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Models of conceptualizing and measuring statistical knowledge for teaching: A critical review

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ABSTRACT

Since it has become necessary for each individual to be statistically literate, statistical education research has taken teachers' competencies into its agenda. The knowledge needed to teach statistics differs from the knowledge needed to teach mathematics since statistics is different from mathematics. Teachers and researchers need to consider these differences and be aware of the challenges of statistics teaching. This article focuses on the nature of statistical knowledge for teaching (SKT). Models of conceptualizing and measuring SKT from research literature were reviewed critically. The strengths and weaknesses of models were discussed. The article concludes with some implications for teacher education and research.

Keywords: statistics education, statistical knowledge for teaching, models of statistical knowledge for teaching, review

INTRODUCTION

In recent years, as a result of the emphasis on the significance of statistical understanding worldwide, a new paradigm shift has been made in statistics instruction and teacher competencies (Ben-Zvi & Makar, 2016; Pfannkuch & Ben-Zvi, 2011). The renewed objectives of statistics instruction aim to improve the statistical literacy of students from different perspectives by focusing on statistical concepts and their relationship to daily life beyond rules, formulas, and some calculations (Bargagliotti et al., 2020; Garfield & Ben-Zvi, 2008). The questions of how to improve statistical literacy can be answered by determining what teachers know, think, or do about statistics, which is the main part of this teaching (Sedlmeier & Wassner, 2008). Because the professional competencies of teachers in the field of statistics directly affect and shape the statistical literacy of students (Callingham et al., 2016). When it is seen that the competencies of teachers in statistics teaching, the knowledge on which they base these competencies, and the extent to which they are related to each other can be seen, the direction of the targeted reforms can be fully drawn. This article aims to review literature related to models of conceptualizing and measuring statistical knowledge for teaching (SKT). The article begins by explaining the emergence of SKT and the various viewpoints on this concept. The next section describes some models of SKT. The model can also be referred to as a framework, structure, or theory in literature. Thus, multiple databases were examined for two different patterns of search terms and potential variations within those patterns ("statistical knowledge for teaching" AND "model" or "framework" or "structure" or "theory"). The term "model" is used in the current study to avoid confusion caused by naming differences. This review covers as much of the available literature as possible. The final section offers implications for teacher education and research on SKT.

Statistical Knowledge for Teaching

If statistics are now to be understood and taught in a wider range with the accumulation of knowledge in modern society, the knowledge of the teacher can be seen as the starting point on this trajectory. Because the knowledge of the teacher is the most important mediator that establishes the association between statistics and the student (Groth & Meletiou-Mavrotheris, 2018). For effective and productive statistics teaching, teachers are expected to have a special knowledge that is far from the deductive structure of mathematics, requires a data-based understanding, and questions the reality within the context (Pfannkuch, 2008). However, ignoring some differences in the nature of statistics and mathematics (Rossman et al., 2006; Shaughnessy, 2019) has dominated the understanding that a teacher who teaches mathematics can easily teach statistics as well. This was also discussed at the joint ICMI/IASE Study Conference. At this conference, different models of knowledge of teacher required to teach statistics were discussed (Batanero, 2011). These discussions point to the realization that since statistics use mathematics, and some common structures are shared between both fields (González, 2016; Groth, 2007), it is clear that models of SKT are generally based

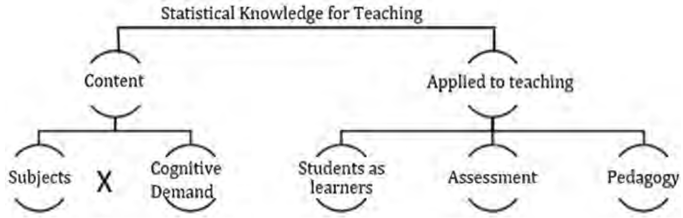

Figure 1. Model for analyzing aspects of statistical knowledge for teaching (Sorto, 2004)

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge	Specialized knowledge	Knowledge of content and students	Knowledge of content and teaching
Thinking	Need for data				
	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					
Dispositions					

Figure 2. Components of knowledge of teacher about statistical thinking & investigating (Burgess, 2006)

on models of mathematical knowledge for teaching (MKT), but they differ in terms of approach the nature of statistics. Thus, the structure of SKT and how it is defined has become a significant matter.

Currently, theories about SKT of teachers are much earlier in their development (Groth, 2007). Gurel (2016) states that the beginning of the knowledge of teacher goes back many years, yet it is still not clear what knowledge explains the statistics teaching. Shaughnessy (2007) pointed out that there is no standard model that allows the evaluation of SKT, while Noll (2008) pointed out that there are few studies investigating what knowledge is necessary and sufficient to teach statistics well. The main notion of all these researchers and the increasing number of scientific research emphasized is that it is quite difficult to define SKT, there is no common model that evaluates SKT in a theoretical framework, and the number and characteristics of the structures focused on in the models may differ. In this research, various models of SKT will be reviewed. The strengths and weaknesses of the models will be elucidated and the complex structure of SKT will be examined. This will enable researchers and teacher educators to gain insight into how they can approach SKT with which model. It will also provide a broad framework in terms of making strong designs about SKT and implementing and evaluating them.

MODELS OF STATISTICAL KNOWLEDGE FOR TEACHING

In her research with middle school preservice mathematics teachers, Sorto (2004) focused on what teachers need to know to teach statistics (**Figure 1**). In this process, Sorto (2004) classified the subjects according to statistical thinking, statistical reasoning, and statistical literacy, which are called cognitive demand, by using the studies of delMas (2002) and Garfield (2002) in addition to the various books, standards, and reports for teachers and students, and defined them as content knowledge. In addition, the knowledge that teachers should have in the teaching process was classified as knowledge of students, pedagogy, and assessment. Knowledge of students requires an idea of the prior knowledge, understanding, and difficulties of students. Knowledge of pedagogy is the ability to reflect all kinds of models, materials, and different teaching strategies into the learning environment, which gives the opportunity to properly construct statistical concepts in the minds of students. Knowledge of assessment requires knowledge of measurement and assessment approaches that reveal the statistical understanding of students.

The model Sorto (2004) is important in that it is the first model to explain SKT. In addition, the fact that the model has been developed by examining many documents gives a general perspective to SKT. However, the development of the model as a result of an instrument and interviews that applied only to preservice teachers independent of the real classroom can be considered a deficiency.

Another model of SKT is by Burgess (2006) (**Figure 2**). The main assumption of this model is that the knowledge of teacher is dynamic and should be evaluated in the context of the real classroom in which the student is involved. Therefore, Burgess (2006) collected data from elementary teachers with an observation technique. He also interviewed each teacher about observations.

This model is based on the models of MKT developed by Ball et al. (2005) and Hill et al. (2004), and the model of statistical thinking developed by Wild and Pfannkuch (1999). From these two models, Burgess proposed to examine SKT with a two-dimensional model.

