Examination of the Abstraction Process of Parallelogram by Sixth-Grade Students According to RBC+C Model: A Teaching Experiment

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Doi: 10.19044/ejes.v10no2a101 https://doi.org/10.19044/ejes.v10no2a101

Submitted: 07 July 2022
Accepted: 05 July 2023
Published: 31 October 2023

Abstract

This study used the RBC+C model to reveal the abstractions of the 6th-grade students in the process of transition to the parallelogram area formula. Also, constructing parallelogram area information was employed as a teaching experiment based on the basic interpretive approach, one of the qualitative research methods. The study participants comprised 9 volunteer sixth-grade students with high, medium and low mathematics success levels in a public school in Istanbul in the 2021-2022 academic year. Four activities prepared by the researcher on triangular, rectangular and parallelograms were used as data collection tools. In the study, activities were recorded on video and then transcribed. As a result of the research, it is seen that students with a high and medium success level in the recognizing and building phases did not have difficulty finding the quadrilateral and triangular areas and guided the students with a low success level. Moreover, it was found that students constructed parallelogram area information. Still, when asked about the area formula, they had difficulty expressing their operations even though they correctly found the area. This is because students process by rote and cannot explain mathematical information logically. In line with these results, it is considered necessary and recommended to organize teaching activities that will allow students to learn meaningfully.

Keywords: Mathematics teaching, parallelogram area, abstraction level, constructivist approach

Introduction

Abstraction means an individual structuring a situation or the information he/she has learned due to mental processes. According to the Turkish Language Society(2022), abstraction " refers to the mental process, dealing with any feature of an object or the relationships between its features alone, and separating the inseparable in reality in thought". Mathematical abstraction is the activity of rearranging mathematical information or problem as a result of mental processes in the individual's learning environment and constructing a new mathematical structure (Hershkowitz, Schwarz & Dreyfus, 2001).

Since concepts are produced at the end of the abstraction process, and mathematical thought and abstraction are inseparable, mathematics is a science of abstraction (Yıldırım, 1988). Abstraction is the process of change that enables the existing situation constructed in memory to be combined with new situations resulting from experiences (Skemp, 1986).

The fact that mathematics is an abstract science and mathematical concepts are obtained due to abstraction (Altun, 2014) reveals the importance of abstraction in examining the formation of mathematical concepts. The inability to observe abstraction has led to much research and the emergence of the Recognizing-Building With-Constructing (RBC) abstraction model, in which observable actions that provide information about the abstraction process are defined by Hershkowitz, Schwarz and Dreyfus (2001). The abstraction process has been defined as recognizing, building with and constructing, and the RBC model name is an acronym (Altun & YILMAZ, 2010). For the abstracted information to become permanent (Memnun, 2011), the model became RBC+C as a result of the addition of the epistemic action of consolidation (consolidation +C) by Dreyfus (2007).

RBC+C works on observable cognitive actions, and the mental processes of the participants are defined based on the actions. It also facilitates the observation of the structures formed in the process (Dreyfus, 2007; Dreyfus & Tsamir, 2004; Tsamir & Dreyfus, 2002). The observable cognitive actions put forward in this model enable the examination of the process of recognizing, building with, constructing and consolidating abstraction.

Recognition from observable cognitive actions refers to using a recognized structure in new situations (Dreyfus, 2007; Hassan & Mitchelmore, 2006; Hershkowitz, Schwarz & Dreyfus, 2001). The act of *building with* is the ability of the individual to associate the mathematical expressions he/she constructed beforehand to reach a solution and achieve the goal by using them (Dreyfus, Hershkowitz & Schwarz, 2001; Tsamir & Dreyfus, 2002). The act of *constructing,* expressed as the process of reorganization and restructuring, can be defined as the construction of new information due to the restructuring of existing information by undergoing partial changes (Bikner-Ahsbahs, 2004). Recognizing, building with, and constructing actions are integrated actions. First is the recognizing action, then the building with and constructing, respectively (Dreyfus, 2007; Dreyfus, Hershkowitz, & Schwarz, 2001). Every newly acquired concept needs to be consolidated. In mathematical concepts, a permanent new structure can only be constructed by consolidation. The act of consolidation can be realized with mental activities as a result of using the information to construct new information by associating it (Dreyfus, 2007; Dreyfus & Tsamir, 2004).

In the literature review, according to the examinations of students' information-constructing processes with the RBC+C model (KALAYCI & Akkaya, 2019; Yeşildere & Türnüklü, 2008; Atıf Karataş, 2021; GÜLER &ARSLAN, 2018; Memnun & Altun, 2012a; Memnun & Altun, 2012b; Türnüklü & Özcan, 2014; Çubukluöz, ADIGÜZEL, Özdemir & Akkaya, 2018; KOBAK-DEMIR & Hülya, 2019; Akkaya, 2010; Çelebioğlu, 2014; Altun & Durmaz, 2013; KAPLAN & Elif, 2015; Schwarz, Hershkowitz & Azmon, 2006), no study was found on how secondary school students constructed parallelogram area information. This study is a unique study that will contribute to the field.

Purpose and Significance of the Study

This study aims to examine the processes of constructing parallelogram area information for sixth-grade students in the context of the cognitive actions of recognizing, building with, constructing and consolidating the RBC+C abstraction model.

