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MENTAL PROCESSING STRATEGIES IN PRIMARY SCHOOL STUDENTS: THE EMPTY NUMBER LINE

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Abstract: The aim of this research is to determine the mental processing strategies used by 3rd-grade primary school students in addition and subtraction using the empty number lines. The model of the research is a case study, one of the qualitative research methods. The participants consist of 143 students; 75 girls and 68 boys, attending the third grade (8-years-old). Content analysis method was used in data analysis. Considering the results of the research, the mental strategies used by primary school 3rd-grade students were gathered in four categories: counting, balancing, first process step and partitioning. It was concluded that the students mostly used the mental processing strategy of using "multiples of 10", which is in the first process category, and this strategy was followed by "counting all" and "partitioning the second number", the least used mental processing strategy by the students was "using equivalent numbers".

Key words: Empty number line, mental processing strategies, primary mathematics

1. Introduction

No physical model can directly demonstrate or explain a concept on its own. The person who will learn the concept should attribute this meaning to the model. Children do this attribution of meaning by making use of the relationships between the objects or the parts of the object presented in the model. In a model, the child may not see a relationship that the teacher sees. This indicates that the child is not yet ready to use the model (Olkun & Toluk-Uçar, 2012). Many teachers only use the model that their students like or understand better. It's a mistake. Concepts should be created by students and these concepts should be applied to models (Van de Walle, Karp & Bay-Williams, 2014). Students should experience the models and discuss the similarities and differences in the representation of the different models.

At an early age, children develop their counting skills (forward and backward) and the meaning of ordinal numbers, by using the number line where each integer is represented by a separate square, and later on by using the number line model 0, 1, 2, 3, 4, ... etc. with equal intervals. Later, children learn to use the number line model in which only the multiples of 10 (0, 10, 20, 30, etc.) are marked (Haylock & Cockburn, 2014). Number lines have numbers whose positions are indicated by signs at regular intervals along the line. The number line encourages children to draw and count step by step with numbers forward or backward (Mosley, 2001). Children sometimes use the number line without starting from 0. In this process, the idea in children develops that the number line is a flexible tool to represent numbers and operate with them. The empty number line actually takes it a step further. An empty number line is a number line that has not been scaled. The numbers in a calculation to be made can be placed at any point as long as their order is preserved. For example, the points representing the number 52, 60, and 100 can be placed at any point on the empty number line, where the number 60 is between 52 and 100 (Haylock & Cockburn, 2014).

Mosley (2001) stated the important skills that children should develop before using the empty number lines as making jumps in different sizes (especially to tens), moving confidently both forward and backward on the number line, spliting and being able to recombine the numbers using the number that completes 10 and then 20. Klein Beishuizen and Treffers (1998) stated that the empty number line

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provides learning support with a higher level of mental activation, a more natural and transparent model for numerical operations, providing support for children to develop more figured, formal and efficient strategies, and especially a model that increases the flexibility of mental strategies. According to Haylock and Manning (2014), because the empty number is actually a draft, they do not need to be drawn to scale. It is a valuable tool that children can use to support their mental work when dealing with addition, subtraction, multiplication or division, or a combination of these operations.

Mental calculation means solving arithmetic problems (addition, subtraction, multiplication, and division) mentally without using a standard written procedure (Altun, 2018; Lemonidis, 2016; Threlfall, 2000; Pesen, 2020). In daily life, mental processing is of great importance (Lemonidis, 2016; Threlfall, 2000). Encouraging students to perform mental operations at an early age is an important step in gaining and developing this ability and using it throughout life (Altun, 2018; Pesen, 2020). Developing activities for mental calculation can lead to an increase in students' understanding and flexibility of working with numbers. Mental processing enables students to think flexibly (Sowder, 1992). The types of mental computation strategies required for flexibility are those in which the operands of the number problem are somehow transformed, perhaps subdivided, or treated in terms of close numbers to make the operations easier to deal with (Threlfall, 2000). Students' use of very different strategies in operations improves their logical analysis skills (Pesen, 2020).

The empty number line is an effective model in terms of providing support to students in verbally expressing the solution steps for the operations, recording the strategies they choose mentally, developing mental calculation strategies and supporting them in using various calculation strategies (Beishuizen, 1999; Bobis, 2007; Rousham, 2003; Foxman & Beishuizen, 2003; Mosley, 2001; Murphy, 2011). It aims to enable students to use their own informal counting and structuring strategies in early-age mathematics activities (Beishuizen & Anghileri, 1998). The empty number line is an important tool to bridge the gap between formal and informal strategies. It is a key part of the progression from formal strategies to high-level, abbreviated strategies. (Murphy, 2011). The empty number line encourages children of all ages to develop mental strategies. In 18+6 operation, 2 is added to 18 and 20 is reached. Then, by adding 4, 24 is reached (18+2) +4 = 20+4 =24 (Mosley, 2001, p.6). Students use the addition, counting, or progression structure of addition on the empty number line. Subtraction, on the other hand, is interpreted as counting down, completing (finding its inverse) or additive inverse (what should be added?) (Haylock & Cockburn, 2014, p.156). Unlike standard written operations in addition and subtraction, the empty number lines can be used to deal with image-based numbers and improve mental processing.