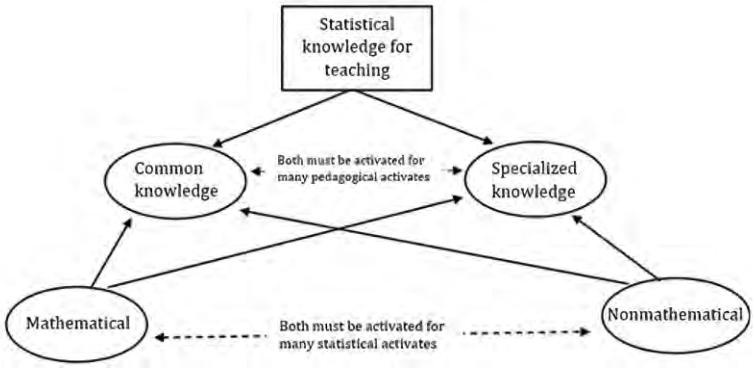

Figure 3. Hypothesized structure of statistical knowledge for teaching (Groth, 2007)

The first dimension includes four components of MKT: common knowledge, specialized knowledge, knowledge of content and students, and knowledge of content and teaching. The other dimension includes a total of eight components defined in the model Wild and Pfannkuch (1999): the five basic components of statistical thinking (need for data, trans numeration, variation, reasoning with models, and integration of statistical and contextual), investigative cycle, interrogative cycle, and dispositions.

In the study of Burgess (2007), four teachers were videotaped and interviewed as they taught statistical investigation in the classroom. In conclusion, SKT of each teacher was analyzed based on this model. In the analyses, classroom events that do not allow for improving the understanding of students are called *missed opportunities*. However, in other studies, he has presented corresponding samples for each cell of the model (Burgess, 2009, 2011). Burgess (2008) examined SKT of two inexperienced teachers who were in the second year of their profession comparatively. From the results, he presented suggestions on the professional development of preservice teachers and teachers.

The model Burgess (2006) is important in terms of defining SKT based in the real classroom. In addition, reaching the data that supports the model with different research makes an important contribution to the validity of the model. However, the fact that it is focused only on statistical thinking, statistical literacy and statistical reasoning not considered can be considered as the limitations of this model.

Groth (2007) described SKT based on the statistical process (formulating a question, collecting data, analyzing data, and interpreting results) highlighted in the *guidelines for assessment and instruction in statistics education report* (Franklin et al., 2007). Growth (2007) narrowed the scope of the components of MKT and focused on content knowledge (common knowledge and specialized knowledge) in the model Hill et al. (2004). He also included components of mathematical and nonmathematical (thought of as statistical knowledge) in his model due to the differences in mathematics and statistics (**Figure 3**). Knowledge of mathematical, while explaining statistical situations, reveals the knowledge necessary for the execution of mathematical operations. Knowledge of nonmathematical, on the other hand, involves being able to express the situations considered in a certain context by using the tools of statistics. Groth (2007) integrated these four components with the statistical process and showed the indicators of each component with illustrative examples.

The model Groth (2007) has made significant contributions to the field in terms of evaluating similar and different aspects of mathematics and statistics, starting from MKT. In addition, the fact that it considers the statistical process shows that the model meets the current recommendations for statistics instruction (Bargagliotti et al., 2020). However, focusing only on content knowledge and offering a theoretical perspective independent of applications in the real classroom can be considered the most important limitation of this model.

Groth (2013) has opened an undergraduate course that focuses on SKT of preservice teachers. In the course he carried out for three years, the preservice teachers read and wrote several articles from teacher-oriented journals, worked on various statistical activities, put forward hypotheses, and solved problems. They also discussed different pedagogical situations and have written down their experiences in the whole process. Thus, Groth (2013) examined the changes in the experiences of preservice teachers and put forward his model (**Figure 4**).

Although the components of the model of MKT developed by Hill et al. (2008) constitute the starting point of the model Groth (2013), the most notable point of this model is that the components of the key developmental understandings (Simon, 2006) and pedagogical powerful ideas (Silverman & Thompson, 2008) addressed in the development of content knowledge and pedagogical content knowledge in the studies aimed at improving MKT are central to explaining the model. Key developmental understandings encompass the cognitive landmarks necessary in the learning of content knowledge. Pedagogical powerful ideas, on the other hand, are seen as the transformation of key developmental understandings into mathematical concepts that are understandable to students. Groth (2013) linked content knowledge–common knowledge, specialized knowledge, and horizon knowledge–with key developmental understandings and pedagogical content knowledge–knowledge of content and students, knowledge of content and teaching, and curriculum knowledge- with pedagogical powerful ideas. The idea of decentering is important in the process of transforming the components of content knowledge into components of pedagogical content knowledge (Groth, 2013). Because the development of pedagogical content knowledge is the result of teachers being able to look beyond their key developmental understandings. According to Groth, in this transformation process, the knowledge of content and students from the components of pedagogical content knowledge is the main element.

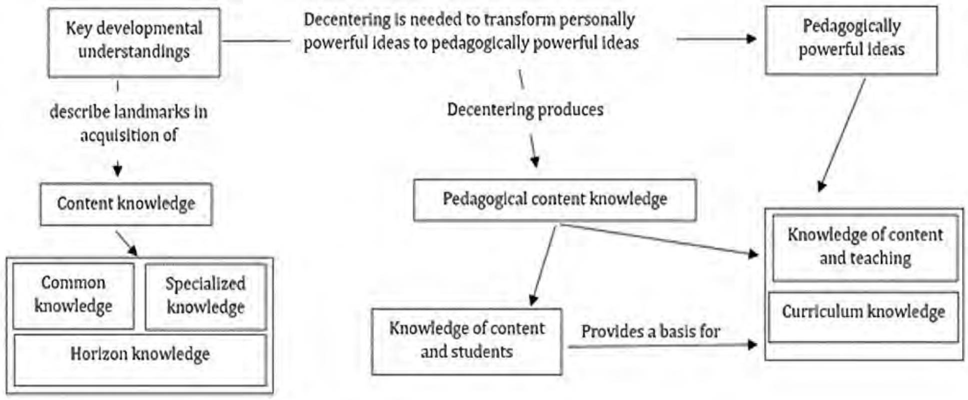

Figure 4. Hypothetical statistical knowledge for teaching elements & developmental structure (Groth, 2013)