In Turkey, much qualitative research is conducted using descriptive analysis, content analysis, etc., to construct new teaching designs and examine existing teaching styles. The abstraction process is to be carried out more actively with these researches about the abstraction of information, examining in depth the sample information abstraction processes. Also, the researchers gain information and experience about the abstraction process and support the students to identify and overcome the points they have difficulty with. This situation shows the need for abstraction studies and reveals its importance. In addition, the 2018 Primary Education Mathematics Curriculum (Ministry of National Education, Board of Education and Discipline, 2018) aims to develop a positive attitude towards mathematics with the students' experiences and develop the ability to construct and build with information.

This study evaluated the construction of the parallelogram area information in the mathematics learning process using the RBC+C abstraction model. The RBC+C abstraction model allows the analysis of students' information learning processes thanks to the recognizing, building with, constructing and consolidation processes, which are observable cognitive actions and facilitates the analysis. Analysis of students' information-constructing processes through the RBC+C model shows the validity and effectiveness of the analysis. In addition, it reveals a need for research examining student achievements, especially in this abstraction, due to the lack of research conducted in Turkey on abstraction in the mathematics literature.

As a result, this study aimed to evaluate the nature of information formation, namely, abstraction during learning parallelogram area information in appropriate learning environments. The research results are also crucial in determining the paths that educators will follow for students' construction of information in parallelogram area learning, and the research is essential in this respect.

Method

Research Model

This study used the process of constructing parallelogram area information as one of the qualitative research methods and the teaching experiment method based on the basic interpretive approach. In the teaching experiment, qualitative data and instructional notes were obtained from video recordings taken in the learning environment for observation and information construction (Knuth & Elliot, 1997).

Participants of the Study

The research was conducted with 9 sixth-grade students studying in a public school in Istanbul. Participants had not yet learned the concept of parallelogram area or done any studies on this subject. Participants were selected among voluntary students who were considered to have high, medium and low mathematics levels. The students' mathematics achievement scores of the two written mathematics exams held during the semester were also influential in determining the students. Their success levels have been presented in detail in Table 1 by giving code names instead of their real names. Codes were used asS_1(Student1), S_2, S_3, S_4, S_5, S_6, S_7, $S_8$and S_9 for the participants in the study.

Table 1. Participants' Codes and Success Levels

	Participant Codes	Mathematics Written Average	Success Levels
Group 1	S_1	97	High
	S_2	73	Medium
	S_3	50	Low
Group 2	S_4	93	High
	S_5	74	Medium
	S_6	50	Low
Group 3	S_7	95	High
	S_8	75	Medium
	S_9	48	Low

In constructing information, the members of the groups were selected heterogeneously according to their different levels of success to provide peer interaction, allow feedback corrections and prevent false learning during the construction of information. The application process was carried out as group work and 2 lessons of 40 minutes each to enable students to think multifacetedly with peer interaction and voice their thoughts. In this way, it was aimed to examine the process of constructing information more clearly and comprehensibly by passing through the students' steps of the RBC+C abstraction process.

Data Collection and Analysis

The research was conducted in a public school in Istanbul in the spring semester of the 2021-2022 academic year. Before the study, the students were told the study's purpose and informed about the subject of the study. The study was conducted as a group of 9 students with high, medium and low success levels.

Four activities prepared by the researcher were used as data collection tools. Activity applications were performed by taking approximately 40 minutes of video recording in the library allocated by the school administration. Students' interactions were observed in constructing the information, and field notes were created. Thus, it was to analyze the study process most accurately and effectively.

Results

In this section, the processes of reaching the parallelogram area information of 3 student groups of 3 students were determined according to the steps of recognizing, building with, constructing, consolidation and action. According to the findings, codes were created with the initial letter of students' names and the X code for the researcher.

Recognizing

In Activity 2, students were given triangular and quadrangular plots of different sizes and asked to explain how they solved by calculating the area separately. Below is the dialogue among the students and the activity carried out in Figure 1.

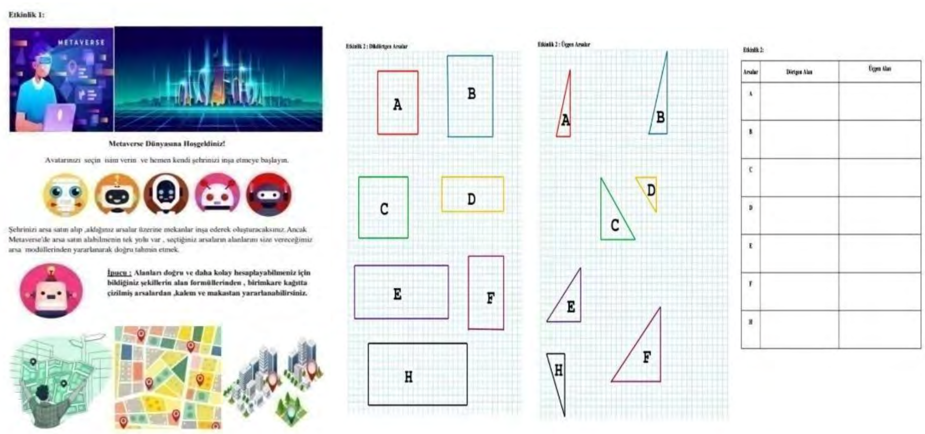

Figure 1.a *Figure 1.b* *Figure 1.c* *Figure 1.d*

Figure 1. *Activities in which Recognizing Action takes place (Figure1.a, Figure1.b, Figure1.c, Figure1.d)*

X: Would you read the Activity 1 sheet aloud, S_4.

