Connotative aspects of epistemological beliefs: A pseudo-longitudinal study with students of different mathematical Programmes of Study

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Various studies have shown that epistemological beliefs affect personal learning and teaching performances. Therefore, epistemological beliefs have become an attractive object of research with different methods of survey. A distinction can be made between denotative and connotative aspects of beliefs, the former being reflected upon, explicit beliefs, whereas the latter being associative and evaluative judgements on (in our case: mathematical) epistemological beliefs. The present study used the instrument Connotative Aspects of Epistemological Beliefs by Stahl and Bromme to collect data from university students in mathematics in the years of 2017, 2018 and 2019. The pseudo-longitudinal data analysis showed 1. that students hold different connotative beliefs regarding the two domains "mathematics at university" and "mathematics at school" regardless their study progress, 2. that the beliefs remain relatively stable within the domains overtime and 3. that – considering the different mathematical programmes of study (e.g., pre-service teachers vs. mathematics majors) – the students' connotative beliefs mainly differ regarding beliefs about the simplicity of mathematical knowledge at school.

Keywords: epistemological beliefs, connotative aspects, development, mathematics, pseudo-longitudinal study

ARTICLE DETAILS

LUMAT Special Issue
Vol 12 No 1 (2024), 15–32

Pages: 18
References: 29

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https://doi.org/10.31129/LUMAT.12.1.2138

1 Introduction

Learners' beliefs about the nature of knowledge and knowing (Hofer & Pintrich, 1997), epistemological beliefs (EB), impact on the choice of learning strategies and information processing, i. e. the integration and acceptance of new knowledge, the comprehension of information, etcetera (Mason & Boscolo, 2004; Buehl & Alexander, 2002; Pintrich, 2002). Therefore, EB eventually affect students' learning outcomes (Schommer, 1993) what might be in terms of grades and rankings crucial for subsequent career options. Furthermore, epistemological beliefs presumably affect teachers' individual teaching styles, i. e. the choices of teaching methods and the subject-specific presentations of knowledge structures and knowledge justifications (Brownlee et al., 2011). The learners adopt the presented knowledge and way of knowing, and thus, teachers' EB indirectly shape their students' EB.

To assess and study EB in a differentiated and comprehensive manner one has to consider the impact of environmental factors on EB in two ways:

1. Environmental factors lead to domain-specific EB (e.g., Stahl & Bromme, 2007; Rott, 2020).
2. Environmental factors promote change and development of EB (e.g., Perry, 1970; Ross & Bruce, 2005).

This study addresses domain-specific EB (1) focusing on mathematics-specific beliefs and further differentiating EB regarding "mathematics as a school subject" and "mathematics as a scientific discipline" as taught in university. This differentiation is made due to the differences in contents, learning and teaching approaches as well as thinking and inquiry methods between subjects taught at school and taught at university (cf. Stengel, 1997; Bromme, 1994). Therefore, one might assume that university students adopt different EB concerning school subjects and related academic disciplines as a result of the leverage effect of the different teaching contents.

Regarding mathematics, "mathematics taught at school and at university [particularly] differ [...] in terms of rigor and in the necessity that is seen for justification" (Dreher et al., 2018, p. 323; see also Beswick, 2012): Mathematics as a scientific discipline (scientific maths) primarily "focuses on the rigorous establishment of theory in terms of definitions, theorems, and proofs" (Dreher et al., 2018, p. 323). The prevalence of the axiomatic-deductive structure results from the mathematical community's request for warrants. Those are presented in journals, books, or lectures as proofs and deductively justified theorems although new concepts and ideas are usually not found in a deductive reasoning process (Ernest, 1999). In mathematics classrooms at school (school maths), new concepts are introduced rather empirically, for example by examining prototypes instead, and reasoning is rather context-related and intuitive than rigorous and abstract (cf. Dreher et al., 2018). Scientific maths usually operate on an abstract level using symbolic mathematical language whereas school maths puts emphasis on the practical benefits of mathematics in everyday life and mainly presents it as a tool to approach and analyse reality (ibid.).

Considering the gap between school math and scientific math, Beswick (2012) emphasizes the key role of mathematics teachers in reducing the gap, stating they can reduce the differences if "they have an appreciation of the nature of mathematics that is akin to that of mathematicians" (p. 129).

Based on this insight, Beswick collected and examined i. a. teachers' EB about the nature of mathematical knowledge. She asked eight mathematics teachers to respond to 26 items on a five-point Likert scale, conducted six semi-structured interviews, and observed teaching sessions. Although Beswick does not claim representativeness of her findings, the examined cases suggest that beliefs of experienced mathematics teachers may differ regarding school maths and scientific maths. New insights into the nature of scientific maths gained while studying at university, do not necessarily transform beliefs about school maths "rather adding up on beliefs from earlier schooling experiences" (p. 145).

Considering the varying educational effects of different study programmes, this study additionally examines the EB of students being enrolled in different mathematical study programmes (e.g., mathematics major studies or mathematics for upper secondary school teaching) regarding school maths and scientific maths.

