whereas, the most prominent was going straight to solve the problem.

Step 3: Carrying out the plan

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)

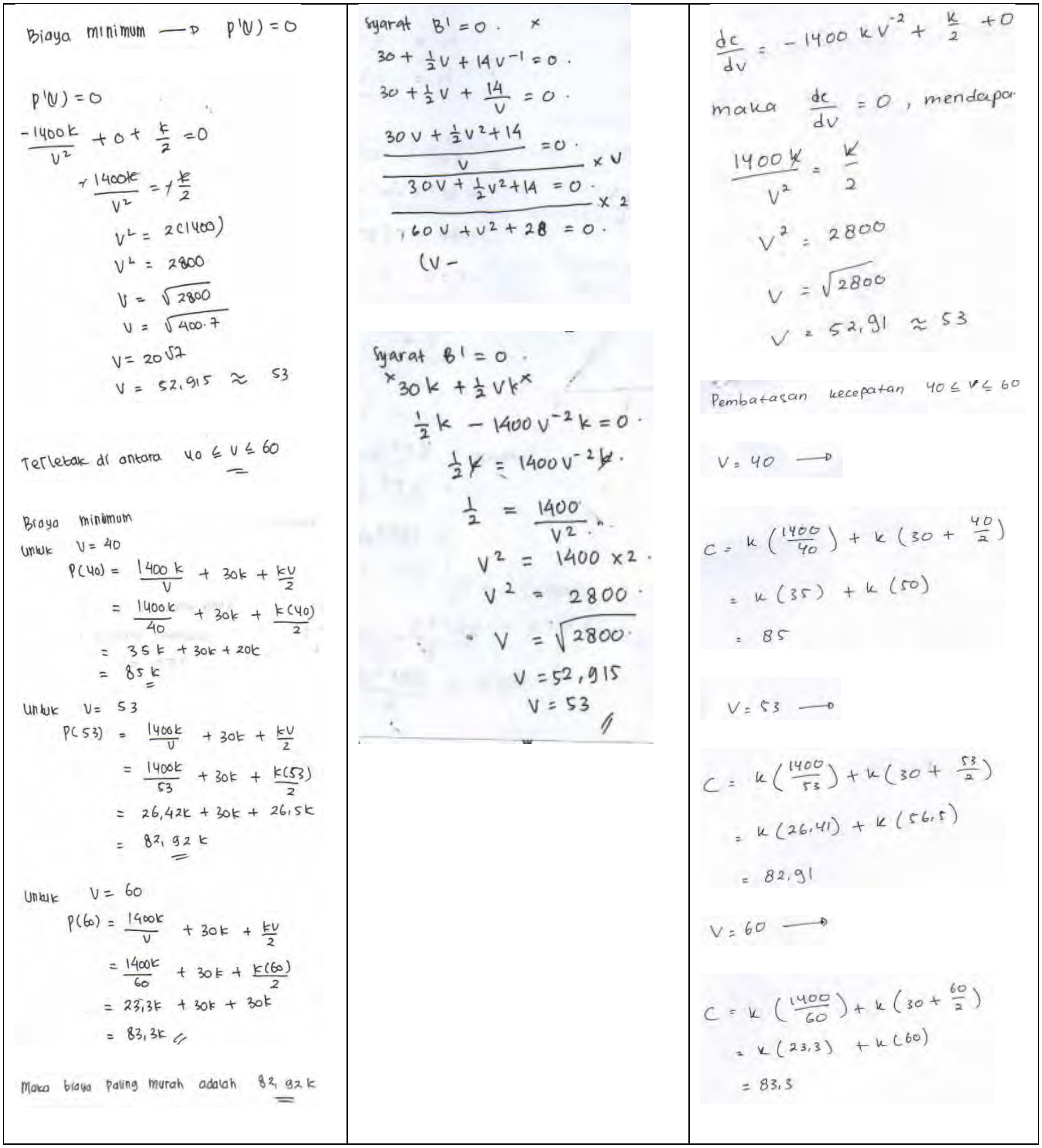


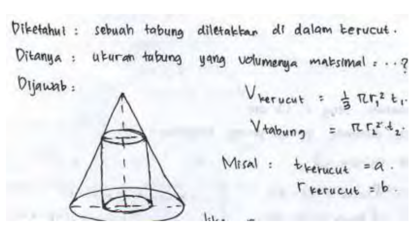
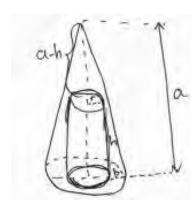
Figure 8: The results of student work on the evaluating (C5) category questions in the problem-solving stage 3

