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Research Article

Relationship between middle school students' metacognition and problem-posing performance: A serial mediation analysis

Kübra Ada Yıldız[1] and Mesut Öztürk[2]

[1]*Bursa Uludag University,Faculty of Education, Bursa, Türkiye (ORCID: 0000-0001-6243-9703)*
[2]*Bayburt University, Faculty of Education, Bayburt, Türkiye (ORCID: 0000-0002-2163-3769)*

This study tests a model that predicts the problem-posing performance of sixth-grade students (independent variable metacognition, mediator variables mathematics and native language course success scores at the end of the year). The research was conducted with the relational model. The results regarding the mediation effect indicated that the mathematics course success scores and the native language course success scores were separately full mediators in the effect metacognition has on problem-posing performance. The results regarding serial mediation effect revealed that the mathematics course success scores and native language course success scores had a full mediating effect on the effect of metacognition on problem-posing performance.

Keywords: Problem posing; Metacognition; Mathematics; Native language; Mediator variable

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1. Introduction

Problem posing is one of the significant concepts of mathematics education, because students' focus on the problem-posing process aids problem solving by showing them how to solve the problem (Cai, 2003). According to Koichu (2020), problem posing can also be considered as a powerful tool for teaching through problem solving and problem solving. The literature regarding problem posing has revealed that problem posing affects conceptual understanding, creativity, problem solving and reasoning skills (Akay et al., 2006; English, 1998; Kar, 2014). Thus, it is important to improve students' problem-posing performance. Studies have indicated that to improve students' problem-posing performance, their metacognition and academic achievements in the mathematics and native language courses should be developed. Also, many studies have shown that these variables are interrelated. For example, Zhao et al. (2019) found a significant relationship between metacognition, the mathematics course achievement score and reading score. They also found that students with higher metacognitive levels have higher performance in problem solving, reading and learning mathematics. Additionally, many studies have revealed that metacognition, mathematics course success or native language course success affect problem posing. However, according to our research, there was no study in literature that reviews all these

Address of Corresponding Author

Kübra Ada Yıldız, Bursa Uludag University, Gorukle Campus 16059 Nilufer, Bursa, Türkiye.

kubrada.16@gmail.com

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variables and examines them as a whole. Yet, evaluating these variables together, which are stated to have a significant effect on problem-posing performance, is considered relevant in terms of organising teaching activities and the learning environment. In this context, this study examines the effects of metacognition, the mathematics success and native language (Turkish) success variables on problem posing together. It is anticipated that the results obtained from the study will provide an interdisciplinary perspective for future studies to develop problem-posing skills.

2. Theoretical Framework

2.1. Problem posing

Problem posing is the ability to create a new problem based on a given situation and certain information (Silber & Cai, 2017). The National Council of Teachers of Mathematics [NCTM] (2000) stated that problem posing is an important component for students' mathematical development. Problem posing requires more comprehensive reasoning and mental processes than problem solving (Çıldır & Sezen, 2011). Thus, problem posing is more beneficial than problem solving in students' internalisation of mathematical concepts and usage of the mathematical language (Akay et al., 2006). Studies have indicated that metacognition, together with intrinsic characteristics (such as motivation, control, willingness and reinforcement), are important factors for students to pose problems. For example, Karnain et al. (2014) emphasised that metacognition is a critical element for problem posing. The authors claim that students with improved metacognitive skills will also have developed problem-posing skills.

2.2. Metacognition and Problem Posing

Individuals develop and regulate certain mental strategies in the process of problem solving and posing. One's thoughts about their own thinking processes and cognition are called metacognition (Flavell, 1979). According to another definition, metacognition is defined as being aware of and controlling the mental activities involved in the individual's perception, recollection and thinking (Brown, 1978). Metacognition consists of two basic dimensions: knowledge of the individual's own cognitive processes (knowledge of cognition) and monitoring and control of cognitive processes (regulation of cognition) (Mazzoni & Nelson, 1998). The knowledge of cognition dimension is defined as the individual's knowledge of how he/she performs cognitive operations. Regulation of cognition is the ability to strategically use metacognitive information to achieve cognitive goals (Brown, 1978).

Metacognition can be defined as the individual's regulation of his/her own cognitive processes. In this context, students with improved metacognition can be expected to be aware of their own learning process and monitor, evaluate and organise their own learning (Öztürk & Kaplan, 2019). Students who are aware of their own cognitive process can have better problem solving and problem-posing skills by developing different ways of thinking and being aware of how they think (Öztürk, Akkan & Kaplan, 2018). Some researchers emphasised the importance of metacognition by stating that it is directly related to problem solving or posing. For example, McCormick (2003) expressed metacognition as the cognitive characteristics that the individual displays in the process of solving the problem. Similarly, while forming a new problem from what is presented in the problem-posing process, it is believed that the individual should know his/her own cognitive processes (knowledge of cognition) and plan, monitor and control cognitive processes (regulation of cognition). It is stated that problem posing requires a much more comprehensive reasoning ability and mental process than problem solving (Çıldır & Sezen, 2011). In this context, it can be assumed that the development of metacognition will lead to the development of students' problem posing skills. For example, Ghasempour et al. (2013) stated that metacognitive strategies are important notions that can help students develop problem-posing skills. Karnain et al. (2014) found that middle school students who do metacognitive planning and monitoring have a higher level of problem-posing skills.

While metacognition may have a direct effect on problem posing, different variables may also play a mediating role in this effect. Mathematics course success and native language course success can be considered two important mediating variables in this regard as students need to use their knowledge of mathematics and express their problem statement clearly in their native language to pose problems. Özgen et al. (2019) stated that students cannot pose problems mainly due to inadequacies in mathematical skills and language use. Since all students participating in this study are native language, native language course success was reviewed as the native language course success.