S_4: Welcome to the world of metaverses! Choose your avatar, name it, and start building your city immediately. You will create your city by buying land and building spaces on the land you buy. However, there is only one way to buy land in Metaverse: to accurately estimate the areas of the lands you choose by using the land modules we will give you. Tip: You can use the area formulas of the shapes you know, the plots drawn on the unit square paper, and the pencil and scissors to accurately and more easily calculate the areas.

S_9: Teacher, now we will calculate the area to buy the land.

X: Yes

S_5: If we do not calculate the areas correctly, we cannot buy the lands. Then we must calculate the areas of all the lands correctly. (laughing)

X: This is our only condition. I hope you will buy all the pieces of land. Well, activity 2 paper has triangular and quadrilateral lands. Calculate the plot areas.

S_2: Teacher, we can do it by counting the boxes.

S_3: I think so too.

S_1: Teacher, I will use the area formulas. Should we use cm or unit while finding the edge lengths?

X: Take the edge lengths as units, guys.

S_8: Will everyone find their areas, teacher?

X: Create your answers first. Then, as a result of the group evaluations, tell me the land areas you found.

Building with

Below is the dialogue among the students and the activity carried out in Figure 2.

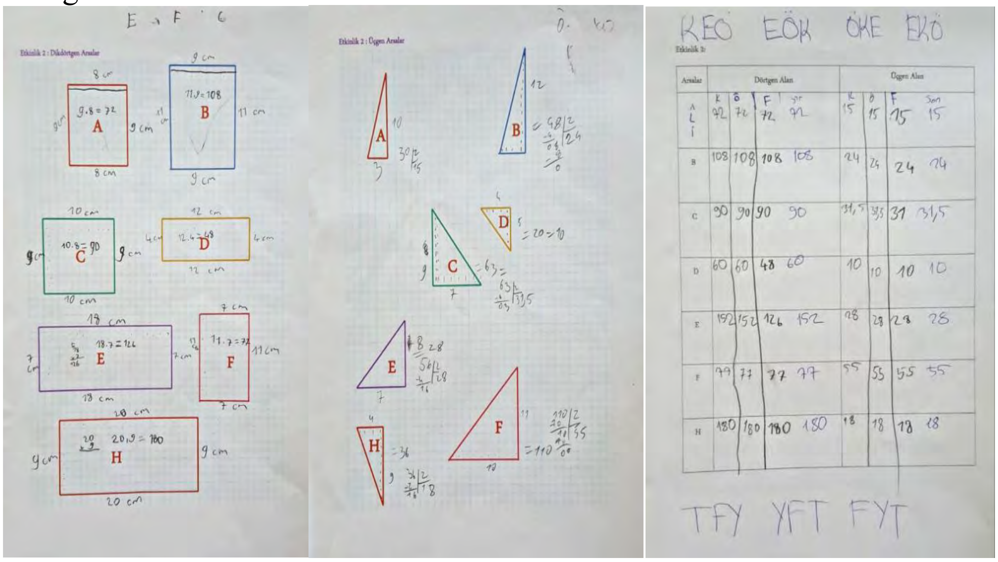

Figure 2.a *Figure 2.b* *Figure 2.c*

Figure 2.*Activities in which the Building with Action takes place (Figure2.a, Figure2.b, Figure2.c)*

Group 1

S_1: I'm done.

S_2: Mine is almost done, too.

S_3: I couldn't calculate some areas.

S_2: Count boxes in rectangular areas.

S_3: What do we do with the triangle? Some of the boxes in the triangle aren't full.

S_1: Friends, let's not bother counting the boxes in the triangle. Just multiply the height of the edge by the edge and divide by 2.

S_2: Let's write the areas as a group.

Group 2

S_4: I found them all.

S_5: I'm on the last question about triangles. What about you, S_6?

S_6: I have more.

S_4: Are your quadrilateral plots finished?

S_6: Yes. I have trouble with the triangles.

S_5: Look, complete the shape. Do you see that it is rectangular?

S_4: Two triangles made a rectangle.

S_5: Just calculate the area of the rectangle and divide it by two.

S_4: Let's write our areas as a group.

Group 3

S_8: If everyone has found an area, let's look at our areas.

S_7: I found them all with area formulas.

S_9: I could not finish it all. You finished it quickly...

S_8: There is nothing when calculating the area; counting boxes is sufficient.

S_9: What do we do about the triangle?

S_7: Multiply those two sides and take half.

When examining the dialogues, it was observed that the students with a high success level completed their information with the feedback of those with a low success level and progressed in the activities this way. It was also seen that the students in the groups generally tried to count the unit boxes while calculating the areas of the quadrilateral plots and the triangular area with the guidance of the students with high success.

X: Okay, guys, we will start with the quadrilateral plot areas. How did you calculate the areas?

