How to deal with and utilize (mathematics (education)) researchers' beliefs

Gero Stoffels

University of Cologne, Germany

This paper addresses the desideratum identified by Törner (2018), that researchers' beliefs are rarely addressed in the research literature dealing with beliefs. For this purpose, firstly a suitable theoretical framework is outlined that links the concept of belief with the research perspectives of researchers. Secondly, examples are given of how beliefs were, can and should be addressed in corresponding research on beliefs. Finally, it is shown in which ways explicating beliefs of mathematics education researchers might made their research, as well as their teaching more effective.

Keywords: researcher beliefs, background theory, methodology, domains of subjective experience

ARTICLE DETAILS

LUMAT Special Issue
Vol 12 No 1 (2024), 33–47

Pages: 15
References: 35

Correspondence:
gero.stoffels@uni-koeln.de

https://doi.org/10.31129/LUMAT.12.1.2140

1 Introduction

Deciding on the topic for my contribution at the MAVI-conference this year, I thought which topic might be interesting for the research community. My dissertation project (Stoffels, 2020) was based on the ÜberPro_WR seminars, which were designed to foster the reflection of students' own beliefs on mathematics during their transition from school to university and comparing them with the beliefs which were held during the transition to formal probability theory in its historical development in the 20th century. During the seminars, I became more and more aware of how important it is to make one's own beliefs on mathematics and probability theory as teacher and researcher explicit in these seminars to promote the students' reflection on their own beliefs. However, making my beliefs explicit does not mean that students were forced to simply adopt these beliefs, but rather to create an awareness of multiple perspectives on mathematics and probability theory. As a result of this observation, I have planned to choose the topic "why you can learn a lot about researchers' beliefs analysing their research on mathematical beliefs" for this contribution. Unfortunately, or rather fortunately, there is already an article by Törner (2018) that deals with similar issues that I had in mind, in particular:

- Describing the state of research: "It should have been pointed out that in research literature dealing with beliefs, researchers' beliefs are often neglected.

This may be due to the assumption that researchers should not be accused of having beliefs in the first place. Beliefs are regarded as features of subordinate teachers, students, parents, educational administrators, and further stakeholders, but not as features of researchers." (Törner, 2018, p. 7)

- Reflecting the state of research and recommendation for further research: "In research literature, this lack of self-reflection is hardly ever mentioned. We believe that this can be regarded as a 'blank spot'." (Törner, 2018, p. 7)
- Indicating (mathematics education) researchers' beliefs influencing educational practice: "This circumstance is tragic since researchers have to be seen as important players in terms of educational change." (Törner, 2018, p. 7)

So instead of raising these issues I decided to tackle them by providing a 'how-to' guide based on theoretical considerations as well as empirical indications how to deal with and utilize ((mathematics (education)) researchers') beliefs[1].

2 How to frame (researchers') beliefs: interaction vs. reflection

There are a lot of different descriptions and definitions of the belief concept in literature (Goldin, 2003; Green, 1971; Grigutsch, Raatz & Törner, 1998; Pajares, 1992; Schoenfeld, 1985; Stoffels, 2020; Thompson, 1992), furthermore a lot of research works mention that there is no consent on a definition of belief (Bräunling, 2017; Pehkonen, 1995; Rolka, 2006). Still, there seems to be no satisfactory answer to these theoretical problems; instead, a lot of research dealing with beliefs focus on exploring beliefs of different bearer groups, e.g., teachers and students (Törner, 2018), or the diversity of beliefs in different mathematical fields, establishing the concept of domain-specific beliefs (Eichler & Erens, 2015; Witzke & Spies, 2016).

An interesting discussion of the "theoretical struggles" is given by Goldin et al. (2009) referring to different perspectives and uses of the term beliefs, stating:

[1] In the title, throughout the article, and even in the term "((mathematics (education)) researcher's) beliefs" marked by this footnote, there is a bracketing notation that at first seems odd. However, it is meant to indicate two things. On the one hand, different belief-bearers, namely, unidentified belief-bearers, researchers in general, mathematicians, and mathematics education researchers, are considered by omitting the bracketed words. On the other hand, this notation is used to illustrate that while beliefs may differ content wise, they do not differ in terms of their structure and development as presented here.

