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Research

Number system conversions in spreadsheets for vocational school students: A case study from instrumental genesis

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Abstract:

Theoretical frameworks for mathematics teaching and learning technology-supported provide a systematic structure in examining the contribution of the tool to conceptual development. This study examines the processes for the use of spreadsheets and the mathematical development of the participants in the tasks for performing the conversions between number systems using the instrumental genesis approach, which deals with transforming a tool into an instrument that will contribute to the conceptual development. In the study, the screen images of the worksheets of the participants, who are at the Department of Computer Technologies Program in a vocational school in Turkey, are analyzed together with the observation notes and evaluation scales prepared based on the outcomes. In the study, while the efforts of the participants to transform the spreadsheet into an instrument are observed, it is seen that their habits of paper-and-pencil experiences and misconceptions lead to an obstacle to transferring their operations to the spreadsheet and hesitations. However, their developments in instrumentalization processes are reflected by the following: they use subjective usage schemes, realize the advantages of spreadsheet functions and features, and create dynamic worksheets through dragging and cell addresses. Nevertheless, it can be stated that the reflections of instrumentalization progress on the instrumentation processes for conceptual development are limited.

Keywords: Instrumental genesis, vocational schools, number systems, spreadsheets, a case studies, content analysis

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INTRODUCTION

Mathematics education is one of the main areas in education that aims for students to gain basic skills for the use of technology for the necessity of the digital age. It is realized that the potential of the use of technology in mathematics education and its contribution to the development of students' mathematical thinking skills (NCTM, 2015). For this reason, there are many studies on the integration of technology into education (Artigue, 2002; Balacheff & Kaput, 1996; Hoyles & Lagrange, 2010; Miles, 2021).

It is stated that technology-supported teaching processes in mathematics education have a complex structure. (Miles, 2021; Drijvers et al., 2010). The prior studies on this subject refer to the opportunities for students to spend more time and focus on the concepts thanks to the convenience of technology (Artigue, 2002). Despite this, some following studies state that didactical obstacles arise by ignoring the relationship between using technology and conceptual understanding. It is notified that there is a tight dialectic between the technical use of tools and conceptual understanding in learning environments in which technological tools are used. Related studies examine the relationships among theoretical frameworks, digital tools, and mathematical backgrounds. Instrumental genesis is one of the main theories used in this field (Hoyles & Lagrange, 2010). The instrumental genesis approach, which deals with the use of technology in mathematics education through the techniques used and conceptual developments, is the theoretical framework of this study (Drijvers et al., 2010; Trouche, 2004).

Spreadsheet as a technology used in various areas of life is one of the benefits of mathematics education with the convenience it provides in terms of presentation, manipulation and recording of data (Abramovich et al., 2019; Bakos, 2022; Marley-Payne & Dituri, 2019). This study examines converting performance on the number systems of computer programing students in a vocational school from the perspective of the instrumental formation.

Mathematics Education and Spreadsheets

Spreadsheets can be defined as an office program that allows the entry of several data and consists of worksheets with cells used for organizing, analyzing, and storing data (Mays, 2015). Using the cell address when creating the formula for the operations to be performed in Spreadsheets allows monitoring the changes of the cells dynamized. This situation contributes to mathematical thinking by creating, manipulating and monitoring the operations with the numbers it contains instantly (Baker & Sugden, 2015). It leads individuals to think algebraic as well as follow change instead of dealing with operations and thinking arithmetic (Ainley et al., 2005; Haspekian, 2014). Sutherland (2007) describes spreadsheets as a new way to mathematical understanding beyond calculations.

Spreadsheets offer the opportunity to update all formulas for each change rather than programing or coding for each function or command. It provides a significant advantage in terms of saving time to monitor changes (Drier, 2001; Sanford, 2018). Students' experience with spreadsheets highlights the feature of being a tool for developing new specific solutions for mathematical concepts. Spreadsheets have the edge over calculators in terms of enabling multiple calculations and dynamiting operations on paper and pencil (Jalbert & Jalbert, 2019). Spreadsheets are important as a learning environment, especially since they enable abstract concepts to be handled in concrete ways with multiple representations such as graphical, numerical, and algebraic (Abramovich et al., 2019; Ainley et al., 2005; Castle, 2021). In addition, spreadsheets give the green light to analytical and in-depth thinking processes for transferring mathematical concepts to a technological environment with the opportunity to design learning environments (Beigie, 2017; Caglayan, 2017; Ozdemir Erdogan & Turan, 2014).

Number System Conversions

Numbers are mathematical objects that have diversified and developed from the first ages to this day with their representations and writing systems. Today, although this representation is usually based on the decimal system, it may also be performed in the binary, hexadecimal number system or with Roman numerals. The sequence of symbols used to represent numbers may correspond to different numbers in number systems. For example, the number of 5 (five) products of the same kind is symbolized as 5 in the decimal system and as 101 in the binary system. It is considerable to use these representations and convert them between them in mathematics.

The binary system is introduced as a different language by those who are interested in computer sciences, which Stewart (2009) describes as a privilege and "there are 10 kinds of people in the world: those who understand binary numerals and those who do not". It is expected that students deal with the conversions between the decimal system and binary systems (4, 8, 16, etc.), the relations between the number systems and the numbers they contain, the arithmetic-algebraic structures, and patterns among numbers. In addition, it is important that the results of division operations and exponential notations rely on the conversions of number systems (*Figure 1*).

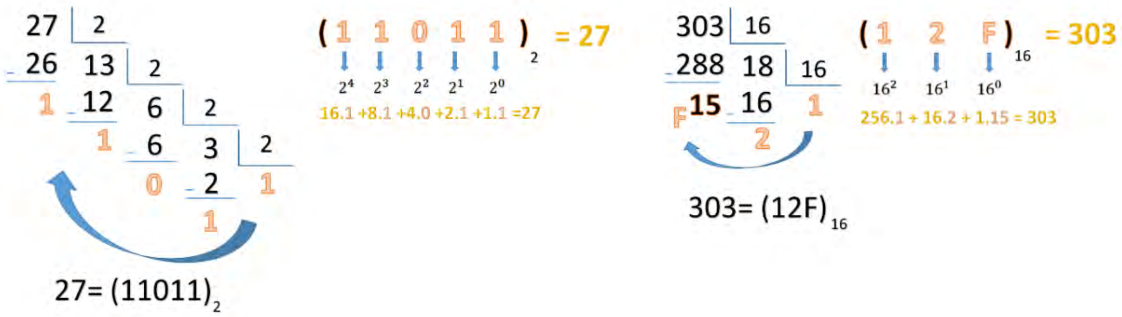


Figure 1. Examples for conversions of number systems

Although students have some difficulties in number systems, in a study with pre-service teachers, it is noticed that the relations between the exponential representations of numbers and the binary system support each other (Melkonian, 2019). In general, studies conducted using a learning environment on technology focus on the skills and motivation for the use of technology as well as try to examine the difficulties experienced by the students in the binary system.