2.3. Relation between Metacognition, Success in Mathematics and Problem Posing

In addition to the individual's ability to pose a problem, it is also important to improve the quality of the established problem. Silver and Cai (1996) reviewed the nature of a problem in the context of 'linguistic', 'mathematical complexity' and 'solvability'. In other words, situations such as solvability of a problem, meaningful solution, understandable language and expression, correct use of mathematical language and high mathematical complexity assert the nature of the problem. It is believed that mathematics success plays an important role in increasing the mathematical quality of the posed problem in this regard because the individual must have sufficient mathematical knowledge about the solution of the problem to pose a solvable problem. At the same time, they must know the mathematical symbols and expressions and use them correctly to convey the problem they have designed properly (Örnek & Soylu, 2021). The mathematical complexity of the problem is undoubtedly shaped in proportion to the internalisation of mathematical knowledge and concepts. As a matter of fact, Türnüklü et al. (2017) stated that problem posing requires students to understand the problems they have solved before, gather mathematical knowledge and internalise mathematical concepts. This situation reveals the necessity of success in mathematics to increase the problem-posing performance. In this context, Özgen et al. (2017) determined that students with high achievement in mathematics are more successful in problem-posing activities. Nicolaou and Philippou (2007) revealed that there is a strong relationship between problem-posing skills of fifth- and sixth-grade students and their success in mathematics. At the same time, some studies determined that lessons designed with a problem-posing approach positively affect students' mathematics success (Akay, 2006; Özdemir & Sahal, 2018). In this respect, it is important to increase students' success in the mathematics course to increase their problem-posing performance.

The problem-posing process, in addition to mathematical knowledge (success in mathematics), requires organising and controlling the information (metacognition). In other words, for students to create a qualified and solvable problem, it is important for them to be aware of their mathematical knowledge and plan, monitor and control their cognitive processes. Thus, it is believed that students' mathematical knowledge should be supported by metacognitive processes for the established problem to be of expected quality. In this regard, literature puts forth the presence of an important relationship between metacognition and mathematics course success (Hassan & Rahman, 2017; Kaya, 2019; Maqsud, 1998). Similarly, Öztürk and Kurtuluş (2017) determined that individuals with a school report with high grades in the mathematics course also have a high level of metacognitive awareness. Additionally, Areepattamannil and Caleon (2013) revealed that there is a positive relationship between the control strategy, one of the metacognitive strategies, and success in mathematics. In other words, according to literature, while metacognition increases success in mathematics lesson, success in mathematics also affects problem-posing performance.

2.4. Relation between Metacognition, Success in Native Language and Problem Posing

Another factor is considered effective in problem posing is success in terms of language and expression. Silver and Cai (1996) stated that the 'linguistic' criterion in problem-posing performance is one of the significant factors that reveal the nature of the problem, because, unlike problem solving, in the process of problem posing, a new problem statement is provided in line with the presented information. This situation requires the individual to be successful in terms of language a problem posing expression as well as understanding what they have read. Özgen et al. (2019) stated that the main reasons for students not being able to pose a problem stemmed from their inadequate mathematical skills and language use. Similarly, Arıkan and Ünal (2013) determined that students with low problem-posing performance cannot use the native language well in their problem statements. Additionally, Özgen et al. (2017) put forth that lessons other than mathematics also had a positive effect on problem-posing activities. He stated that students' success particularly in understanding what they read, writing and expressing their thoughts in Native language lesson will also increase their success in problem-posing activities. As the researchers stated, the knowledge of using the native language, with features such as linguistic, wording and expression, along with mathematics knowledge, is also important in problem-posing performance.

In the process of posing a problem, students make metacognitive adjustments in line with the information and instructions presented in the question (Ghasempour et al., 2013). For the students to pose a problem that is of expected quality, they should understand the presented information and the intention of the questions correctly and perform cognitive planning. Even if students who do not understand what they have read sufficiently can pose a problem by following a good metacognitive process, the problem they pose may not be in accordance with the information and instructions presented. In addition, it is also important that the problem statement created by the students is clear and understandable (Silver & Cai, 1996). In this case, it is thought that the knowledge of using the native language is also necessary for students to pose problems with the desired quality by using their metacognition. In studies conducted on the relationship between metacognition and success in the native language course, it is stated that metacognitive skills have a positive effect on reading and comprehension (Bonds et al., 1992; Çetinkaya & Erktin, 2002). Similarly, Tice (1991) determined that metacognitive skills in the sixth grade increased reading, reading comprehension, and writing competencies. Yıldız (2015) revealed that students' metacognitive awareness levels directly affected their native language course success scores. In this context, according to the studies in the literature, while metacognition increases success in the native language course, native language course success also affects the problem-posing performance. Since the research was carried out with native language students, this study considered the success in the native language course as the knowledge of using the native language.

Literature indicates that success in the two relevant disciplines, mathematics and native language, is related to metacognition and problem-posing ability. In addition, many studies reveal the existence of a significant relationship between individuals' success in mathematics and ability to use the native language (Albayrak & Erkal, 2003; Jordan et al., 2002; Reusser & Stebler, 1997; Räty et al., 2004; Taşdemir & Taşdemir, 2011). Güneyli et al. (2010) determined that success in the native language course is mostly related to success in the mathematics course.

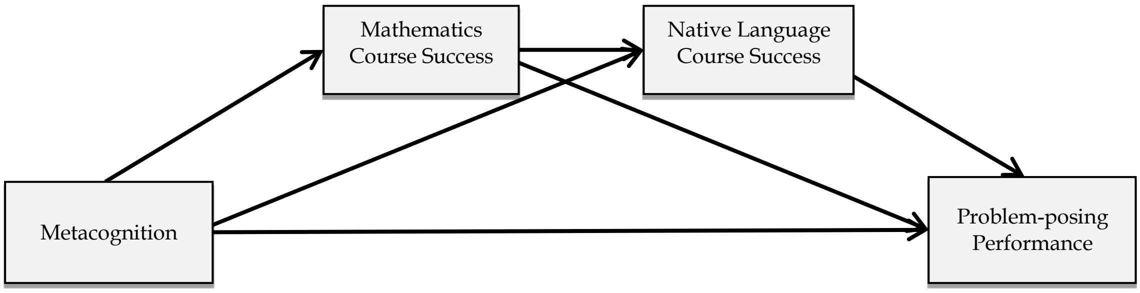
3. Literature Review

According to the literature review, problem solving has been studied extensively (Artz & Armor-Thomas, 1992; Desoete et al., 2001; Öztürk et al., 2018), but the number of studies on problem posing has increased in recent years. The samples of problem-posing studies in literature mostly comprised pre-service teachers and eighth-grade students. In this context, it can be said that the sample of this study conducted with sixth-graders differs from previous studies in terms of grade level. At the same time, to determine the problem-posing performance of students within the scope of the research, the study is based on the learning domains of natural numbers and operations. Natural numbers form the basis of the learning process in relation with other numbers such as integers, decimals and rational numbers. Additionally, the situations that require four operations in natural numbers are also used in other learning areas such as algebra and geometry and measurement. In this context, operations with natural numbers are relevant in that they constitute the foundation for sub-learning and other learning areas. It is believed that problem-posing studies to be carried out in this sub-learning area will provide support to other learning areas for students.