S_1: Teacher, we can reach the area by counting boxes in the rectangle.

S_8: Or we could multiply the short and the long side.

X: It could be both. Did you do the same thing in the triangular?

S_9: Teacher, I had a hard time calculating the triangular plots, but I succeeded when

S_7 said that multiplying the horizontal and vertical sides in the triangle and taking half of them can calculate the area.

S_5: When we completed the rectangle in triangles, I saw that two identical triangles made a rectangle and divided it by two.

X: Good idea. If we do not think of a formula, let's produce such a formula ourselves. As far as I understand, all groups have calculated their areas. Let's see who gets which lands...

Constructing

To construct the parallelogram area information, the students were given the Metacrociand Activity 3 paper containing three questions. The following is the dialogue between the students and the activity carried out in Figure 3.

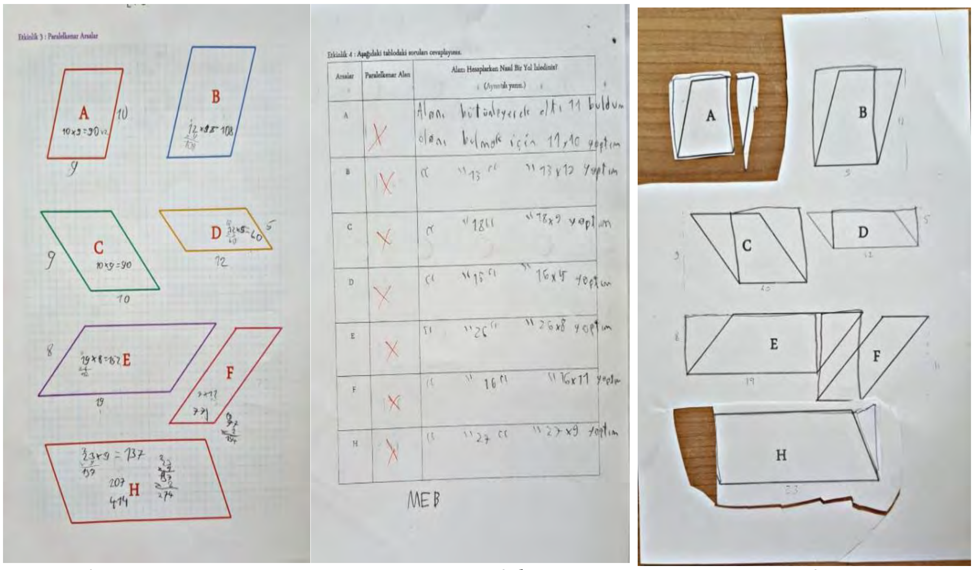

Figure 3.a *Figure 3.b* *Figure 3.c*

Figure 3. *Activities in which the Constructing Action takes place (Figure 3.a, Figure 3.b, Figure 3.c)*

X: In this lesson, we will buy new land. I will distribute your sketches. See the lands you can buy.

S_9: Teacher, this is a different shape.

X: Yes, S_9. Last year, we learned about this shape and its features.

S_2: It looks like a rectangle.

S_1: It is not rectangular, but it is a parallelogram.

X: You are right, S_1. This shape is called a parallelogram.

S_4: Teacher, we have never calculated the area of such a shape.

X: Do you remember the activity paper you read to the class? It said, "You can use the area formulas of the shapes you know, the plots drawn on the unit square paper, and the pencil and scissors to calculate the areas accurately and more easily."So try to see which groups can buy which lands.

Students first calculated the areas using the trial and error method while finding the area of parallelogram lands.

Group 1

S_1: Let's count the boxes again.

S_2: I say we do that.

S_3: Let's calculate the areas.

Group 2

S_4: We do not know the parallelogram area calculation.

S_5: We will count boxes, then there is no other way.

S_6: Let's find it and write it down.

Group 3

S_8: Let's count the boxes.

S_7: We have to be careful about the corners.

S_9: Look at how many units we're going to get! That's a tough one.

When examining the dialogues among the students in the groups, it was noteworthy that the students with a high success level completed their information with the feedback they gave and thus progressed in the activities.

X: Can everyone calculate the areas?

S_1: Can we count the boxes, teacher?

X: How will you count the boxes that are not full in the corners? Won't it be difficult?

S_4: Teacher, this is how we found the areas we calculated last time.

S_8: Yes, teacher. We do not know how to calculate the area.

X: Calculate your results and see which group will buy which lands with this method.

It was observed that the students tried to use the method of counting the unit boxes they used in Activity 2 to calculate the parallelogram area and had difficulty counting the unit boxes.

S_8:We finished as a group.

S_2: So we calculated it.

S_5: We're done, too.

X: Bring the paper on which you wrote the land areas. Which groups were able to buy which lands in Metaverse?

The Activity 3 paper was collected, and the answers given by all groups for the parallelogram areas were examined.

X: Children, all fields were found to be incorrect. Unfortunately, nobody can buy land at the moment. Let's discuss it together. How can you calculate the field that the box-counting method did not work for you?

S_3: Teacher, the corners caused problems.

S_4: What else can we do?

S_8: If you say that corner boxes make 1/2 or 1/3 units of boxes, we can calculate it.