In stage 3, indicator 1 (I5), students A1, A2, and A3 had received the minimum fee. Minimum cost provided that $p'(v) = 0$, and all correct answers were $52{,}91 \approx 53$. A1 and A3 were able to determine the speed limit, $40 \leq v \leq 60$, so three critical points could be easily found such as 40, 53, and 60. In addition, A2 did not specify a speed limit. Whereas, A1 and A3 had incorporated a tipping point into the shipping cost equation, resulting in shipping costs of $85\text{k}, 82{,}92\text{k},$ and $83{,}3\text{k}$. Finally, A1 and A3 concluded a minimum fee of $82{,}92\text{k}$. Then in stage 3, indicator 2 (I6), all students failed to write down explicitly in their answers. Moreover, the results of the interviews showed that students had already known the flow of solving the existing problems. This stage focused on looking for critical points in the mathematical model. However, students failed to write it down since they thought that it was unnecessary. In fact, the most prominent was going straight to solve the problem.

Stage 4: looking back

In stage 4, indicator 1 (I7) was carried out by students but explicitly unwritten in their answer. In stage 4 indicator 2 (I8), only A1 attempted to find other alternative solutions to the questions. In stage 4 indicator 3 (I9), A2 students did not undergo this stage, while A1 and A3 only attempted to find the minimum cost. A1 and A3 failed to list the rate at which shipping costs to a miles away city will be the cheapest. In fact, they should had written the cost per mile in the general equation, namely $\frac{C}{k} = 1400\, v^{-1} + \frac{v}{2} + 30$. In the other hand, students should had written down the cost per mile in the general equation, namely C/k=1400 v^(-1)+v/2+30. Finally, the result of the interview supports the finding such as students A1 and A3 only wrote down the cheapest cost values but had not answered the question about finding the rate position to get the cheapest cost values. In addition, students also did not try to determine or ensure that the answers written are correct. The next problem is a problematic category: "Find the size of a vertical circular tube with the largest possible volume that can be placed in a vertical circular cone!" The outcomes of the student's efforts are listed below.

Stage 1: *understanding the problem*

High skill (A1)	Medium skill (A2)	Low skill (A3)
		

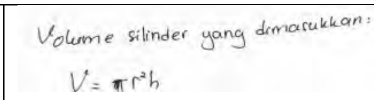
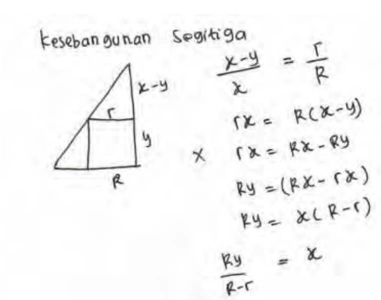
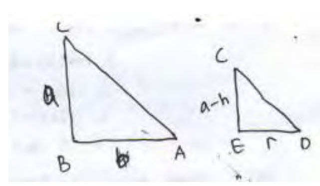
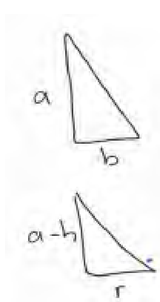
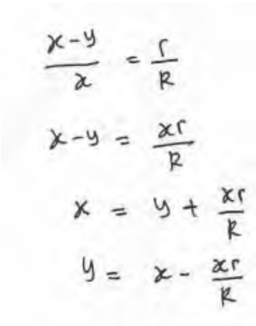
		
Translation : Given : There is a tube inserted into the cone Asked: Maximum tube volume Answer : Cone radius = R Tube radius = r Cone height = x Tube height = y	**Translation :** Given : A tube is placed inside the cone Asked: What is the maximum volume of the tube? Answer :	**Translation :** Introduced cylinder volume

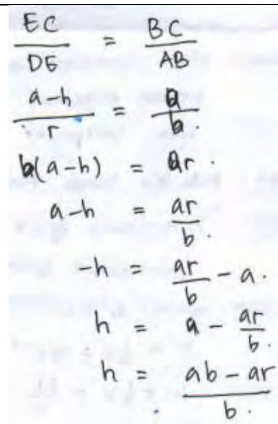
Figure 9: The results of student work on the problematic questions in the problem-solving stage 1

