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Investigation of Gifted Students' Mathematical Proving Processes

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Abstract: The "proving process" was considered as the stages that should be exist in a proof and in this study, it was aimed to investigate the mathematical proving processes and opinions about proof of gifted students. Case study, one of the qualitative research methods, was used in the study. The research was carried out with the students determined by criterion sampling method. The attendance of students to Science and Art Education Centre, the data obtained from the Proof Interview Form and the opinions of mathematics teachers constituted the criteria of the criterion sampling method. As a data collection tool, the Proof Interview Form and the Proof Clinical Interview Form were used. "Proof Clinical Interview Form" prepared by the researcher was applied to examine the students' proving processes. The data obtained from the clinical interviews were analysed with the descriptive analysis. Students were generally able to examine the problem situation and formulate the conjecture, but they did not determine the appropriate strategies, perform the necessary actions and summarize clearly while proving.

Key words: Mathematical proof, Proving, Proving process, Gifted student.

1. Introduction

Proof is one of the concepts that all mathematicians agree on its importance (Tall, 2002). The concept of proof is used to demonstrate the accurateness of theorems that are based on axioms (Bloch, 2011). In its most general sense, mathematical proof is the demonstration that a proposition is absolutely true (Hilbert et al., 2008). The general purpose of mathematical proof constitutes the structure of knowledge, which is one of the basic parts of mathematics (Hemmi, 2010). Understanding the concepts in mathematics, explaining the reasons and revealing the existing relationships is may possible with proof. Because mathematical proof includes making predictions, associating concepts, revealing relationships between concepts and topics, verifying mathematical expressions and generalizing new information (Schabel, 2005). In addition, it has an important role in learning the origin of knowledge, understanding the inter-operational relations and making sense of the result. So, it is an important tool in reducing rote learning, as it enables learning concepts by establishing a cause-effect relationship (Pekşen Sağır, 2013). Thus, mathematical proof will enable students to reconstruct the knowledge learned in mathematics.

While the idea of proof was used only for geometry in the past, but this situation has changed and proof has become the main goal of the mathematics curriculum and has become an acquisition that should be taught to all students in all mathematics lessons (Coe and Ruthven, 1994). It was recommended by the National Council of Teachers of Mathematics (NCTM, 2000) that students should start proving from the first years of their education. It is important for students to know the basic concepts of proof and the differences between the concepts and to be able to prove them so that they can understand the math projects that have been done before and prepare mathematics projects themselves (Öztürk et al., 2017). Because through proofs, it will be ensured that students understand the basis of the mathematical knowledge they are expected to learn and that the learning is permanent. But most of the students generally avoid proving because of the low level of understanding and structuring mathematical proof (Recio and Godino, 2001). In addition, they do not know where to start proof and how to use the conceptual information and definitions while proving (Öztürk, 2017). Thus,

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their proving skills and experience remain low. In order to understand the reason for the problems faced by students while they proving, it is necessary to examine the stages that should be exist in a proof (Öztürk et al., 2017).

1. 1. Theoretical Framework

It is clear that professional mathematicians and students have different proving processes. Mathematicians can prove in a more detailed, in-depth and conscious way (Boero, 1999) but this is not the same case for students. Heinze and Reiss (2004) thought that the students lost control in proving, so the proving process should be handled by dividing into small parts for the students. They developed the Boero model (Boero, 1999), which examines the stages of mathematicians' proofs, in order to examine the stages of students' proofs. According to the model they adapted, Heinze and Reiss tried to determine the skipped or not skipped stages when they were proving. Heinze and Reiss stated the proving process, which Boero described as six stages, as five stages for students in school mathematics and these stages were summarized in below.

The first stage was determined as an *examination of the problem situation*. At this stage, the students are expected to exploration of the problem situation and the generation of a possible conjecture. So, the first stage consists of the identification of different types of arguments that give support for the plausibility of this conjecture. In the case of the geometry lessons this means that the investigation of a drawing plays a role.

The second stage was determined as a *formulation of the conjecture*. At this stage, students are expected to identify the propositions of the conjecture according to the shared textual conventions that by determining the hypothesis and conclusion. So, the second stage consists of the precise formulation.

The third stage was determined as a *determining of the appropriate strategies*. At this stage, students are expected to develop strategies suitable for the actions that need to be taken to reach the result, and to decide which action to take. So, third stage consists of identification of appropriate arguments for the validation of the conjecture and a rough planning of a possible proof strategy. This stage may divide into for subcategories for the analysis: (i) the reference to the assumptions, (ii) the investigation of the assumptions, (iii) the collection of further information, and (iv) the generation of a proof idea.

The fourth stage was determined as a *performing the necessary actions*. At this stage, students are expected to take the necessary actions based on the strategy they determined in the previous stage. So, fourth stage consists of the combination of arguments into a deductive chain that more or less constitutes a sketch of the final proof. Thus, the rough plan of the proof is ensured by mathematical arguments. This stage can be performed pure verbally or together with some written remarks. Although the students correctly determine which operation to take, the mistakes made at this stage may prevent them from reaching the result.

The fifth stage was determined as a *summarizing*. This stage is the last stage of the proving process in school mathematics. At this stage, there is a retrospective general evaluation. If all the steps are stated and a summary of the process is available, it can be said that this step has passed successfully.

Developing these stages is important for creating suitable environments for students to understand proving and increasing the quality of education. Although the stages are named in a certain order, there is no requirement of linearity between the stages. Below, for stages, examples were given through direct quotations based on the data obtained as a result of the analysis of the clinical interviews conducted during the implementation. The italic parts indicated the stages students had gone through in the proving.