X: Try different methods. I reminded you at the beginning of the activity. You can use the fields of pencils, scissors or shapes you know while calculating the area.

S_2: Teacher, I found it. This shape is the side of the rectangle, so we multiply the horizontal and vertical sides to find the area. The area of the land is 72 square units.

X: That shape is not a rectangle, S_2. However, we can say that they are rectangular. Look, they are not perpendicular to their corners. However, you calculated the area of land A correctly.

S_5: Will we multiply the short and long sides like the rectangle?

X: No, honey. We do not multiply two sides. S_2 thought the shape changed form and made the edges vertical. However, we cannot do this.

S_1: What are we going to do then, teacher?

X: A little tip for you. S_2 is right about one thing. We can obtain a rectangle in this way. But how?

S_4: I found it; we can complete it like this.

X: You can add an extra triangular area to your parallelogram.

S_2: If we draw height from the lower corners, it makes a rectangle.

X: When you do not count a triangle area at that time, you add the triangle area you produce. If the area of these two triangles is equal, it happens. If you want, try cutting.

S_1: I found it, teacher. Look, I cut the triangular part. If we add it to this side, it becomes rectangular; we calculate its area.

X: The 1st group took all the parallelogram plots, children. Group 1 said that if we create a perpendicular from the lower corners, cut the triangle inside the parallelogram, and add it upside down from the other lower corner, we will get a rectangle and calculate the area, so show this in the figures you have.

S_8: How did Group 1 do it?

X: S_1 and S_2, show your friends how you did it.

At the end of the erroneous results of all groups, the teacher gave a clue to guide the students correctly. With the clue given by the teacher, the students constructed the information correctly using the trial and error method; that is, they realized the constructing stage. At this stage, high and medium-successful students noticed the field relationship faster than low-successful students.

Consolidation

To consolidate the parallelogram area information, the students were given the Metacrociand Activity 4 paper containing three questions. The following is the dialogue among the students and the activity carried out in Figure 4.

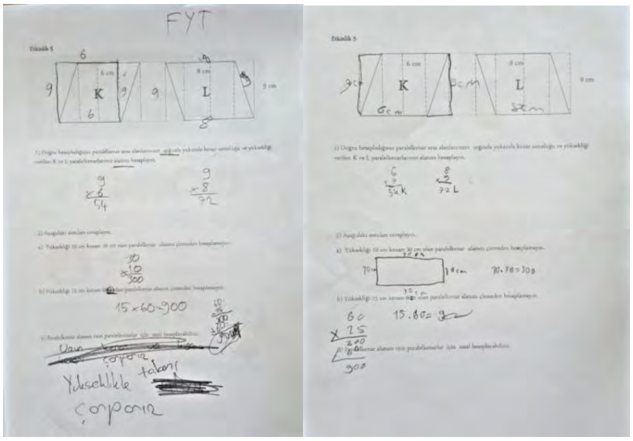

Figure 4.a *Figure 4.b*

Figure 4. *Activities in which the Consolidation Action takes place (Figure 4.a, Figure 4.b)*

X: S_2, what did you say the parallelogram looks like?

S_2: Rectangle, teacher.

S_1: In fact, we said quadrilateral.

X: What are the similarities between the parallelogram and the rectangle, S_2?

S_2: The opposite sides are parallel and equal in length.

X: Right, that's how I explained it when we were learning the features of the quadrangles. You guys now have an idea of the parallelogram area. Let's do the Activity 4 paper as a group.

Students were allowed to review the similarities between the parallelogram and the rectangle and then started the activity. All student groups quickly answered the first two questions in Activity 4 with the guidance of students with high and medium levels of success.

X: Everyone can calculate the areas of the parallelograms whose height and edge are given, but the areas and drawings aren't given. So tell me, what is your field formula for all parallelograms?

S_1: We multiply the horizontal and vertical edges.

X: No, we don't multiply two sides. You produce the vertical length. It has a name.

S_3: We multiply the perpendicular by the length.

X: What else can we say if we don't call it perpendicular?

S_1: Right, right angle.

X: It was even derived from English.

S_1: High, I mean height!

X: Yes. Then which ones are you multiplying?

S_1: We multiply the length of the side by the height.

X: A random edge?

S_1: No, a certain edge.
X: What is it?
S_2: Horizontal edge?
X: No.
S_2: Short side?
X: No.
S_4: We multiply the long side.
X: No. Which side do you multiply by height each time?
S_8: Vertical edge.
X: No
S_2: We multiply the height by its bottom side.
S_5: Yes, you're right.
S_2: We multiply the height and the horizontal side.
X: In the triangle area, we also asked what we multiply by height?
S_1: Area of the triangle= $\frac{h.a}{2}$ and h is the height.
X: Yes, h is the height. Well, what about a?
S_5: Base! It is the base, namely the side which intersects the height. We multiply the base and height.
X: Yes, S_5, we multiply them. We multiply the height and base, and instead of the base, we can even say the edge is ere the height intersects.

All students answered the question "How can the area of all parallelograms be calculated?" in a shorter time in the Activity 4 paper. At the end of the activity, all students constructed and consolidated their parallelogram area information.