Grigutsch et al. (1998) have already examined study-programme related differences of mathematical beliefs in the context of surveying "mathematical world views". They used a questionnaire completed by 310 teachers that asked to agree on different statements about the nature of mathematics on a four-point Likert scale. The statements described mathematics to be either a formal-coherent system, a process determined by activity, a schematic toolbox or formula set and expressed opinions on its range of applicability, amongst others. The researchers compared the data of different teacher degree courses. Most of the students agreed on the significance of rigor in mathematics and on a wide range of applicability of mathematics. They also considered mathematics to be a process-driven discipline. But most of the students denied mathematics being reduced to a schematic toolbox or a formula set except for students of lower secondary teaching who expressed a significantly higher rate of agreement on that schematic view of mathematics. Grigutsch et al. (1998) summarized that the mathematical world views, and thus, the mathematical beliefs, of students in different teacher-training programmes, do not differ substantially.

Besides building on the research done by Grigutsch et al. (1998) – considering not only EB of pre-service teachers but mathematical EB of science students as well –, this study tracks the development of EB in both domains, school maths and scientific maths, for three years of study at university, too, and thereby considers the second aspect of environmental factors (2). Research findings either characterise the development of EB in a normative way, describing a development from naïve towards sophisticated EB (e.g., Perry, 1970; Kuhn, 1991), or in a quantitative way, finding EB to be rather stable or unstable in the course of time (e.g., Charalambous & Philippou, 2003; Green, 1971). Liljedahl et al. (2012) attribute the inconclusive results about the stability of EB – being perceived as more stable or "more susceptible to change" – to the lacking common definition of belief stability. Several researchers even have assumed that both characteristics of beliefs are not necessarily "mututally exclusive" and e. g., suggest that EB consist of core and peripheral belief aspects (Green, 1971; Kaasila et al., 2005), with the latter ones being more changeable and the former ones being more stable.

The present studies neglect the normative evaluation of the development of EB (with a distinction between naïve and sophisticated EB) and focusses instead on the quantitative evaluation of EB development, i. e., assesses the stability of mathematical EB during studies at university. In respect of the inconsistent research findings on belief stability, this article examines a specific aspect of EB, the connotative aspect of EB, which is explained in the following. Thus, it continues the theoretical approach of distinguishing different belief aspects, this, tries to shed light on the inconsistencies of research of findings about belief stability, and provides a new perspective on the nature mathematical EB.

2 Theory

2.1 Denotative and connotative aspects of epistemological beliefs

Stahl and Bromme (2007) take the different components of EB (e.g., Green, 1971) and their influence in forming beliefs into account and differentiate between connotative and denotative beliefs. This terminology is inspired by linguistics in the sense that a connotative meaning of a word is an associated, usually culturally shared meaning in addition to its denotative meaning. The denotative content is the precise, propositional, literal sense of a word.

Accordingly, connotative aspects of epistemological beliefs (CEB) denote associative-evaluative assumptions about the nature of knowledge which tend to be spontaneous, more emotional, and personal (Stahl & Bromme, 2007). In terms of mathematics, for example, connotative judgments about mathematics are stimulated "when a student is asked whether he or she generally thinks that mathematical knowledge is rather certain or uncertain" (Rott et al., 2015, p. 40) and no further context is given. On the contrary, denotative aspects of epistemological beliefs (DEB) encompass explicit, reflected-upon knowledge about the nature of knowledge and often are less contextual (Stahl & Bromme, 2007) and can be grouped into naïve and sophisticated DEB (Rott, 2020). Despite of the suggested distinction, Stahl and Bromme (2007) do not assume CEB and DEB to be strictly separable from each other.

2.2 The CAEB (Connotative Aspects of epistemological beliefs) – an instrument by Stahl & Bromme

Based on Osgood and Snider's semantic differential (1969), Stahl and Bromme (2007) developed an instrument to measure the Connotative Aspects of Epistemological Beliefs (CAEB). Osgood et al. (1957) originally used the semantic differential as a quantification method for "affective meanings".

Stahl and Bromme (2007) collected 24 pairs of opposing adjectives to be judged on a 7-point Likert scale for each contrastive pair, describing EB in the dimensions: "(a) [...] simplicity of knowledge (knowledge consists of simple facts vs. it is a complex network of information), (b) [...] certainty of knowledge (knowledge is certain vs. it is tentative), and (c) [...] source of knowledge (knowledge is objective and observable vs. it is subjective and constructed)" (p. 775). For the purpose of validation, they tested their items in two studies with more than 1000 participants each and identified 17 stable adjective pairs via factor analysis which could be summed up under the two factors "Texture" describing beliefs about the structure and accuracy of knowledge and "Variability" describing beliefs about the stability and dynamics of knowledge. In those studies, the CAEB proved to be sensitive enough to detect differences in students' CEB about different domains.

2.3 CAEB adaptation and further findings by Rott, Leuders, & Stahl

Since several research results indicate that there are domain-dependent EB (De Corte, Op't Eynde, & Verschaffel, 2002; Hofer, 2000), Rott et al. (2015, 2017) aimed to measure CEB about mathematics on a discipline-specific level.