Theoretical Framework: Instrumental Genesis

In a learning environment supported by technology, learners should make sense of the conceptual background of the techniques they use to display their conceptual development. Instrumental genesis examines the relationship between the tool and the learning process, the techniques developed by the learners in the use of tools, and their conceptual development (Trouche, 2004). Instrumental genesis consists of the following dimensions: instrumentalization, instrumentation, tool(artifact)-instrument, and schemes.

While an artifact(tool) is a material or abstract structure to be used for a task; an instrument is a subjective process developed on this tool by the learner. A schema is a solution created through an instrument (Drijvers & Trouche, 2008). There are two dimensions of instrumental genesis: instrumentalization expresses the usage schemes that contribute to the transformation of the tool into an instrument, and instrumentation emphasizes the conceptual schemes formed during the task (*Figure 2*).

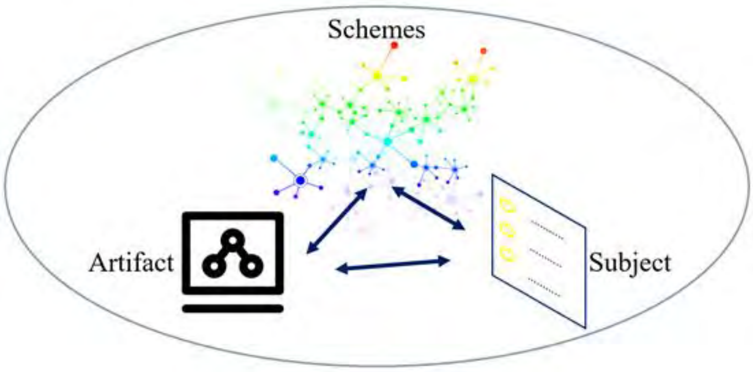


Figure 2. The Conversion of a Tool to an Instrument (Drijvers & Trouche, 2008)

Schemas can be revealed with the techniques formed by customizing the general features of the tool (Drijvers et al., 2013). Learners can transform the same tool into different instruments in terms of their experience. Therefore, it is important whether the tools are suitable for the task and the individual's background (Drijvers & Trouche, 2008). It is expected that the schemes that lead to the transformation of the tool into an instrument include the techniques preferred in the use of the tool and the mathematical background on which it is based. These schemes can be monitored using the processes of instrumentalization and instrumentation (*Figure 3*).

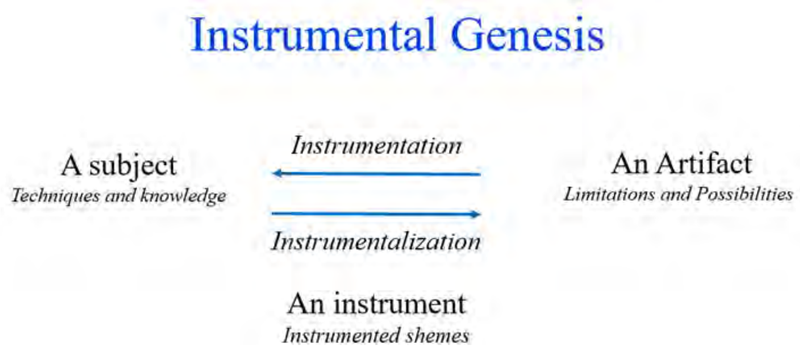


Figure 3. The processes of instrumental genesis (Drijvers & Trouche, 2008)

While *instrumentalization* is the learners' mental paths that lead to solutions by personalizing the instrument, *instrumentation* is the relationship between mathematical structure and its reflections on the formation of the instrument (Drijvers & Trouche, 2008). Instrumentation is a learning process that learners develop through the relationship of task and tool. Instrumentation is not a purely mathematical/conceptual development. It should be examined with instrumentalization together (Trouche, 2004). Goos et al., (2009); express conceptual schemes of geometric features created by dragging as an example of instrumentation, while they express the techniques of how to drag geometric structures in the dynamic geometry environment as instrumentalization. Gueudet and Trouche (2011) emphasize the necessity of determining the conceptual components of the task and the features used in the transformation of the tool into an instrument to reveal these processes.

Purpose of the research

This study examines converting performance on the number systems of computer programing students in a vocational school from the perspective of the instrumental formation. In our study, the use of spreadsheets by students of computer programing in a vocational school is examined from the perspective of the instrumental genesis approach within the framework of the tasks given to number systems. Based on this, we determine the research questions;

- What are the using schemes of the participants on spreadsheets?

- What are the reflections of the participants' use of spreadsheets in number system conversion tasks on conceptual understanding?

METHOD

Participants

In the study, the participants were determined as two girls and two boys, on a voluntary basis, among the second year students of the department of computer programing at a vocational school at a state university in Turkey. The participants encounter both spreadsheets and the binary number system in vocational courses and mathematics.

Data Collection Tools

Data collection tools of the research are the following: (1) screen recordings of students while performing the tasks, (2) audio recordings taken from the lessons, and (3) observation notes taken by the researcher. Relationships between audio recordings and students' actions and thoughts are tried to be revealed as well as supported by observation notes to increase the validity and reliability of the research data by using more than one data collection tool (Corbin & Strauss, 1990).

Data Analysis

In the research, the tasks to be presented to the students in the sessions consisting of 45 min per week, a total of four weeks were designed to appropriate instructional planning. Participants studied individually on the same spreadsheet program (Excel) in the same environment with different computers that had the same hardware. Explanations about the sessions and tasks followed in the research processes are given in *Table 1.*

Table 1

The feature of Seasons and Tasks

Seasons	**Tasks**	
Season 1	**Task 1:** transfers the process of finding the equivalent of a binary system of a two-digit number to the worksheet. Form a readable worksheet, describe the functions and features you use.	
	Key features of the task	Transferring and solving the task from papers to spreadsheets without interfering with students
Season 2	**Task 2:** finds the binary system equivalents of 40, 72, 99, and 2 in a worksheet. Use appropriate functions such as =base, =mode, =value, =concatenate, and =quotient. Explain the features of the spreadsheet you used and the process of creating the worksheet, along with the reasons.	
	Key features of the task	Discussion on functions and features of spreadsheets
Season 3	**Task 3:** presents the binary and hexadecimal equivalents of sequences 1, 2, 4... in a worksheet using appropriate functions and features of spreadsheets. Develop a format that will reveal the relationships between the numbers. If there are different results between the worksheet and the paper you formed, explain the reasons.	
	Key features of the task	Evaluation in terms of mathematical processes of the formed worksheets
Season 4	**Task 4:** presents the binary and hexadecimal equivalents of the sequence 1, 4, 16… in a worksheet using appropriate functions and features of spreadsheets. Develop a format that will reveal the relationships between the numbers. Explain how you use the functions and features with reasons.	
	Key features of the task	Evaluation in general of student development processes

The tasks were given to the participants in a written text. The answers given by the participants were not limited to screen recordings; they were recorded with audio recordings and observation notes during the tasks.