> - They want me to prove Euclid's theorem, but I need to use the Pythagorean theorem in proof. Let me draw a right triangle and write what is given on this figure. *(He drew a triangle and investigated drawing. Here, the student understood the problem situation and drew. So, the first stage was completed)*

-This is $a, b>0$ what we have and we will reach $a/b + b/a \geq 2$ by starting from $a, b>0$. *(This sentence indicates that the student determined the hypothesis and conclusion, so he completed the second stage)*

- We know that the square root of a number is always positive. *(A reference to the assumptions)* If I square root both sides, we may find equation. *(A generation of the proof idea. So, this sentence indicates that the appropriate strategy was tried to be determined and the third stage was completed)*

-The size of the arc seen by the interior angles of the triangle is equal to twice the angle. Since the circumference of the entire circle is 360°, the sum of the inside angles of the triangle is 180°. *(This sentence indicates that the student did the necessary operations regarding the process and the fourth stage was completed)*

- If I had used Thales' theorem here, I would have reached the same result because parallelism also gives same. But I noticed similar triangles in the question and using similarity I proved that the ratio is 2/3. *(This sentence indicates that the fifth stage was completed because the student had summarized the process by reviewing what has been done)*.

1. 2. The Aim and Importance of the Research

In this paper, the mathematical proving process of gifted students was examined by considering the stages of Heinze and Reiss (2004). For the meaning of giftedness many definitions have been made to explain and culture and traditions have led to the emergence of different definitions on this subject. Insomuch that Gagne (2003), Gardner (2003), Sternberg (2003), Renzulli (2005) and Tannenbaum (2003) have theories that consider giftedness as different components. In this study were accepted students who were defined as gifted according to Science and Art Education Centre in Turkey. Science and Art Education Centers are institutions that provide additional education after school, which were opened to enable specially talented and gifted students to use their individual talents at the highest level. Only primary school students are nominated to this center by their classroom teachers. These candidate students are then taken to group screening exam and those who score at or above the specified criteria are taken into individual examination. By individual review they start additional education in these centers according to the determined general mental ability areas. According to these centre, who learn faster than their peers, who are creative-productive, who have special academic ability, who can understand abstract ideas, and who have a high level of performance in mathematics, are accepted as gifted. Different definitions of giftedness have also affected educational practices. In order to ensure and support the development of gifted students, it is necessary to prepare suitable educational environments and educational programs for students, and to consider that these students have different developmental characteristics. It is possible to see gifted students as mathematicians of the future, as gifted individuals can acquire skills such as thinking, questioning, making inferences, and scientific research skills (Tortop, 2016), such as creating hypotheses, designing experiments and analysing models (Sriraman, 2005). In addition to the programs prepared in accordance with the characteristics of gifted students, differentiated education programs can be prepared in Science and Art Education Centre and proof studies can be included in Science and Art Education Centre, gifted students receive trainings related to research and project preparation. Proof allows these students to uncover relationships, make predictions, associate concepts, validate statements, generalize new information, and reconstruct the knowledge they have learned. It is expected that gifted students will be successful in mathematical proving, considering these students' characteristics. Gifted students are individuals who can understand proof and can make basic proof (Öztürk et al., 2017).

Comprehensive examination of the proof which has such an important role in the comprehension, development and meaning of mathematics will be beneficial in terms of exploring the ways of understanding and reasoning in students. One of the main research topics of mathematics education is to find out whether mathematical proof is used by students to justify mathematical propositions, or to what extent they are used by students (Rodd, 2000). So, proof and proving have come to the fore in mathematics education research in recent. Thus, new ways for proof teaching and learning are tried to be revealed by researches (Hanna, 2000).

In the accessible sources on proof and proving, there were studies conducted with students, teacher candidates or teachers. These studies were conducted to determine the views of students, teacher candidates and teachers about proof, their attitude, proving processes, learning difficulties related to proof, and proof schemes (Recio and Godino, 2001; Knuth, 2002; Almeida, 2003; Raman, 2003; Harel and Sowder, 2007; Keçeli Bozdağ, 2012; Güler, 2013; Karahan, 2013; Yılmaz, 2015; Çontay, 2017; Barak, 2018; Polat, 2018; Yıldız, 2019). In addition, there were limited number studies about proof with gifted students (Sriraman, 2005; Lee, 2005; Yim, Song and Kim, 2008; Lee, Park and Jung, 2009; Uğurel, Moralı, Karahan and Boz, 2016; Öztürk et al., 2017). However, there were no study investigates the proving process of gifted students by considering the stages of Heinze and Reiss at accessible sources.

It may be important to determine the proving process in order to raise the reasoning and math skills of gifted children to a higher level. In this respect, the aim of the study was determined as investigate the gifted high school students' mathematical proving processes and opinion about proof. For this purpose, answers to the following questions were sought:

1. What are the opinions of gifted students towards proving?
2. What are the indicators of the proving competency of gifted students?

2. Method

2. 1. Research Design

This study, which aimed to investigate the mathematical proving processes of 11th grade students attending the Science and Art Education Centre, was designed according to the case study model, one of the qualitative research methods.

2.2. Participants

The study group of the research consists of five students selected among twenty-five 11th-grade science high students who were diagnosed with giftedness from Science and Art Education Centre in the central district of Gaziantep province in Turkey. The reason why the 11th-grade level was preferred is that 11th and 12th grade students' level of proving is higher than the students at the lower grade and the 12th-grade students do not attend Science and Art Education Centre properly for studying the university exam.

Criterion sampling method, which is one of the purposive sampling methods, was used to determine the study group. The duration of students' attendance to Science and Art Education Centre, the data obtained from the Proof Interview Form and the opinions of mathematics teachers constituted the criteria of the criterion sampling method. According to the answers given to the Proof Interview Form; the five students who were able to define theorems and proofs correctly and performed above average in mathematics lessons for the opinion of their teachers were included in the clinical interview. Pseudonyms were given to the students as S1, S2, S3, S4 and S5.

S1, S2, S3, S4 and S5 had been Science and Art Education Centre students since primary school. S1, S2 and S5 studied on various projects at Science and Art Education Centre and S1 participated in the "Game 2016" project organized by the Intelligence Foundation of Turkey in the 6th grade. Among all students, S1 stated that he worked on proof at school but did not work on proof in Science and Art Education Centre; S2 stated that he loved to prove, but he did not work on proof in the lessons, he was working on proof because he was preparing for the Olympics; S3, S4 and S5 stated that they did not work on proof at school, only if the students asked, their teacher did proof work on the subject.