Literature determines that studies on problem posing are mostly qualitative, while quantitative studies are insufficient. Existing quantitative studies were found to be mostly empirical studies based on problem-posing activities (Bae & Park, 2016; Lee & Han, 2018; Oh & Jeon, 2018; Yıldız, 2014). It was observed that a limited number of relational screening studies aimed at determining the variables related to posing problems (Çelik & Yetkin-Özdemir, 2011; Ganioğlu & Cihangir, 2019; Karnain et al., 2014). Conducting research to determine the variables that affect problem-posing performance is essential for designing activities to improve problem-posing performance. When studies regarding the relationship between metacognition and problem-posing performance, specifically, were examined, very few studies were found (Karnain et al., 2014). However, in the process of posing a problem, it is believed that individuals should become aware of their mental activities and ensure the control of these activities. For this reason, it is significant to make a detailed examination by revealing the direct and indirect relationships between students' metacognition and their problem-posing performances. Karnain et al. (2014) examined how students use their metacognitive skills in the problem-posing process. As a result of the study, it was determined that students who combined the planning and monitoring dimensions of metacognition had a higher level of problem-posing skills. Ghasempour et al. (2013) stated that metacognitive strategies are influential in developing students' problem-posing skills. Yıldız (2014), in a study conducted with pre-service mathematics teachers, concluded that problem-posing practices significantly increased their metacognitive awareness. At the same time, it was determined in many studies in literature that these two concepts are related to native language and mathematics lessons.

The literature review determined that there are significant relationships between metacognition, problem-posing ability, native language course success and mathematics course success. This study evaluates the relationships between these concepts as a whole. According to the studies conducted, it is believed that native language and mathematics course success scores may have a mediating effect on the relationship between students' metacognition and their problem-posing performances. Similarly, Hassan and Rahman (2017) examined the mediating role of metacognitive awareness in the effect of problem solving skills on mathematics success in middle school students. This study determines that problem solving and metacognitive awareness were effective in mathematics success and that metacognitive awareness had a mediating effect. It is believed that a similar relationship is applicable to problem posing as well as problem solving skills. However, unlike the study of Hassan and Rahman (2017), in this study, the effect of metacognition on the problem-posing ability was examined by considering the mediating role of mathematics success. Because the ability to pose a problem requires the individual to know and organise his/her own cognitive processes while creating a new problem based on the information presented. In this respect, it is believed that metacognition improves problem-posing skills. At the same time, situations such as the posed problem having strong mathematical aspects, being solvable, having understandable language and expressions require the individual to be successful in mathematics and native language. Therefore, this study examines the effect of metacognition on problem-posing performance as well as that of mathematics and native language success. In this context, this study examined the serial mediation effect of sixth grade students' academic success in mathematics and native language courses on the relationship between metacognition and their problem-posing performances. For this purpose, Figure 1 presents the prediction model tested in the study.

Figure 1
Serial mediation effect prediction model



The following hypotheses have been tested analyse this prediction model:

H_1: The metacognition of sixth grade students affects their problem-posing performance.

H_2: The mediating effect of mathematics course academic success in the relationship between the metacognition of sixth-grade students and their problem-posing performance is significant.

H_3: The mediating effect of native language course academic success in the relationship between the metacognition of sixth-grade students and their problem-posing performance is significant.

H_4: The serial mediation effect of academic success in mathematics and native language courses, respectively, is significant in the relationship between sixth-grade students' metacognition and problem-posing performance.

H_5: The serial mediation effect of academic success in native language and mathematics courses, respectively, is significant in the relationship between sixth-grade students' metacognition and problem-posing performance

4. Method

4.1. Research Model

This study was conducted with a correlational research, one of the quantitative research designs. The correlational research is a model that examines the existence of co-change (relationship) between two or more variables and reveals the degree of the relationship, if any (Fraenkel et al., 2012). It is difficult to talk about the presence of a cause-effect relationship between variables in relational research. However, in correlational studies using different complex methods, some inferences can be made about the cause-effect relationship between variables (Fraenkel et al., 2012). One of these methods is the structural equation model. Structural equation modelling enables testing the mediating variable models created to examine the indirect effect between variables and examining causal relationships. The structural equation model is superior to other models in terms of examining the causal relationships (Tüfekçi & Tüfekçi, 2006). Since this study examines the mediating effect of native language and mathematics course academic success in the relationship between the metacognition of middle school students and their problem-posing performance, causal relationships and mediating effects should be presented. Thus, this study used the correlational research, which includes the structural equation model, and examined the total and direct effect and the indirect effect caused by the influence of mediator variables in the relationship between students' metacognition and their problem-posing performances.

4.2. Sample

Operations with natural numbers form the foundation of the learning process for other numbers. In this regard, students' performance in problem posing, which requires four operations with natural numbers, will lay the groundwork for the problem-posing performance for other learning areas. In the middle school mathematics curriculum, the learning goals for posing problems related to operations in natural numbers are presented in the sixth grade (Turkish Ministry of National Education [MoNE], 2018). For this reason, sixth-grade students were chosen by considering the mathematics curriculum in the selection of the sample. At the same time, it is known that sixth-grade students have prior experience in problem posing, considering that problem-posing outcomes for operations with natural numbers are also present at the primary school level.

This study was conducted with 121 volunteer sixth grade (12 years old) students. The sample of the study was formed according to typical case sampling, one of the purposeful sampling methods. In this regard, first, schools in the city centre, which do not carry any extraordinary features and show an ordinary typical case in terms of size, physical state and equipment facilities, were determined. Then, official permissions were obtained and students who volunteered to participate in the study were determined through consultations with school principals. The activity was carried out with the students for one hour (60 minutes) outside of school hours. All the participants are native language and speak native language as their native language. All students study at public schools. While 53 (43.8%) of the students are female, 68 (56.2%) are male. At the same time, considering the socio-economic level of the students' families; 17 (14.04%) are from low-income families, 85 (70.25%) are from families with average income, and 19 (15.70%) are from high-income families.