For this purpose, Rott et al. (2015) had 230 respondents complete the CAEB twice, once with "mathematics as a school subject" (school maths), the second time with "mathematics as a scientific discipline" (scientific maths) in mind. Rott et al. (2015) focussed on epistemological judgements about the certainty of mathematics and found ten items that could be subsumed under the factor "Certainty" via factor analysis. They also collected denotative judgments of students about the certainty of mathematical knowledge and a two-way ANOVA supported the distinction of connotative and denotative judgments. In comparison to DEB, the collected CAEB-data reveal nothing about the degree of reflection upon the claimed beliefs or the sophistication of the students' beliefs.

Rott et al. (2017) repeated the survey with 147 students (105 1st and 42 4th semester students). A factor analysis showed that three factors could be distinguished: Certainty/ Texture, Simplicity and Variability of mathematical EB. Using this three-factor-based model, Rott et al. (2017) compared CEB about school maths vs. CEB about scientific maths. They found that the students judged school maths to be significantly easier and more superficial than scientific maths. Moreover, the trend could be observed that scientific maths was perceived to be more tentative and variable. School maths, on the other hand, was judged to be more organized, but also to be more inaccurate. Looking at the study progress, more advanced students (in the 4th semester) rated school maths to be significantly more tentative and more structured and scientific maths to be significantly easier compared to the judgement of the first-year students.

3 Research objective

The focus of this article lies on the analysis of CEB as portrayed in the introductory section, and thereby continues the research done by Rott et al. (2017), trying to answer the following main question about the domain-dependent nature of CEB: How do CEB about school maths and scientific maths develop during three years of bachelor's degree at university? Or put more precisely: In what sense differ CEB about mathematical knowledge regarding school maths and scientific maths in different semesters of study?

Such differences in CEB regarding the mentioned domains have already been hypothesized by Rott et al. (2017) based on a small sample and are probable due to the different representation modes of mathematical knowledge in school and in university (see *Introduction*). As freshmen are not accustomed to scientific maths, their CEB about this domain might shift in the course of their studies. Furthermore, it may well happen that the students – especially those that are enrolled to become teachers – reassess their beliefs about school maths over time as they gain new experiences, new knowledge and new skills. Rott et al. (2017) have found that first semester students and 4th semester Bachelor students differ regarding their EB about school maths and scientific maths (see *Theory*).

To comprehensively analyse CEB, the mentioned research question includes and combines the two main environmental factors that determine the nature of mathematical CEB, namely 1) the domain-specific formation of EB and 2) the development of EB over time (see *Introduction*). Tracking the development of CEB is reasonable as a static snapshot might not reflect the overall nature of CEB. And surveying general, not domain-specific CEB might not represent the mathematics-specific nature of CEB.

In fact, we further investigate the differences in domain specific CEB and the beliefs' development with regard to the students' different programmes of study. This allows an even more differentiated insight into the impact of environmental factors on CEB as students of different programmes of study are trained considerably differently, i.e., they have to meet different educational requirements and specialise in different fields. For example, pre-service teachers for upper secondary schools attend the same mathematics courses as students of the study programme "Bachelor of science". Pre-service teachers for primary school and for lower secondary school", on the contrary, usually attend less demanding university courses in terms of mathematical skills and knowledge. Pre-service teachers for primary schools do not choose to study mathematics voluntarily; it is a mandatory part of their curriculum. Accordingly, the different educational requirements might account for the slight differences in beliefs found by Grigutsch et al. (1998) between students of different teaching programmes and might account for intraindividual differences in CEB as well (see chapter *Introduction*). Unlike students majoring in mathematics, all pre-service teachers learn about the didactics of mathematics in addition to university mathematics and remain connected to school maths as they go through practical training sessions at school during their studies. These curricular activities might affect the pre-teacher's beliefs about school maths and induce change of CEB in this domain over time.

Whereas many surveys focus on mathematics teachers' EB (while still in university training or working professionally), none to little surveys consider EB of students majoring mathematics compared to students in teacher training. In this respect, this study, e.g. extends the research of Rott et al. (2017).

4 Method

This study is part of the project "Learning the Science of Mathematics" (LeScMa), in which students' skills in mathematical critical thinking as well as DEB and CEB have been assessed (Rott, 2020); here, we focus on the latter. For the assessment of mathematical CEB during academic studies, the CAEB was presented to university students attending different mathematical programs of study at the University Cologne at the beginning of the winter terms in 2017, 2018, and 2019, respectively (cf. Schreck et al., 2023). 1774 students completed the questionnaire (601 male, 1086 female, 87 students did not specify their gender): 580 students in 2017 (mean age 20.36, SD 3.06), 397 in 2018 (mean age 22.03, SD 3.26), and 797 in 2019 (mean age 21.71, SD 2.95). 84 individuals participated in all three rounds of survey, 279 individuals participated twice either in 2017 and 2018, 2018 and 2019, or 2017 and 2019, and 1495 were single participants. Participants created pseudonyms which allowed to track single and multiple participation. 365 of the respondents were preparing to become upper secondary teachers, 127 students were lower secondary teachers, 412 students were primary teachers, 340 students were teachers for special needs, 428 of the respondents were students of the Bachelor of Science degree (mathematics majors). 150 did not specify their field of study.