2.3. Data Collection Tools

Clinical interview was chosen as the data collection method because it was aimed to examine the processes of mathematical proof of gifted students in depth. Before the clinical interviews, the Proof Interview Form was used to consult the opinions of 11th grade students attending Science and Art Education Centre regarding their pre-learning and readiness related to proof. The process of collecting the data of the study took approximately three months. The interviews were conducted by the researcher and the data obtained were recorded with a tape recorder. Depending on the qualitative research design in which the research was designed, the Proof Interview Form was used to determine the sample of the study, and the Proof Clinical Interview Form was used to examine the proving process in depth.

2.4. Proof Interview Form

The Proof Interview Form was about students' knowledge and opinion about the concepts of theorem and proof. Because taking students' opinions about proof would guide them to interpret the proving processes in a more detailed and accurate manner. So, the Proof Interview Form consisting of three open-ended questions prepared by the researcher was directed to the students for the opinions on their pre-learning and readiness for proof.

In addition, expert opinions were taken from high school mathematics teachers, mathematicians and Science and Art Education Centre teachers for the reliability and validity studies of the Proof Interview Form. The necessary corrections were made according to the feedback received from the experts, and the Proof Interview Form was made ready for the implementation (Appendix 1).

2.5. Proof Clinical Interview Form

For the clinical interview, proof questions were prepared on the determined subjects by taking the opinions of Science and Art Education Centre teachers and by following a few lessons of the students in Science and Art Education Centre. There was no learning outcome about proof in 9-11th grade curriculum but suitable questions were determined on the students' textbooks for revealing the stages of proving process.

Firstly, 11 proof questions were prepared. However, considering the expert opinions and pilot study results, five proof questions, which were not suitable for the purpose of the study and proving process, were removed from the Proof Clinical Interview Form. After all, the six proof questions in the Clinical Interview Form were arranged as three mathematics and three geometry proof questions (Appendix 2). Thus, it was aimed to examine the proving processes of the students in both mathematics and geometry questions.

2.6. Implementation Process

The implementation process of the research was handled as pilot implementation and actual implementation. The pilot implementation was carried out before starting the actual implementation in order to solve the problems that may be encountered in the actual implementation of the study.

The pilot implementation of the study was conducted with 11th grade students of a science high school. The pilot study was conducted with fifteen students determined by the suggestions of the school's mathematics teacher. At the beginning of the pilot study, the Proof Interview Form was applied to the students. Clinical interviews were conducted with six students who were determined voluntarily, taking into account the data obtained from the Proof Interview Form at pilot study. As a result of pilot study, it was observed that the Proof Interview Form applied during the pilot study could be used in the actual practice. In addition, the Proof Clinical Interview Form in pilot, five questions that could be solved with similar methods, were not suitable for the purpose of the study, and the proving process could not be examined. Therefore, these questions were removed from the Proof Clinical Interview Form for actual implementation. So, six proof questions in the Proof Clinical Interview Form were determined for the actual implementation, three of which are algebraic and three of which are geometric proof questions.

The actual implementation of the research was carried out with twenty-five students attending the 11th grade of a Science and Art Education Centre in Gaziantep. The Proof Interview Form was applied to the students at the end of the first term. Firstly, Students who were able to define the theorem and proof correctly, who proved in their lessons and who were willing to prove in their lessons were evaluated from the Proof Interview Form. After all, the five most successful students were selected for clinical interview.

A date was determined for clinical interviews for each student, and one-on-one interviews were held at the Science and Art Education Centre at the beginning of the second semester. Each interview lasted approximately 60 minutes.

2.7. Data Analysis

A case study, which is a very common model in qualitative studies, was used. Qualitative data in the study were obtained from interviews with students. The qualitative data set of the study was created by converting interview forms and audio recordings into separate text documents. Descriptive analysis technique was used to analyse these qualitative data.

The Clinical Interview Form applied in this study, in which the proving processes of gifted students were examined, was presented to the opinion of mathematics education experts and high school mathematics teachers, and the external validity of the study was tried to be achieved. In addition, expert opinion was obtained from high school mathematics teachers regarding the mathematical correctness and logical understanding of the proof questions. According to the feedback from the expert opinion, necessary changes were made and the Clinical Interview Form was made ready for application.

The intercoder reliability was examined in the analysis of the data obtained from clinical interviews. The researchers encoded the research data independently. The resulting codes are grouped as "similar codes" and "dissociated codes". According to Miles and Huberman (1994), "Agreement Percentage = Agreement / (Agreement + Disagreement)" is calculated for coder reliability. The Miles and Huberman compliance percentage calculated according to the formula given within the scope of this research was obtained as 0.88. Accordingly, the overall fit between the two researchers is high. In this way, intercoder reliability was provided in the study.

3. Findings

3.1 Findings Obtained from the Proof Interview Form

In the Proof Interview Form, which was applied to totally twenty-five gifted 11th-grade students, it was asked about their knowledge of the theorem and proof, whether they proved in lessons, and their opinions about proving in the lessons.

There were three students who could explain both the concepts of theorem and proof, two students who could explain only the concept of theorem, and thirteen students who could only explain the concept of proof. So, students could explain the concept of proof, but they could not explain the concept of theorem.

Fifteen students stated that they did prove in mathematics and geometry lessons. In addition, some students stated that only their teachers proved and that they took part only as listeners in the lessons rather than proving. It was stated that proofs were made on trigonometric identities, rules of congruence and similarity, analytical geometry rules and some elements of the triangle. So, students could prove in geometry lessons mostly and remember geometry proof.

Twenty-one students stated that proving should be in the lessons, but four students declared that they were hesitant to prove because they did not know what to do in the proof questions, and therefore they had difficulty in proving. The answers of some students were given below with direct quotations.

> - We do not learn a mathematical formula without proof. My interest increases when we learn where the formula has come. I think mathematics lessons should definitely be taught with proof.
>
> - I think it is better to do proof. Permanence increases during the learning phase.
>
> - When you see the proof, the subject becomes clearer.
>
> - It is absolutely necessary to prove, to learn the basics.
>
> - I do not know how and where I start proving. So, proof is boring and not necessary.
>
> - Because of I do not know what to do, I hesitate proof study.