4.3. Instruments

In the process of collecting research data, the 'Metacognition Scale' and 'Problem-posing Test' were used. For the mathematics and native language course academic success scores, students' end-of-the-year success score averages related to these courses were considered.

4.3.1. Metacognition scale

The Metacognition Scale developed by Yıldız et al. (2009) was applied to determine students' metacognition. The TOPSIS method was used to determine the study's metacognition scale and frequently used to determine the ideal option using certain criteria among various objects (Boran et al., 2009). The TOPSIS method is one of the multi-purpose decision-making methods that allow making the best choice among alternatives (Özdemir, 2014). TOPSIS is a simple and easy-to-understand method because it does not contain complex algorithms and mathematical models. Since the results obtained in the TOPSIS method are evaluated according to their proximity to +1, the results are easy to interpret. Within the scope of the study, scales for measuring the metacognitive awareness of middle school students were scanned. Five scales were determined as a result of the scanning. These scales were considered as alternatives. The scales considered are as follows: First alternative [A1] developed by Çetinkaya and Erktin (2002), second alternative [A2] adapted to Turkish by Irak (2012), third alternative [A3] adapted to Turkish by Aydın and Ubuz (2010), fourth alternative [A4] developed by Yıldız et al. (2009) and fifth alternative [A5] adapted by Karakelle and Saraç (2007). To evaluate these alternatives, the criteria of reliability coefficient (G1), lowest correlation in factor loadings (G2), explained variance (G3), number of participants (G4) and number of questions (G5) were used. The researchers assigned the weights for these criteria as .30, .25, .20, .15 and .10, respectively, according to the importance level of the criteria. The TOPSIS method evaluation results found that the distance values of the alternatives were .10, .12, .51, .88 and .62, respectively. In this context, the closest calculated value to +1 was [A4]. This situation indicates that the ideal scale for determining middle school students' metacognition is the scale developed by Yıldız et al (2009).

The scale developed by Yıldız et al. (2009) includes two basic components: 'knowledge of cognition' and 'regulation of cognition', and eight dimensions belong to these components. The scale, which has a 4-point likert type structure, contains the levels of 'always', 'often', 'sometimes' and 'never'. The scale development study involved 426 students at the primary education level. The study conducted explanatory and confirmatory factor analysis to determine the construct validity. The corrected item total correlations of the scale were between 0.49 and 0.81. The Cronbach alpha internal consistency coefficient for the whole scale comprising 30 items was .96. The Cronbach alpha internal consistency coefficients determined by Yıldız et al. (2009) for the dimensions in the scale are as follows: Explanatory information (α= .93), methodological knowledge (α= .85), conditional knowledge (α= .79), planning (α= .78), control (α= .74), cognitive strategies (α= .76), self-assessment (α= .64) and self-monitoring (α= .69).

To determine the reliability of the data collected within the scope of the study, there was a re-examination of the Cronbach alpha internal consistency coefficient of the metacognition scale. The internal consistency coefficient of the scale, which was applied to 121 students studying in the sixth grade, was calculated as .94. Thus, it was determined that the metacognition scale was applicable at the sixth grade in line with the aims of the study. Two of the items included in the scale are: 'When answering questions, I check whether I am doing it right' and 'When I encounter a problem, I think of many solutions and choose the best'.

4.3.2. Problem-posing test and assessment rubric

The problem-posing test developed by Ada and Öztürk (2019) was used to measure students' performance in different problem-posing situations. The test included 10 questions that require four operations with natural numbers. The questions were prepared based on five different problem-posing situations put forward by Stoyanova and Ellerton (1996) and rearranged and improved by Christou et al. (2005). In the test, there are two questions for each of the 'free', 'arrangement', 'transferring', 'comprehension' and 'choice' problem-posing situations. To determine whether the questions adequately reflect different problem-posing situations, expert opinion was obtained. At the same time, balanced distribution was provided for the type of operation by including questions that require each of the four operations in natural numbers within the scope of the test. For the questions to be suitable for the level of students, activities and exercises for the learning goals in the existing textbooks were considered. Accordingly, content validity was increased by including items suitable for the purpose of the test. In addition, researchers paid attention to the items and instructions in the test to be comprehensible. It was determined that the Cronbach alpha reliability coefficient (α= .73) calculated for the internal consistency of the problem-posing test was sufficient (Büyüköztürk, 2002). In addition, the item discrimination and item difficulty indexes of the test were calculated. Although the subgroup and upper group t values of the items were at least 3.91 and at most 14.08, it was determined that the distinctiveness of each item between the lower group and the upper group was significant at the $p<.01$ level. It was determined that the values related to the item difficulty index ranged between .21 and .67 and were mostly valued at around .50. It was determined that the values obtained were appropriate and sufficient for the item difficulty index and that each item was distinctive at a significant level. Two questions in the problem-posing test are presented as examples.

Example question 1:

Form a problem that will require the following procedure during its solution.

$(150-70)\ x\ 4$

Example question 2:

	Number of people	Fee for the Theatre (Per person)
Student	60	6 TL
Teacher	9	11 TL

Choose what you want from the information presented in the table above and set up a problem that requires only **multiplication** and **subtraction** in its solution.
(Note that the solution requires **both** *multiplication and subtraction* processes)

A rubric developed by Ada and Öztürk (2019) consisting of two parts was used to evaluate the performances obtained from the problem-posing test. The answers to the questions in the first part are categorised as blank (0 points), not a problem (0 points), and a problem. The answers that are found to be problems are evaluated according to the dimensions and criteria in the second part. The second part of the rubric consists of the 'linguistic', 'mathematical complexity' and 'solvability' dimensions. In the linguistic level dimension, the criteria of 'comprehensibility of the problem', 'mathematicality' and 'compliance with the information and instructions given in the question' are included. In the dimension of mathematical complexity, there exist the criteria of 'structure of the problem' and 'originality'. In the solvability dimension, 'the quality of the data and the solvability' criterion is examined. The rubric, comprising three dimensions and six criteria, includes evaluation items corresponding to certain scores for each of the criteria.