The students completed the adapted CAEB questionnaire in 10-15 minutes during lecture time; participation was voluntary. The CAEB asks the students to position themselves on a 7-point Likert scale between two opposing adjectives describing two opposing epistemological beliefs. The semantic differential format combined with limited response time should ensure that the students judged the positions on an associative-connotative basis. 24 adjective pairs were given in total and the students responded to the CAEB twice, with regard to first school maths, and to second scientific maths. In a previous study by Groß Ophoff et al. (in prep.), the adjective pairs in the adapted CAEB-version could be classified into the factors “Texture/ Certainty”, “Variability”, and “Simplicity”.

Knowledge in the domain of "mathematics as a school subject" is:

		1	2	3	4	5	6	7	
1	simple	❑	❑	❑	❑	❑	❑	❑	complex
2	stable	❑	❑	❑	❑	❑	❑	❑	unstable
3	dynamic	❑	❑	❑	❑	❑	❑	❑	static

Figure 1. Excerpt of the adapted CAEB-questionnaire to survey CEB regarding school maths and alternatively regarding scientific maths.

The first factor, “Certainty/ Texture”, is a mixture of concepts about the nature of knowledge and about the nature of knowing operationalizing EB about the accuracy and safeguard of knowledge. The factor “Variability” represents beliefs about the stability and dynamics of knowledge (see Figure 1: the adjective pairs “stable vs. unstable” as well as “dynamic vs. static”). According to the analysis of CEB about the knowledge in Educational Sciences compared to Mathematics (as a common subject in teacher education) in a previous project, the originally proposed dimension Simplicity (Stahl & Bromme, 2007) could also be identified (Groß Ophoff et al., in prep.; see Figure 1: the adjective pair “simple vs. complex”). Therefore, the same psychometric structure, that had been validated via confirmatory factor analysis ($\chi^2 = 423.673$; $df = 239$; $\chi^2/df = 1.8$; CFI = .048; RMSEA = .918), was applied in this analysis: A multifactorial variance analysis was conducted for the self-reported beliefs about the “Certainty/Texture”, “Simplicity”, and “Variability” in the two separate domains school maths and scientific maths as dependent variables. The ratings about school maths and scientific maths were treated as repeated measurements as they were surveyed with parallel questionnaires at the same time of measurement. Furthermore, the three study programmes (Bachelor of Science, Mathematics for teaching at the upper secondary school, Mathematics for teaching at other German school forms) were used as independent variables. Academic progress was included as a covariate.

5 Results

Looking at the total sample (i.e., students of different study programmes and different semesters), school maths and scientific maths are perceived as two separate domains of knowledge (see Table 1; cf. Schreck et al., 2023). The largest discrepancy is found with the assessment of the factor “Simplicity” of knowledge. Correspondingly, school maths is judged to be significantly simpler (mean value (MV) 3.41 vs. 6.4) and more superficial (MV 3.52 vs. 6.23) than scientific maths. Judgments about school maths and scientific maths also slightly differ regarding the certainty, acceptance, precision, and confirmability of knowledge. Scientific maths is judged less certain (MV 3.26 vs. 2.8), less stable (MV 3.6 vs. 2.96), and more disputed (MV 3.48 vs. 2.83) than school maths, whereas scientific math is more precise (MV 2.41 vs. 3.07) and better confirmable (MV 2.28 vs. 2.65).

Table 1. Mean values of the item-ratings in the semantic differential of the CAEB-questionnaire. A wide cross section of all students that were surveyed in 2017, 2018, 2019. The adjective pairs are clustered into the three factors “Certainty/ Texture”, “Simplicity” and “Variability”.

Factor	**Likert-scale rating options**		**School maths**	**Scientific maths**
	1 vs. 7		**Mean (Standard Error)**	
1. Texture/ Certainty	stable	unstable	2.96 (0.034)	3.6 (0.045)
	confirmable	unconfirmable	2.65 (0.039)	2.28 (0.036)
	exact	vague	3.1 (0.035)	2.86 (0.044)
	absolute	relative	3.55 (0.034)	3.4 (0.04)
	precise	imprecise	3.07 (0.035)	2.41 (0.033)
	definite	ambiguous	2.6 (0.031)	2.92 (0.038)
	accepted	disputed	2.83 (0.037)	3.48 (0.04)
	certain	uncertain	2.8 (0.032)	3.26 (0.036)
2. Simplicity	simple	complex	3.41 (0037)	6.4 (0.026)
	superficial	profound	3.52 (0.039)	6.23 (0.03)
3. Variability	dynamic	static	4.56 (0036)	3.82 (0.044)
	flexible	inflexible	4.51 (0.36)	4.29 (0.041)

With regard to their study programmes, the students were sorted into three groups (students of the Bachelor of Science degree, pre-service teachers for upper secondary school, other pre-service teachers including pre-service teachers for lower secondary schools and for primary schools) to analyse study group specific CEB as well (cf. Schreck et al., 2023). Regarding school maths (see Figure 2), students aiming at the Bachelor of Science degree find school maths less confirmable (MV 3.04) and slightly vaguer (MV 3.38) while at same time more static (MV 4.75) than pre-service teachers of mathematics (see Figure 2). Pre-service teachers for upper secondary school judge school maths to be quite certain with the lowest rating of all students (MV 2.58), as well as well accepted (MV 2.63). They take fairly similar views on the stability (MV 2.68 vs. 2.8), flexibility (MV 4.4 vs. 4.32) and precision of mathematical knowledge (MV 3.26 vs. 3.39) at school as students enrolled in the “Bachelor of Science” study programme, whereas sharing similar judgements about the exactness (MV 3.02 vs.