Content analysis was used in the study (Corbin & Strauss, 1990). The content analysis from the perspective of instrumental genesis focuses on the processes of instrumentalization and instrumentation. While the themes given in *Table 2* in terms of functions and properties of spreadsheets for instrumentalization were discussed in Table 3 regarding the instrumentation process. These content analysis themes reveal the subjects' schemes defined within the framework of the instrumental genesis.

Table 2

Processes on Participants' Use Spreadsheets (Instrumentalization)

Themes	Tasks and outcomes on the instrument
Task 1: Examining preliminary knowledge about number systems	
1. Readability	a. Use worksheets effectively. b. Prepare readable worksheets. c. Fix the columns and rows for base, quotient, divisor,, and reminder.
2. Skill on spreadsheets	a. Use cell addresses along with =value, =mode, =concatenate, =base excluding dragging.
3. Skills on Office programs	a. Realize the arrangements of columns, rows and cells.
Task 2: Examining the functions of spreadsheets with their features by conceptual learning	
1. Dynamize cells	a. Dynamize cells for manipulations.
2. Use cell addresses	a. Type cell addresses correctly. b. Use right cell addresses for functions, use the syntax correctly.
3. Type functions	a. Use functions of =quotient, =mode, =concatenate, =value and =base effectively.
4. Dragging	a. Realize vertical and horizontal dragging using fixing ($). b. Realize that fixing ($) aims to perform more operations with minimum typing.
5. Skills on Office programs	a. Use the features of spreadsheets. b. Check the results relied on cell addresses.
6. Dynamize	a. Dynamize the cells for multiple manipulations.
Task 3: Examining the conversions of bases on multiple manipulations	
1. Readability	a. Form a readable worksheet that indicates comparisons of number systems.
2. Dragging	a. Drag and drop for multiple manipulations.
3. Dynamize	a. Dynamize the worksheet for multiple manipulations.
Task 4: Examining knowledge and skills on number systems using spreadsheets	
1. Readability	a. Prepare a readable worksheet.
2. Use functions and developing	a. Realize that the functions of =quotient and =mode should be dragged separately. b. Realize the integration of the functions =quotient, =mode, =base, =concatenate, and =value.
3. Dragging	a. Use dragging by patterns and functions, avoid manual typing.
4. Use cell addresses	a. Prefer the cell addresses and explain the reasons why cell addresses are used.

Table 3

Conceptual Processes on Conversions of Number Systems (Instrumentation)

Themes	Tasks and outcomes on mathematical processes
Task 1: Examining preliminary knowledge about number systems	
1. On the number systems	a. Reflect the task given on a paper on the spreadsheet by using pre-knowledge on number systems.
2. The purpose of mathematical use of functions	a. Realize the reason for the use of functions. b. Check the operations on the paper through =base. c. Explain the relations between spreadsheets and number systems (typing reverse, removing zeros).
Task 2: Examining the functions of spreadsheets with their features by conceptual learning	
1. Place value	a. Express the effect of zeros on place value by =concatenate and =value.
2. The relation between mode-remainder	a. Use =mode in terms of its mathematical aspect (remainder).
3. The relation between conversions of number systems and division operations	a. Realize the status of the divisor, quotient, dividend, and reminder for the conversions.
4. The relation between number systems and spreadsheet functions	a. Realize the mistakes in the use of functions and features of spreadsheets by checking results.
Task 3: Examining the conversions of bases on multiple manipulations	
1. The relations between place values and bases	a. Express the relation between bases and place values.
2. Patterns	a. Express the relation between bases and numbers in them.
3. The relation among the number systems	a. Realize the relationships between binary and hexadecimal systems on spreadsheets. b. Compare the results of conversions from binary and hexadecimal bases to decimal bases.
Task 4: Examining knowledge and skills on number systems using spreadsheets	
1. Patterns	a. Express the relation between bases and numbers in them by multiple manipulations. b. Realize the results and reasons of consecutive operations that are not appropriate for the pattern. c. Express the relations among bases, place values, and numbers.
2. Use of functions	a. Handle mathematical aspects of =quotient, =mode and =base on spreadsheets.
3. The relation among the number systems	a. Realize the status of zero for various number systems on spreadsheets.
4. Place value	a. Handle the effect of the status of zero for place values by =mode.

The data collected in the research were analyzed according to the determined themes and outcomes. It is presented through the notes on participants' progress prepared for each session and the evaluation scales developed for the first and last sessions.

Ethical considerations

In this study, all rules stated to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions stated under the title "Actions Against Scientific Research and Publication Ethics", which is the second part of the directive, were not taken.

Ethical review board name: Isparta Uygulamalı Bilimler Üniversitesi Bilimsel Araştırma ve Yayın Etik Kurulu

Date of ethics review decision: 16.11.2021

Ethics assessment document issue number: 72/01

RESULTS

The Processes of Using Spreadsheets

The processes of participants for the use of spreadsheets are addressed with four components: readability, using functions, cell addresses, dynamizing, and dragging.

Readability

In the first task, it is seen that participants try to transfer the structures they have built in the paper-and-pencil to spreadsheets instead of focusing on functions used for transformation in number systems. In *Figure 4*, it is seen that participants who can perform operations related to conversions in the number systems in a paper-and-pencil environment create more readable worksheets.

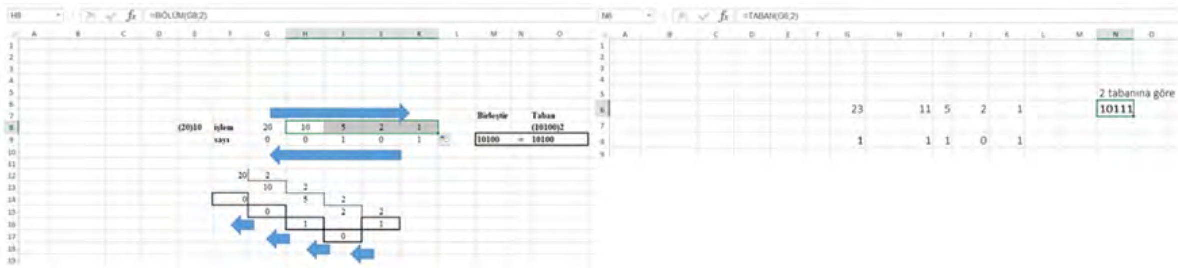


Figure 4. Task 1 in terms of readability by Duygu and Cenk

While participants without a good grasp of the conversion algorithm in the paper-pencil environment are able to use the =base function in the spreadsheet environment, they are hesitant about the accuracy of the results of the operations. The readability of their pages is low. Some participants' structures do not change in the following tasks. According to the findings, the readability of worksheets is more dependent on the location of the variables, the cell in which each value is involved in the operation.