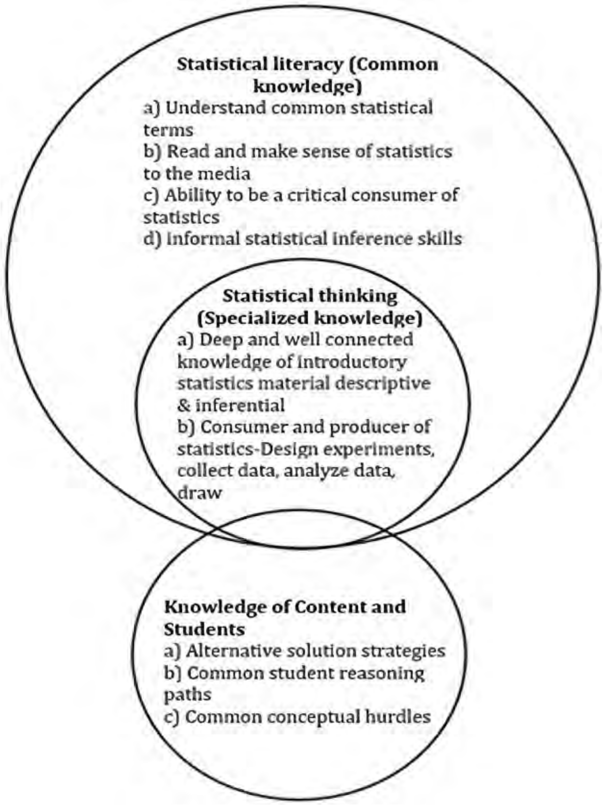

Figure 5. Model of statistical knowledge for teaching (Noll, 2007)

The model Groth (2013) is very significant in terms of reflecting the developmental structure of SKT and including components different from the components in the models of MKT. However, there is no clear knowledge of how this model, which is based on the Groth (2007) model, handles the distinction between statistics and mathematics. In addition, the model is not very useful for evaluating SKT of teachers by using the survey research methodology.

Noll (2007) focused on SKT of graduate teaching assistants (TAs). She proposed a model of SKT related to sampling concepts by using the model of MKT (Ball et al., 2001) (**Figure 5**). Statistical literacy, statistical thinking, and knowledge of content and students are like parts of a connected spiral. Statistical thinking is included in statistical literacy as it requires being both a consumer of statistics and a producer of statistics (Noll, 2007). In addition, certain characteristics of the knowledge of content and students overlap with statistical literacy and statistical thinking. According to Noll (2007), the evolution of knowledge of TAs in one component triggers the use of their knowledge in other components. Thus, this spiral works both downwards and upwards. For example, in ascending order, classifying questions or solution strategies of students can force a TA to think more deeply about the statistical subject, improving statistical content knowledge (Noll, 2007).

The model Noll (2007) differs from other models in that it attempts to explain SKT at the graduate grade. However, although the model is significant in terms of focusing on statistical literacy and statistical thinking concepts, there are uncertainties about how this knowledge is shaped in the real classroom. This model was developed by task-based web surveys and interviews applied to TAs. In addition, the fact that the study focused only on the knowledge of students and that no evaluation was made for other components related to SKT can be considered as limitation of the model. On the other hand, the fact that the model only addresses knowledge of teacher for sampling concepts constitutes an important deficiency in terms of reflecting other key ideas of statistics.

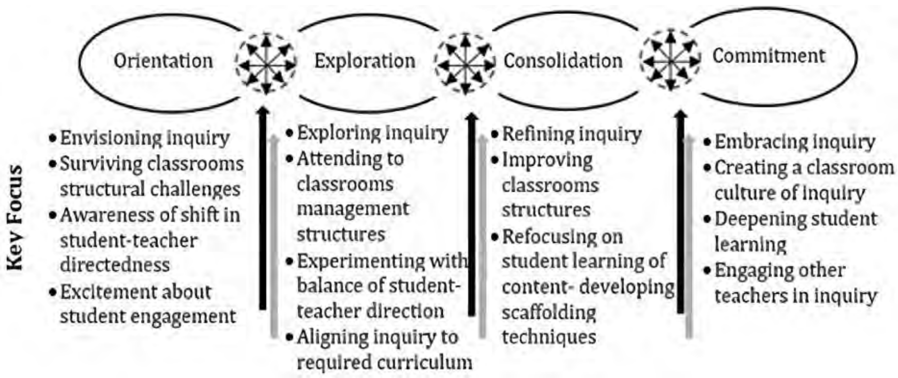

Figure 6. A model of learning to teach statistical inquiry (Makar & Fielding-Wells, 2011)

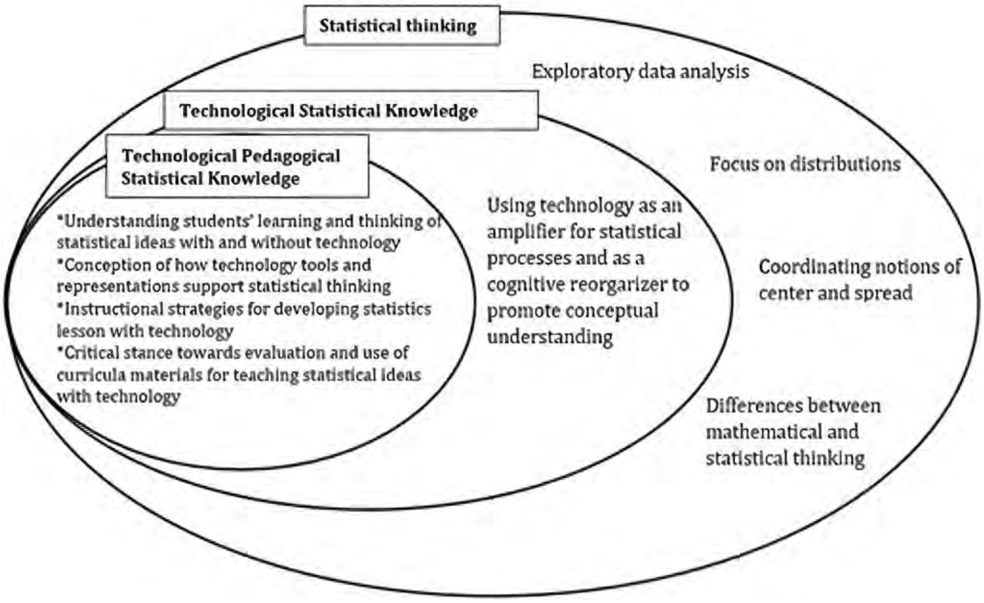

Figure 7. Model for developing technological pedagogical statistical knowledge of teachers (Lee & Hollebrands, 2008)

Makar (2008) planned a four-year project based on the investigation cycle of Wild and Pfannkuch (1999) within the model of statistical thinking to construct the model of learning to teach statistical inquiry. In the first year of the project, based on design-based research methodology, was worked with five teachers. As a result of professional development courses and interviews, experiences of teachers in teaching statistical inquiry were revealed in four cycles. These consist of cycles of orientation, exploration, consolidation, and commitment. Later, Makar (2010) conducted a longitudinal study using this model with 23 elementary teachers. Teachers participated in professional development programs, where there were applications related to statistical inquiry during the process. Interviews with teachers provided insight into the challenges of statistical inquiry. As a result of their two longitudinal studies, Makar and Fielding-Wells (2011) proposed solutions on how to turn these challenges into advantages in ensuring the professional development of teachers. They also specified the key indicators for the model proposed by Makar (2008) (**Figure 6**).