From primary school to university, emphasis should be placed on improving mental processes and skills, and individuals who will shape the future of countries should be trained (Threlfall, 2000). Understanding and managing mental computation strategies is essential for students' mathematics education (Barrera-Mora, Aarón Reyes & Mendoza-Hernandez, 2018). Some researchers suggest encouraging instructional environments in which the use of mental computation strategies precedes the learning of paper-and-pencil strategies (Reys et al., 1995; Gómez, 2005). When the studies on the empty number line are examined, it is seen that the use of the empty number line is an effective tool to record the solution ways of the students and to make sense of their thinking processes (Altun, 2002; Beishuizen, 1999; Beishuizen & Anghileri, 1998; Bobis, 2007; Gervasoni, Brandenburg, Turkenburg, K & Hadden, 2009; Gravemeijer 1994; Klein & Beishuizen, 1998; Murphy, 2011; Treffers 2001; Treffers & De Moor 1990; Van den Heuvel-Panhuizen 2001). When the researches are examined, it is stated that the empty number line model is a useful tool for us to see the strategies used by children in addition and subtraction with a visual representation. The most important reason that prompted the researcher to carry out this research was to reveal the mental processing strategies used by the students by using an empty number line as a model. At the same time, by revealing which different mental processing strategies students use; during the organization of classroom activities in mathematics lessons, it is considered important to conduct the research in terms of giving the classroom teachers ideas that a solution can be reached by using different mental strategies for an operation and including exercises in the student workbooks in which the empty number line is used to solve the operation. The aim of this research is to determine the mental processing strategies used by primary school 3rd-grade students in addition and subtraction using the empty number lines.

2. Method

2. 1. Research Model

The model of the research, which aims to determine the mental processing strategies used by primary school 3rd-grade students in addition and subtraction using empty number lines, is a case study from qualitative research methods. Qualitative research is concerned with how and why the behavior to occur. It describes how people interpret what they experience (Merriam & Tisdell, 2016). Case study is an qualitative approach in which the investigator collects in-depth information about a real-life, a bounded system (a case) or multiple bounded systems over time through multiple sources of information (observation, interview, audio-visual material, documents, and reports) and that involves in-depth and longitudinal examination of the analyzed data (Creswell, 2016; Glesne, 2012).

2. 2. Participants

The participants of this research consists of 143 students (8-year- old), 75 girls (52%) and 68 boys (48%), attending the third grade in two different public schools in Ankara, Turkey in the 2020-2021 academic year, maximum diversity sampling, one of the purposeful sampling methods. The maximum diversity method aims to explore and identify the main themes that encompass a large number of differences associated with the event or phenomenon under study (Neuman, 2014). Two schools with different socioeconomic structures were determined by the researcher. Among the schools, the participants in the classes with the most differentiation were selected. The mathematical success patterns of the participants in the classes were determined by examining the mathematical success scores of the previous years.

2. 3. Data Collection Tool

In this study, the "empty number line worksheet" developed by the researcher was used as a data collection tool. The worksheet consisting of 10 items including 5 addition and 5 subtraction was developed to determine the mental processing strategies of the students by examining the primary school mathematics curriculum, teacher guidebooks, student textbooks, supplementary workbooks, and related literature. Davis analysis was carried out by taking expert opinion from 3 mathematics education experts and 2 classroom teachers. The suggestions given by the experts were examined and the items in the worksheet were reviewed and necessary corrections were made. Content validity indexes (CVI) were determined in line with the data obtained from the experts. Davis (1992) grades expert opinions on the technique as (1) “The item is appropriate”, (2) “The item needs some correction”, (3) “The item should be seriously reviewed” and (4) “The item is not suitable”. In this technique, the content validity index (CGI) of 0.94 was obtained by dividing the number of experts who ticked options (1) and (2) by the total number of experts. If the CGI is above 0.80, it is accepted that the content validity result of the questions in the scale is appropriate (Davis, 1992). A pilot study was conducted on 26 students before the study in order to determine the suitability and intelligibility of the questions for the students' levels. In the examinations made as a result of the pilot study; It was determined that the questions were appropriate and understandable for the students' levels.

2. 4. Data Collection Process

The data were collected during the period when the effects of the Covid-19 epidemic decreased and the formal education took place 2 days a week in March. School administrators and teachers at the schools where the application would be made were informed about the study and the process by the researcher. Necessary consents were obtained from the participants and their families to participate in the study. Considering that the empty number line is not included in the program and is not fully known by the classroom teachers, a pre-application information interview was held with the classroom teachers about the definition of the empty number line, its differences from the number line, how it is used in addition and subtraction. In line with the information provided, the classroom teachers were asked to inform the students about the empty number before the worksheets were distributed to the students. In order to prevent the mental strategy used by the students from being affected by the strategy used by the teacher, it was requested from the classroom teachers not to make sample operations. After the necessary information was given to the students by the classroom teachers, the "empty number line worksheet" was distributed to the students and they were asked to answer the questions during 1 class hour. Classroom teachers contributed to the collection of data by sending the worksheets containing the answers of the students to the researchers. During the data collection process, the researcher in the classroom; It has not been found in order to avoid the conditions and distraction caused by the pandemic conditions.