In stage 1, indicators 1 (I1) of students A1 and A2 had written correctly what was known in the problem. A3 student could not find out the information from the questions and what to do to solve the problem. Through the consideration, finally, the lecturer gave direction to A3 regarding the purpose of the questions and the stages of solving the questions. In stage 2, indicator 2 (I2) is not written explicitly in the answer. *The results of interviews with A1, A2, and A3 showed that students A1 and A2 were able to analyze and focus questions by writing them well. In A3, students can analyze the questions and focus on questions from the questions given, so they can visualize cones and tubes and exemplify their sizes but are not given information on the size problems that have been written.*

Stage 2: ***devising a plan***

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
		





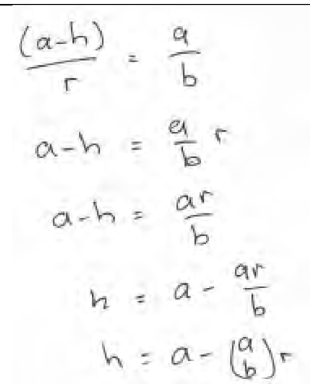
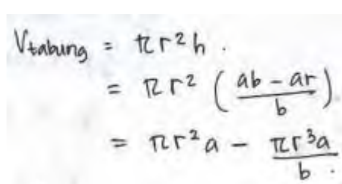


Figure 10: The results of student work on the creating (C6) category questions in the problem-solving stage 2

Stage 2 indicator 1 (I3) students A1, A2, and A3 had found a relationship between the known and the unknown. Students A1 and A3 were able to write the visualization of the shapes in question and their measurements (variables). While, student A2 wrote down the visualization of the image only, but the size was quite clear. A2 jot down several alternative sizes of the radius of the tube and cone with three alternatives, such as a). radius of cone = radius of tube consequently t tube = 0 consequently v tube is 0. b). The radius of the tube = 0, then t tube = t cone consequently v tube = 0. c). $0 < r\ tube\ < r\ cone$, the tube would experience a certain volume on the third point, which is an alternative answer. Stage 2 indicator 2 (I4) was not written explicitly in the answer. The results of interviews A1, A2, and A3 showed that all of them were able to find and relate what was known to the problem and find problems to find solutions using the concept of triangular congruence. They thought about how to solve the problem, unfortunately it was not written down because it was unnecessary. They thought that focusing more on the solution of the problems is more important.

Stage 3: ***carrying out the plan***

High Skill (A1)	Medium Skill (A2)	Low Skill (A3)
		