> "Beliefs are highly subjective and vary according to the different bearers. Thus, observers of a specific situation may refer to quite different beliefs. [...] Our goal is to be able to apply the flexible construct of beliefs to various situations pertaining to mathematics education". (Goldin et al., 2009, p. 4)

Furthermore, they describe four classes of aspects of beliefs (Goldin, Rösken & Törner, 2009, p. 4) which are the ontological aspects (referring to a "belief object"), enumerative aspects (a "content set" of mental states or experiences connected to beliefs), normative aspects (how conscious the belief bearer is about the activated belief) and affective aspects of beliefs ("emotional feelings, attitudes and values" attached to the belief).

It seems that these attributes and aspects of beliefs are commonly accepted, especially as they are relatively general in nature. In relation to Törner's (2018) original desideratum regarding the beliefs of mathematics education researchers and this paper, one can conclude, assuming the premise, that the beliefs of researchers are not fundamentally different in nature from those of other bearers of beliefs (e.g., students or teachers), the following framing of researcher's beliefs might work. It simply adds the word (mathematics (education)) researcher to Goldin's et al. (2009, p. 4) description:

1. (Mathematics (education)) researchers' beliefs are highly subjective.
2. Beliefs vary according to the different (mathematics (education)) researchers.
3. (Mathematics (education)) researchers observing specific situations may refer to quite different beliefs.

Especially the third attribute shows the difficulties in the research on beliefs, as "observing specific situations" means, that during/after observations (mathematics (education)) researchers identify, probably a better word may be 'assign', beliefs to the observed bearer of beliefs while referring to their own beliefs. So, it seems as if there are multiple levels of beliefs, beliefs about beliefs, and so on.

Before these statements are elaborated further and explained by relating beliefs and belief-systems to the theory of "domains of subjective experience", some concrete examples of these statements will be given here.

For the highly subjectiveness of (mathematics (education)) researchers' beliefs, I will give two examples, an explicit and an implicit one. The explicit example is given by Grigutsch et al. (1998, pp. 13-14)) who stated that their qualitatively identified four aspects of "mathematical worldview" ('schema', 'formalism', 'process' and 'application') may have their origin in their own worldview. Implicitly the subjectiveness of

beliefs can be stated by the various catalogues of aspects or belief-systems, which can be found in literature, based on previous experiences and backgrounds of the authors (Ernest, 1989; Grigutsch et al. 1998; Beswick, 2012). For the second and third claim I want to give a paper (Heyd-Metzuyanim, 2019) as an illustrative example, because it discusses different background theories, which are based on different beliefs about whether internal or "mental" constructs are fruitful for addressing different research questions in mathematics education. Or, as Heyd-Metzuyanim (2019, p. 7) states:

> "as exemplified in the two studies reviewed above, studies of beliefs and identity tend to crossover and deal with aspects that belong, according to the above suggestion, in the *others' camp* [emphasized by G.S.]".

Goldin et al. (2009, p. 3) referring by their construction of "constitutive elements of a structural framework guiding our understanding of beliefs" on Hilbert's (1899) approach to axiomatization in his "Foundations of Geometry" as implicit defining mathematical concepts in comparison to the classical definitions of "points" and "lines" by Euclid. The similarity, according to the authors, is that the constitutive elements of beliefs they propose provide a framework for discussing different perspectives on beliefs, just as Hilbert's implicit definitions in his axiomatics can do for explicit definitions in the context of geometry. In the following I will proceed analogously to particle physics and show in which way established concepts as "society of the mind" (Minsky, 1988) used by mathematics education and mathematics educational theories like "domains of subjective experience (DSE)" (Bauersfeld, 1983) can be a basis for the conception of belief in order to address the problem of researchers' beliefs of beliefs.

Similar theoretical issues to Goldin et al. (2009) are stated in Stoffels (2020) in the context of "Auffassungswechsel", which can be translated as "change of belief systems", in the transition from school to university. Considering similar to Goldin et al. (2009) (a) beliefs and belief-systems are subjective, which means they are internal, and (b) therefore it might be the case that observed participants may refer to different beliefs even in similar situations, following question arose: in which ways can a researcher indicate whether the observed participant refers to one, multiple, different, or similar beliefs? The idea solving this problem, that has guided my work, is that researchers *do not identify beliefs in bearers*, but rather *they attribute certain beliefs to them as observers of their activities in their environment*. This may look at first

glance like a mere shift of the problem, but in the following it will become clear how this can be used productively for research on beliefs.