The concordance coefficients of Krippendorff Alpha between raters (2 raters) regarding the dimensions of the rubric were valued between .55 and .80. In this context, when the interrater reliability was evaluated, it was determined that there was a 'high' level in the solvability dimension, a 'medium' level in the mathematical complexity dimension, and a 'weak but acceptable' level in the linguistic dimension. Looking at Cohen's Kappa coefficient values, it was determined that the linguistic dimension (κ= .54) had 'medium', the mathematical complexity k dimension (κ= .75) and the solvability dimension (κ= .80) had significant reliability. In this context, it is understood that the rubric's compatibility and reliability values between raters are appropriate and sufficient. The correlation coefficient (.99) between raters for rubric was determined close to perfect and significant. Additionally, the correlation coefficient between raters in all dimensions and criteria of the rubric is also highly significant. In this context, it is understood that there is consistency between the two raters (Ada & Öztürk, 2019).

4.4. Data Analysis

The first stage of data analysis examined the consistency between different raters for the reliability of the scores obtained from the problem-posing test. Accordingly, of the 121 students in the sample of the study, 24 students were randomly selected. The problem-posing test answered by these students was evaluated by two expert raters. The Pearson Product-Moment Correlation Coefficient (.97) between different raters regarding the problem-posing test was found to be high. Accordingly, the scores obtained from the problem-posing test within the scope of the study were reliable.

After testing the rater reliability of the problem-posing test in the study, univariate and multivariate normality cases were examined on all variables within the scope of the study. For the case of univariate normality, kurtosis and skewness coefficients of the continuous variables were examined. The skewness coefficient (SC) and kurtosis coefficient (KC) values of the variables were (SC= −.23; KC= −.63) for problem-posing test scores, (SC= −.32; KC= −.96) for the metacognition scores, (SC= −.67; KC= −.53) for the native language course academic success scores and (SC= −.42; KC= −.98) for the mathematics course academic success scores. In this context, it can be said that the univariate normal distribution assumption is met for the continuous variables. Multivariate normality was analysed by calculating Mahalonobis distance coefficients. Pearson and Hartley (1958) stated that the Mahalonobis distance values for the four predictive variables should be within the limit of 18.47. As a result of the examinations, for the four predictor variables, the min. (.63) and max. (17.38) Mahalonobis distance values show that the multivariate normality assumption is fulfilled. In the third stage of data analysis, correlation coefficients were calculated to determine the relationship between variables. In the fourth stage, the direct and indirect effects of the independent variable on the dependent variable were investigated. In this relationship, the PROCESS macro developed for SPSS and based on Bootstrap sampling was used to determine the significance of the mediating variables. During this analysis, bias corrected bootstrap confidence intervals (5000 resampling) were calculated asthe 99% confidence level. At the same time, the statistical significance of mediating effects was examined with the Sobel test.

Mediating variable analysis can be performed by performing a series of regression analyses as well as the PROCESS macro. However, since the indirect effects can not be examined with regression analysis, the explained variance for mediation effects is found to be low. However, in multivariate model analysis, the explained variance is very important. While one purpose of these analyses is to examine how much of a difference a one-unit change in each of the independent variables will cause, the other is to determine how much of the change in the dependent variable is explained by these independent variables (Dursun & Kocagöz, 2010). In this context, the PROCESS macro was preferred for the data analysis of the study, as it has a feature that determines confidence intervals based on Bootstrap sampling, examines the indirect effects in the relationship between variables and provides ease of operation in the analysis process.

Within the scope of the study, while metacognition was the independent variable (X), problem-posing performance was considered the dependent variable (Y). The mediator variables (M) are students' success in native language and mathematics lessons.

5. Results

5.1. Relationships between Problem-posing Performance and Metacognition, Native language Course and Mathematics Course Academic Success Scores

Table 1 presents the relationships between middle school students' metacognition, problem-posing performances, Native language course academic success scores and mathematics course academic success scores.

Table 1
Correlation coefficients between variables

		Bivariate Correlation				*Descriptive Statistics*		
		1	2	3	4	*n*	*Mean*	*SD*
1	Metacognition	–				121	96.5	16.16
2	Problem-posing Performance	.31**	–			121	90.6	41.65
3	Native language Course Success	.37**	.53**	–		121	76.1	14.67
4	Mathematics Course Success	.41**	.45**	.87**	–	121	70	20.24

Note. *p<.05, **p<.01

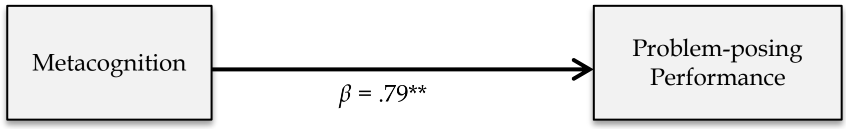
When Table 1 is examined, a positive meaningful relationship between students' metacognitions and problem-posing performance (r =.31, p <.01), native language course academic success score (r =.37, p <.01), and mathematics course academic success (r =.41, p <.01) is noted. At the same time, there is a positive, significant and moderate relationship between students' problem-posing performance, native language course academic success score (r =.53, p <.01), and mathematics course academic success score (r =.45, p <.01). It was determined that the relationship between students' native language and mathematics course academic success scores (r =.87, p <.01) was high, positive and significant. In this context, it can be said that there is a significant and positive relationship between all variables.

5.2. Mediating Effect of Mathematics Course and Native language Course Academic Success Scores on the Relationship between Metacognition and Problem-posing Performance

In this section, first, the total effect of students' metacognition on problem-posing performance was examined. Then, the mediating role of the mathematics and native language course academic success scores in the effect of metacognition on problem-posing performance was discussed separately. Finally, the significance of the serial mediating roles of the two mediator variables was examined. Figure 2 presents the findings regarding the total effect of metacognition on problem-posing performance.

Figure 2 shows that the total effect of students' metacognition on their problem-posing performance is significant (β = .79, p <.01). In other words, metacognition is a significant predictor of problem-posing performance. In this regard, the first hypothesis of the research was accepted. The mediating role of students' mathematics course success in the effect of metacognition on problem-posing performance was examined, and the findings obtained are given in Table 2.