2. 5. Data Analysis

In the research, the data collected by using the "empty number line worksheet" were analyzed with the content analysis technique. Data analysis in qualitative research involves the preparation and organization of data for analysis, then coding the data and categorizing it by assembling the codes, and finally presenting the data in figures, tables or discussion (Creswell, 2016) Coding and analyzing data is an analytical step. Organizing coding hierarchically is part of the analysis process (Gibbs, 2007; Glesne, 2012).

The data obtained from the students participating in the study were organized by labeling them as S1, S2, … S143. The operations on the empty number lines in the worksheet, which is the data source, are examined. At the stage of creating the categories and codes in which the mental processing strategies used by the students will be evaluated; domestic and foreign literature were examined, expert opinions of 3 mathematics education experts and 2 classroom teachers were used. Under counting categories; counting all, counting forward by 10s, counting backward by 10s, rhythmic counting, using reference numbers codes; under balancing categories; increase and decrease, completion/adding, decrease and increase codes; under first process step categories; using multiples of 10, using multiples of 5, changing the order of operations, using equivalent numbers codes; under partitioning categories; partitioning of the second number, partitioning of both numbers and partitioning of the first number codes have been created. According to the answers given by the students, the categories, the codes related to the categories, the frequency and percentages, distribution showing the number of students (F) and percentages of students (%) belonging to the mental processing strategy used by the students were determined.

To ensure reliability, randomly selected samples from student worksheets were analyzed at different times and the results were compared. The most useful method to increase reliability in qualitative research is member checking (Gibbs, 2007; McMillan, 2000; Glesne, 2012). In this study, a second researcher was provided to encode the data and review the codings in order to ensure coder reliability while conducting content analysis. Miles and Huberman (1994) reliability formula (Reliability = Consensus / (Agreement + Disagreement) x 100 was used to calculate the consistency between the two analyzes and the consistency value was determined as 91.57%. This shows that the results of the research are reliable. According to Miles and Huberman (1994); if the reliability calculations are over 70%, it is considered reliable for the research. In addition, all the data obtained at the end of the research were reviewed by external controllers who were not familiar with the study, and the reliability of the research was tried to be increased. In order to ensure the credibility of the results of the analysis of the data, the photographs of the students' answers are included in the results section.

3. Findings

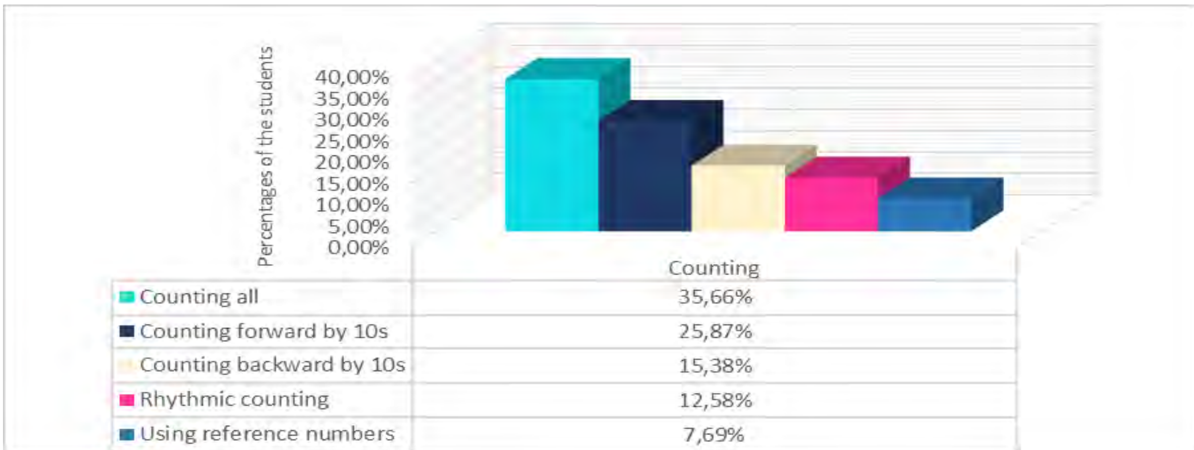
In the research, by examining the answers given by the students to the worksheets, the mental processing strategies used by the 3rd grade students in the addition and subtraction operations using the empty number line were determined. The categories according to the answers given by the students, the codes related to the categories, the frequency distribution indicating the number of students (F) and percentages (%) belonging to the mental processing strategy used by the students, and examples of student answers are given below. The mental processing strategies used by primary school 3rd grade students in addition and subtraction using empty number lines are given in Table 1.