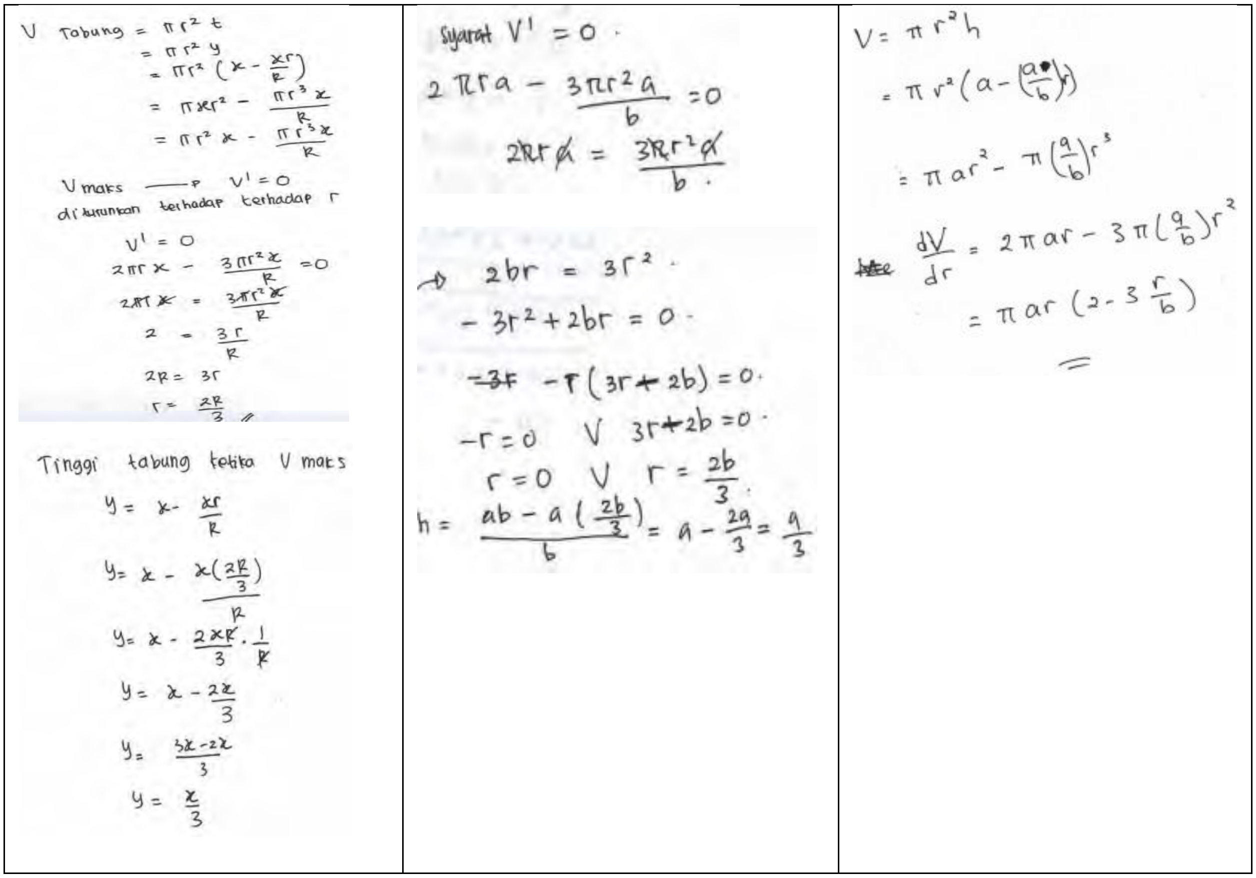


Figure 11: The results of student work on the creating (C6) category questions in the problem-solving stage 3

In stage 3, indicator 1 (I5), students A1, A2, and A3 used a similarity triangle to find the radius ratio to the tube's height. A1, A2, and A3 figured out that the ratio of height to the radius, and all of them were correct, i.e., $h = a - \frac{a}{b}r$. In this stage, A1, A2, and A3 attempted to find the volume of the cylinder $v = a\pi r^2 - \pi\frac{a}{b}r^3$. Then, A1, A2, and A3 tried to find the maximum volume with $v'(r) = 0$ that was $v'(r) = \pi ar\left(2 - \frac{3}{b}r\right)$. However, only A1 and A2 produced a stationary point $r = 0$ or $r = \frac{2b}{3}$ and A3 only up to $v'(r)$. On the other hand, A1 and A2 did not determine the critical point that should have 3, namely $r = 0$ or $r = \frac{2b}{3}$ and $r = b$. Finally, A1 and A2 tried to find the tube size $r = \frac{2R}{3}$ and the tube height $y = \frac{x}{3}$. In stage 3, indicator 2 (I6) was not written explicitly in the answer. These results indicated that A3 failed to solve the problem properly. Then, an interviews with A3 students was carried out and it showed that students were unable to construct their thoughts to relate to the concepts they had in solving problems. In stage 3 indicator 2 (I6), students A1 and A2 could determine solutions to problems and write down solutions to the problems. Unfortunately, student A3 did not explicitly wrote the answers. Based on the interviews result, it showed that students were unable to construct their thoughts to relate the concepts they had in solving problems, so that the answer was not written.

Stage 4: looking back

In stage 4, indicator 1 (I7) was carried out by students although it was not explicitly written in the answer. In stage 4 indicator 2 (I8) there was only A1 who find other alternative solutions, while A2 and A3 did not try to find alternative explanations. Stage 4 indicators 3 (I9) both A1 and A2 underwent this step and made the exact answer without considering to write down the answer.

DISCUSSION

The first question showed that students A1 and A2 answered the questions accurately. In answering the questions, both students A1 and A2 had carrried out many problem-solving stages but they failed to write the answers explicitly as in indicator I4. Students did not write down the strategic plan to solve the problem. This finding was in line with a study conducted by (Özdemir, Furkan, and Çelik, 2021). According to the result, it would affect the accuracy of answering questions. Students who plan strategies appropriately will carry out the problem-solving stages well (Ersoy, 2016; Eichmann et al., 2019). In fact, A1 student was the only one who attempted to seek alternative answers to the first question (indicator I7). While, student A2 and A3 did not carry out indicator activities I7 because they did not have enough time to accomplish and could not think about indicator I7. For A3 students, they were unable to complete the answers due to the constrained processing time that runs out. All stages had been undergone according to the analysis results on the easy category questions, but some indicators missed. It happened because students were not get used to answer the questions using problem-solving steps. Thus, it is necessary for the students to get used to work on non-routine questions (Riastini & Mustika, 2017; Özdemir, Furkan, and Çelik, 2021) and problem-solving with students (Purnomo et al., 2014; Ersoy, 2016; Gogo et al., 2021; Purnomo et al., 2022).