Figure 2
Total effect of metacognition on problem-posing performance



Note. **p< .01

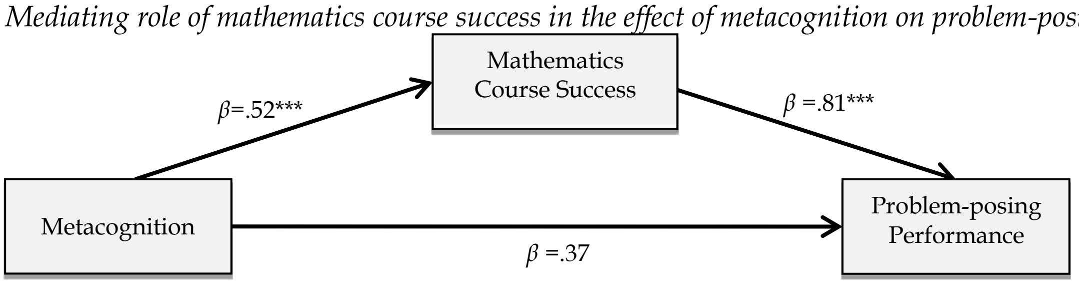
Table 2
Mediating role of mathematics course success in the effect of metacognition on problem-posing performance

	Outcome variables			
	→ M (Mathematics course success)		→ Y (Problem posing)	
Predictor variables	*β*	*SE*	*β*	*SE*
Constant	20.29*	10.24	−2.09	20.79
X (Metacognition)	.52***	.11	.37	.23
M (Mathematics C.S.)			.81***	.18
R^2	.17		.22	
F	24.21***		16.95***	
Bootstrap	Metacognition → Mathematics course success → Problem posing			
Indirect effect	K^2= .16 *β*= .42 SE= .12 %99 CI [.14, .74]			

*Note.**p<.05; **p<.01; ***p<.001; n=121; CI: Bootstrap lower and upper confidence interval. SE: Standard error. K^2: Fully standardised effect size. Bootstrap resampling= 5000. Non-standardised beta coefficients (*β*) were reported.

According to Table 2, the effect of metacognition on academic success in the mathematics course, which was determined as the mediator variable, was significant (*β*= .52, p <.01). At the same time, students' academic success in the mathematics course has a significant effect on problem-posing performance (*β*= .81, p <.01). Additionally, the indirect effect of students' metacognition on their problem-posing performance through their mathematics course success is also significant (*β*= .42; CI= [.14, .74], SE= .12). After examining the mathematics course success determined as a mediator variable, it was found that metacognition did not have a significant effect on problem-posing performance (*β*= .37, p >.01). In other words, while the total effect of students' metacognition on their problem-posing performance and the indirect effect through mathematics course success are significant, the direct effect of mathematics course success is not significant. In this case, it is observed that the mediator effect of mathematics course academic success is statistically significant and plays a full mediating role in the effect of students' metacognition on their problem-posing performance. The fully standardised indirect effect size (K^2= .16) of students' metacognitions on their problem-posing performance was determined to be significant and at a high level. In addition, as a result of the Sobel test, it was found that the academic success of students in mathematics course played a full mediating role (Sobel z = 3.29, p <.01) on the effect of metacognition on their problem-posing performances. The second hypothesis of the research was approved in line with these findings. Figure 3 presents the confirmed mediating effect model.

Figure 3
Mediating role of mathematics course success in the effect of metacognition on problem-posing performance



Note. ***p<.001

Within the scope of the study, as another mediating effect model, the mediating role of the native language course academic success in the effect of students' metacognition on their problem-posing performance was examined. The findings obtained are presented in Table 3.

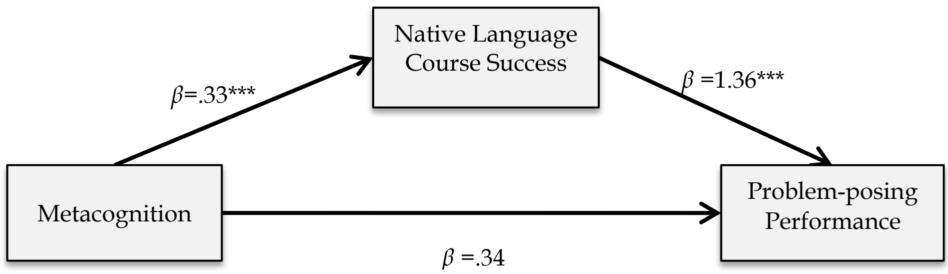
Table 3
Mediating role of native language course success in the effect of metacognition on problem-posing performance

	Outcome variables			
	→ M (Native language course success)		→ Y (Problem posing)	
Predictor variables	*β*	*SE*	*β*	*SE*
Constant	44.10***	7.58	−45.45*	22.14
X (Metacognition)	.33***	.08	.34	.22
M (Native language C.S.)			1.36***	.24
R^2	.13		.29	
F	18.29***		24.29***	
Bootstrap Indirect effect	Metacognition → Native language course success → Problem posing K^2= .17 *β*= .45 SE= .12 %99 CI [.19, .80]			

*Note.**p* <.05; ***p* <.01; ****p* <.001; *n*=121; CI: Bootstrap lower and upper confidence interval. SE: Standard error. K^2: Fully standardised effect size. Bootstrap resampling= 5000. Non-standardised beta coefficients (*β*) were reported.

Considering the findings in Table 3, the effect of students' metacognition on native language course academic success, which is determined as the mediator variable, is significant (*β*= .33, *p* <.01). In addition, it was determined that students' academic success in the native language course had a significant effect on problem-posing performance (*β*= 1.36, *p* <.01). In addition, the indirect effect of students' metacognitions on their problem-posing performance through their native language course success is also significant (*β*=.45; CI= [.19, .80], SE= .12). When the native language course academic success was examined, it was determined that metacognition did not have a significant effect on problem-posing performance (*β*= .34, *p* >.01). In other words, while the total effect of metacognition on problem-posing performance and its indirect effect through native language course success are significant, the direct effect that occurs upon examining native language course success is not significant. In this case, it was determined that students' academic success in the native language course played a full mediating role in the effect of students' metacognition on their problem-posing performance. The fully standardised indirect effect size (K^2=.17) of students' metacognition on the problem-posing performance was determined to be significant and at a high level. In addition, this mediation model is supported by the results obtained from the Sobel test (Sobel *z* =3.43 *p* <.01). In line with the findings obtained, the third hypothesis of the research was approved. Figure 4 presents the confirmed mediation model.