One of the expectations in Task 1 is to ensure the readability of the worksheets while monitoring the changes by dynamizing the cells. However, it is seen that the lack of developing and dynamiting in spreadsheets, such as the inability to write numbers in reverse and not removing unnecessary zeros (0), are reflected in the readability of the worksheets (*Figure 5*).

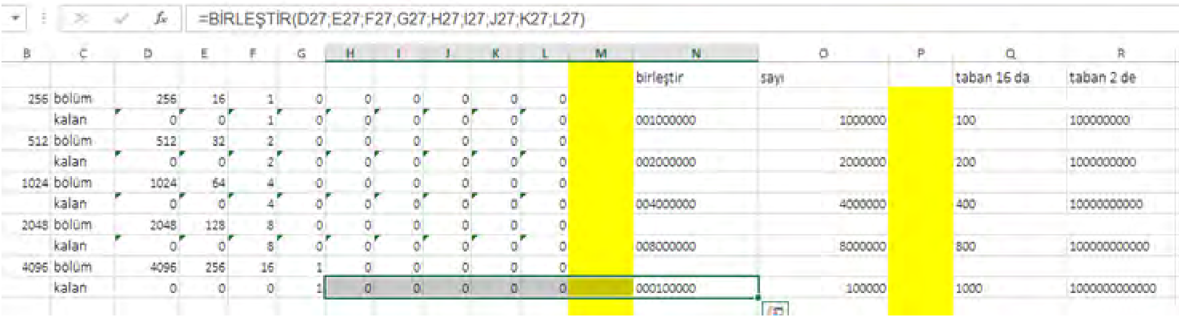


Figure 5. The status of zero (0) to the place values, Task 3 by Ada

Using functions

Depending on the conversion tasks in the number systems discussed in the research, the use of functions =quotient, =mode, =base, =value and =concatenate is more noticeable. It is seen that participants prefer general functions such as =base instead of specific functions such as =Dec2bin. *Figure 6* indicates which function participants prefer in which seasons.

	=quotient				=mode				=base				=value				=concatenate			
	S1	S2	S3	S4	S1	S2	S3	S4	S1	S2	S3	S4	S1	S2	S3	S4	S1	S2	S3	S4
A	√	√	√	√	√	√	√	√	√	√	√	√		√	√		√	√	√	√
B	√	√		√		√		√	√	√	√	√			√			√	√	√
C			√	√			√	√	√	√	√	√		√	√	√		√	√	√
D	√	√	√	√		√	√	√	√	√	√	√		√	√	√	√	√	√	√

√: realized ▇ : no run ▒ : not notified in the seasons

Figure 6. Analysis of students' use of function

Ada and Duygu are the participants who regularly use the functions in the tasks. While Ada does not use =value only in the last task, Duygu does not use =mod in the first task. Ada preferred manual entries in the last task, while Duygu, on the other hand, performed division operations manually using arithmetic symbols (=, /) without using a function in the first task. Cenk uses =base and =value regularly from the first tasks, while he especially starts to use =quotient, =mode and =concatenate throughout the process. Cenk expresses that he believes that the use of the relevant functions facilitates the operations as well as the operationsin this way.

In terms of using functions, Berk exhibits a different behavior compared with other participants throughout the research. He uses =base throughout all tasks. While he starts to use =concatenate from Task 2, he does not use the other functions regularly. Berk focuses on the results and does not consider other functions so that they are not necessary. He thinks that they only appeal to the visual. For this reason, he uses =base consistently because of its role in the results, contrary to other functions. Cenk and Berk continue to write their results manually instead of using =quotient and =mode. They declare that they do not need these functions because of the accuracy of their operations and they can perform manual entries more easily. However, participants who gain experience in these functions apply them more frequently in the following seasons because of their facilities. Nevertheless, it is observed that the participants have difficulty abandoning their usage habits and mistakes on the functions and features of spreadsheets.

4.Ada: Yeah, I think I had a mistake in =*concatenate* again. …I should have written it backward.

4.Berk: …because it is going to be wrong (He explains why he does not use cell address and dragging).
I can see more easily through this way (manually), but I am not sure about the results in the other (cell addresses).

Here, Cenk said he could not remove doubts about the use of spreadsheets and the accuracy of the results throughout the four-week period.

2.Cenk: …I am sure that my numbers are correct, I did =concatenate correctly, but it is different according to the result of =base. I do not understand where I have made the mistake.

4.Cenk: … I did not make a mistake in the operation, but this number is very large, is there a wrong thing?

It is seen that all participants showed improvement. While there are participants who do not prefer =mode, which is one of the basic functions for number systems, in Task 1 there are no participants who do not use any relevant functions in Task 4. From the first study, all participants used =base. Throughout the research, the participants constantly compare their own predictions with the results of the operations they obtain based on the functions on the spreadsheet. In this context, it is observed that some participants recognize that they have some misconceptions about the results after using the function.

Using cell addresses and dynamizing

It is important for the purposes of the research that cells become dynamic for calculations using cell addresses. *Table 4* indicates whether participants used cell addresses in the tasks.

Table 4

Participants using cell addresses in the tasks

Using cell addresses	Task 1	Task 2	Task 3	Task 4
Ada	✓	✓	✓	✓
Berk	-	✓	-	✓
Cenk	-	-	✓	✓
Duygu	✓	✓	✓	✓

While some participants who had problems writing cell addresses encountered false results, some could not reach any results (*Figure 7*).