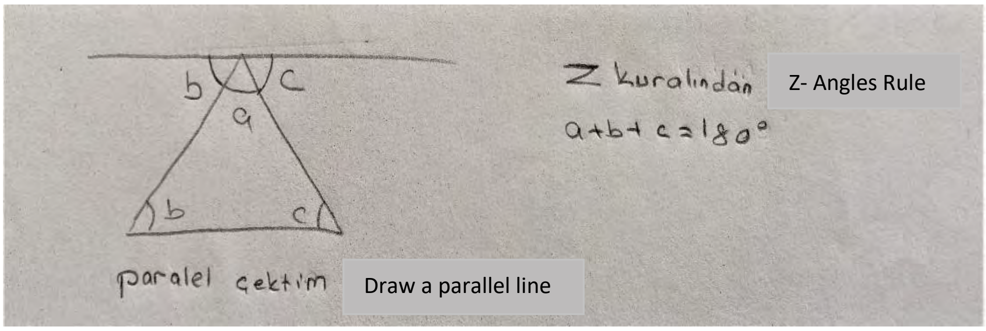
Based on the quotations, it has been stated that although proof is boring for some students it is beneficial for most students in terms of understanding the subject learned better, increasing the permanence of the knowledge and learn the taught rules together with the reasons.

3.2. Data Obtained from Proof Clinical Interview Form

3.2.1. Findings and Comments for the First Question. In this question, students were asked to prove that the sum of the dimensions of the interior angles of any triangle is 180˚. The two students were able to pass all the stages of the model. Below, the stages of the proving process for this question were explained as an example by giving the direct quotations of S5 from the interviews with the students.

> S5: In the question, it is wanted me to show that the sum of the angles of a triangle is 180˚. Let me draw a triangle *(he drew a triangle with interior angles named as a, b and c in Figure 1)*. I draw parallel to an edge in the triangle I drew. With this parallel and Z rule, a straight angle is formed on the parallel as well. As a result, $a+b+c=180˚$, so I proved the sum of the angles in the triangle. Here if I draw parallel from another edge the same result will be found.

Figure 1. *S5's solution for the first question and text' translation*



As can be seen from the quote above, S5 explained the problem situation and drew the triangle. In addition, he formulated it mathematically and expressed the conjecture. In order to solve the problem, he chose the necessary operations by drawing parallel to any side on the triangle he drew. On the other hand, using straight angle may show the reference to the assumptions. Then, he performed the necessary actions and calculated. At the end of, S5 explained the whole process again and summarized what he used, how he proved it and why he did it.

The stages of the proving process of all students regarding the question were indicated in Table 1. In table, if the student completed stage in true, it was presented with the sign of '+'; if he/she completed stage in wrong, it was presented with the sign of '-'. But if there is no finding for the stage it was presented with gap.

Table 1. *Stages of the Proving Process for the first question*

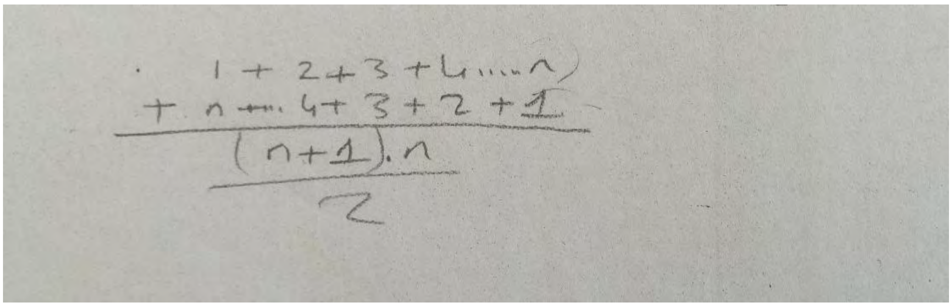
Stages		S1	S2	S3	S4	S5
Examination of the problem situation		+	+	+	+	+
Formulation of the conjecture		+	+	+	-	+
Determining appropriate strategies	The outer angles of the triangle	+		+	-	
	The inner angles of the square	+				
	The property of circle		+			
	The concept of parallelism					+
Performing the necessary operations		+	+	+	-	+
Summarizing			+			+

According to findings, five students could examination of the problem situation and determined different strategies to prove and performed the necessary operations. Although S4 could examine the problem situation and determined appropriate strategies, he could not formulate and not express the theorem clearly. Although he did not know what to show, he determined a strategy and performed necessary action. Even though he determined strategy and performed action, he could not reach the true result. So, passing the stages is not enough to prove it right. This shows that the stages may not be passed sequentially. In addition, S2 and S5 summarized the solution of the problem.

3.2.2. Findings and Comments for the Second Question. In this question, students were asked to prove the formula for the sum of consecutive numbers from 1 to n. Two students were able to pass all the stages of the model. Below, the stages of the proving process for this question are explained by giving direct quotations S4 from the interviews with the students.

> S4: I know how to prove the sum of the numbers from 1 to n if n is equal to 1 or greater than 1. We will do it like this. Let's write the numbers in the form of $1+2+3+\ldots+n$. Let's write the numbers up to the n in reverse and add them one after the other. The sum of the two terms in each row is $n+1$. Since there are n terms, the sum is $(n+1)n$ but since I added it twice, it would be half the total so I would have proved it.

Figure 2. *S4's solution for the second question*



As can be seen from the quote above, S4 examined the problem situation, formulated the conjecture clearly. To prove the problem, S4 wrote the given sequence of consecutive numbers using Gauss method, and then he summed the same consecutive number sequence by writing it backwards. In addition, S4 stated that the sum of both terms equals $n+1$ and that the sum of $n(n+1)$ constitutes the result and mentioned that half of the sum is equal to the sum of consecutive series of numbers, since he summed up the same series of numbers twice. S4 chose the necessary operations and summarized the steps he followed and completed his proof.

The stages of the proving process of all students regarding the question are shown in Table 2.