Figure 4
Mediating role of native language course success in the effect of metacognition on problem-posing performance



Note. ****p*<.001

Within the scope of the study, the serial mediating role of academic success in the mathematics and native language courses in the effect of metacognition on problem-posing performance was examined. In this context, the findings obtained regarding the serial mediating role of mathematics course success and native language course success, respectively, on the effect of metacognition on problem-posing performance are presented in Table 4.

Table 4

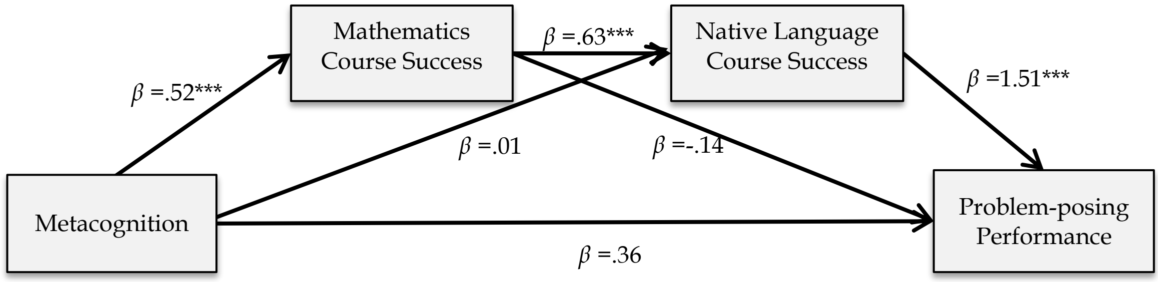
Serial mediating role of mathematics and native language course success in the effect of metacognition on problem-posing performance

	Outcome variables					
	→ M_1 (Mathematics C.S.)		→ M_2 (Native language C.S.)		→ Y (Problem posing)	
Predictor variables	*β*	*SE*	*β*	*SE*	*β*	*SE*
Constant	20.29*	10.24	31.38***	4.11	−49.61*	24.35
X (Metacognition)	.52***	.105			.36	.22
M_1 (Mathematics C.S.)			.63***	.04		
M_2 (Native language C.S.)					1.51***	.45
R^2	.17		.76		.29	
F	24.21***		182.19***		16.15***	
Bootstrap Indirect effect	Metacognition → Mathematics C.S. → Native language C.S. → Problem posing K^2= .19 *β* = .49 SH= .17 %99 CI [.13, 1.06]					

*Note.***p* <.05; ***p* <.01; ****p* <.001; *n*=121; CI: Bootstrap lower and upper confidence interval. SE: Standard error. K^2: Fully standardised effect size. Bootstrap resampling= 5000. Non-standardised beta coefficients (*β*) were reported.

Considering the findings in Table 4, the effect of students' metacognition on academic success in the mathematics course, which is determined as the first mediator variable, is significant (*β*= .52, *p* <.01). Additionally, the effect of students' academic success in the mathematics course on native language course academic success, which is determined as the second mediator variable, is significant (*β*= .63, *p* <.01). In other words, in this serial mediation model, mathematics course success is the predictor of native language course success. At the same time, it was determined that students' native language course success has a significant effect on problem-posing performance (*β*= 1.51, *p* <.01). In addition, the indirect effect of students' metacognition on their problem-posing performance through their mathematics and native language course success, respectively, is significant (*β*= .49; CI= [.13, 1.06], SE = .17). However, after examining mathematics course success and native language course success, which were determined as mediator variables, it was determined that metacognition did not have a significant effect on problem-posing performance (*β*= .36, *p* >.01). In other words, while the total effect of students' metacognition on their problem-posing performance and the indirect effect through their mathematics and native language course success are significant, the direct effect that occurs when the mediator variables are examined is not significant. In this case, it was determined that the effect of students' metacognition on their problem-posing performance was significant, and the serial mediation effect of students' academic success in the mathematics and native language courses, respectively, was significant and played a full mediating role. At the same time, the full standardised indirect effect size (K^2= .19) of students' metacognition on their problem-posing performance was found to be significant and at a high level. The result obtained from the Sobel test also supports this finding (Sobel z= 2.76, *p* <.01). In line with the findings obtained, the fourth hypothesis of the research was approved. Figure 5 presents the determined serial mediation model.

Figure 5
Serial mediating role of mathematics and native language course success in the effect of metacognition on problem-posing performance



Note. *p<.05, **p<.01, ***p<.001

In the effect of students' metacognition on problem-posing performance, the serial mediation effect of native language and mathematics course academic success, respectively, was also examined. The findings obtained are presented in Table 5.

Table 5
Serial mediating role of native language and mathematics course success in the effect of metacognition on problem-posing performance

	Outcome variables					
	→ M_1 (Native language C.S.)		→ M_2 (Mathematics C.S.)		→ Y (Problem posing)	
Predictor variables	*β*	*SE*	*β*	*SE*	*β*	*SE*
Constant	44.10***	7.58	−30.19***	6.19	−49.61*	24.35
X (Metacognition)	.33***	.08			.36	.22
M_1 (Native language C.S.)			1.15***	.07		
M_2 (Mathematics C.S).					−.14	.33
R^2	.13		.77		.29	
F	18.29***		192.60***		16.15***	
Bootstrap	Metacognition → Native language C.S. → Math C.S. → Problem posing					
Indirect effect	K^2= −.02 *β* = −.05 SE= .12 %99 CI [−.41, .28]					

*Note.**p <.05; **p <.01; ***p <.001; *n*=121;CI: Bootstrap lower and upper confidence interval. SE: Standard error. K^2: Fully standardised effect size. Bootstrap resampling= 5000. Non-standardised beta coefficients (*β*) were reported.

According to Table 5, when students' metacognition is examined in relation to their problem-posing performance, there is no significant serial mediation effect between native language and mathematics course academic success (*β* = −.05; CI= [−.41, .28], SE= .12).The fifth hypothesis of the research was dismissed in line with the findings.

6. Discussion and Conclusion

This study examined the direct and indirect relationships between sixth-grade students' metacognition and their problem-posing performance, and determined the mediating roles of students' academic success in the mathematics and native language courses in this relationship. The study differs from previous studies in that it deals with the variables that affect problem-posing skills together. Additionally, the fact that this study prioritises causal relationships makes the results obtained valuable.

