A New Method For Solving Electrostatics Problems

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ABSTRACT

In this article we show that the electrostatic field intensity of a uniformly charged straight line equals that of the corresponding arc of a circle charged with the same linear density. This new method greatly simplifies the calculation of the electrostatic field of a system consisting of uniformly charged straight lines.

Keywords: Electrostatic field calculations, intensity, charged line and arc of circle

INTRODUCTION

In this paper, a method that simplifies the calculation of the electrostatic field intensity for a system of uniformly charged straight lines is described. The simplification is achieved by replacing the field created by a charged straight filament with the corresponding equivalent field of a charged circular arc. The method makes it easy to find the magnitude and direction of the electrostatic field intensity at a given point in the plane.

METHODOLOGY

Consider the problem of obtain the electric field Intensity in a point O_1 (Figure 1). A charged thread with a linear charge density $\lambda > 0$ consists of two parts: 3/4 circle (1) of radius R and a semi-infinite straight line (2). Required to determine the Intensity of electric field in the center of the circle, point O_1.

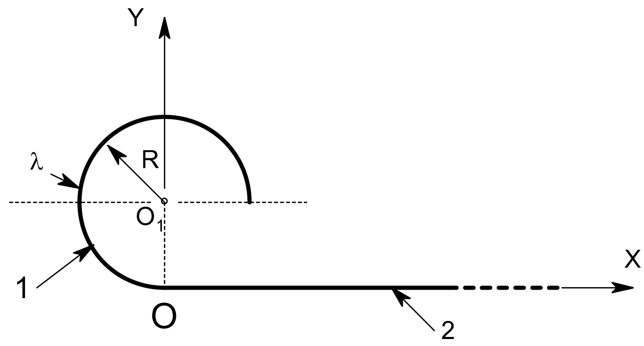

***Figure 1**. To The Calculation Of Semi-Infinite Charged Thread And A Quarter Of A Circle*

The standard solution obtained on the basis of the principle of superposition (sum of the fields of a circle and a semi-infinite line) and described in various textbooks gives: $\vec{E}_{O_1} = 0$ (Ohanian, 1985). It is easy to see that the same result can be obtained if, instead of the considered scheme in Fig. 1, we pass to a charged thread in the shape of full circle. In other words, the fields created by a semi-infinite charged thread and a quarter of a circle have the same intensity at point O_1. Moreover, it can be shown that the element dx of the straight thread creates the same field as the element dl of the circle, Fig. 2a. The electric field. Intensity generated by the element dx:

$$dE = \frac{\lambda \cdot \mathrm{Cos}^2 \varphi \cdot dx}{4\pi \cdot \varepsilon_0 \cdot R^2}, \tag{1}$$

Where,

$$x = R \cdot tg\varphi, \quad dx = \frac{R \cdot d\varphi}{\mathrm{Cos}^2 \varphi} \quad . \tag{2}$$

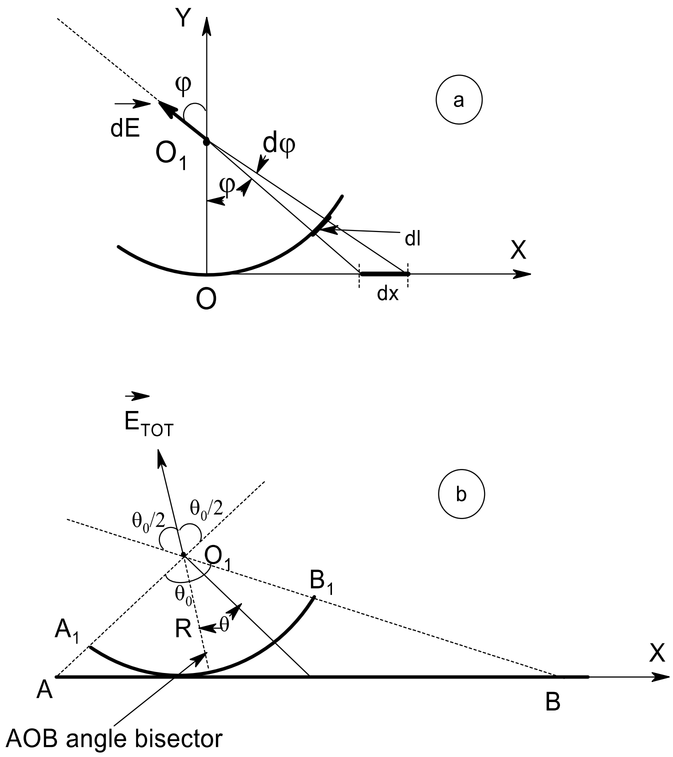

***Figure 2**. Calculation Schemes*

Substituting (2) in (1), we obtain:

$$dE = \frac{\lambda \cdot d\varphi}{4\pi\varepsilon_0 \cdot R}, \tag{3}$$

Intensity of the field created by the arc element dl is equal to the same value as in expression (3). Summarizing the result, we can say, that Intensity of the field, created by the charged segment AB at point O_1 is equal to the Intensity of the field, created by the corresponding charged arc A_1B_1. Due to the symmetry of the electric field with respect to the bisector of the angle AO_1B, Fig. 2b, resulting field Intensity at point O_1:

$$E_{TOT} = \frac{\lambda \cdot \mathrm{Sin}(\theta_0 / 2)}{2\pi\varepsilon_0 \cdot R} \tag{4}$$

The direction of the vector $\vec{E}_{TOT}$ coincides with the direction of the bisector of angle AO_1B formed by two rays, as shown in Figure 2b.

EXAMPLES

1. Semi-infinite charged thread, Figure 3.

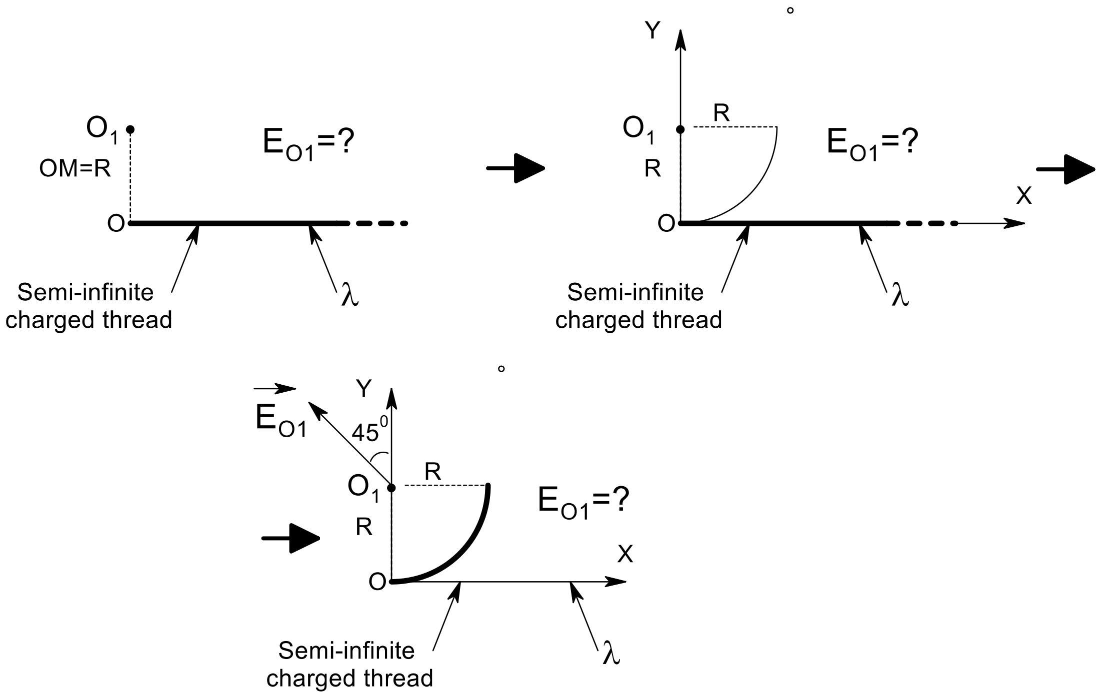

***Figure 3**. To Calculation Of Semi-Infinite Charged Thread*

$$E_{O_1} = E_{TOT} = \frac{\lambda \cdot \mathrm{Sin}(\pi / 4)}{2\pi\varepsilon_0 R} = \frac{\lambda \cdot \sqrt{2}}{4\pi\varepsilon_0 R} \tag{5}$$

2. Infinite charged thread

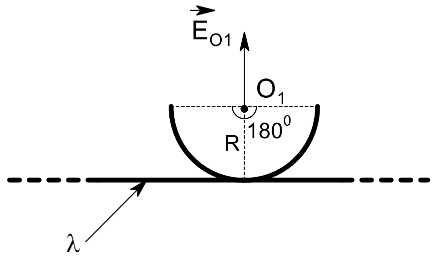

Figure 4. *Infinite Charged Thread*

$$E_{O_1} = E_{TOT} = \frac{\lambda \cdot \mathrm{Sin}(\pi/2)}{2\pi\varepsilon_0 R} = \frac{\lambda}{2\pi\varepsilon_0 R} \tag{6}$$