Table 2. *Stages of the Proving Process for the second question*

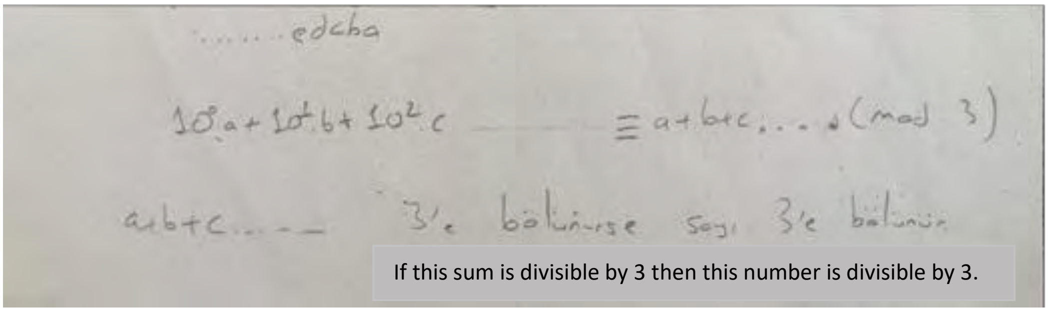
Stages		S1	S2	S3	S4	S5
Examination of the problem situation		-	+	-	+	+
Formulation of the conjecture		-	+	-	+	+
Determining appropriate strategies	Gauss method				+	+
	The concepts of term number and arithmetic mean		+			
Performing the necessary operations			+		+	+
Summarizing			+		+	

According to findings, S2, S4 and S5 were able to examine the problem situation, formulate the conjecture, determine appropriate strategies to prove it, and performed the necessary operations. However, S2 and S4 could use the same method in a different question by summarizing their procedures. S1 and S3 could not explain the problem situation and the theorem. S1 and S3 tried to produce a strategy for the operations they had to do without questioning the meaning of the expressions in the given theorem, but they could not prove the theorem. Moreover, in this question, all the students realized that although they used the addition formula for consecutive numbers a lot, but they did not think how the formula was obtained.

3.2.3. Findings and Comments for the Third Question. In this question, the students were asked to prove that the rule of dividing any integer by 3 is multiple of 3 or three multiples of the number's sums of numerical values. In this question, because students passed maximum 4 stages, the proving process did not completed fully. Below, the stages of the proving process for this question were explained by giving direct quotations from the interviews of S2.

> S2: It is wanted a proof that the rule a number can be divided by 3. I look at what's left of modular arithmetic. If I choose …*edcba*, the expansion of this number continues as $10^0a+10^1b+10^2c+\ldots \equiv a+b+c+\ldots\ (mod3)$ so, if the sum of numbers is divided by 3, the number is divided by 3 completely.

Figure 3 *S2's solution for the third question and text' translation*



As seen in the quote above, S2 examined the problem situation and formulated the conjecture. He said that he can use modular arithmetic to prove the problem and mentioned that if the sum of remains from the equivalents of $\ldots 10^2 \equiv 1$, $10^1 \equiv 1$, $10^3 \equiv 1$, which the powers of 10 form in mode 3, is a multiple of 3, the number can be divided by 3 completely. S2 completed the proof by performing the necessary procedures. However, in the proving process, S2 did the operations by considering only the positive numbers. This situation stems from the student's carelessness. As a result, although S2 showed the proof, he completed the process incompletely because he ignored the rule of dividing negative numbers by 3.

The stages of the proof process of all students regarding the question are indicated in Table 3.

Table 3. *Stages of the Proving Process for the third question*

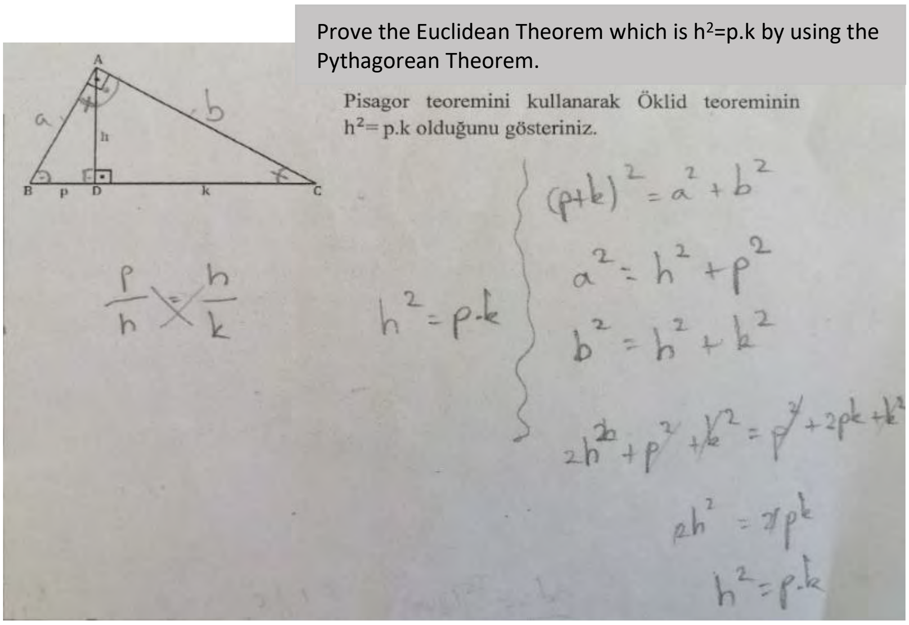
Stages		S1	S2	S3	S4	S5
Examination of the problem situation		+	+		+	+
Formulation of the conjecture		+	+		+	+
Determining the appropriate strategies	Using the analysis method					+
	Using the modular arithmetic method		+			
Performing the necessary actions			-			-
Summarizing						

According to the findings, S1, S2, S4 and S5 could formulate the conjecture by examining the problem situation, but S2 and S5 determined appropriate strategies and did the necessary operations to prove. S2 and S5 also summarized the methods they used by associating them with different problem situations. However, S2 and S5 chose their operations without considering negative numbers during the proving process. This situation may be the reason for the students' lack of knowledge on this subject or it may be the result of their carelessness. In addition, S1, S4, and S5 used special examples for proving and S1 and S4 could not use mathematical language. S1, S3, and S4, who could not complete the proof, only memorized the divisibility rules and used them for the question. But, at the end of the interview, the S4 said that he wanted to learn the proof of the divisibility rule. So, by the proof studies although students cannot prove a theorem, they may be curious about a proof and increase their motivation for proving. As a result, no student could pass all the stages of the model for this question.