In the *orientation cycle* of this model, which is based on the Makar (2008) study, the first experiences of teachers regarding the statistical inquiry are revealed. In this cycle, teachers are expected to think about statistical inquiry, put forward problems, carry out classroom practices, and work out curriculum subjects. *The exploration cycle* includes the actions of teachers regarding the situations that will arise after acquiring their first experiences of how the statistical inquiry will be carried out in the classroom. In this cycle, teachers can help students with decision-making, collaborative and individual study. In addition, when necessary, it may apply the teaching strategies aligned with the current situation. *In the consolidation cycle,* teachers are expected to take a big picture of statistical investigation. The actions of teachers in this cycle are of a nature that focuses on and supports the learning of students. *In the commitment cycle,* teachers can make the statistical investigation part of their teaching and help other teachers.

This model has unique in that it describes knowledge of teachers of the teaching of statistical investigation with developmental cycles. However, the fact that it focuses only on statistical investigation and is not considered statistical literacy, and statistical reasoning can be considered as the limitations of this model.

Lee and Hollebrands (2008) developed a model specific to statistics teaching by drawing on technological pedagogical content knowledge (TPCK) model of Koehler and Mishra (2005) and Niess (2005) (**Figure 7**). This model illustrates the layered development of technological pedagogical statistical knowledge (TPSK) with a foundation focused on the statistical thinking of teachers.

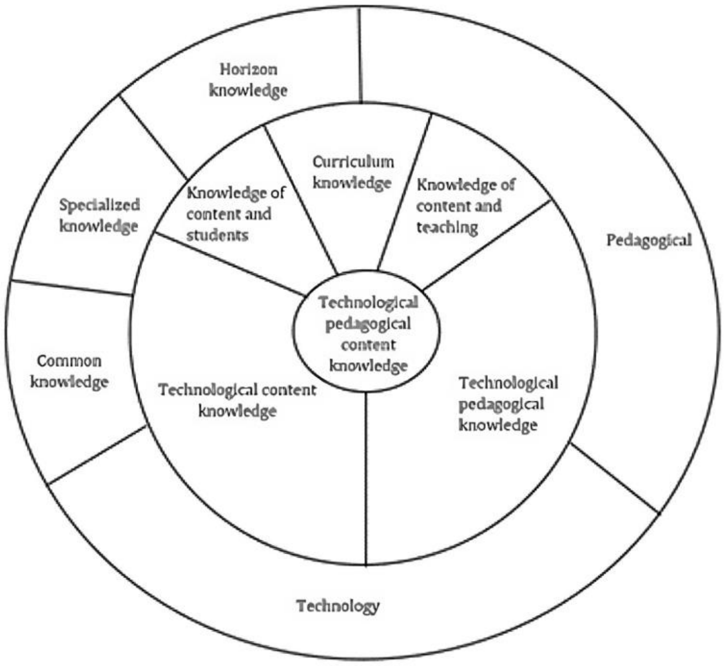

Figure 8. Components of knowledge & competencies of teacher (Wassong & Biehler, 2010)

According to Lee and Hollebrands, the development of technological statistical knowledge and statistical knowledge is important for the development of TPSK, but this may not be enough for teachers to have a specialized TPSK. Statistical thinking as foundational content knowledge includes knowledge of the different processes of statistical thinking so that both preservice teachers and teachers become active learners of statistics and practitioners of statistical practices. These processes are discussed concerning the four big ideas (exploratory data analysis, focus on distributions, coordinating notions of center and spread, and differences between mathematical and statistical thinking). Technological statistical knowledge is the calculation of measurements (e.g., average, spread) of data, the creation of graphs, and the use of these graphs as well as the knowledge of the advantages of technology in the association of multiple representations. TPSK, on the other hand, includes pedagogical information on the statistical thinking of students and the role of technology in the design and implementation of instructional content.

In their study, Lee and Hollebrands (2011) made some changes to the model they had previously developed. Accordingly, they defined the outermost layer as the statistical knowledge necessary to involve teachers in statistical thinking. Five aspects of statistical knowledge are adapted from the model Wild and Pfannkuch (1999). These are engaging in statistical thinking, recognizing the need for data, trans numerating, considering variation, reasoning with models, and integrating the context.

This model is important in that it is the first model to emphasize the significance of technology for the knowledge of teacher required for the teaching of statistical subjects. However, in classrooms, where it is not feasible to make use of technology, it remains limited in terms of the evaluation of the knowledge of teacher.

Another model that integrates technology into the knowledge of teacher is by Wassong and Biehler (2010). Wassong and Biehler (2010) developed a model that integrates the model of MKT developed by Ball et al. (2008) with the model of TPCK by Niess et al. (2005) and Mishra & Koehler (2006) (**Figure 8**). The outermost ring shows three types of knowledge for the teacher: content, pedagogical, and technology. Content knowledge includes common knowledge, specialized knowledge, and horizon knowledge. In the middle ring is the new knowledge that is formed by the combination of the knowledge in the outer ring: Pedagogical content knowledge, technological content knowledge, and technological pedagogical knowledge. Pedagogical content knowledge includes knowledge of content and students, curriculum knowledge, and knowledge of content and teaching. The innermost circle, TPCK, is considered a combination of these three pieces of knowledge. Thus, a model emerges in which different knowledge is combined from the outermost to the center and new knowledge is formed.

This model offers a significant fortune to the literature in terms of seeing technology as a part of the knowledge of teacher. But it is not very appropriate to use technology in cases, where it is not possible in every classroom. In addition, it is seen that the model has a very theoretical structure. Wassong and Biehler (2010) tested and concretized this model by analyzing a piece of the book by Garfield and Ben-Zvi (2008). This creates uncertainty in how the model will be reflected in the application dimension.

The StatSmart project (Callingham & Watson, 2007) is a large-scale project involving 17 schools, 50 teachers, and over 4,000 students in three different states of Australia. The project aims to increase the statistical literacy of students in middle and high school by offering a learning program that ensures the professional development of teachers. Thus, to monitor teacher and student changes in the project, valid and reliable measurement tools, interviews, and classroom observations were used. These measurement tools, which are developed for teachers and applied once a year, are in line with the components proposed by Shulman (1987). As a result of the StatSmart project, various studies have been produced that define the components or levels of pedagogical content knowledge of teachers related to statistics (Callingham & Watson, 2011; Watson et al., 2008, 2009).