Conclusion

In this study, the processes of constructing parallelogram area information of sixth-grade students were examined within the framework of the cognitive actions of the RBC+C abstraction model. As a group, students were enabled to realize their learning processes, exchange information with each other and realize the process by performing peer learning.

As a result of the research, it is seen that students with a high and medium success level in the recognizing and building phases did not have difficulty finding the quadrilateral and triangular areas and guided the students with a low success level. It is an indication of the students' action of recognizing and using erroneous calculations of the quadrilateral and triangular land areas in Activity 2. During the constructing phase, it was observed that students with high and medium success levels constructed parallelogram area information by applying the trial and error method and provided peer education to students with low success levels. Calculating the area by converting the parallelogram to a rectangle indicates that the construction actions have occurred. The duration of the constructing action varied according to each group, and it was observed that the process of students with a high level of success in the groups was better internalized. It is thought that the later formation of the parallelogram area formula by those at the other level may be due to the lack of necessary preliminary information. When asked about the parallelogram area formula during the application, the students had difficulty expressing their operations even though they found the area correct. The reason for their difficulty is that the students performed the operations by rote without knowing why they were performed and had difficulty explaining and sharing mathematical information logically (TOYGAN, GÖK & CANCAN, 2019). The first steps of the RBC+C model, that is, the recognizing, building with and constructing processes, are essential for the consolidation step. In the consolidation, the newly constructed information was expected to be consolidated. In Activity 4, the students reached the parallelogram area formula by consolidating the information with the questions.

In the RBC+C model, recognizing, building with, constructing and consolidation processes are integrated, not consecutively. Many studies have shown that the abstraction process is integrated, and the recognizing process is the basis of abstraction (ÖZGÜL & Kaplan, 2016; Hershkowitz, Hadas, Dreyfus & Schwarz, 2007; Yeşildere, 2006).

As a result, it can be stated that the RBC+C abstraction process is effective in constructing information. In addition, the findings obtained from the four unstructured activity observations showed that since the process of constructing information was carried out in the case of group work, there was peer learning among the students, and thus, equivalent learning was obtained. An in-depth examination of the process of constructing information of the students during the activities with the RBC+C model allowed them to understand in which process or action the students had problems structuring the information and constructing parallelogram area information. It is thought that identifying the problems experienced by the students will help overcome the problems by drawing attention to these problems. RBC+C abstraction model can be a theoretical framework for research on detecting and explaining the observable cognitive actions of the student in the conceptual learning process of acquisition (Baki, 2018). This theoretical framework can also enable researchers and mathematics teachers to gain information and experience in the field of sixth-grade parallelograms and contribute to more meaningful and effective learning and teaching of mathematics subjects.

Recommendations

- In line with these results, it is considered necessary and recommended to organize teaching activities that will allow students to learn meaningfully.
- The fact that the students with a low achievement level make explanations in an unambiguous and self-confident manner using mathematical language is a positive reflection of their work in groups during the implementation process. Based on this situation, it would be beneficial for students with medium and low levels to conduct group studies with students with different success levels.
- Similar studies can be used to create a theoretical framework for abstraction.

Acknowledgements: This paper was presented as an online presentation at the III. KARABAKH International Congress of Modern Studies in Social Sciences and Humanities June 9, 2022.

Human Studies: All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

Funding Statement: The authors did not obtain any funding for this research.

Data Availability: All the data are included in the content of the paper.

Conflict of Interest: The authors reported no conflict of interest.

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APPENDIX 1
Activity 1: Metaverse Lands

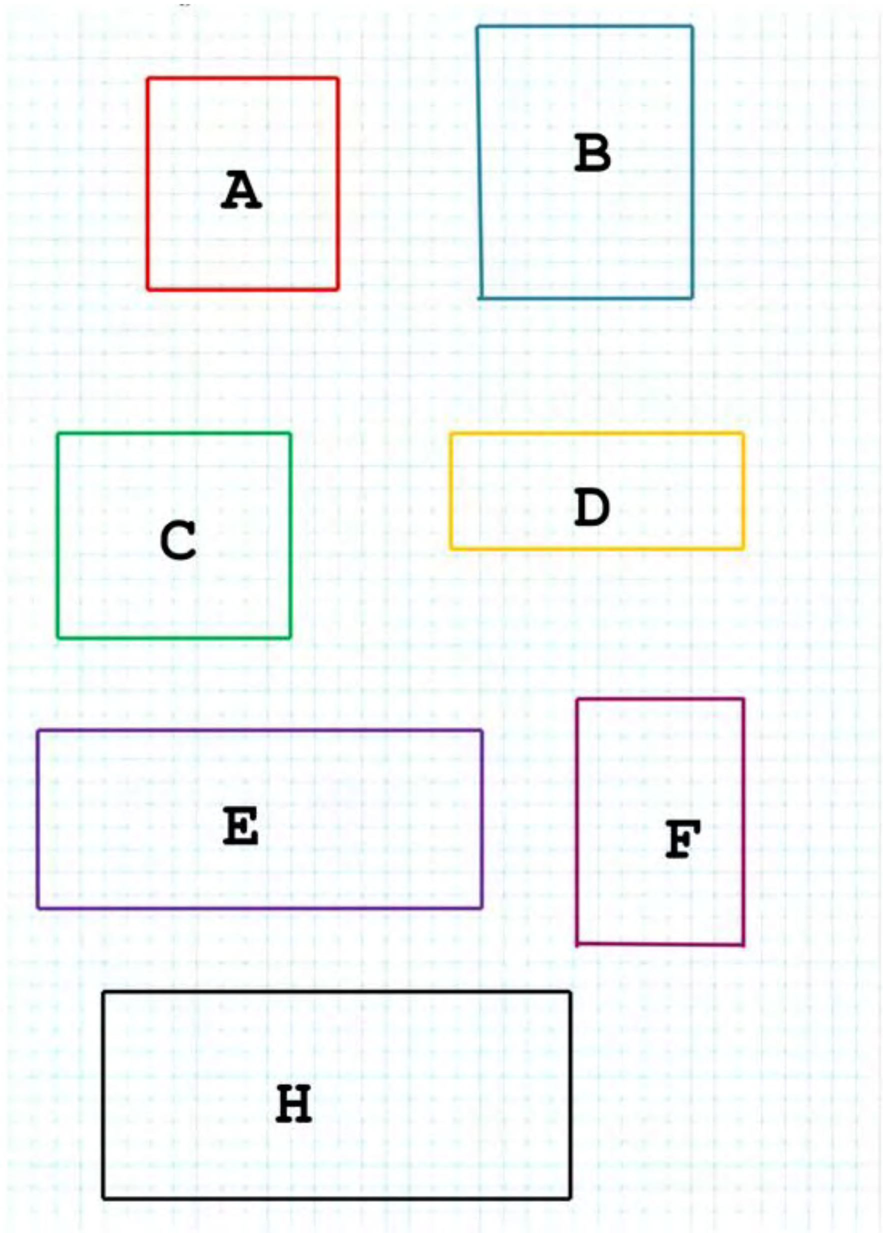
Welcome to the world of metaverses!