Still, researchers want to talk about assigning beliefs to observed subjects based on their activities, which may be guided by cognitive, affective, or behavioural processes (Liljedahl & Oesterle, 2014). Thus, according to such an interpretation of the belief concept, a theory is needed that explains the activities of the subjects in such a way, that it:

- allows the identification of different beliefs since mathematics educational research has made great progress in this area,
- can depict the above stated aspects of beliefs (Goldin et al., 2009), and finally.
- can form a basis for (inter-)active processes such as reflecting and sharing beliefs or doing/having a change of beliefs.

A good candidate for such a theory is offered by Bauersfeld's (1983) approach of subjective domains of experience (DSE). This insight is not fundamentally new since Pehkonen already described that in:

> “Germany, researchers usually speak instead of beliefs (Vorstellungen) and conceptions (Auffassungen) on "subjective theories" (e.g., Bauersfeld, 1983; Jungwirth, 1994; Tietze, 1990), and the central term to be used there is "a subjective experience domain" (Bauersfeld, 1983).” (Pehkonen, 1995, pp. 10–11)

A similar overview can be found in Grigutsch et al. (1998).

A new perspective can be established by using the DSE model as a suitable basis for the concept of belief and not as a mere similar concept (Stoffels, 2020). Accordingly, at this point I would like to first give a short overview on Bauersfeld’s (1983) conception of DSE, before I show that Goldin et al.'s (2009) aspects of beliefs can be found in the conception of DSE. Then I will give a definition of beliefs based on the DSE model, which allows explaining the reflection of beliefs as a relevant process for addressing beliefs. Finally, the issue of researchers’ beliefs on beliefs will be discussed.

The research in the 1980s and 1990s by Bauersfeld and his research group can be subsumed under the paradigm of Interactionism, which was also influenced by a long term cooperation with Paul Cobb (Cobb & Bauersfeld, 1995). In his working group several theoretical approaches were discussed how to shape this interactionist perspective. In this article I want to focus on two complementary foundations for the interactionist perspective Bauersfeld mentioned himself in his 1983 article “Domains of

Subjective Experiences as the Basic Issue for an Interactive Theory of Mathematics Learning and Teaching". Bauersfeld states (1983, p. 40, translated by G.S.):

> "The DSE model allows for the clarifications of the concepts of abstraction, transfer, and illustration [...]. In particular, it allows a differentiated description of mathematical learning via the formation of new DSE and the linking of existing DSE. The frame model leads to a more precise description of institutionalized communication processes, in particular through terms like 'working interim', 'frame conflict', the phenomenon of 'down-modulating', etc."

This juxtaposition shows Bauersfeld's assessment of the DSE model as an individualistic model. Stoffels (2020) has shown through a theoretical analysis based on an enactivist paradigm, that by considering the shared domain of experience of interactants, this individualistic limitation can be resolved, which is also important for this article. Apart from this extension of the DSE model in interactions, this article follows Bauersfeld's general conception of DSE, which includes the following main ideas (translated by G.S.):

- every subjective experience is domain-specific, i.e., a subject's experience is divided into DSE that are activated in the respective situations. (Bauersfeld, 1985, p. 11)
- the totality of DSE presents itself in an agglomeration of non-hierarchically ordered DSE - the "society of mind" (Minsky, 1988). The DSE compete for activation, the more effectively, the more frequently they are reactivated or the more intensively they have been formed. (Bauersfeld, 1985, p. 12)
- the crucial basis for the formation of a DSE is the subject's actions and the context of meaning he or she constructs, or more precisely, their formation in social interaction. (Bauersfeld, 1985, p. 14)
- since experience is total, a DSE includes various elements. Bauersfeld (1983, 1985) proposed a list of specific elements capable of being extended: knowledge, mathematical habitus, procedural knowledge, emotions, values, I-identity, etc.