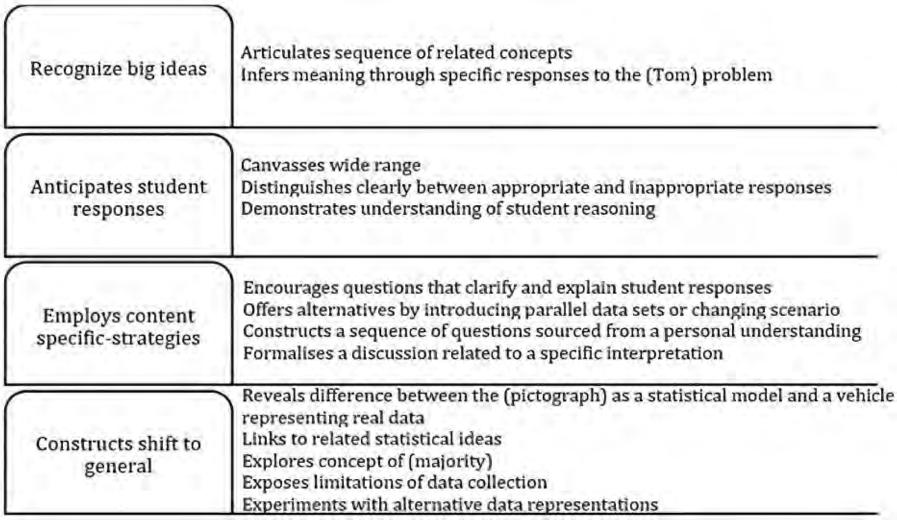

Figure 9. Model for a refinement of pedagogical content knowledge (Watson et al., 2009)

Watson et al. (2008) tried to determine the statistical competencies of 42 teachers with the help of the profiling instrument Watson (2001). As a result of the study, the pedagogical content knowledge of teachers was defined by three hierarchical levels (low, middle, and high): *At the low level*, teachers are partially successful in predicting the responses that students to questions and how the questions will be used in the classroom. However, teachers are deficient in forming responses to student responses that require proportional reasoning. *At the middle level*, teachers anticipate appropriate and inappropriate student responses to the question of graphs, identify mistakes, and come up with suggestions. However, it is seen that these suggestions for the use of the questions in the classroom contain only one single response. In addition, student responses that require proportional reasoning are also responded to using mathematical content knowledge. *At the high level*, teachers are successful in explaining student responses that require proportional reasoning, even though they still have difficulties in planning classroom activities associated with responses from students, and in explaining appropriate and inappropriate responses to questions presented by the media.

The study of Watson et al. (2008) is significant in that it is the first model that hierarchical levels SKT. However, since the levels have indicators for the components of content knowledge, knowledge of content and students, and knowledge of content and teaching, it does not allow for a multifaceted evaluation of SKT. In addition, the levels are not general and focus on some skills, such as proportional reasoning. This creates a limitation in the search for knowledge of teacher for all skills of statistics.

Watson et al. (2009) expressed pedagogical content knowledge of statistics of teachers with four nonhierarchical components (**Figure 9**). It is seen that the components of recognizing the big ideas, anticipating student responses, and employing content-specific strategies in the model Watson et al. (2009) are parallel to the components of content knowledge, knowledge of content and students, and knowledge of content and teaching in the model Ball and colleagues, respectively. This model is very significant in terms of focusing on big ideas in statistical education. In addition, the pedagogical content knowledge of teachers differs from other SKT studies in terms of considering it as a combination of content and pedagogical knowledge. However, the model can be considered limited because it was developed only for a problem with the pictograph.

By improving the study of Watson et al. (2008), Callingham and Watson (2011) have made more specific levels of pedagogical content knowledge of teachers. In the study, 45 teachers were asked 12 questions from the instrument Watson (2001) at the middle and end of the first year of the project. The first group of questions is about anticipating possible responses from students, and the second is about how teachers will use these questions in the classroom. The questions in the third group are about how teachers will respond to the real responses from students. At the end of the research, the pedagogical content knowledge of statistics of teachers was defined as four hierarchical levels (aware, emerging, competent, and accomplished) from basic to advanced: *At the aware level*, teachers produce a single response as to whether students' responses are appropriate or not. Although statistical understanding of teachers is not detailed, they are unable to provide appropriate suggestions in explaining understanding of students. *At the emerging level,* teachers use their statistical knowledge to explain appropriate or inappropriate responses of students. However, suggestions of teachers are more general and inclusive than contextual and subject-specific. In addition, although the teachings carried out by teachers at this level are of good quality, this performance is seen as low in the field of statistics. The explanations of teachers *at the competent level* are better in traditional and accustomed statistical subjects. Teachers can only give appropriate responses to questions they are familiar with. Finally, the *accomplished level is the* high level, reflecting the pedagogical content knowledge of teachers. At this level, teachers offer suggestions for appropriate and inappropriate responses from students and demonstrate a student-centered understanding by relating their instructional explanations to the field of statistics.

The increase in pedagogical content knowledge levels of teachers shows that they both produce more complex and relevant responses using their statistical knowledge and evaluate understanding of students effectively (Callingham & Watson, 2011). The levels presented by Callingham and Watson (2011) have more general indicators for assessing knowledge of statistics of teacher. However, these levels are also limited only by competencies related to components of content knowledge, knowledge of content and students, and knowledge of content and teaching.

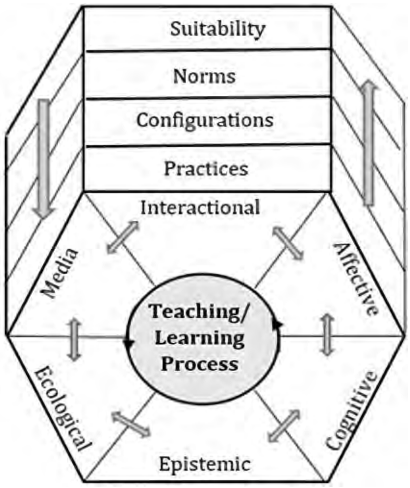

Figure 10. Components and levels of knowledge of teachers (Godino et al., 2011)

Godino et al. (2008) define pedagogical content knowledge of statistics of teachers following the dimensions of the onto-semiotic approach to mathematics education. This approach consists mainly of instructional dimensions based on epistemological, cognitive, and social constructivism (Godino, et al., 2007). In this context, Godino et al. (2008) put forward their model by including 55 preservice elementary teachers in a formative cycle. Of the components of this model, the component of epistemic emphasizes the instructional meaning considered and applied to a statistical subject (e.g., emphasis on different meanings of randomness). The component of cognitive requires knowledge of the level of development of students, understanding, and difficulties and mistakes in the subject. The component of affective includes knowledge about the attitude, emotions, and motivation of students on the subject. The component of interaction includes the teacher-student relationship and the organization of classroom discourse. The component of media includes educational tools and technological resources necessary for teaching. The component of ecology, on the other hand, requires knowing the relationship of the subject to the curriculum, statistical themes, and social, political, and economic situations associated with teaching. Later, Godino et al. (2011) added the levels at which teachers could be used to characterize their pedagogical content knowledge of statistics (**Figure 10**).