3.02), confirmability (MV 2.72 vs. 2.47) and the dynamic nature (MV 4.47 vs. 4.5) of mathematics at school with students of other teacher training programmes. On the contrary, these latter pre-service teachers judge school maths most unstable (MV 3.13), confirmable (MV 2.47), disputed (MV 3.0) and uncertain (MV 2.93) but at the same time most inflexible (MV 4.68) of all study groups. All student groups rate the absoluteness (MV 3.61 vs. 3.46 vs. 3.56) and definiteness (MV 2.68 vs. 2.49 vs. 2.59) of school maths nearly the same while differing on the factor "Simplicity of knowledge":

Students of the Bachelor of Science degree rate school maths to be the easiest (MV 2.54) and most superficial (MV 2.73), followed by the pre-service teachers for upper secondary school who rate it second easiest (MV 2.97) and second most superficial (3.28). Other preservice teachers cannot really decide on judging it rather easy or rather complex (MV 4.01), rather superficial or rather profound (MV 3.98).

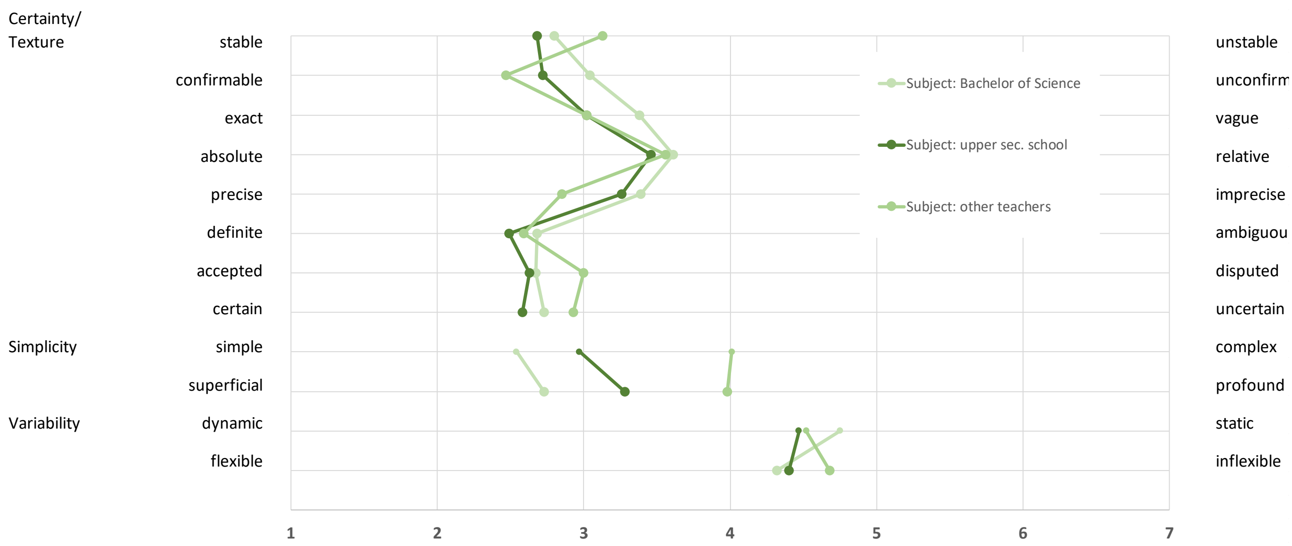

Figure 2. Item-rating "mathematics as a school subject" in the semantic differential of the CAEB-questionnaire. Ratings grouped by study programmes.

The judgements about scientific maths tend in the same direction regardless the students' study programme and the considered item-factor (see Figure 3; cf. Schreck et al., 2023). And yet, students of the study programme "Bachelor of Science" rate items regarding the factors "Certainty/ Texture" and "Variability" the lowest, which means that they judge scientific maths the most stable (MV 3.34), precise (MV 2.27), confirmable (MV 2.12), accepted (MV 3.18), certain (MV 3), dynamic nature (MV 3.56), and the most flexible (MV 4.05). The students of all study groups find scientific maths quite complex (e.g., Bachelor of Science MV 6.38) and profound (e.g., Bachelor of Science MV 6.24). Pre-service teachers apart from the pre-service teachers for upper secondary school are somewhat doubtful of the validity of mathematical knowledge at university (item “accepted vs. disputed” MV 3.68).