Table 1. *The mental processing strategies used by primary school 3rd-grade students in addition and subtraction using the empty number lines*

Category	Code	F	%
Counting	Counting all	51	35,66
	Counting forward by 10s	37	25,87
	Counting backward by 10s	22	15,38
	Rhythmic counting	18	12,58
	Using reference numbers	11	7,69
Balancing	Increase and decrease	33	23,07
	Completion/adding	22	15,38
	Decrease and increase	16	11,18
First process step	Using multiples of 10	72	50,34
	Using multiples of 5	23	16,08
	Changing the order of operations	19	13,28
	Using equivalent numbers	9	6,29
Partitioning	Partitioning of the second number	47	32,86
	Partitioning of both numbers	28	19,58
	Partitioning of the first number	14	9,79

When Table 1 is examined, there are the categories and codes of mental processing strategies used by primary school 3rd-grade students in addition and subtraction using the empty number lines. It is seen that the code "using multiples of 10" in the category of the first process step was used by 72 students, and it was the code with the highest percentage (50,34%) among all categories. It was concluded that this code was followed by the codes of "counting all" in the category of counting, and "partitioning of the second number" in the category of partitioning. It was determined that the "using equivalent numbers" code, which is included in the first process step category, has the least percentage (6,29%) among all students. Graph 1 shows in detail how many students used the counting category and codes from mental processing strategies.

Graph 1. *Counting category and codes from mental processing strategies used by the students*



When Graph 1. is examined, there are codes belonging to the category of counting, one of the mental operations strategies used by students in addition and subtraction using the empty number lines. It was concluded that the "counting all" code in the counting category was used by 35,66% primary school third grade students, and it was the code with the highest percentage value in this category. It was concluded that this code was followed by "counting forward by 10s" used by 25,87% of the students, "counting backward by10s" used by 15,38% of the students, "rhythmic counting" used by 12,58% of the students and "using reference numbers" with the lowest percentage in the counting category used by 7,69% of the students. The examples of the strategies used by the students in the counting category; counting all (Figure 1), counting forward by 10s (Figure 2), counting backward by10s (Figure 3), rhythmic counting (Figure 4), using reference numbers (Figure 5) are shown.

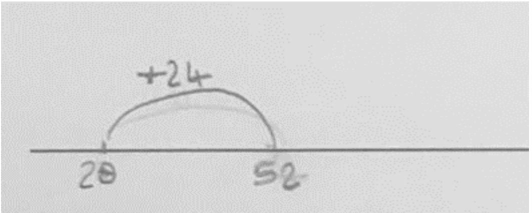


Fig. 1 Student 37 (Counting all)

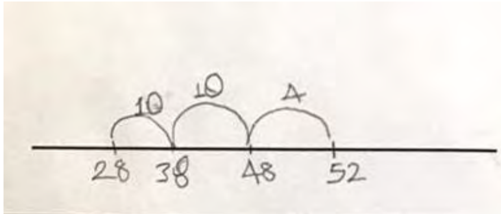


Fig. 2 Student 86 (Counting forward by 10s)

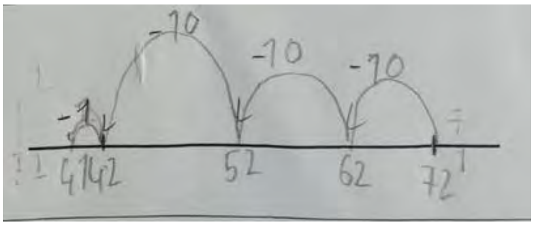


Fig. 3 Student 14 (Counting backward by 10s)

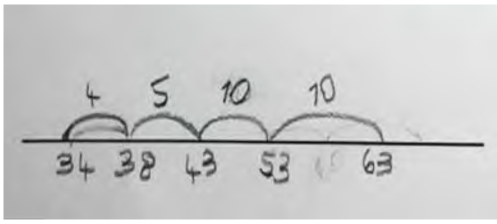


Fig. 4 Student 73 (Rhythmic counting)

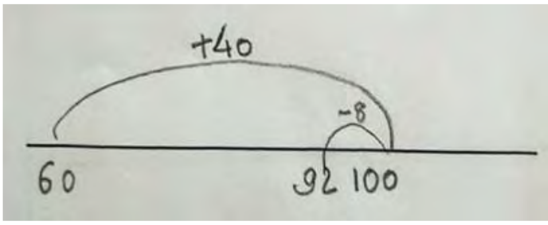
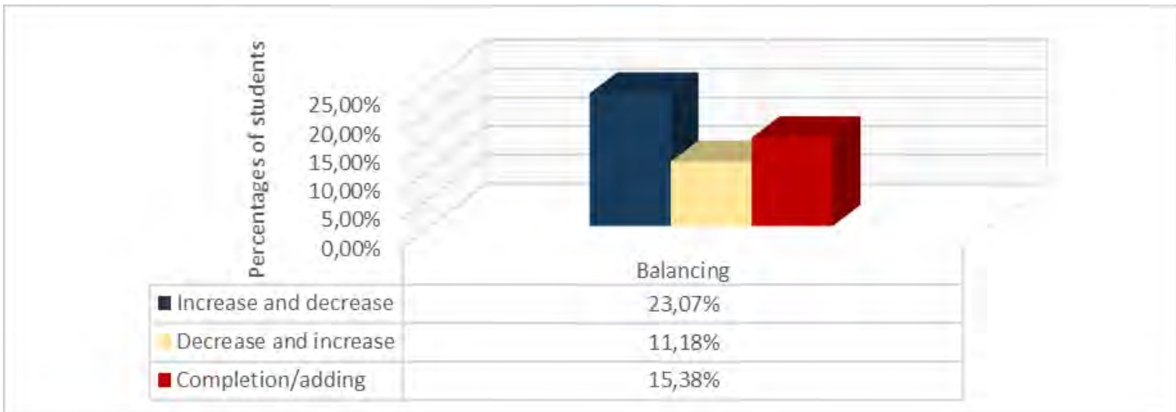