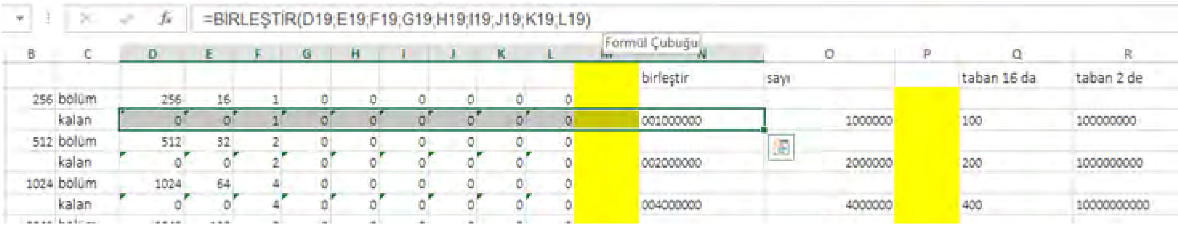


Figure 7. The screenshot on the mistakes to type the functions Task 3 by Ada

Ada and Duygu used their cell addresses from Task 1. On the other hand, some participants continue entry operations manually in Tasks 2 and 3 (Berk and Cenk). It is seen that participants who continue manual entries are deprived of multi-processing skills as well as not being able to follow or notice their mistakes (*Figure 8*).

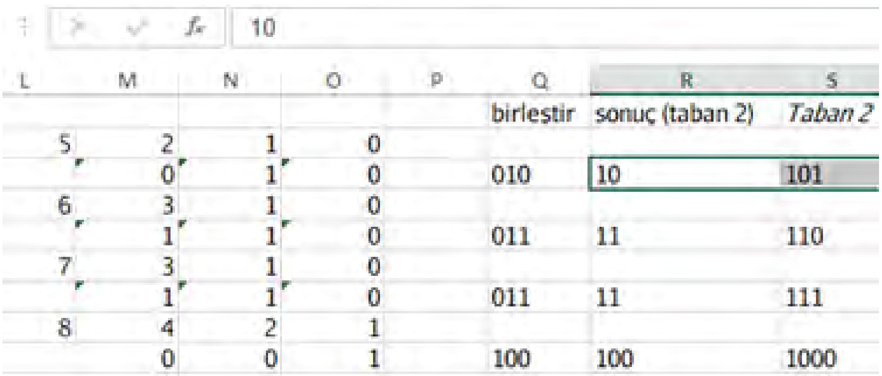


Figure 8. The screenshot of the reflections of manual typing Task 3 by Berk

In contrast, it is observed that the participants who are sure about the use of cell addresses leave their habit of controlling their operations on paper and pencil. In the last task, everyone uses cell addresses properly except Berk. While Ada and Duygu provide the dynamism of their worksheets, Berk and Cenk impair the dynamism of their worksheets by manually correcting the errors. In general, it is seen that using cell addresses, which is not preferred at the beginning, is preferred in the following tasks due to the convenience of dynamism and the necessity of performing multiple operations.

Dragging for multiple manipulating

the participants who do not prefer the use of functions and cell addresses could not realize dragging and fixing (*Table 5*).

Table 5

Participantsusing dragging in the tasks

Dragging	Task 1	Task 2	Task 3	Task 4
Ada	✓	✓	✓	✓
Berk	-	-	-	✓
Cenk	-	-	✓	✓
Duygu	✓	✓	✓	✓

In Task 1, there is no situation where dragging or fixing can be used. From Task 2, the participants start to benefit from dragging with the guidance of the researcher. However, the discussions among the participants show their hesitations about whether they are doing the dragging correctly or not.

> 2.Ada: I have used all the functions, that is right, but one zero are missing! Did I do the dragging wrong?

On the other hand, it is observed that some participants who use =mod and =base also tend to rewrite functions for each row instead of dragging.

> 2.Berk: …I wrote separately for each, nevertheless, there was a mistake…..

It is also seen that the participants bring their past experiences to the tasks. For example, Cenk prefers copy paste instead of dragging. As a reason, he expresses that he has no control over dragging and that he is not sure of the results.

> 2.Cenk: I used copy paste for the functions of =concatenate and =base because when I pulled it down (dragging) different numbers appeared. Sometimes it can be wrong…

However, Cenk benefits from dragging and fixing along with other functions in Task 3. Duygu benefits from dragging in Tasks 3 and 4 for cells and when entering powers of two (*Figure 9*).

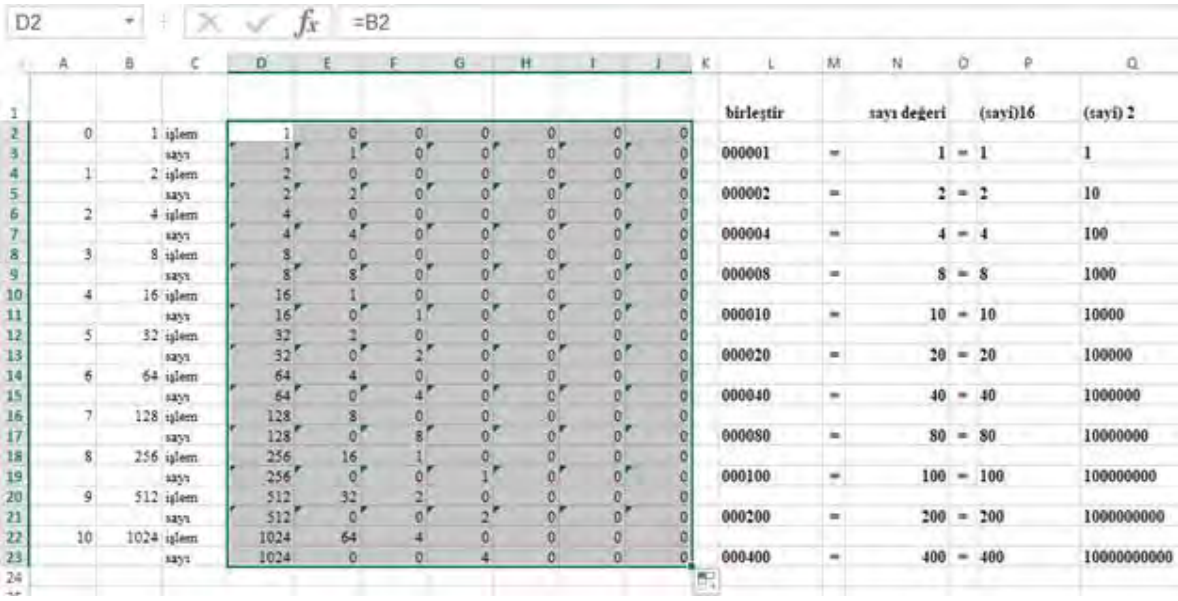


Figure 9. Examples of horizontal and vertical dragging Task 3 by Duygu

4.Duygu: …If I do not do that (talking about dragging both rows together) the process takes longer, I delete the in-between now (means the cells which are not related to the result)

Conceptual Processes Intended for Converting Number Systems

Examining the purpose of using functions and their relationships with the conversions of number systems

In Task 1, the schemes developed by the participants while transferring their paper-pencil operations to spreadsheets are examined. It is seen that Duygu can express operational reflections of functions she uses in spreadsheets.