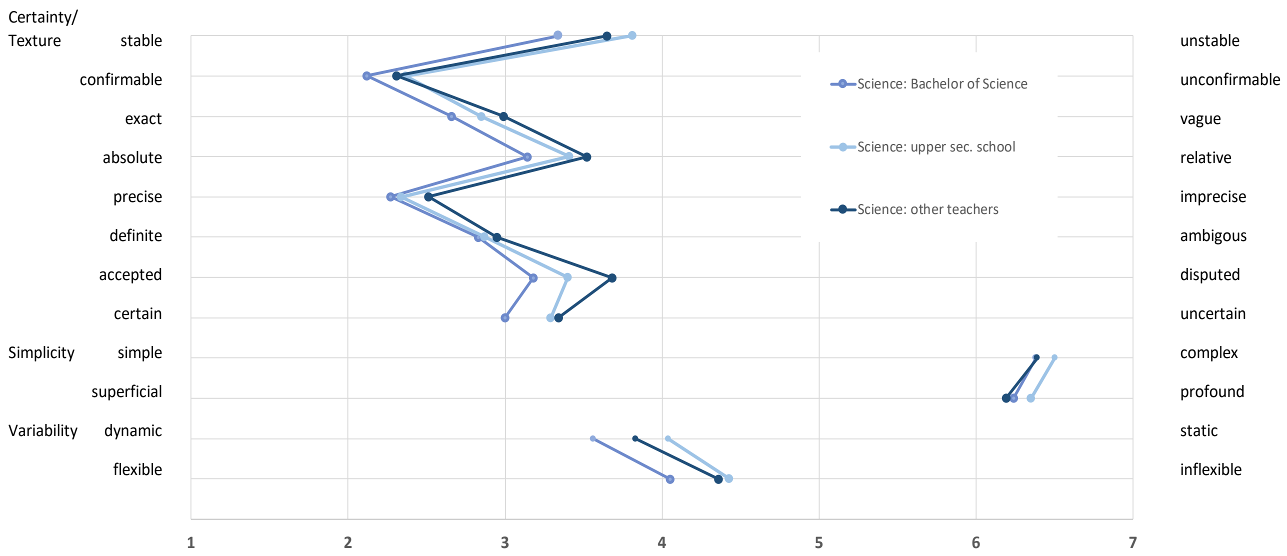

Figure 3. Item-rating “mathematics as a scientific discipline” in the semantic differential of the CAEB-questionnaire. Ratings grouped by study programmes.

To trace possible developments during bachelor’s degree, the participants—regardless of their programmes of study—were sorted into three groups: 1st and 2nd semester, 3rd and 4th semester, as well as 5th and 6th semester (students with a higher semester count were discarded for this analysis) (N=1493; cf. Schreck et al., 2023). The data do not suggest great change in connotative judgements about school maths within the first three years of university studies (see Figure 4). The greatest shifts occur regarding the factor “Simplicity” whereby school maths is rated to be slightly simpler (MV 3.2 vs. 3.37) and more superficial (MV 3.28 vs. 3.74) by 5th and 6th semester students compared to 1st and 2nd semester students.

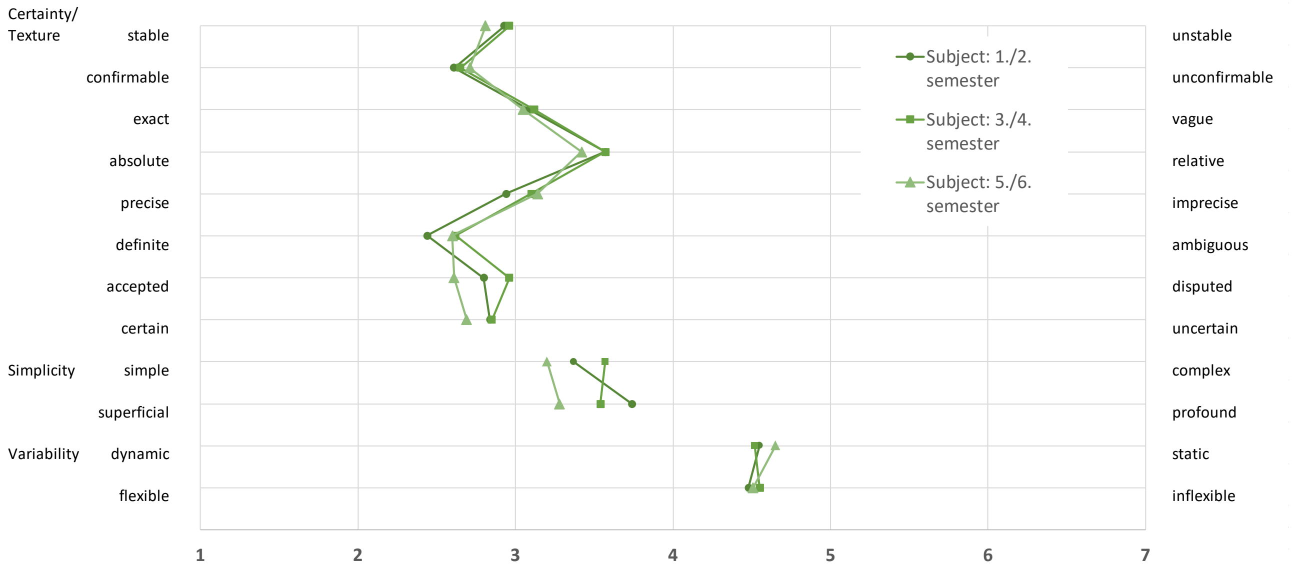

Figure 4. Item-ratings on school maths in the semantic differential of the CAEB-questionnaire. Ratings grouped by semester of study.