The level of practice includes the actions of students in statistical problem-solving and the understanding of the teacher of the subject and the support of learning. The level of configuration is the control of statistical objects and thinking stages in the learning process. The level of norms includes all kinds of rules, habits, and conventions that affect the order of the working environment. The level of didactic suitability includes objective criteria for developing and evaluating the learning-teaching process. Godino et al. (2011) stated that the statistics of the components and levels used in the model are necessary for learning and teaching stages. This model differs from other models in that it tries to explain SKT by considering the onto-semiotic approach in mathematics education. However, the fact that the model remains theoretical and that there is not enough knowledge about how it will be reflected in the application dimension can be considered as the limitation of this model. It is also not clear whether the levels in the model have a hierarchy.

González (2011) proposed a model that examines the teaching knowledge of high school mathematics teachers with a focus on the concept of variability. He built the model on four bases. The first is that the model is based on components from the study of Ball et al. (2008), one of the models of MKT. The second is that the components of MKT have been modified in the model to reflect the nature of statistics. In this context, he defined common knowledge as statistical literacy. Third, the beliefs about teaching and learning statistics of teachers should be considered. Fourth, to ensure the development of the model, activities that reflect understanding of teachers of the concept of variability should be put forward. In this context, depending on these four bases, he has developed a measurement tool with activities related to variability and applied it to teachers. In line with the responses to this measurement tool, he defined 12 indicators for assessing SKT of teachers. Thus, it has formed an eight-component model of SKT, including six components from the model Ball et al. (2008), beliefs, and understandings of variability. In his study, González (2014) redefined his model in terms of these dimensions by making the distinction that this model he proposed has two different dimensions, cognitive and affective (**Figure 11**). In this context, he stated that the cognitive dimension can be expressed by the six components of Ball et al. (2008), while the affective dimension can be reflected by two components as beliefs about teaching and learning statistics of teachers and their understanding of variability.

It is thought that it is important for this model to emphasize the distinction between the fields of statistics and mathematics with the concept of statistical literacy and to define SKT by focusing on the concept of variability, which constitutes the essence of statistics. However, it is focused only on the concept of variability and is also limited in terms of reflecting teacher knowledge of other concepts of statistics. In addition, the components in the model were revealed by questions directed to teachers. The fact that the classroom practices carried out by the teachers are not included in this process gives limited knowledge about how the pedagogical content knowledge is reflected.

Batur (2021) claimed that a new model should be developed in that the current models are not sufficient in characterizing SKT of teachers today. She built this claim on three main reasons: Firstly, statistical literacy is a special concept that forms the basis for the development of many skills, such as statistical reasoning and statistical thinking (Sabbag et al., 2018). Perhaps it has the emphasis on statistical literacy been as severe as it is as today throughout history (Bargagliotti et al., 2020). However, it is

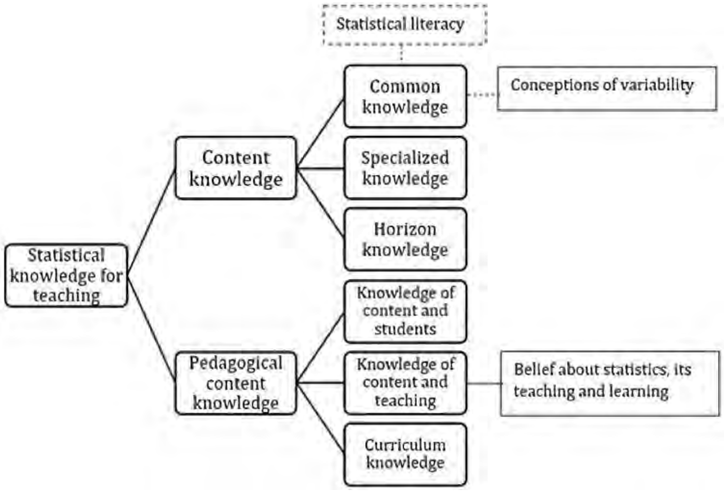

Figure 11. Proposed conceptual model for statistical knowledge for teaching (González, 2014)

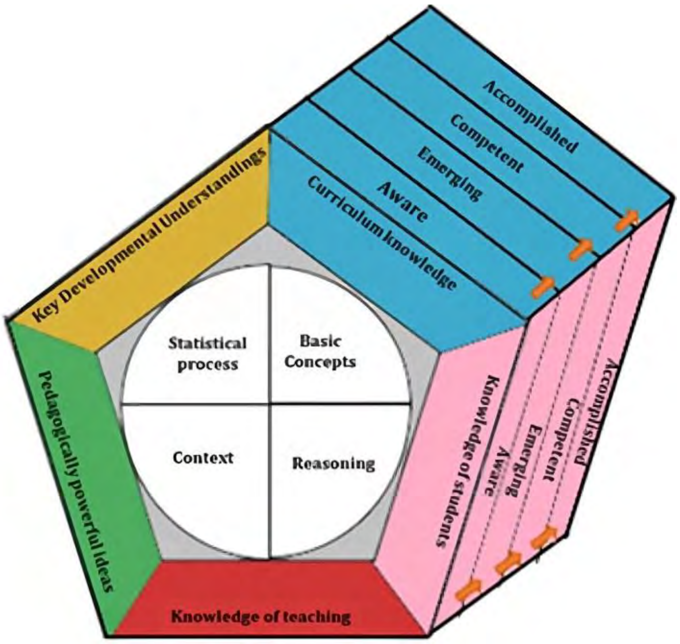

Figure 12. Hypothesized model of statistical knowledge for teaching (Batur, 2021)

noteworthy that the tendency for statistical literacy in current models is limited and this concept is approached from a narrow perspective. Statistical literacy is generally handled in parallel with the content knowledge in the models (González, 2014; Noll, 2007; Sorto, 2004). However, we see from the study of González (2014) that teachers who are not statis tical literate are no longer likely to evaluate the solution method of their students and to anticipate the difficulties. This shows that statistical literacy for well-planned teachings today is a common competence that directs not only the content knowledge but also the pedagogical content knowledge of teachers (Batur, 2021). In addition, statistical literacy includes all the competencies of individuals to effectively manage each stage of the statistical process, to understand statistical information in different contexts, to interpret this information by using the basic concepts, terms, and symbols of statistics, and to evaluate it critically (Batur, 2021). Therefore, this concept is very significant in terms of reflecting the distinction between statistics and mathematics and understanding why statistics-specific knowledge of teacher should define. Secondly, Burgess (2007) states that defining SKT depending on only a few components would constitute a serious limitation. However, when the models are examined, it is noticeable that in general, knowledge of content and students and knowledge of content and teaching are investigated as common components. To understand the holistic nature of SKT, it is significant to focus on the many components that differ from each other (Groth, 2013). The revision of these components, especially following conditions today, is important in terms of considering the changing and developing statistical and pedagogical needs (Batur, 2021). On the other hand, approaching SKT only in terms of components or levels constitute an important limitation for its evaluation on a large scale. Today, it has gained significance to reveal the competence of teachers regarding these types of knowledge as well as the types of knowledge they have about teaching statistics. Therefore, there is a need to popularize models that contain both components and levels. Thirdly, today, to raise statistical literate students, teachers need not only to graph data but also to teach different ideas of statistics. However, it is seen that the knowledge of teacher is defined in some models through a single concept (e.g., variability and sampling). Considering that the concepts related to statistics are increasing in the curriculum of countries, modern teachers need to have a holistic knowledge of statistics.