1.Duygu: What we mean by the operation is division indicated =quotient in excel. =concatenate is used the remainders but in reverse. =base gives the result directly.

It can be said that =base is used to control the results the entire research. At the end of the tasks, participants realize that the functions of =quotient, =mode, =concatenate are reflections of the steps of the division process used in number systems. However, it is seen that they confuse the purpose of use of the functions for the operations in the number systems.

1.Ada: Here is already = quotient, we are using =mode for the remainder, we also control it through =base, but the numbers seem reversed here, I think I made a mistake in =concatenate.

2.Berk: Yes, but I do not understand why it happens. (for the same results of 6. and 7.) I wonder if I have mistyped =base?

3.Ada: …yes I noticed the mistake (did not write backward)

Ada expresses the reason why the remainders are written in reverse, but she thinks the error relies on the use of =concatenate. In contrast, Berk and Cenk could not say that they should write the remainder in reverse. So they think that the error relies on the use of =base. In general, participants miss the use of the remainder but realize the error when checking the result by =base.

2.Ada: I have used all the functions, that is right, but one zero are missing! Did I do the dragging wrong?

…or do I use the wrong number (the cell)? Because they all have the same mistake (no last digit), =concatenate could not be wrong, I wrote it correctly.

3.Ada: …Yes, I noticed the mistake (did not write =concatenate in reverse. It was reflected in the number value) … it is clear from the base what the number is.

Ada sees the same error in all results and has a correct idea of the source of the error. In this process, Ada handles the results in spreadsheets with together the conversion stages she does in a paper-and-pencil. Since Ada does not start the =mode from the previous number (cell) in Task 2, she reaches improper operations and results. However, she corrects this false in Task 3. She prefers manual entries for multiples of two, but she makes a mistake in the results due to the position of zero in the digits because she uses =value and =concatenate together (*Figure 10*).

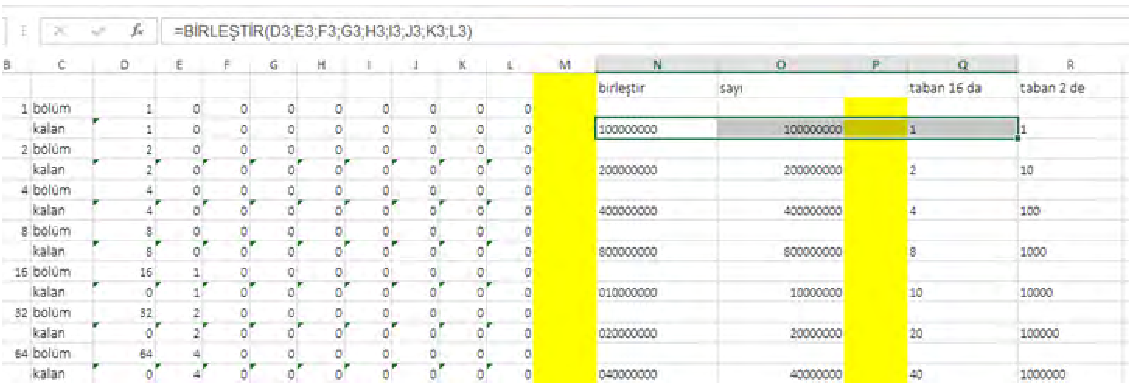
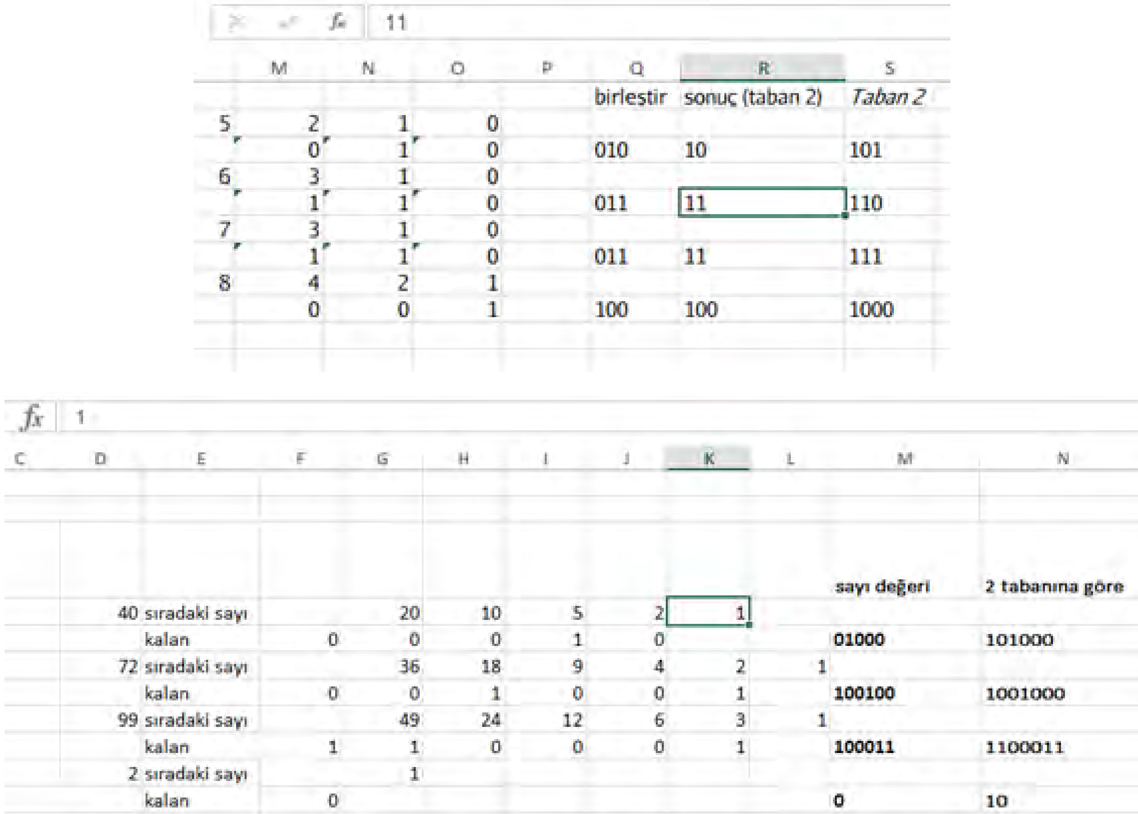


Figure 10. The screenshots of mistakes on the functions of =value and =concatenate, Task 3 by Ada