3.2.4. Findings and Comments for the Fourth Question. In this question, the students were asked to show that the square of the height of the triangle is equal to the product of the lengths formed at the base, using the Pythagorean Theorem, based on the right triangle given. All students were able to pass all the stages of the model. Below, the stages of the proving process for this question are explained by including direct quotations of S3 from the interviews with the students.

> S3: I am asked to prove Euclid theorem $h^2 = p \cdot k$ by using the Pythagorean Theorem. I can do it using similarity. In a right triangle, it is equal to $a^2+b^2=c^2$ Then, I will write the Pythagorean equations in triangles. Still, I would have proved it.

Figure 4. *S3's solution for the fourth question and text' translation*



The stages of the proving process of all students regarding the question are shown in Table 4.

Table 4. *Stages of the Proving Process for the fourth question*

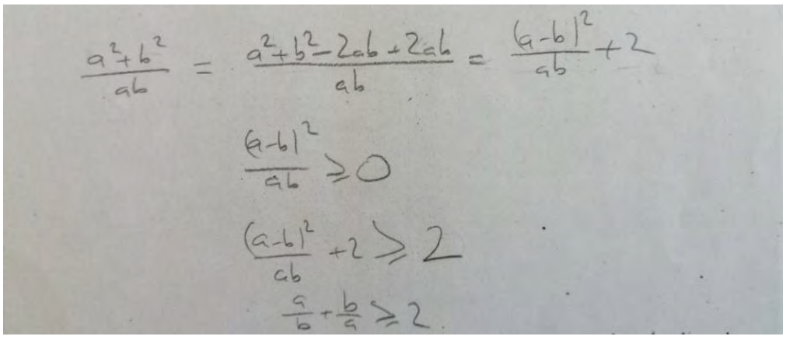
Stages		S1	S2	S3	S4	S5
Examination of the problem situation		+	+	+	+	+
Formulation of the conjecture		+	+	+	+	+
Determining the appropriate strategies	Using the Pythagorean theorem	+	+	+	+	+
	Using similar triangles		+	+		
Performing the necessary operations		+	+	+	+	+
Summarizing		+	+	+	+	+

All five students could formulate the conjecture by examining the problem situation, determined appropriate strategies to prove it, and did the necessary operations. In addition, S2 and S3 proved the problem in two different ways, using both the Pythagorean Theorem and the similarity theorem. However, S3 defined the Pythagorean Theorem as $a^2+b^2=c^2$ without determined which of its perpendicular sides. While this situation shows that the student generalized the definition of the Pythagorean Theorem by accepting these side lengths for all the right triangles he encounters, it may also indicate that the student had difficulties in mathematical language and notation. In this process, the students proved Euclid's theorem using the Pythagorean Theorem and could make associations between the subjects. The students summarized the procedures and completed the proof. Moreover, S2 and S3 added a new strategy to the appropriate strategy that they had previously determined during the summarization stage.

3.2.5. Findings and Comments for the Fifth Question. In this question, students were asked to prove that if the values of real numbers *a* and *b* are greater than 0 then the value of $a/b + b/a$ is equal to or greater than two. Only S2 could pass all the stages of the model. Below, the stages of the proving process for this question are explained by including direct quotations of S2.

> S2: I need to show that the expression is greater than/equal to 2 according to the given *a* and *b* values. In this question, I first equate the denominator in the expression $a/b + b/a$. I add and subtract the term $2ab$ to the $(a^2+b^2)/ab$ expression I find. Its share here equals to $(a-b)^2+2ab$ I divide the numerator by the denominator. The calculation must be $(a^2+b^2)/ab \geq 2$ because *a* and *b* must be greater than zero. Its numerator is also always positive because it is the square of a number. In this case, the expression given is equal to or greater than 2.

Figure 5. *S2's solution for the fifth question*



As it can be understood from the quotation, S2 examined problem situation and formulated the conjecture. He also tried to reach the conclusion by accepting that the values of *a* and *b* are greater than zero, that is, he expressed the theorem. For proving, he made denominator equalization in the expression on the left and tried to show that the expression is greater than or equal to 2 by using the rule of addition and subtraction. While performing the operations, he completed the operation by considering that the numbers *a* and *b* were in positive situation and proved that $a/b + b/a \geq 2$. S2 assumed that the numbers *a* and *b* were greater than zero and explained the procedures he made and summarized his process.

The stages of the proof process of all students regarding the question are shown in Table 5.

Table 5. *Stages of the Proving Process for the fifth question*

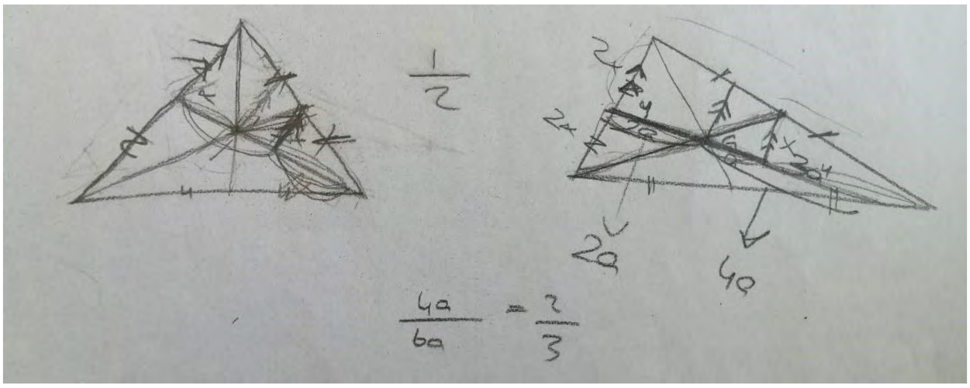
Stages		S1	S2	S3	S4	S5
Examination of the problem situation		+	+	+	+	-
Formulation of the conjecture		-	+	-	-	-
Determining the appropriate strategies	Using the trial method	-		-		
	Using the denominator equalization method		+	-	-	-
	Using squaring method	-				
Performing the necessary operations		-	+		-	
Summarizing			+			