Batur (2021) put forward a hypothetical model that describes SKT based on the reasons listed above (**Figure 12**).

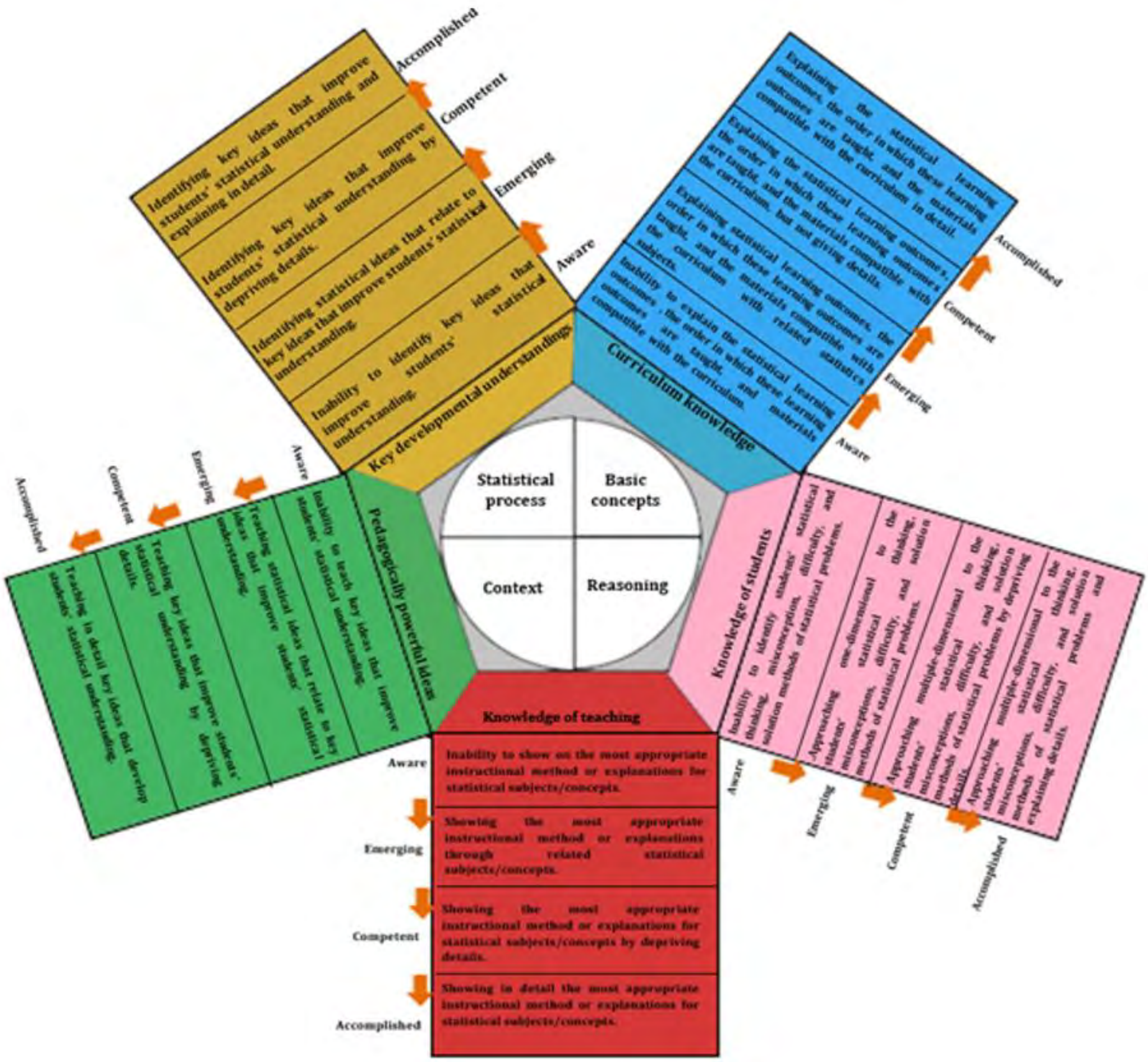

Figure 13. Model of statistical knowledge for teaching (Batur, 2021)

According to the model Batur (2021), statistical literacy is a prerequisite for the teacher to be able to effectively use SKT. Therefore, it is at the core of the model. Statistical literacy is the representation that statistics is a field in which it differs from mathematics, that is, it has its thinking system. Batur (2021) adapted the components of the statistical process, basic concepts, context, and reasoning in the model Ozmen (2015) to examine statistical literacy in a wide range. In this model, skills of statistical thinking and reasoning are included in the concept of statistical literacy. The components SKT, which are blended with statistical literacy, are the main elements of the model. Batur (2021) argued that the components in the model Groth (2013) (key developmental understandings, curriculum knowledge, knowledge of content and students, knowledge of content and teaching, and pedagogically powerful ideas), go beyond the components in the model Ball and colleagues, allow the evaluation of SKT from different perspectives. Batur (2021) expanded and redefined the indicators of these components. Moreover, with the help of levels (aware, emerging, competent, and accomplished) whose theoretical background is based on the model Callingham and Watson (2011), development of teachers can be better seen, and their levels can be compared.

Batur (2021), who assumes statistics as a theoretical structure that is the synthesis of three different elements (statistical literacy, SKT, and levels of statistical pedagogical content knowledge), has carried out a two-stage study to support this structure with experimental data. First, she developed an instrument for the different statistical ideas at the middle school level and the components in the hypothetical model. In the first stage of the study, she identified indicators in terms of levels of the components included in the hypothetical model from 11 responses of teachers to the instrument along with interviews (**Figure 13**). To test whether the model of SKT is effective in reflecting the desired feature, the instrument was reapplied to 20 teachers in the second stage of the study. Thus, Batur (2021) confirmed the model and found that this model was sufficient to evaluate SKT.

The model Batur (2021) is very significant in terms of being the most current model that explains SKT. This model differs from other models in that it is a synthesis of the models developed in statistics education and includes comprehensive components and hierarchical levels related to the nature of statistics. It also has a unique structure in that it reflects the situations specific to the nature of statistics with statistical literacy and sees statistical literacy as a necessary competence for all components of SKT.

The model Batur (2021) also has a general structure in that it focuses on different concepts that are key to the understanding of statistical subjects and concepts. However, this model, like some other models, has several limitations in terms of being developed as a result of an instrument and interviews independent of the real classroom. Batur (2021), who is aware of this limitation, has made suggestions at the end of her study that the teaching process cannot be observed due to the COVID-19 pandemic and that future research should focus on this situation.

A general synthesis of the structure of the models of SKT, described in detail above, with whom and how they are developed, and how they reflect the difference between statistics and mathematics is given in **Appendix A**.