3. Two semi-infinite mutually perpendicular charged thread.

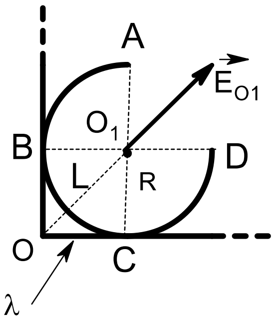

Figure 5. *Semi-Infinite Charged Thread*

$$\vec{E}_{O_1} = \vec{E}_{\cap AB} + \vec{E}_{\cap BC} + \vec{E}_{\cap CD} = \vec{E}_{\cap BC} \tag{7}$$

$$E_{O_1} = E_{TOT} = \frac{\lambda \cdot \mathrm{Sin}(\pi/4)}{2\pi\varepsilon_0 R} = \frac{\lambda}{2\pi\varepsilon_0 L} (V/m) \tag{8}$$

4. Two semi-infinite charged thread with arbitrary angle between them.

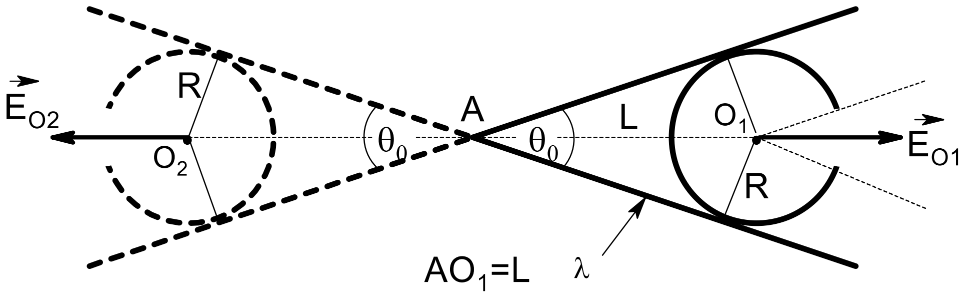

Figure 6. *Semi-Infinite Charged Thread With Arbitrary Angle Between The*m

$$E_{O_1} = E_{TOT} = \frac{\lambda}{2\pi\varepsilon_0 R}\mathrm{Sin}(\theta_0/2) = E_{O2} \tag{9}$$

Interestingly, for a given value of L, the result does not depend on the angle θ_0.

5. Field on the continuation of the charged segment.

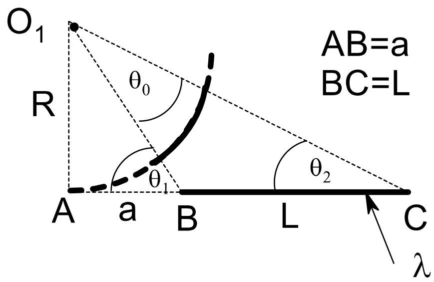

Figure 7. *Field On The Continuation Of The Charged Segment*

$$E_{O_1} = E_{TOT} = \frac{\lambda}{2\pi\varepsilon_0 R}\mathrm{Sin}(\theta_0/2) = \frac{\lambda}{2\pi\varepsilon_0 R}\mathrm{Sin}(\frac{\theta_1-\theta_2}{2})\,(V/m) \tag{10}$$

$$E_A = \frac{\lambda}{2\pi\varepsilon_0} \lim_{\substack{R\to 0\\ \theta_1\to 0\\ \theta_2\to 0}} \frac{1}{R}\mathrm{Sin}\left(\frac{\theta_1 - \theta_2}{2}\right) = \frac{\lambda}{4\pi\varepsilon_0} \lim_{\substack{R\to 0\\ \theta_1\to 0\\ \theta_2\to 0}} \frac{\theta_1 - \theta_2}{R} =$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \lim_{\substack{R\to 0\\ \theta_1\to 0\\ \theta_2\to 0}} \frac{1}{R}(\mathrm{Sin}\theta_1 - \mathrm{Sin}\theta_2) = \frac{\lambda}{4\pi\varepsilon_0} \lim_{\substack{R\to 0\\ \theta_1\to 0\\ \theta_2\to 0}} \frac{1}{R}(tg\theta_1 - tg\theta_2) = \qquad (11)$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \lim_{R\to 0} \frac{1}{R}\left(\frac{R}{a} - \frac{R}{L+a}\right)$$

Finally:

$q = \lambda \cdot L$.

$$E_A = \frac{\lambda}{4\pi\varepsilon_0} \cdot \frac{L}{a(L+a)} \quad , \qquad (12)$$

CONCLUSION

From the given description and examples, it is seen that the proposed method for calculating the intensity of the electrostatic field created by charged straight filaments in a plane is significantly simpler than the standard method for solving such problems. The results of this work will be useful for teachers and students of courses such as electricity and magnetism.

REFERENCES

Ohanian H.C., (1985). *Physics*, W.W. Norton and Comp.: New York.