Nearly the same is true for the ratings on scientific maths, given by students of the three groups in different semesters (see Figure 5; cf. Schreck et al., 2023): The connotative judgements about mathematics at university shift surprisingly little during studies. The greatest change can be seen regarding judgements about the certainty and acceptance of mathematical knowledge. Thus, 5th and 6th semester perceive "mathematics at university" as more accepted (MV 3.35 vs. 3.54) and more certain (MV 3.09 vs. 3.39) than 1st and 2nd semester students.

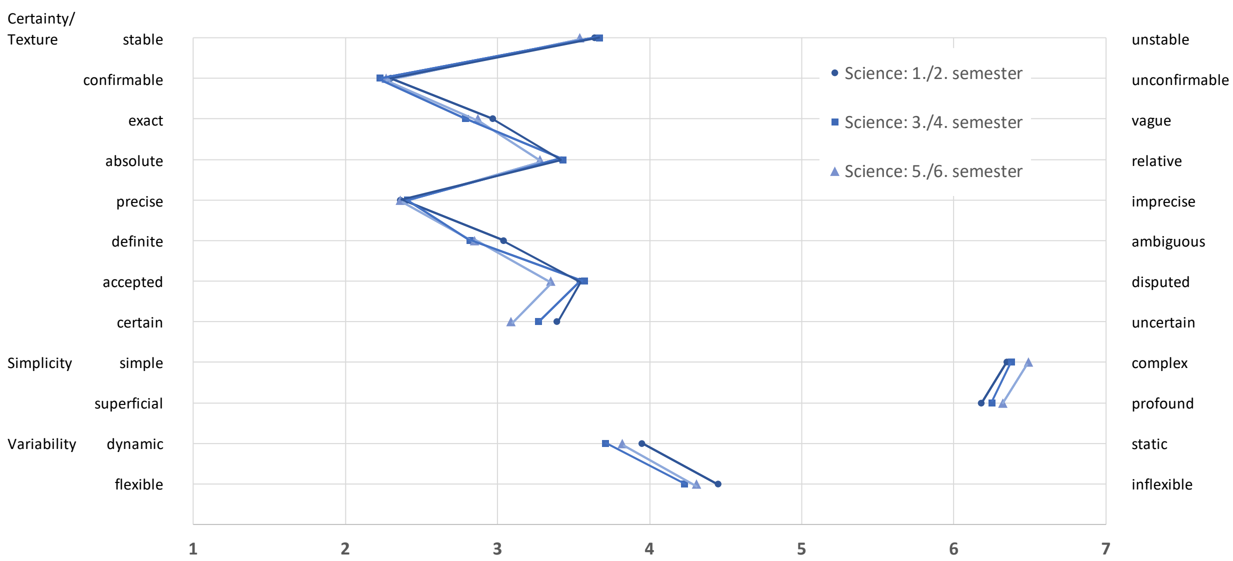

Figure 5. Item-ratings on scientific maths in the semantic differential of the CAEB-questionnaire. Ratings grouped by semester of study.

Table 2. The multi-dimensional variance analysis with repeated measures found statistically significant main effects for the self-reported beliefs ($F(1,1566) = 513.738$, $p \leq .05$; $\eta^2 = .396$; Wilk's $\Lambda = .604$), for the two domains school maths and scientific maths ($F(1,1566) = 571.572$, $p \leq .05$; $\eta^2 = .267$; Wilk's $\Lambda = .733$), and for the between-subjects factor "study programme" ($F(1,1566) = 176.853$, $p \leq .05$; $\eta^2 = .087$) (see Table 2).Item factors vs. programme of study vs. domain: The largest discrepancies concerning the two domains school maths and scientific maths can be found with the factor "Simplicity" regardless the programme of study. The judgements regarding the other two factors do not differ much.

Belief factor	study programme	mathematics as school subject MV and SD	mathematics as science MV and SD
Certainty/ Texture	Bachelor of Science	3,0 (0.9)	2,8 (1.2)
	Upper sec. school	2,9 (0.9)	3,0 (1.2)
	Other schools	2,9 (0.9)	3,1 (1.1)
Simplicity	Bachelor of Science	2,6 (1.2)	6,3 (1.0)
	Upper sec. school	3,1 (1.3)	6,4 (1.0)
	Other schools	4,0 (1.2)	6,3 (1.0)
Variability	Bachelor of Science	4,5 (1.3)	3,8 (1.4)
	Upper sec. school	4,4 (1.3)	4,2 (1.6)
	other schools	4,6 (1.2)	4,1 (1.4)