In Table 1 the description of DSE by Bauersfeld (1983) is deconstructed for depicting corresponding specific elements of DSE for each aspect of belief by Goldin et al. (2009). Probably the most important and most frequently referred property of DSE is its domain specificity, which Bauersfeld (1983, p. 28, translated by G.S.) describes as "the 'domain' is less universal than a world. Just the limitedness and particularity separate the domains of subjective experience (by short as DSE) from each other", which

gives the possibility of assigning different and even contradicting beliefs to one person.

Table 1. Deconstruction of the DSE concept (Bauersfeld 1983, p. 17, 28, 56) for a comparison to the aspects of beliefs proposed by Goldin et al. (2009, p. 3)

Aspects of belief	Domains of Subjective Experience
Ontological aspects: *belief object*	Perspectives and functions of DSE [Bauersfeld refers with these concepts on Lawler's (1981) microworlds, G.S.]
Enumerative aspects: (subjective) *content set* of various possible perceptions, characteristics, suppositions, philosophies, and/or ideologies, which are often simply referred to as beliefs, or better, belief states.	The mathematical habitus is a specific element of DSE.
Normative aspects: Beliefs are highly individualized, means that the elements of the content set possess different weights that are attributed to various perceptions or assumptions.	The designation [of a DSE by a researcher, G.S.] contains the reference to the 'subject' as bearer.
Affective aspects: beliefs are interwoven with affect – emotional feelings, attitudes, and values	[The concept DSE] focus 'total experience' and not only knowledge. The non-cognitive dimensions of motor skills, procedural knowledge, emotions, evaluations, identity, etc. are specific elements of DSE.

Considering the distinction between DSE as situated in the subject and beliefs as assigned to the subjects by an observer (this could be a researcher) the following definition of beliefs can be given:

> A belief system (cf. Figure 1, dotted lines) refers to different domains of subjective experience (cf. Figure 1, filled shapes) that contain the same or similar perspectives and functions for the subject (cf. Figure 1, black ellipse and rounded rectangle). The clustering of reconstructed domains of subjective experience into belief systems on the basis of an identified sameness or similarity is done by an observer of the subject. This identification can be described as belief systems are clusters of domains of subjective experience. One way to specify this observation is to state that belief-systems of a subject form equivalence classes of domains of subjective experience of the subject. (Stoffels, 2020, p. 153, translated by G.S.)

The identified beliefs are therefore observer-related, which refers to the paradigm of enactivism (Maturana & Varela, 2008; Steinbring, 2015) in this conception. This does not mean that assigning beliefs is purely subjective by the observers. For

example, there may be a mode of assigning beliefs by researchers according to methodological and content criteria that the scientific community considers as adequate. Examples might be the use of certain Likert-scalable items and an associated factor analysis, or a qualitative content analysis using theoretically grounded categories. This is for example the belief of researchers, that the four aspects provided by Grigutsch et al. (1998) are a reasonable choice or that the methodology given in this article is adequate. This belief can be reconstructed by showing the adaption of the aspects and methodology by other researchers (Schukajlow, Rakozy & Pekrun, 2017; Rolka, 2006).

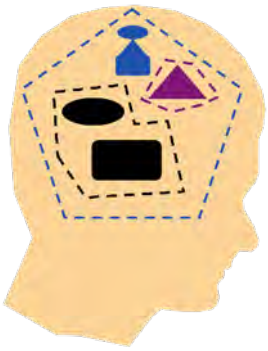

Figure 1. Diagram of different DSE's (black, purple, and blue filled figures) within the "society of the mind" (entire yellow polygon) and different indicated beliefs (dashed polygons). (cf. Stoffels, 2020, p. 154)

So, for the identification of beliefs an *interaction* between observer and subject is necessary. Of course, only those beliefs can be identified by the observer that he is aware of himself. Thus, one can speak of a *reflection* of one's own beliefs, in which the observer focus at his own activated perspectives and their functions. He thus interacts in a certain way with himself, which productively turns the previously identified theoretical issues of the researcher's beliefs about beliefs into a prerequisite for identifying beliefs. The required awareness of beliefs is implicitly shown by the fact that the observing researcher can take (different) perspectives into account, name them and thus indicate them. This can be also illustrated by Grigutsch et al. (1998), as they reflected their own beliefs on mathematics for getting an idea of possible beliefs on mathematics, they might be able to reconstruct for their participants in the study. In terms of DSE, this is only possible if the observer has at least one superordinate DSE whose perspectives allow the observation of subordinate DSE (Bauersfeld 1985, p. 40). In Figure 1, this is illustrated with the blue-filled figure, which diagrammatically represents a superordinate DSE that enables perspectives on the black- and purple-

filled subordinate DSE. For example, considering Grigutsch et al. (1998) again, if they would have not a DSE for comparing the different aspects of beliefs on mathematics, they would not be able to compare this different aspects, but might be only able to activate the different beliefs in different situations.