It is seen that participants could not reflect their mathematical knowledge on spreadsheets because they had difficulties using it. Although Berk uses more functions according to the directions of the researcher, he has difficulty demonstrating his mathematical knowledge using spreadsheets. For example, he enters the results manually rather than using the =value function. Similarly, he tends to rewrite the function in the corresponding cells of each number instead of dragging to find their binary equivalents. Berk, who can make the conversion in number systems in the paper-pencil environment and realizes the errors he makes in his worksheet, is unsuccessful in correcting these errors on the spreadsheet. When examining the effects of the students' usage schemes in the tasks on understanding the operations and reaching the correct results, Cenk does not adopt to find the remainder through =quotient and =mode, he performs these operations in his mind and recorded them manually. He uses =value, =concatenate, and =base as control tools. He could not realize the cause of the error because he forgot to write the last remainder manually (*Figure 11*).



fx 11

	M	N	O	P	Q	R	S
					birleştir	sonuç (taban 2)	*Taban 2*
5	2	1	0				
	0	1	0		010	10	101
6	3	1	0				
	1	1	0		011	11	110
7	3	1	0				
	1	1	0		011	11	111
8	4	2	1				
	0	0	1		100	100	1000

fx 1

C	D	E	F	G	H	I	J	K	L	M	N
										sayı değeri	2 tabanına göre
		40 sıradaki sayı		20	10	5	2	1			
		kalan	0	0	0	1	0			01000	101000
		72 sıradaki sayı		36	18	9	4	2	1		
		kalan	0	0	1	0	0	1		100100	1001000
		99 sıradaki sayı		49	24	12	6	3	1		
		kalan	1	1	0	0	0	1		100011	1100011
		2 sıradaki sayı		1							
		kalan	0							0	10

Figure 11. The screenshots of mistakes due to manual typing Task 2 by Berk and Cenk

Duygu matches the number systems and their remainders in the conversions in spreadsheets. For this matching, the knowledge presented by the researcher in Tasks 2 and 3 is discussed with the participants. Duygu's statements mirror these discussions. Duygu uses the =mod function, which she does not use in Task 1, and in the following tasks she also expresses its mathematical equivalent (*Figure 12*).

3.Duygu: Since the numbers are multiples (powers) of 2, I first wrote down the powers and then converted the cells to powers of 2….For example, since the base is equal to 16. I guess it is the same as base 10. =mode shows the reminder, so we always used it. I do not know another formula for the remainder.

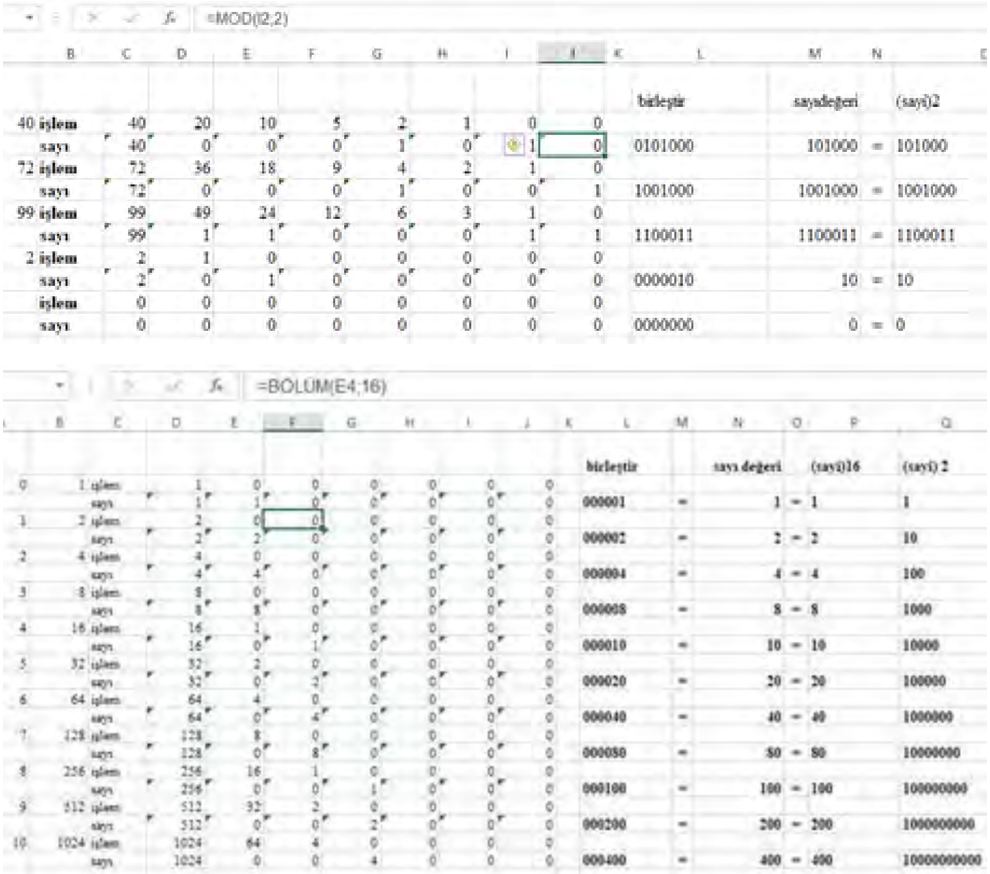


Figure 12. The screenshots of the functions of =mode ve =quotient, Task 2, and Task 3 by Duygu

Examining patterns on the conversions of number systems

The tasks include discussions about the relationships between the number systems and the numbers it contains and the patterns between the number systems.

3.Ada: … could it be like this? As the numbers are doubled, the results are multiplied by ten (in the binary system), but I did not understand the growth of 16.

3.Cenk: …yes, I saw it. As bases get smaller, numbers get bigger and zeros increase.

3.Duygu: …For example, since the base is equal to 16. I guess it is the same as base 10.

The participants exhibited behaviors of following and expressing the relationships between the numbers used in the conversions. Ada, who examines the relationship between the binary and decimal systems, senses that one of every two consecutive numbers in the binary system is a multiple of ten. However, since the same situation has a larger range in the hexadecimal, she could not comment on it. Cenk emphasizes the relationship between the number systems and the numbers they contain. Duygu, unlike others, enters the power of numbers by dragging. Besides, her explanations of the number systems and the relationships between the numbers they contain show that she produces hypotheses about their conceptual structures. It is seen that Duygu can interpret the situation of reaching multiples of 10 like Ada, while Cenk has difficulties in this regard. Berk reflects the rule in his mind regarding the pattern between the 2 and 16 systems in his explanations (*Figure 13*).