Although S1, S2, S3 and S4 could examine the problem situation, but only one of these four students (S2) could formulate the conjecture clearly. S1, S3 and S4 tried to solve the theorem without focusing on the relationship between hypothesis and conclusion. Here the students were expected to accept the hypothesis as *a, b*>0 and reach the conclusion as a/b + b/a ≥2. But they did not reach the expression a/b + b/a ≥2. They accepted that a/b + b/a ≥2 is true and only they calculated from a/b + b/a ≥2. Therefore, in the case of the problem they encountered, it was observed that the students ignored which one is the hypothesis or the conclusion, just tried to operate. S1 and S4 coded students also could not complete their operations and stated that they reached an expression loop equal to the given. On the other hand, S1 and S3 used the trial method and thought about the process only through special examples. This situation led us to the conclusion that the students performed the operations in the questions based on memorization while doing this proof. On the contrary, S2 completed all stages of the proving process and even explained the operations they made with their reasons. Thus, it can be said that other students, except the S2, could not express the theorem and could not complete the process because they had difficulties in mathematical language and notation. As a result, only S2 successfully completed the proving process in this question.

3.2.6. Findings and Comments for the Sixth Question. In this question, students were asked to prove that the distance from the center of gravity to a corner in a triangle is equal to 2/3 of the median length drawn from the same corner to the opposite side. Three students were able to pass all the stages of the model. Below, direct quotations of S4 from the interviews with the students are given and the stages of the proving process for this question are explained.

> S4: I am asked the ratios that the center of gravity creates. The center of gravity was the cutting point of the angle bisectors. Was it the bisector or the median? I can reach this result by drawing. (*He drew triangles, indicating both the bisector and the median in Figure 6*) The intersection point of the bisectors forms the inner tangent circle. The intersection point of the medians is the center of gravity. So, it wants me to show that the ratio in the center of gravity is 1 to 2. I can prove it by similarity because there is a similarity. I prove it using Thales's theorem. (*He showed two similar triangles on the triangle he drew*) It has a ratio of 1 to 2 due to its edges. Let's name the lengths because of this ratio. Also, if the length we call y is equal to 3*a* due to the butterfly theorem, then the length drawn from the corner to the center of gravity will be 4*a*, all of which will be 6*a*. Its ratio is equal to 2/3. I proved it by similarity.

Figure 6. *S4's solution for the sixth question*



As seen in the quote above, S4 examined the problem situation and formulated the conjecture. S4 was unsure whether the center of gravity was the cut-off point of the medians or the cut-off point of the bisector. However, S4 made this clear by creating and drawing a median and bisector in different triangles. S4 concluded that the cutting point of the bisectors is the center of the inner tangent circle, and the cutting point of the medians is the center of gravity from drawing. As a consequence, S4 learned the concepts of median and bisector in triangles in a meaningful way. S4 showed the similarity on the triangle he drew and also expressed the relationship between the lengths due to the similarity of the two triangles. S4 followed the necessary procedures, summarized the operations he did and completed proving.

The stages of the proving process of all students regarding the question are shown in Table 6.

Table 6. *Stages of the Proving Process for the sixth question*

Stages		S1	S2	S3	S4	S5
Examination of the problem situation		-	+	+	+	+
Formulation of the conjecture		-	+	+	+	+
Determining the appropriate strategies	Using area relationship		+			
	Using Thales theorem				+	+
Performing the necessary operations			+		+	+
Summarizing			+		+	+

S2, S3, S4 and S5 could examine the problem situation. All of them draw triangle for proof. In addition, they formulated the conjecture clearly. S2, S4 and S5 identified a solution to the problem, performed the necessary operations, summarized the operations they performed and completed their proof. S4 and S5 used Thales's theorem while proving, and S2 reached the conclusion by approaching from a different perspective and writing the area relation on the triangles formed by the medians. S1 and S4 had a dilemma about whether the center of gravity was the cut-off point of the medians or the cut-off point of the bisector. And then while S4 reached the conclusion by establishing a relationship between what he learned by drawing, there was no definite conclusion about the S1 coded student's knowledge on this subject. As a result, S2, S4 and S5 completed all the proving process stages but only S1 could not complete any stages related to the proving.

The stages that five gifted students in the six proof problems were shown in Table 7.

Table 7. *Students' Proof Process Stages for all questions*

Questions	Stages	S1	S2	S3	S4	S5		Questions	Stages	S1	S2	S3	S4	S5
1.	1	+	+	+	+	+		4.	1	+	+	+	+	+
	2	+	+	+	-	+			2	+	+	+	+	+
	3	+	+	+	-	+			3	+	+	+	+	+
	4	+	+	+	-	+			4	+	+	+	+	+
	5		+						5	+	+	+	+	+
2.	1	-	+	+	+	+		5.	1	+	+	+	+	-
	2	-	+	+	+	+			2	-	+	-	-	-
	3		+		+	+			3	-	+	-	-	-
	4		+		+	+			4	-	+		-	
	5		+		+				5		+			
3.	1	+	+		+	+		6.	1	-	+	+	+	+
	2	+	+		+	+			2	-	+	+	+	+
	3		+			+			3		+		+	+
	4		+			+			4		+		+	+
	5								5		+		+	+

S2 completed the proving process as the most successful one and S3 as the least successful one. It could be said that the easiest question was the fourth one and the hardest question was the fifth one, according to the answers given by the students. Because in the fifth question, since they could not determine the hypothesis and the conclusion correctly, they used the wrong strategy and made wrong operations.