An answer to the question of this paragraph's heading 'how to frame (researchers') beliefs' is then, that there is no need for a new or different frame, rather it seems to be more important making the beliefs explicit in the research work by reflecting own researcher's perspective as enablers of research dealing with beliefs (cf. Heyd-Metzuyanim, 2019).

3 How to fill the "blank spot" of research on researchers' beliefs: reconstruction vs. construction

Törner (2018, p. 7) comes to the conclusion, that there is a lack of self-reflection leaving out the self-reflection of the researchers, which can be "regarded as a 'blank spot'". Rather, I think there is a 'blind spot' in the literature which looks like as if there is a blank spot. So, in this paragraph I want to give some examples of how researchers' beliefs were, can and should be addressed in corresponding research on beliefs. So that researchers' beliefs becoming the object of belief.

One of the more classic examples I already used in the previous paragraphs can be found in Grigutsch's et al. (1998, pp. 13–14) three reasons why they considered the well-known four aspects ('schema', 'formalism', 'process' and 'application') of mathematical world views. In this paper I want to focus on the first part of the first reason as it neither refer to their theoretical analysis (reason 2) nor their empirical results (reason 3):

> "We believe that these four aspects are the central and strategic elements of mathematical worldviews. Possibly this is an expression of our own worldview. But multiple observations – which are certainly also selectively guided by our own attitudes – showed us that thinking about mathematics and mathematical teaching often occurs in these four dimensions." (Grigutsch et al., 1998, p. 13, translated by G.S.)

Using the perspectives of the previous paragraphs we can *reconstruct* multiple beliefs by indicating that Grigutsch et al. (1998) activated a superordinate DSE, which allowed them to address four different perspectives on mathematics, respectively doing mathematics, which may be originated from subordinate DSE only allowing separate perspectives on each aspect.

One has to speak of ‘may’ here, because due to the same conditionality, which Grigutsch et al. (1998) address in this quote, one's own evaluation is also determined by one's own experiences and perceptions. Also, it is part of Bauersfeld's (1983) concept of DSE, that DSE can only be reconstructed interpretively and incompletely, because of its situatedness in the subject. This is true, even if the observer is the subject him-/herself, insofar as the DSE cannot be recognized ‘completely’, since this would require another superordinate DSE, which the subject needs to activate as a reflecting observer.

Presumably, all research dealing with beliefs in mathematics education addresses some perspectives on mathematical concepts or activities, decides which dimensions of beliefs are in scope of the work, or in which ways bearers of beliefs may interact based on their beliefs. These are implicitly the researcher's beliefs in their research. The ‘blind spot’ can thus be resolved by looking closely, in this case by reconstructions, similar to the given example by Grigutsch et al. (1998). Specifically, through an interpretative explication of the perspectives addressed by the researchers, which can then result in an indication of researchers' beliefs by an observer – the readers of the research work or the author(s) of the research work themselves.

Now that we have seen that one way of filling the "blank spot" is to reconstruct researchers' beliefs from existing research literature, the question naturally arises whether one can and should also make one's own beliefs as a researcher explicit, which means *constructing* researchers’ beliefs. Of course, every explication can only be done under the limitations already described. But addressing one's own beliefs can of course be realized in a similar way Grigutsch et al. (1998) have done in their article. So, one's own beliefs can and should be made explicit as well as the presumed limits of these beliefs. Still, another problem might be drawing a distinction between beliefs and knowledge (Pehkonen & Pietilä, 2003).