Choose your avatar, name it, and start building your own city right away.

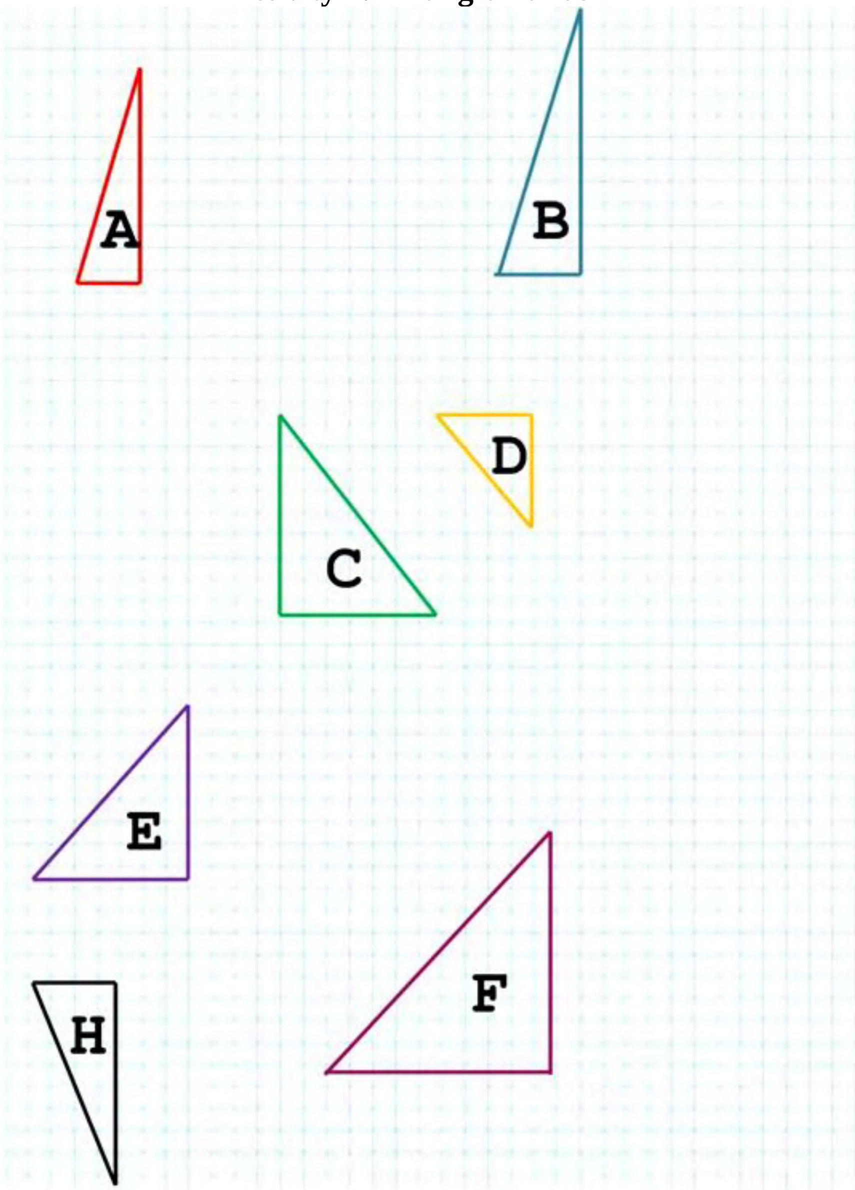
you buy. However, there is only one way to buy land in Metaverse, and it is to accurately estimate the areas of the selected lands by using the land modules we will give you.