Fig. 5 Student 52 (Using reference numbers)

How many students used the balancing category and codes from mental processing strategies are shown in detail in Graph 2.

Graph 2. *Balancing category and codes from mental processing strategies used by the students*



When Graph 2 is examined, there are codes belonging to the balancing category, which is one of the mental operations strategies used by the students in addition and subtraction using the empty number lines. It was seen that the "increase and decrease" code used by 23,07% students was the most used strategy in this category. It was concluded that the "completion/adding" code was used by 15,38% students, and the "decrease, increase" code was the strategy with the lowest percentage in the balancing category using by 11,18% students. The examples of the strategies used by the students in the balancing category; increase and decrease (Figure 6), decrease, increase (Figure 7), completion/adding (Figure 8) are shown.

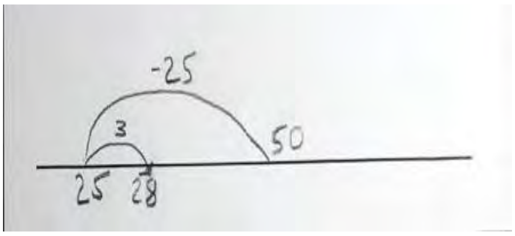


Fig. 6 Student 103 (Increase, decrease)

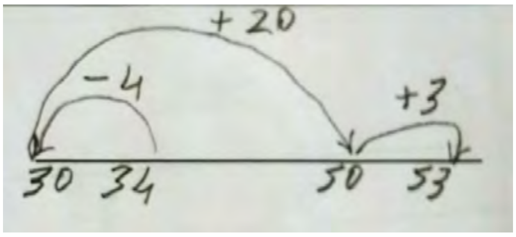


Fig. 7 Student 40 (Decrease, increase)

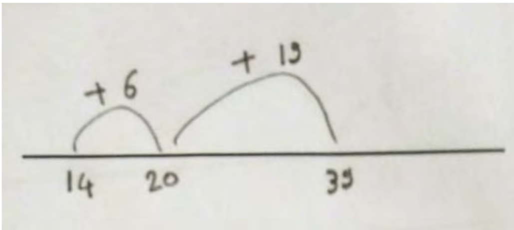
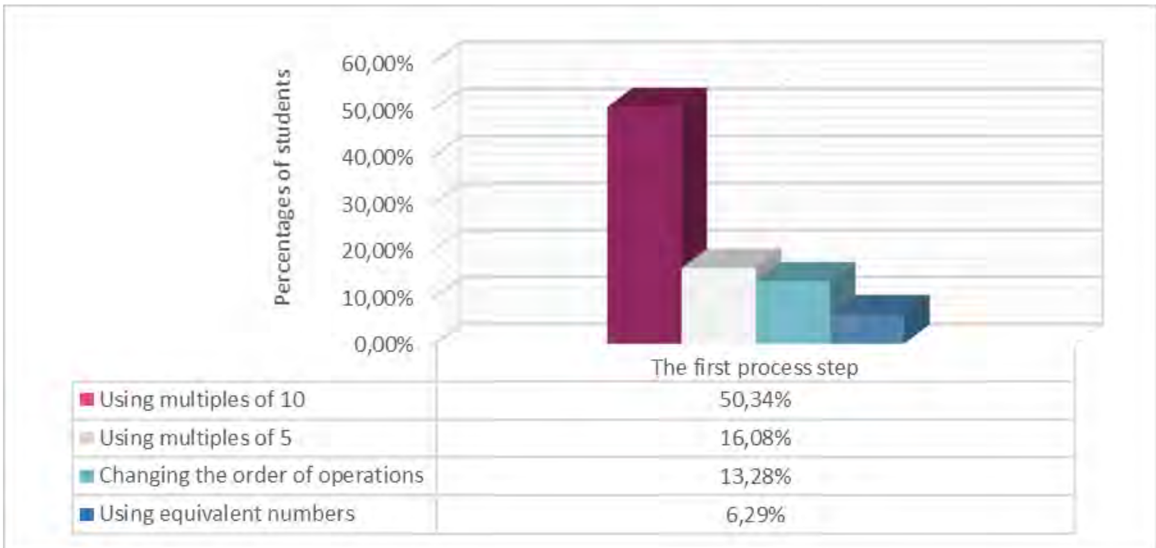


Fig. 8 Student 85 (Completion/adding)

How many students used the first process step category and codes from mental processing strategies are shown in detail in Graph 3.