Furthermore, significant, but rather small interaction effects emerged for

- domain vs. programme of study ($F(2,1565) = 41.772$, $p \leq .05$; $\eta^2 = .051$; Wilk's $\Lambda = .949$)
- connotative judgements vs. programme of study ($F(4,3130) = 23.824$, $p \leq .05$; $\eta^2 = .030$; Wilk's $\Lambda = .942$)
- domain vs. connotative judgements ($F(2,1565) = 397.252$, $p \leq .05$; $\eta^2 = .337$; Wilk's $\Lambda = .663$)
- domain vs. connotative judgements vs. academic progress ($F(2,1565) = 5.4$, $p \leq .05$; $\eta^2 = .007$; Wilk's $\Lambda = .993$)
- domain vs. connotative judgements vs. programme of study ($F(4,3130) = 41.981$, $p \leq .05$; $\eta^2 = .051$; Wilk's $\Lambda = .901$)

No significant effects were identified for the covariate "academic progress" ($F(1,1566) = .001$, $p > .05$) or the interaction effects of CEB vs. academic progress ($F(2,1565) = .169$, $p > .05$; Wilk's $\Lambda = 1$) or domain vs. academic progress ($F(1,1566) = 3.392$, $p > .05$; Wilk's $\Lambda = .998$).

6 Discussion

We conclude about the nature of CEB that all students regardless of their study programme or academic progress hold different beliefs about school maths and scientific maths (cf. Schreck et al., 2023). Therefore, we assume that students perceive these domains as separate domains of knowledge. Especially, differing beliefs about the simplicity of knowledge in both domains (see Table 1 & 2) indicate the discrepancy. The causes of such a domain-sensitivity of specific CEB need to be discussed and further investigated: One plausible explanation for the domain-sensitivity of the belief factor "Simplicity of knowledge" in the present study is that the given research design esp. responding to the questionnaire twice successively – once with school maths and once with scientific in mind –, might have enhanced a contrast effect in respect of the ratings on the simplicity of mathematical knowledge in the two domains. One practical way to mitigate such a probable contrast effect of the two questionnaires might be to ask the students to respond to the questionnaire twice separately – regarding school maths and scientific maths – with larger time lags in-between. At least, students *of different study programmes* particularly judge the simplicity of mathematical knowledge at school differently (see Figure 2) which could reflect a selection effect taking place with the choice of the study programme at beginning of studies. That means that students who are enrolled in the mathematically more demanding study programmes assumably found mathematics at school to be comparably simpler than their classmates having a natural affinity for mathematics. Another reason might be that pre-service teachers are more preoccupied with learners' difficulties with school maths as they learn about those difficulties in practical training sessions at school and in the field of didactics during their studies.

Looking at the development of beliefs, CEB about school maths and scientific maths prove to remain relatively stable within the domains respectively throughout the course of the participants' bachelor's programme (see Figures 4 & 45) which means particularly that the anticipated shift in beliefs about scientific maths and school maths did not occur over time. Therefore, we assume that the CEB of the students were quite resilient to the students' current social, emotional context or surrounding environment at the given times of measurement. Besides, considering environmental stimuli for belief change, the little shift in judgements, e.g., about the belief factor "Variability", might result from little to none discourse about mathematics at the boundaries of knowledge during school education as well in the first academic years. Accordingly, Ross and Bruce (2005) claim that there must be great environmental stimulus to induce change in beliefs over time, at least in terms of pre-service teachers.

Appropriately, besides the shown temporal stability of CEB, it would be interesting and useful to investigate the stability of *denotative* aspects of epistemological beliefs as well, to examine whether and in which manner structural aspects of beliefs contribute to the claimed simultaneous maintenance of stable and flexible beliefs, for example.

Unlike school maths, scientific maths is rated to be highly complex and profound by students of all semesters and study programmes (see Figures 3 & 35). These divergent CEB regarding scientific maths might result from the discrepancies between mathematics teaching in school and at university (see Introduction). Thus, difficulties with the subject matter at university and the corresponding CEB about the simplicity of scientific maths may arise from the fast pace of progression in lectures and seminars, the huge amount of study matter, greater complexity of the subject matter, the high level of abstraction of advanced mathematics, the continuous demand for rigor and proof in lectures and seminars, the students' own responsibility for their learning progress, necessary skills regarding self-organisation and time management, etc.

A limitation of this study is that due to the CEB, a new aspect of EB in educational research, comparisons to previous studies on beliefs from the respective literature might fall short.

Finally, even though data was gathered in three consecutive years, the study at hand is not a longitudinal study in the narrow sense, i. e. tracing the EB of individual students from the 1st to the 3rd to the 5th semester of their bachelor's degree. Instead, we use a pseudo-longitudinal or panel approach, to have a large enough number of participants to interpret the quantitative data. The analysis of the actual longitudinal data (cf. Schreck et al., 2023) confirmed the results described above: 1) different judgement of school maths and scientific maths in general, 2) different judgements by different study groups especially regarding the simplicity of school maths, 3) domain-wise stability of the judgements over time.

Note

This article has a slight overlap with the article "Studying mathematics at university level: a sequential cohort study for investigating connotative aspects of epistemological beliefs" published in the International Journal of Mathematical Education in

Science and Technology (2023), as both report on the same project..

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