The medium category questions illustrated that there were no stages of problem-solving written explicitly in the answers, such as indicator I4. Students A1 and A2 searched for alternative solutions (indicator I8), while A3 did not even think about alternative answers. At this category, student A3 still faced difficulty answering questions and did not have enough time accomplish the questions. All students did not correctly carry out the Looking Backstage, especially indicator I9. They were quite unsure about what was being asked in the question. Finally, all answers found a tipping point where the cost wasminimal. In the question, it was clear that what wasbeing asked was a general mathematical model so that the prices were minimum. This happened due to weak problem-posing skills (Science & Soyba, 2018; Tabak, 2019) and inappropriate problem-solving processes (Rohmah & Sutiarso, 2018). The conclusion was that re-checking what was asked and the answer given was prominent thing. In addition, students must accomplish and understand questions thoroughly.

Faced on the problematic questions, it was found that student A3 failed to understand the questions. It could be obviously seen from how they deal with and accomplish the problem. The information in the question coud not be described adequately. Student A3 did not understand the questions (Yeni et al., 2020) and were not get used to solve questions with high difficulty category questions. So the new A3 students could get information from the questions and the flow of solving the questions. In the second stage, all students did not write their answers explicitly (indicator I4). In indicator I8, A1 students was the one to carry out this stage, while A2 and A3 did not perform it well. It happened because A2 and A3 had difficulty in dealing with the questions. Besides, time limit was also become their consideration. From the explanation, it could be concluded that there was a need for habituation in working on questions in the high difficulty category (Abdullah et al., 2015). Therefore, it is needed to assist students in analyzing problems by modeling the situation into a mathematical model (Varaki & Earl, 2006; Yasa & Karatas, 2018; Schukajlow et al., 2018). For that reason, using mathematical modeling can also help students in planning problem-solving strategies.

Again, in this finding it was found that students still experienced difficulties at all stages of Polya (Abdullah et al., 2015; Yayuk & Husamah, 2020; Pardiansyah et al., 2021). There were several indicators of the stages of problem-solving. Indicators that were not carried out including 1). determine the method used to solve the problem, 2). the existence of alternatives to get different results, 3). check the accuracy of answers to questions, 4). modeling the problem into a mathematical model situation. There were still errors when planning for the completion and implementation of the project (Pardiansyah et al., 2021). Students should expressed how the planning process in their minds when solving problems, for example, by telling the steps in their minds. For instance, performing prior planning such as outlining the information provided in mathematical form and implementing strategies during the processes and calculations (Yayuk & Husamah, 2020).

Polya's problem-solving design stages included the Problem Posing Learning Model (PPLM) (Örnek & Soylu, 2021), IDEAL problem solving (Bransford & Stein, 1984; Bransford & Stein, 1993) and Wilson's problem solving (Wilson et.al, 1993) and problem-solving evaluation design (Gebel & Kuzle, 2020). Based on these findings, Polya's problem-solving stages could be modified to be appropriate to solve problems with the HOTS type. The characteristics of HOTS-type questions are problems in complex situations (Andin & Aziz, 2019) that cannot be translated directly. Solving HOTS questions require a more thorough problem-solving stage and the proper steps. The locations of solving the Polya problem require modifications to be used to solve HOTS questions. Errors in student solutions are in the last stage of trial solving. In general, students need to defend the chosen plan at this stage. Alternative solutions would minimize errors by improving the stages of problem-solving (Wee, 2007) and modeling the problem into a mathematical situation. In the findings of this study, the Polya stages need to be developed must be improved by inserting mathematical modelling stages and returning answers to existing problems. Problem-solving designs need to be made so that problem solving, especially HOTS type questions, can be adequately solved. Four factors served as the foundation for the new problem-solving phases' design, including 1). to find the root of the problem in the field, 2). Literature review related to difficulties in solving problems, 3). Stages of problem-solving that have developed, and 4). Mathematization process and mathematical modeling. The study results showed that it was necessary to design a problem-solving step with the characteristics of solving HOTS questions. In solving the HOTS problem, a need for additional stages were strongly necessary. For instance, when expressing the situation mathematically and comprehending how to apply math to real-world situations or issues. The steps of problem-solving were developed by various experts, including Dewey (1910), then Polya (1945), Mason, Burton & Stacey (1982), Schoenfeld (1985), Wilson et al. (1993), and Yimer & Ellerton (2010). Based on the analysis, it can be concluded that the stages had several advantages and disadvantages. In addition, the results of the research can be used to add new problem-solving stages. The mathematization process consists of four steps, including 1). Horizontal mathematization: organizing, reflecting, compiling problems, and identifying aspects of the problem mathematically to find the rules or relations; 2). Vertical mathematization: formalizing and abstracting mathematical concepts to give birth to mathematical concepts; 3). Applying in different situations: apply in various problems and cases, and 4). Back to reality: back to the real problem. The illustration below shows how the phases of problem-solving are laid out.