Graph 3. *The first process step category and codes from mental processing strategies used by the students*



When Graph 3 is examined, there are codes belonging to the first process step category, which is one of the mental strategies used by students when adding and subtracting using the empty number lines. It was concluded that the code "using multiples of 10" in this category was used by 50,34% students to be the most used code in both the first process step category and all categories. It was concluded that the code "using multiples of 5" in the first process step category was used by 16,08% students, the code "changing the order of operations" was used by 13,28% students, the code "using equivalent numbers" was used by 6,29% students and was the least used mental processing strategy in both the first process step category and all categories. The examples of the strategies used by the students in the first process step category; using multiples of 10 (Figure 9), using multiples of 5 (Figure 10), changing the order of operations (Figure 11), using equivalent numbers (Figure 12) are shown.

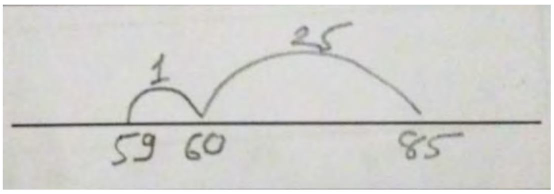


Fig. 9 Student 84 (Using multiples of 10)

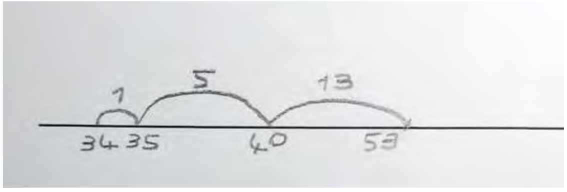


Fig. 10 Student 16 (Using multiples of 5)

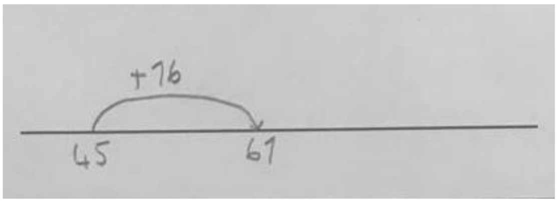


Fig. 11 Student 25 (Changing the order of operations)

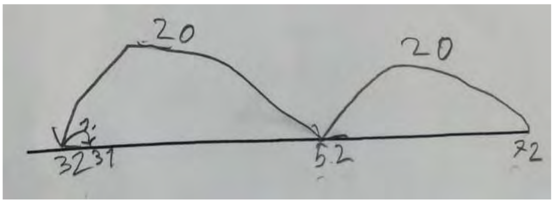
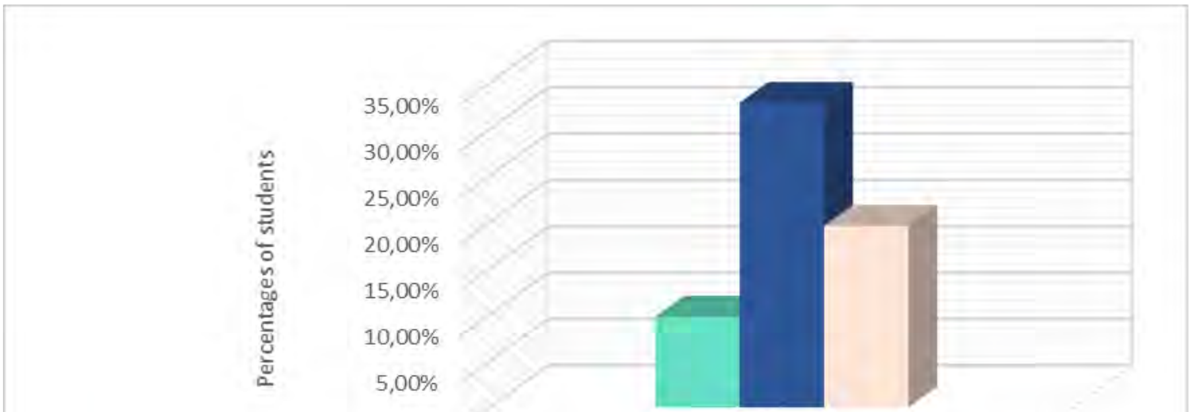


Fig. 12 Student 68 (Using equivalent numbers)

How many students used the partioning category and codes from mental processing strategies are shown in detail in Graph 4.