=TABAN(A22;16)

A	B	C	D	E	F	G	H	I	J	K	L
									sayı	sonuç (taban 16)	sonuç (taban 2)
1	sayı	0									
	kalan	1							1	1	1
4	sayı	0									
	kalan	4							4	4	100
16	sayı	1	0								
	kalan	0	1						10	10	10000
64	sayı	4	0								
	kalan	0	4						40	40	1000000
256	sayı	16	1	0							
	kalan	0	0	1					100	100	100000000
1024	sayı	64	4	0							
	kalan	0	0	4					400	400	10000000000
4096	sayı	256	16	1	0						
	kalan	0	0	0	1				1000	1000	1000000000000
16384	sayı	1024	64	4	0						
	kalan	0	0	0	4				4000	4000	100000000000000
65536	sayı	4096	256	16	1	0					
	kalan	0	0	0	0	1			10000	10000	10000000000000000
262144	sayı	16384	1024	64	4	0					
	kalan	0	0	0	0	4			40000	40000	1000000000000000000
1048576	sayı	65336	4096	256	16	1	0				
	kalan	0	0	0	0	0	1		100000	100000	100000000000000000000

Figure 13. The screenshot on the pattern of number systems Task 4 by Berk

Despite using =base in Task 1, Berk could not reach the correct solutions. In Task 2, he makes sense that there might be a pattern among the numbers and he shows patterns by explaining why in Task 4. In the operations directed to examine the effect of the position of zero in the numbers, participants' usage schemes mirror their thoughts. Cenk could not notice the mistakes in the solution he makes manually in Task 1. Therefore, he doubts the accuracy of the result by =base. On the contrary, in Tasks 3 and Task 4 he deletes the unnecessary zeros written. According to his observation notes, Cenk reflects this improvement on the other operations in his worksheets. It shows that he acts consciously in these choices (*Figure 14*).

=BİRLEŞTİR(Q21;P21;O21;N21)

L	M	N	O	P	Q	R	S	T	U
1	bölüm	0	0						
	mod	1	0			01	1	1	1
2	bölüm	0	0						
	mod	2	0			02	2	2	10
4	bölüm	0	0						
	mod	4	0			04	4	4	100
8	bölüm	0	0						
	mod	8	0			08	8	8	1000
16	bölüm	1	0	0					
	mod	0	1	0		010	10	10	10000
32	bölüm	2	0	0					
	mod	0	2	0		020	20	20	100000
64	bölüm	4	0	0					
	mod	0	4	0		040	40	40	1000000
128	bölüm	8	0	0					
	mod	0	8	0		080	80	80	10000000
256	bölüm	16	1	0	0				
	mod	0	0	1	0	0100	100	100	100000000

Figure 14. Effect of zero (0) for number systems, Task 3 by Cenk

The research cannot answer the questions about the relationships between the elements of the division operation and the numbers contained in the number systems.

4.Berk: . …there was a rule (pattern) among the results, but I could not see a rule between the remainder and the first number (converted number).

4.Duygu: Yes, there is a rule (the remainder should be less than the base) but how can we show it here?

CONCLUSION

In this study, It is examined the process of transforming the spreadsheet, which the participants know as an office application, into a mathematical instrument. It is thought that it is considerable to examine technology-supported teaching processes in the field of education. (Mishra & Koehler, 2006). Particularly in mathematics education, the interaction between the participants' tool use and conceptual understanding in the technology environment can be examined with the Instrumental Genesis Approach. One of the results we obtained is that the instrumentalization processes progress differently for each participant despite some common points (Baker & Sugden, 2015; Trouche, 2004).

In the process that starts on a blank worksheet with a question, the readability of the worksheets of the participants show the participants' tool-using skills and mathematical skills have differences. The participants who are good at the algorithm between number systems could reflect the formation of the transformation better in the worksheet (determining the concepts, phasing the solution process, adding images, coloring, etc.). However, it is seen as a substantial result that they are not willing to change their practices and the worksheet structures are not changed in the following tasks, although conceptual development was observed in the students during the process (van Dijke-Droogers, Drijvers & Bakker, 2021).

To realize the conversions of number systems, there is a combination of some functions including =base. Participants could develop their use of functions, correct their deficiencies, and add new uses for themselves in the process. Nevertheless, it should be noted that the use of functions by some of them is not systematic. On the contrary, all the participants use =base regularly throughout the research to check their results; therefore, it is seen that =base has become an instrument as a control tool for all of them.

The feature of dragging is critical for both operations based on formulas in cell addresses used in instant manipulations and updating these operations and values for a dynamic worksheet (Bozkurt & Uygan, 2020; Jalbert & Jalbert, 2019). This feature is essential to transform a tool into an effective instrument because it reflects the potential of the spreadsheet. Although the participants use both cell addresses and functions-features of spreadsheets in the first task scarcely, a similar development is observed in their use of them. In particular, it is observed that some participants who recognize their results in the worksheets are not false leave their habit of using paper pencils. This is important in terms of showing the change in usage schemes of the participants (Drijvers et al., 2013).

In this study, dragging was the most difficult progress for the participants in the instrumentalization process. When the participants do not reach the expected result, they first think that there may be an error due to dragging. It is thought that this reaction indicates that they hesitate to use features they are not good at (Baker & Sugden, 2015). This leads the participants who describe the use of dragging as "suspected" to enter the values manually. However, some participants who realize the benefits of dragging in terms of saving time and ease can form using schemes by dragging both horizontally and vertically (Ainley et al., 2005; Drijvers & Trouche, 2008).

The final step of the number conversions in the spreadsheet is to obtain the binary and hexadecimal equivalents of the number sequences given in the decimal system and to discover the patterns in these number sequences. At this step, in which number conversions can be made by all participants, relations on binary and hexadecimal number systems such as size, number digit expansion, and multiple/power of the number systems are partially established. In addition, it is concluded that the participants can organize their usage schemes related to cell addresses and dragging based on these relationships. On the other hand, another result is that understanding the effect of the position of the zeros (0) in the place value obtained during the conversions closely affects the use of =value and =concatenate (Haspekian, 2005; Marley-Payne & Dituri, 2019).

As a result, this study reveals how the instrumentalization and instrumentation processes of the instrumental approach affect each other and how they actually have an intricate relationship. According to the results, we suggest,

- It may need more time to observe the transformation of tool into instrument. In studies based on this approach, the observation period can be kept longer, especially depending on the characteristics of the participants.

- The results show that participants develop their usage schemes in the process. In the field of education, especially mathematics education, regular use of technology should be given importance instead of target use for research.

- A spreadsheet is a useful tool that can expose the relationships between numbers and formulas.

Therefore, it is appropriate to examine it in terms of the Instrumental Genesis Approach. Similar studies can be carried out for the use of this dynamic teaching approach at different grade levels in teaching mathematics concepts and forming instruments from tools.

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