Despite of that, it still seems as if mathematics education researchers' beliefs are still not recognized widely in the mathematics education literature. I do not think this is due to a lack of reflection in the community, but rather that a culture of openness and appreciation of different beliefs should prevail as well as being visible in research articles. Balacheff (2008) develops an interesting scientific program that addresses these aspects. His starting point is the notion of proof in the mathematics education community. After a detailed reconstruction of different perspectives on proving and mathematical proofs he concludes the diversity in this field. A central part of his scientific program lays in the “elicitation of theoretical commonalities and divergences,

and possibly turn them into questions" (Balacheff, 2008, p. 511). This idea can be effective through continuous asking and answering reflective impulse questions during the research process on the investigated objects of belief from the perspective of the researchers–in case of Balacheff (2008)–beliefs about proofs. Specifically, the researcher assigns beliefs to him-/herself by this constructive process and opens another level of scientific discourse.

4 How to utilize mathematics (education) researcher's beliefs: Research vs. teaching (an explicit approach)

On a theoretical level it is interesting to think about (mathematics (education)) researchers' perspective, for example regarding differences and similarities to other bearers of belief. Still, it is in question, how these considerations can lead to scientific progress in mathematics education. I belief an answer to this question needs to address the utilization aspect of (mathematics (education)) researcher's beliefs.

In this context, the following uses of reflected beliefs in the research process seem to be particularly relevant:

- To raise awareness of one's own beliefs about the investigated belief object:
  - explicating the limits of one's own research perspectives on these belief objects, as Grigutsch et al. (1998) did in their focus on four aspects of mathematical worldviews,
  - identifying hastily assumed commonalities or differences of different approaches, as Networking of theories as research practice allows (Bikner-Ahsbahs & Prediger, 2014),
  - overcoming mistakenly deadlocked beliefs about concrete belief objects, as Kolmogorov (1956) did, for example, through his construction of a formal-abstract concept of probability,
- to become aware of one's own beliefs about ways of constructing beliefs,
  - deciding whether the chosen methods or timeframes of research are adequate regarding the investigated belief object, e.g., the change of beliefs (Stoffels, 2020),
  - documenting and reflecting one's own development of beliefs and making it become one's own paradigmatic example for belief changes (Altrichter & Holly, 2005),

- to become aware of one's own differentiation between one's own beliefs as (mathematics (education)) researcher and beliefs of other belief bearers:
    - evaluating if and which differences might exist to beliefs of other bearers (Törner, 2018),
    - Evaluating if one has too high or too low expectations towards mathematical learners and teachers, who of course have had different experiences and to whom correspondingly different beliefs can be assigned (Törner, 2018).

Before I discuss the use of (mathematics (education)) researchers' beliefs in teaching I will at this point explicate one of my own beliefs about (mathematics (education)) researchers' beliefs: (mathematics (education)) researchers' beliefs do not differ principally from non (mathematics (education)) researchers' beliefs.

This belief may originate directly from the conceptualization of beliefs described in the first section based on the DSE approach by Bauersfeld (1983, 1985). This means, that the research findings and recommendations for teaching regarding teachers' beliefs may be transferred directly. However, a distinction could possibly lay in the beliefs of how mathematics education can or should be learned or taught. This, admittedly, is a field that has hardly been considered so far, but in which important questions about beliefs of mathematics education presumably arise. Not only regarding mathematics and its teaching and learning, but also regarding their own discipline. Such a perspective on mathematics education can be used, for example, to organize mathematics educational knowledge by explicating beliefs, e.g., for the teaching and learning of calculus (Dilling, Stoffels & Witzke, 2024).

With these preliminary remarks in mind, I would like to emphasize the following uses of mathematics education researchers' beliefs:

- to enable discourses in teaching and to reveal the discourse basis on the teacher side,
- to stimulate multiple perspectives on mathematics, mathematical objects as well as mathematics education,
- to reveal reasons for ways of working in mathematics education, and last but not least,
- to be a role model for learners in making them aware of their own beliefs.

5 Final remarks

While writing this article once again I realized how difficult it is to become aware of one's own beliefs–in this case about the beliefs object “(mathematics (education)) researchers' beliefs”–and to be willing to bring them up for discussion.