IMPLICATIONS FOR TEACHER EDUCATION & RESEARCH

The main purpose of this article is to examine the nature of SKT and to review the different aspects of the various models that have been developed. The main implications for practice and research from this article are discussed below.

Notably, efforts to expand the role of statistics and probability in curricula have grown, particularly since the 1990s. This situation has forced our mathematics teachers, who have little or no experience to teach the concepts of statistics (Shaughnessy, 2019). Thus, the essential knowledge of teachers to teach statistics and probability started to be questioned as well (Watson et al., 2008). The research indicates that statistics educators and researchers from around the world are unable to agree on SKT and there are numerous models in the literature. From different models examined in this article, it is evident that SKT is a complex construct that requires not only a range of components but also developmental stages (**Appendix A**). Teaching statistics is not a competency that every mathematics teacher has (Batur, 2021). Most teachers with strong experience in mathematics cannot integrate the subjects of statistics into their classes. (Watson, 1998). Preservice teachers must graduate from the university prepared to teach statistics if they are to be well-informed educators. The models of SKT documented in this article offer a picture of SKT of teachers, and provide a base for designing, implementing, and evaluating instruction.

Begg and Edwards (1999) concluded that most teachers interpret the teaching of statistics and mathematics the same. However, since statistics and mathematics are distinct disciplines, SKT differs from MKT. Most models of SKT have used adaptations of MKT model (e.g., Burgess, 2007; González, 2014; Groth, 2007, 2013; Noll, 2007; Wassong & Biehler, 2010). For example, some models of SKT refer to content knowledge in MKT models as statistical literacy. Other models of SKT combine various statistical constructs (e.g., statistical process) with components of MKT (**Appendix A**). Thus, it is crucial to acknowledge the difference between SKT and MKT when preparing teachers.

Teaching is a social process; it involves countless interactions between students and their instructors (Shaughnessy, 2019). Therefore, it is important to how SKT of teachers is shaped in the real classroom (Burgess, 2007; Gurel, 2016). However, most models of SKT have been developed theoretically or as a result of written documents and interviews (**Appendix A**). As a result, it is more difficult to see how SKT is applied. Thus, more research is needed to validate the models that have already been proposed for SKT.

It is significant that most statistical knowledge of teachers is restricted to teaching some concepts (e.g., average, and variability) (Groth, 2009; Gurel, 2016; Vermette & Savard, 2019). However, in reform-oriented statistics instruction, it is aimed that the student can easily see and understand the relationships between concepts and based on these relationships, repeatedly question the important ideas and processes underlying the events (Bargagliotti et al., 2020). Teachers should have a general knowledge that relates statistics to one another and continuously analyses them instead of only teaching the average or graph due to the universal nature of statistics instruction. However, some models explain SKT by reducing it to a single concept (e.g., variability, sampling) (**Appendix A**). From this viewpoint, we believe that models that enable an analysis of knowledge of teacher of various statistical concepts are more significant in measuring SKT.

It is clear that the studies to determine SKT mostly involve preservice teachers (Chick & Pierce, 2008; Groth, 2013; Santos & Ponte, 2015; Sorto, 2004). However, studies conducted with preservice teachers do not provide very realistic results in showing the current situation and may only be useful in making some assumptions. Based on this justification, it is clear that research has been concentrated on determining SKT of teachers who are personally involved in the profession recently (Batur, 2021; Engledowl, 2017; González, 2014; Gurel, 2016; Vermette & Savard, 2019). As a result, more models dealing with SKT have been developed, from elementary to higher education. Thus, most of the models focused on in this article were created with data obtained from teachers responsible for teaching statistics at the school level (**Appendix A**).

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APPENDIX A

Table A1. A general synthesis of models for statistical knowledge for teaching

Models	Structure			Participants	Sources of data	Difference between statistics & mathematics
	Components					
	Content knowledge	Pedagogical content knowledge	Developmental stages			
Sorto (2004)	Subjects×cognitive demand	Students as learners, assessment, & pedagogy	-	Preservice middle school teachers	Instrument & interview	Cognitive demand (statistical literacy, statistical reasoning, & statistical thinking)
Burgess (2006)	Common knowledge & specialized knowledge	Knowledge of content & students & knowledge of content & teaching	-	Elementary teachers	Observation & interview	Investigative cycle, interrogative cycle, types of thinking, & dispositions
Groth (2007)	Common knowledge & specialized knowledge	-	-	-	-	Mathematical, nonmathematical, & statistical process
Noll (2007)	Common knowledge (statistical literacy) & specialized knowledge (statistical thinking)	Knowledge of content & students	-	Graduate teaching assistants	Task-based web survey & interview	Sampling concepts, statistical literacy, & statistical thinking
Lee and Hollebrands (2008)	-	-	Layers (statistical thinking, technological statistical knowledge, & technological pedagogical statistical knowledge)	Teachers	A module on teaching data analysis & probability with technology	Statistical thinking
Makar (2008)	-	-	Cycles (orientation, exploration, consolidation, & commitment)	Elementary teachers	Observation & interview	Investigative cycle
Watson et al. (2008)	Content knowledge	Knowledge of content & students, & knowledge of content & teaching	Levels (low, middle, & high)	Teachers (elementary to high school)	Instrument	-
Watson et al. (2009)	Recognize big ideas	Anticipates student responses, employ content-specific strategies, & constructs shift to general	-	Middle school teachers	Interview	-
Wassong and Biehler (2010)	Common knowledge, specialized knowledge, horizon knowledge, & technological content knowledge	Knowledge of content & students, curriculum knowledge, knowledge of content & teaching, technical pedagogical knowledge, & technological pedagogical content knowledge	-	-	Document analysis	-
Callingham and Watson (2011)	Content knowledge	Knowledge of content & students & knowledge of content & teaching	Levels (aware, emerging, competent, & accomplished)	Middle school & high school teachers	Instrument	-
Godino et al. (2011)	-	Epistemic, cognitive, affective, media, interactional, & ecological	Levels (practices, configurations, norms, & suitability)	-	-	Epistemic facet
González (2011)	Common knowledge (statistical literacy), specialized knowledge, & horizon knowledge	Knowledge of content & students, knowledge of content & teaching (beliefs about statistics teaching & learning), & curriculum knowledge	-	High school mathematics teachers	Instrument	Conceptions of variability & statistical literacy
Groth (2013)	Key developmental understandings (common knowledge, specialized knowledge, & horizon knowledge)	Pedagogically powerful ideas (knowledge of content & students, knowledge of content & teaching, & curriculum knowledge)	-	Preservice elementary teachers	Writing prompts, observation, & discussion	Mathematical, nonmathematical, & statistical process
Batur (2021)	Key developmental understandings	Pedagogically powerful ideas, knowledge of students, knowledge of teaching, & curriculum knowledge	Levels (aware, emerging, competent, & accomplished)	Middle school mathematics teachers	Instrument & interview	Statistical literacy (statistical process, context, basic concepts, & reasoning)