<u>Tip :</u> You can use the area formulas of the shapes you know, the plots drawn on the unit square paper, and the pencil and scissors to calculate the areas accurately and more easily.

Activity 2: Rectangular Lands



Activity 2: Triangle Lands

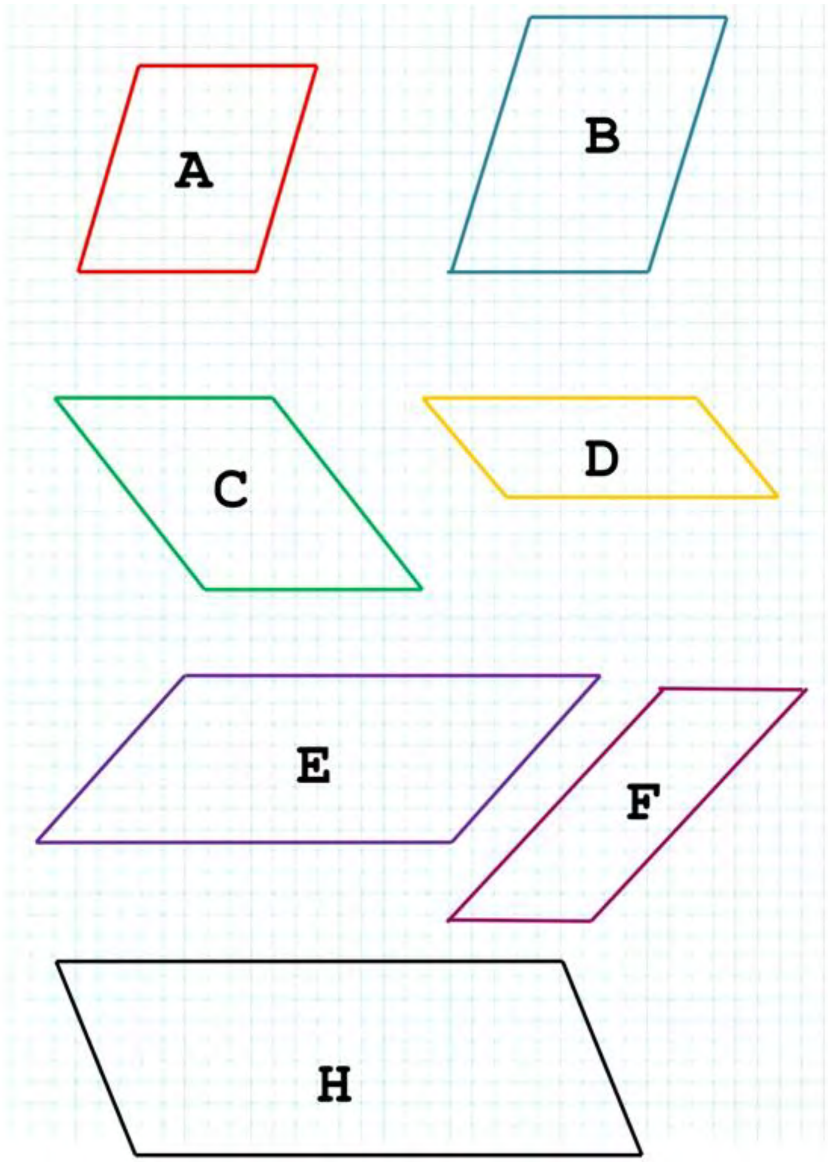


Activity 2: Measuring areas of triangle and rectangular

Answer the questions in the table below.

Lands	Rectangular area	Triangular area
A		
B		
C		
D		
E		
F		
H		

Activity 3: Parallelogram Lands

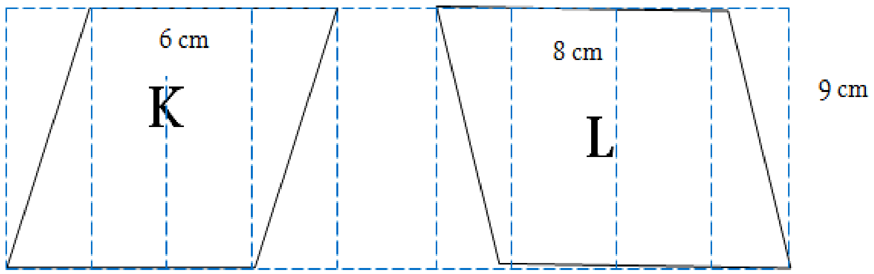


Activity 3: Measuring the area of a parallelogram

Answer the questions in the table below.

Lands	Parallelogram area	What path did you take when calculating the area? (Write in detail.)
A		
B		
C		
D		
E		
F		
H		

Activity 4: Problems about the parallelogram area



1) Calculate the area of the K and L parallelograms given above in the light of your correctly calculated parallelogram land areas.

2) Answer the following questions.
a) Do not calculate the parallelogram area with a height of 10 cm and an edge of 30 cm without drawing it.

b) Do not calculate the parallelogram area with a height of 15 cm and an edge of 60 cm without drawing it.

3) How can we calculate the area of the parallelogram?