For me still, the most striking example of this difficulty and inner conflict, which arises in such an undertaking of addressing one's own beliefs, can be found in Kolmogorov's (1956) "Foundation of probability theory". Kolmogorov explicates his view on mathematics by a comment to the reader in his footnotes belonging to paragraphs “§1 Axioms” [2] and “§2 The relation to experimental data” [1] :

> “ [2] The reader who wishes from the outset to give a concrete meaning to the following axioms, is referred to §2.” (Kolmogorov, 1956, p. 2)

> “ [1] The reader who is interested in the purely mathematical development of the theory only need not read this section, since the work following is based only upon the axioms in §1 and make no use of the present discussion. [...]” (Kolmogorov, 1956, p. 3)

Kolmogorov's inner struggle offering a formal formulation of probability theory can be seen in footnote 2, where he offers the reader a concrete interpretation in an empirical context. This is somehow in conflict with his objective formulating a formal-abstract foundation of probability theory, which he states in footnote 1.

I hope this 'how-to' guide to (mathematics (education)) researcher's beliefs may be helpful to focus on this ‘blind spot’ and not to lose sight of it in the future.

References

Altrichter, H., & Holly, M. L. (2005). Research Diaries. In B. Somekh & C. Lewin (Eds.), *Research methods in the social sciences* (pp. 24–32). SAGE Publications.

Balacheff, N. (2008). The role of the researcher's epistemology in mathematics education: an essay on the case of proof. *ZDM*, *40*(3), 501–512. https://doi.org/10.1007/s11858-008-0103-2

Bauersfeld, H. (1983). Subjektive Erfahrungsbereiche als Grundlage einer Interaktionstheorie des Mathematiklernens und -lehrens [Domains of Subjective Experience as the Basis of an Interaction Theory of Mathematical Learning and Teaching]. In H. Bauersfeld (Ed.), *Analysen zum Unterrichtshandeln: Vol. 2. Lernen und Lehren von Mathematik* (pp. 1–57). Aulis-Verlag Deubner.

Bauersfeld, H. (1985). Ergebnisse und Probleme von Mikroanalysen mathematischen Unterrichts. [Results and issues of microanalyses of mathematical teaching]. In W. Dörfler, & R. Fischer (Eds.), *Empirische Untersuchungen zum Lehren und Lernen von Mathematik*, *4*, 7–25.

Bikner-Ahsbahs, A., & Prediger, S. (2014). *Networking of theories as a research practice in mathematics education. Advances in Mathematics Education*. Springer. https://doi.org/10.1007/978-3-319-05389-9

Bräunling, K. (2017). *Beliefs von Lehrkräften zum Lehren und Lernen von Arithmetik [Teachers' beliefs about the teaching and learning of arithmetic]. Freiburger Empirische Forschung in der Mathematikdidaktik*. Springer. https://doi.org/10.1007/978-3-658-15093-8

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational studies in mathematics*, *79*, 127–147. https://doi.org/10.1007/s10649-011-9333-2

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Psychology Press.

Dilling, F., Stoffels, G., & Witzke, I. (2024). Beliefs-oriented subject-matter didactics: Design of a seminar and a book on calculus education. LUMAT: International Journal on Math, Science and Technology Education, 12(1), 4–14. https://doi.org/10.31129/LUMAT.12.1.2125

Eichler, A., & Erens, R. (2015). Domain-Specific Belief Systems of Secondary Mathematics Teachers. In B. Pepin & B. Roesken-Winter (Eds.), *Advances in Mathematics Education. From beliefs to dynamic affect systems in mathematics education* (pp. 179–200). Springer International Publishing. https://doi.org/10.1007/978-3-319-06808-4_9

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of education for teaching*, *15*(1), 13–33. https://doi.org/10.1080/0260747890150102

Goldin, G. A. (2003). Affect, Meta-Affect, and Mathematical Belief Structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Mathematics Education Library. Beliefs: A Hidden Variable in Mathematics Education?* (Vol. 31, pp. 59–72). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_4

Goldin, G. A., Rösken, B., & Törner, G. (2009). Beliefs – No Longer a Hidden Variable in Mathematical Teaching and Learning Processes. In J. Maasz & W. Schlöglmann (Eds.), *Beliefs and Attitudes in Mathematics Education* (pp. 1–18). BRILL. https://doi.org/10.1163/9789087907235_002

Green, T. F. (1971). *The activities of teaching. International Student Edition*. McGraw-Hill Kogakusha, Ltd.

Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellung gegenüber Mathematik bei Mathematiklehrern [Beliefs about mathematics among math teachers]. *Journal für Mathematik-Didaktik*, *19*(1), 3–45. https://doi.org/10.1007/BF03338859

Heyd-Metzuyanim, E. (2019). Dialogue between Discourses: Beliefs and Identity. *For the Learning of Mathematics*, *39*(3), 2–8.

Hilbert, D. (1899). *Grundlagen der Geometrie [Foundations of Geometry]*. BG Teubner.

Jungwirth, H. (1994). Erwachsene and Mathematik - eine reife Beziehung? [Adults and mathematics - a mature relationship?]. *Mathematica Didactica*, *17*(1), 69–89.

Kolmogorov, A. N. (1956). *Foundations of the theory of probability* (2d English ed.). Chelsea Pub. Co.

Lawler, R. W. (1981). The progressive construction of mind. *Cognitive Science*, *5*(1), 1–30.

Liljedahl, P., & Oesterle, S. (2014). Teacher Beliefs, Attitudes, and Self-Efficacy in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 583–586). Springer Netherlands. https://doi.org/10.1007/978-94-007-4978-8_149

Maturana, H. R., & Varela, F. J. (2008). *The tree of knowledge: The biological roots of human understanding* (Rev. ed., 8 [print.]. Shambhala.

Minsky, M. (1988). *Society of mind*. Simon and Schuster.

Pajares, M. F. (1992). Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct. *Review of Educational Research*, *62*(3), 307–332. https://doi.org/10.3102/00346543062003307

Pehkonen, E. (1995). *Pupils' view of mathematics: Initial report for an international comparison project. Research report / Department of Teacher Education, University of Helsinki: Vol. 152*. University of Helsinki.

Pehkonen, E., & Pietilä, A. (2003, February). On relationships between beliefs and knowledge in mathematics education. In *Proceedings of the CERME-3 (Bellaria) meeting* (pp.1–8).

Rolka, K. (2006). *Eine empirische Studie über Beliefs von Lehrenden an der Schnittstelle Mathematikdidaktik und Kognitionspsychologie[An empirical study on teachers' beliefs at the interface of mathematics didactics and cognitive psychology]*. PhD thesis, University of Duisburg-Essen, Duisburg, Germany.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. *ZDM*, *49*, 307–322. https://doi.org/10.1007/s11858-017-0864-6

Steinbring, H. (2015). Mathematical interaction shaped by communication, epistemological constraints and enactivism. *ZDM*, *47*(2), 281–293. https://doi.org/10.1007/s11858-014-0629-4

Stoffels, G. (2020). *(Re-)Konstruktion von Erfahrungsbereichen bei Übergängen von empirisch-gegenständlichen zu formal-abstrakten Auffassungen [(Re-)constructing domains of experience during transitions from empirical-concrete to formal-abstract belief-systems : a theoretical foundation, historical reflections and an intervention for the transition from school to university]*. universi - Universitätsverlag Siegen. https://doi.org/10.25819/ubsi/5563

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. *(Keine Angabe)*. http://psycnet.apa.org/psycinfo/1992-97586-007

Tietze, U. P. (1990). Der Mathematiklehrer an der gymnasialen Oberstufe: Zur Erfassung berufsbezogener Kognitionen [The mathematics teacher at the upper secondary school level On the assessment of job-related cognitions]. *Journal Für Mathematik-Didaktik*, *11*(3), 177–243. https://doi.org/10.1007/BF03340096

Törner, G. (2018). Are Researchers in Educational Theory Free of Beliefs: In Contrast to Students and Teachers? —Is There an Overseen Research Problem or Are There "Blank Spots"? In B. Rott, G. Törner, J. Peters-Dasdemir, A. Möller, & Safrudiannur (Eds.), *Views and Beliefs in Mathematics Education* (pp. 1–8). Springer International Publishing. https://doi.org/10.1007/978-3-030-01273-1_1

Witzke, I., & Spies, S. (2016). Domain-Specific Beliefs of School Calculus. *Journal Für Mathematik-Didaktik*, *37*(S1), 131–161. https://doi.org/10.1007/s13138-016-0106-4