4. Conclusion, Discussion and Suggestions

The students, who were from different science high schools' students and attended the same Science and Art Education Centre and had the same opinion and knowledge about proof, were successful at different stages in the proving process. This may be due to school education or their proof work outside of school. Because the successful ones stated in the Proof Interview Form that they tried to prove outside the classroom and wondered how the rules they encountered were obtained. So, creating suitable environments for students to perform proof studies in mathematics or geometry lessons can increase their success in this subject. In addition, gifted students stated that they were hesitant about proof questions because they did not know how and where to start the proof in the Proof Interview Form. This result is also similar with the results of Moralı et al. (2006), Karaoğlu (2010), Yılmaz (2015) and Polat and Akgün (2016) with typical development students. The reasons why students do not know how and where to start a proof may be related to their not knowing the conceptual knowledge that should be used in the process of proving and not knowing how they should be used.

Considering the results of the Proof Clinical Interview Form, the gifted students could generally examine the problem situation and the gifted students who could examine the problem situation and formulate the conjecture completed the next stages comfortably. They determined wrong strategies and could not prove in the 5th question because they did not realize what they had to prove when they could not identify the hypothesis and the conclusion. Therefore, gifted students could not prove when they could not formula the conjecture clearly. This result is also in line with the studies of Uğurel and Moralı (2010), Demircioğlu and Polat (2016) and Polat (2018) conducted on students with typical development. The stages where gifted students had the most difficulty and made a lot of mistakes were the stages of determining appropriate strategies and performing the necessary actions. In these steps, they perform operations based on rote and try to complete the proof by trial method. So, the gifted students had deficiencies in pre-knowledge and mathematical knowledge. This situation is in line with result of Arslan (2007), Aylar (2014), Pekşen Sağır (2013), Polat (2018), Uğurel and Moralı (2010), Güler (2013), Yılmaz (2015), Yıldız (2019), Demircioğlu and Polat (2016), Baker (1996), Boero (1999), Özer and Arıkan (2002), Harel and Sowder (2007), Güler et al. (2011) conducted on students with typical development and Sriraman (2005) conducted with gifted students. They also stated that the students did not prove when they used the trial method, but only confirmed the theorem. Although some study groups thought that they completed the proof by accepting the trial method (Almeida 1996; Boero, 1999; Harel and Sowder, 2007; Güler et al., 2011; Pekşen Sağır, 2013), gifted students realized that they did not. In addition, most of the time was spent in the stages of determining the appropriate strategies and performing the necessary actions. This result was similar to the study of Heinze and Reiss (2004). Moreover, when the appropriate strategy to be determined for reaching from hypothesis to conclusion is given ready-made as in the 4th question, it has been seen that the proving is done without difficulty. The gifted students used summarize and generalization skills at least. In addition, they had deficiencies in mathematical language and notation. It could be because the students have not structured knowledge and used operations based on rote. This result is in line with some study of Moore (1994), Selden and Selden (2007), Uğurel and Moralı (2010), Güler (2013), Pekşen Sağır (2013), Yılmaz (2015), Polat (2018), Aylar (2014) and Arslan (2007) conducted on students with typical development. In addition, the students stated that after this study, their motivation to prove increased, they would try to prove the statements given as a formula in their lessons and if they could not prove, they would research and learn. This result is due to the characteristics of gifted students being curious and willing. As a result, gifted students used the examination and formulation skills mostly, but summarizing and generalization skills at least in proving process. Moreover, they made the most mistakes in the steps of determining the appropriate strategies and performing the necessary actions.

If differentiated teaching is done in proof teaching in line with the interests and wishes of gifted students, more successful results can be obtained by eliminating the students' deficiencies in this subject. Considering that the proof education is suitable for differentiated teaching and that the gifted students in this study also want to study proof, students can be provided with appropriate learning environments in proof teaching. So, in Science and Art Education Centre, necessary arrangements should be made in the curriculum prepared for gifted students and proof teaching should be included in the curriculum, taking into account the cognitive development levels of the students.

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Appendix 1.

Proof Interview Form

Dear Students,

The form below has been prepared to obtain your opinions related to your pre-learning for mathematical proof and your views on your readiness for proof. Answer three open-ended questions. Please, answer all questions and do not leave them blank. The data obtained from this research will be used in scientific research. Your name or other personal information will not be disclosed or declared at the time of publication of the findings and results. Please read the questions carefully and try to answer them objectively and meticulously. Thank you for your contribution.

Researcher: XXXXXXXX

1) What do you think the words theorem and proof mean? Explain.

2) Do you perform proof studies in math or geometry lessons? If so, on what subjects have you proved?

3) What do you think about whether proving practices should take place in your lessons or not?

Appendix 2.

Proof Clinical Interview Form

Name and surname:

Dear Students,

This clinical interview is prepared to examine the proving processes of gifted students. Since every answer you give is important for this research, I am sure that you will answer all questions sincerely. The data obtained from this interview will be used in scientific research and all the information you provide will remain confidential. Thank you for your contribution in advance.

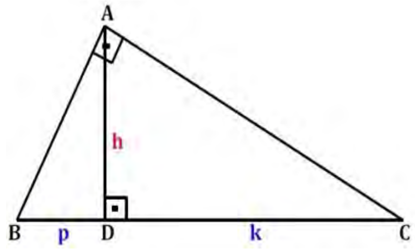
Researcher: XXXXXXXXXX

1- Prove that the sum of the dimensions of the angles of a triangle is 180 °.

2- For the natural number $n \geq 1$; prove that $1 + 2 + 3 + \cdots + n = \frac{n.(n+1)}{2}$

3- Prove that in order for any integer to be divided by 3 without a remainder, the sum of the numeral values of the digits of the number must be 3 or a multiple of 3.

4-



Prove the Euclidean Theorem which is $h^2=p.k$ by using the Pythagorean Theorem.

5- For *a, b* > 0, prove that $\frac{a}{b}+\frac{b}{a} \geq 2$

6- Prove that the distance from the center of gravity to a corner in a triangle is equal to $\frac{2}{3}$ of the median length drawn from the same corner to the opposite edge.