Graph 4. *Partitioning category and codes from mental processing strategies used by students*



	Partitioning
Partitioning of the first number	9,79%
Partitioning of the second number	32,86%
Partitioning of both numbers	19,58%

When Graph 1.4 is examined, there are codes belonging to the partitioning category, which is one of the mental strategies used by students in addition and subtraction using the empty number lines. Among the strategies used by the students in the category, it was determined that the "partitioning of the second number" code was used by 32,86% students and had the highest percentage in the category. It was determined that the "partitioning of the first number" code was used by 9,79% students and was the code with the lowest percentage in the category. It was concluded that "partitioning of both numbers", another code in the category, was used by 19,58% students. The examples of the strategies used by the students in partitioning category; partitioning of the second number (Figure 13), partitioning of both numbers (Figure 14), partitioning of the first number (Figure 15) are shown.

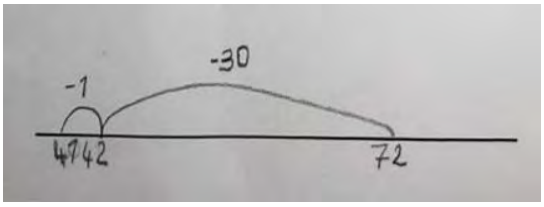


Fig. 13 Student 07 (Partitioning of the second number)

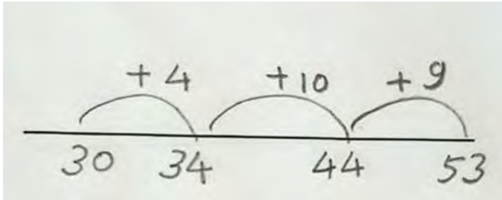


Fig. 14 Student 123 (Partitioning of both numbers)

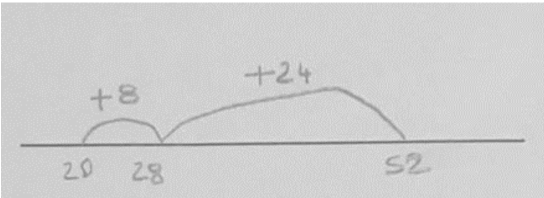


Fig. 15 Student 36 (Partitioning of the first number)

4. Conclusion, Discussion and Recommendations

When the results of the research are examined, it is seen that the correct use of the empty numbers by students is a flexible tool for determining different mental processing strategies, representing numbers and performing operations with them. According to Altun (2002), children can act quite freely and use their own strategies if they use the empty number line in performing operations. In the experimental studies of Klein and Beishuizen (1998) on the primary 2nd-grade students performing operations on the empty number line while performing operations, it was determined that the use of empty numbers is an effective teaching way in teaching the operations. Treffers (2001) suggested the use of the empty number lines as an instructive model at an early age and states that the model should be used not only to operate with small numbers below 20 but also for mental operations with large numbers up to 100. Gervasoni et al., (2009) concluded that the empty number line is useful in providing proof, understanding the calculation strategy and thinking processes of the students in their research, in which the empty number line is used to monitor and reflect the mental strategies of the 3rd and 4th-grade students. According to Beishuizen and Anghileri (1998), the choice of strategy by writing down the empty number line and the reflecting monitoring on the choice of strategy in class discussion is important to encourage progress towards higher-level strategies. In operations using the empty number line, it provides a good opportunity for strategy selection, comparison with strategies used by other students, and for discussion and reflection on which students' strategies involve the least steps or are the smartest-perfect.

It was concluded that the most used mental operation strategy in the counting strategy used in addition and subtraction operations using the empty number line is counting all. Within the counting strategy, it was determined that the students also made counting forward to 10s, counting backward to 10s, using reference numbers and rhythmic counting. It is important for children to see numbers and number relationships so that they can make connections between symbols and various images and their spatial representations at hand. According to Bobis (2007), the strong connections between the empty number line and the intuitive mental strategies of young children are clearly seen in the form of counting all, counting on or counting down, which naturally tends to focus on the counting strategies first when trying to solve number problems up to 100. According to Van de Walle, Karp and Bay-Williams

(2014), while the children were solving the "74 - 112" problem, they may say "Firstly, I added 6 to 74. Then, I added 2 more tens to 80 to get 100 because 100 is the reference number.” How numbers relate to these special numbers is an important step in the development of students' sense of numbers. The use of number lines is an effective way to see these relationships with reference numbers. Beishuizen (1999) points out that the empty number lines are important for inducing greater variety and flexibility in children's solution strategies and calculation procedures, when students become familiar with all number positions up to 100, they can go very easily (forward) by making leaps such as 15, 25, 35 and so on and 82, 72, 62 (backward). In their study, Gervasoni et al. (2007) concluded that 30% of students who started the last year of primary education had underdeveloped arithmetic reasoning strategies, which is an important indicator of mathematical proficiency. They emphasize the importance of using models that develop children's reasoning strategies for computation rather than using rote procedures (algorithms) or counting-based strategies.