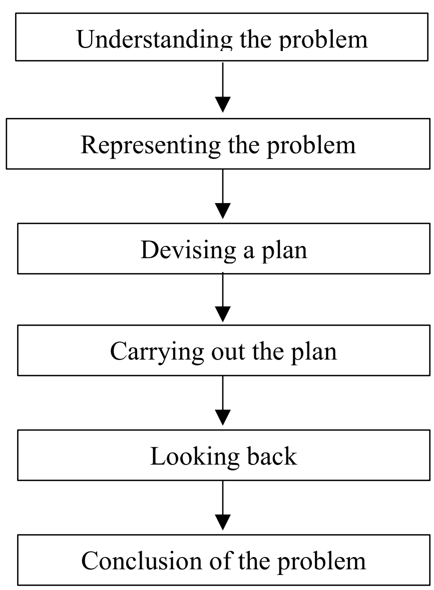


Figure 12: Design new problem-solving stages

Based on the analysis, the stages of the mathematization process can be included in the new problem-solving steps—the addition of solving stages in the 2nd stage, such as; representing the problem. In the last stage, there is also an additional stage, namely conclusion of the problem. The design results of the solving stage consist of 6 steps. Each stage of problem-solving consists of indicators. The design of the problem-solving stage consists of 6 stages and 11 indicators. More specifically, each indicator in the problem-solving stage can be described in Table 3.

Step	Problem-Solving Step	Indicator Description
Step 1	Understanding the problem	• Analyzing the problem by identifying what is known and asked • Focusing on the problem
Step 2	Representing the problem	• Presenting statements in the form of pictures or example variables • Finding the relationship between known data to find the unknown
Step 3	Devising Plan problem solving	• Determining the method used to solve the problem • Looking for various alternative solutions to problems
Step 4	Carrying out the plan	• Executing the solution plan and checking every Step • Determining solutions to problems and writing solutions or answers to problems
Step 5	Looking back	• Looking for alternatives get different results • Checking the accuracy of answers to questions
Step 6	Conclusion of the problem	• Returning the results of the answers to the context of questions or everyday life

Table 3 lists the phases of problem-solving and provides an indicator description.

A design for a problem-solving stage is created based on the analysis. Problem-solving stages using six groups of techniques, such as 1). understand the problem, 2). representing the problem, 3). devising plan problem solving, 4). carrying out the plan, 5). looking back, and 6). conclusion of the problem. The limitation of this study is that the research sample is still in the scope of mathematics education study program students. Subsequent further research will be more sophisticated when it is carried out in other study programs and at high school level. Future research can also be conducted to test the effectiveness of the problem-solving model stage design. By conducting the effectiveness test, a better problem-solving stage design will be obtained.

CONCLUSION

Based on the elaboration in the previous part, it can be clearly concluded that several indicators in the stages of problem-solving are not performed well. In indicator I4 for instance, all students did not write a work plan for solving the problem. Indicator I4 was also not carried out perfectly that caused less-than-optimal completion result. However, in indicator I8 only high-ability students encouragingly attempted to find other alternative answers. In fact, indicator I8 is important to be carried out by students to stimulate other alternative answers and avoid the wrong answers. Whereas, all students did not perform well in indicator I9 (medium category questions). It is as important as the previous one, this indicator needs to be performed by students to match between questions and answers. This study come to new paradigm that the design of problem-solving stages now can be categorized into 6 steps, including 1). understand the problem, 2). representing the problem, 3). devising plan problem solving, 4). carry out the plan, 5). looking back, and 6). conclusion of the problem. In addition, this study focuses that there is a need to develop problem-solving stages. By adding the stages of the problem-solving model, students can cover the shortcomings of the previous problem-solving steps. This new problem-solving stage, in addition, will be much more necessary to solve problems with the HOTS category.

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