It was seen that the increase-decrease strategy used by the students in the balancing mental processing strategy was used more than the decrease-increase and complement/add strategies. It was observed that the students using the increase-decrease strategy first reached a higher number than the result by using a larger number in the operations, and then reduced it by the amount they increased in order to reach the correct answer. According to Haylock and Cockburn (2014), mental strategies such as balancing or using multiples of 10 as the first process step are effectively supported by the use of the empty number line in addition and subtraction. In the 52-29 operation, by balancing, 30 is subtracted and 22 is reached. Then, 1 is added to the result 23, and in the 34 + 28 operation, firstly, 30 is added to 34 and then 2 is subtracted, and 62 is reached. In the complement/addition strategy, on the other hand, in operations that require subtraction, for example, in the operation 100- 63, it is useful to think of the distance between 100 and 63 as finding the difference. The key numbers in this process are 63, 70, and 100. In the process, starting at 63, we can ask, "What do we need to add to complete 63 to 100?". On the empty number line, we can solve it by first adding 7 to 70, then adding 30 to reach 100. We find that 100 – 63 is equal to 7+30=37. Murphy (2011) states that when children are asked to calculate 83-39, there may be several solutions at three levels; For example, one child may count backwards in small steps (83, 82, 81, 80…), another child may use a structured approach (83 - 30, 53, 53 - 9, 44), or a child may use a balancing or 'jump further' approach. Beishuizen and Anghileri (1998) asked some children in a project class to calculate mentally the answer "26 + 17 = n" and then represent their thoughts on an empty number line. When the answers were examined, it was seen that one of the students found it easier to add 16 to 20 (20+16+4+3) than to add 10 to 26.

According to the results of the research, it was concluded that the most used strategy among the different mental processing strategies used by the students was to use the multiples of 10 as the first process step. Students using this strategy are in the process of reaching the nearest ten as the first process step. It was also determined that the students used multiples of 5, changed the order of operations and used equivalent numbers as the first process step. According to Haylock and Cockburn (2014), adding 7 to 28 in the problem "28 + 7" can be solved in two steps, first adding 2 and then adding 5. In this operation, 30 is used as the first process step. Gravemeijer (1994) examined how the problem "65-38" was calculated by different children on the empty number, and it was seen that some of the students started from 65 to reach 35 by 10 each. In the last step, it was seen that they reached 27 by counting down 8, some of them reached 40 from 38 as the first process step, then reached 60, and finally reached 65 by taking 5 more steps. He concluded that while some of the students first subtract 40 from 63 and then add 2, in other words, they make balancing, some of them first subtract 20 from 65 and then reach 38 subtracting 7 and calculate the difference. He stated that students' self-confidence increased as a result of being free in operations. Mosley (2001) ın the "34+79" operation, the places of the numbers are changed as the first process step and then, with the number 79 is started and 80 is reached adding 1 to this number. Then three tens are added (90, 100, 110) and 3 is added at the end to get 113. According to Altun (2002), it was observed that in the "42 + 43 =?" operation, children reached the conclusions "40 + 40 = 80", "2 +3 = 5" and "80 +5 = 85" by using equivalent numbers.

It was observed that the students using the partitioning strategy performed operations by dividing the numbers into parts. In this strategy, it was determined that the partitioning of the second number was the most used mental processing strategy. It was concluded that the partitioning of both numbers was used more than the partitioning of the first number. According to Haylock and Cockburn (2014), the second number is partitioned (divided) on the empty number line, and the number 25 is divided as "20+5" in the "33+25" operation, and the operation is transformed into "33+20+5". On the number line, firstly, adding 20 to 33 and reaching 53, then adding 5 and reaching 58, in the problem "27+28", firstly, adding 20 to 27, then adding 3 and 5 and reaching 55, in the problem "78+45", adding 40 to 78 and then, as a result of adding 2 and 3, reaching the result of 123 are examples of partitioning strategy. Mosley (2001) stated that students using the partitioning strategy split and recombine numbers as "7+3+2" in the "7+5" operation or "28+10-1" in the "28+9" operation. Bobis (2007) stated that students who have effective mental strategies usually use partitioning strategies in pieces of 10. According to Murphy (2011) the partitioning strategies (15+18, 10+5+10+8, 10+10+5+8, 20+13) in which numbers are treated separately as tens and ones are based on a formal view of the place value. This separation process is a complex idea that is not fully understood by children. Foxman and Beishuizen (2003) reviewed British children's computational strategies in their study and concluded that children using sequential computational strategies (the first summed whole pattern and the second number partitioned) outperformed children who used both number partitioning strategies. This may be because sequential strategies take fewer steps and avoid migration issues.

It is seen that performing operations using the empty number line reveals the mental thinking processes of the students. Which strategy the students adopt while performing the operations and how they reach the result can be determined by recording them on the empty number line. It is seen that using the empty number line is important for understanding mental processing and for the development of mental processing. Recording children's thinking strategies in such a visual way can be used in the classroom for effective teaching by discussing and sharing mental strategies. Students can explain the mental processing strategies they use on the empty number line to their friends and teachers. Thus, the use of the empty number line can become a powerful tool to improve communication and learning in the classroom. Therefore, not only the level of thinking but also the thinking errors that may occur can be seen. This research was conducted with third grade (8 years old) students. Research can be conducted on the use of empty number lines in different age groups and grade levels.

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