Correlation analysis showed that there was a positive and significant relationship between problem posing, metacognition, academic success in the mathematics course and academic success in the native language course. It was determined that there is a positive and significant relationship between students' metacognition and problem posing in the bivariate relationships between variables. In the literature, it was found that there is a positive and significant relationship between metacognition and problem posing. For example, Ghasempour et al., (2013) stated that there is a relationship between metacognition and problem-posing performance. Karnain et al. (2014) found that students who combined the planning and monitoring dimensions of metacognition had a higher level of problem-posing skills. In this respect, it can be said that the finding of a positive and meaningful relationship between metacognition and problem posing is in line with the literature. The correlation analysis conducted for the relationship between students' metacognition and their academic success in the mathematics course showed that there was a positive and significant relationship between these variables. Öztürk and Kurtuluş (2017) found that individuals with high mathematics school report grades have a high level of metacognitive awareness. Areepattamannil and Caleon (2013) revealed that there is a positive relationship between the control strategy, one of the metacognitive strategies, and success in mathematics. Many studies on the subject revealed the existence of a significant relationship between metacognition and mathematics course success (Hassan & Rahman, 2017; Kaya, 2019; Maqsud, 1998). In this context, it can be said that the result obtained regarding the existence of a positive and significant relationship between metacognition and mathematics course success is in support of the literature. The bivariate correlation showed that there was a positive and significant relationship between students' metacognitions and their native language course academic success. Previous studies on the relationship between metacognition and native language course success indicate that metacognitive skills have a positive effect on reading and comprehension in particular (Bonds et al., 1992; Çetinkaya & Erktin, 2002; Tice, 1991), and on native language course academic success in general (Yıldız, 2015). Similarly, a positive and significant relationship was found between problem-posing performances and native language course academic success and mathematics course academic success in the study. The literature states that students' problem-posing performance varies according to both their mathematics course success (Nicolaou & Philippou, 2007; Özgen et al., 2017) and their good usage of the native language language (Arıkan & Ünal, 2013; Özgen et al., 2017). In this respect, it can be said that the findings of the study are in line with the literature. In addition, within the scope of the study, a strong relationship was found between the students' native language course and mathematics course academic success. Güneyli et al. (2010) determined that native language course success is relatedthe mostto mathematics course success. At the same time, there are many studies in the literature that reveal the existence of a significant relationship between individuals' ability to use their native language and their success in mathematics (Albayrak & Erkal, 2003; Jordan et al., 2002; Reusser & Stebler, 1997; Räty et al., 2004; Taşdemir & Taşdemir, 2011). In this regard, the relationship between the two important disciplines of mathematics and Native language course success is in line with the literature. The fact that the variables that have a positive relationship with problem-posing performance in the study also have a positive relationship with academic success in mathematics indicates that the findings of the study are consistent.

The analysis of the mediator variables showed that the students' metacognition significantly explained their performance in problem posing. In other words, as the metacognition of the students increases, their problem-posing performance improves, or as their metacognition decreases, their problem-posing performance weakens (causal effect). Many studies in the literature determined that students' metacognition is related to problem posing (Ghasempour et al., 2013; Karnain et al., 2014). Analyses conducted to examine the indirect effect revealed that students' success in the mathematics course played a full mediating role in the effect of metacognition on problem-posing performance. In other words, the effect of students' metacognition on their problem-posing performance is explained by their academic success in the mathematics course. In many studies on metacognition, it was determined that metacognition increased the success in the mathematics course (Areepattamannil & Caleon, 2013; Hassan & Rahman, 2017; Kaya, 2019; Maqsud, 1998; Öztürk & Kurtuluş, 2017) and the mathematics course also had an effect on problem posing (Nicolaou & Philippou, 2007; Özgen et al., 2017). In this regard, it can be said that this indirect effect, which is revealed through the success in the mathematics course, is in support of the literature.

Another result regarding the mediator variable analysis is that native language course success plays a full mediating role on the problem-posing performance of metacognition. In this respect, it can be said that the effect of students' metacognition on their problem-posing performances is explained by their native language course academic success. The literature indicates that metacognition is effective on success in Native language (Yıldız, 2015). In addition, many studies determined that metacognitive skills have a positive effect on reading and comprehension and increase the ability to use the native language (Bonds et al., 1992; Çakıroğlu, 2007; Çetinkaya & Erktin, 2002; Tice, 1991). Also, Arıkan and Ünal (2013) determined that students with a low problem-posing performance cannot use the native language well in their problem statements. In this context, it can be said that native language course success and its mediation role between metacognition and problem-posing performance is in line with the literature.

The results of the serial mediation analysis showed that students' academic success in the mathematics and native language courses, respectively, played a serial mediating role in the effect of metacognition on problem-posing performance. At the same time, in this model, it was determined that the indirect effect of metacognition on the serial mediating role of the success in the two courses and problem-posing performance was higher than the indirect effect of the courses mediation roles separately. In the serial mediation model presented, the fact that mathematics and mother tongue courses are mediating variables, respectively, can be interpreted from different perspectives. This may be due to the fact that students become more successful problem solvers when their mathematical knowledge is supported by their native language skills. Because, in the problem-posing process, a new problem statement is put forward in line with the given ones. This situation primarily requires the individual's knowledge of mathematics, and this knowledge must be supported in terms of language and expression. As a matter of fact, Özgen et al. (2019) stated that the main reasons for students not being able to pose a problem stemmed from their inadequate mathematical skills and language use. In addition, dealing with mathematics improves individuals' systematic thinking and reasoning skills. It is thought that these skills also contribute to the student's skills such as better understanding of what he reads, making correct inferences and determining the main idea to be given. In this respect, it is thought that the skills developed with mathematics also support the skills of using the mother tongue, and this situation positively affects the problem-posing performance of the students. In the literature, there are findings that suggest that increasing students' metacognition will increase their success in the mathematics course (Hassan & Rahman, 2017; Kaya, 2019; Maqsud, 1998), success in the mathematics course may affect native language course success in various ways (Albayrak & Erkal, 2003; Güneyli et al. 2010; Jordan et al., 2002; Reusser & Stebler, 1997; Räty et al., 2004; Taşdemir & Taşdemir, 2011), and native language success will also affect problem posing (Arıkan & Ünal, 2013; Özgen et al., 2017; Silver & Cai, 1996). In this context, it is understood that the results obtained are in support of the literature.

7. Recommendations

The study observed that metacognition was effective in the problem-posing performance of students, but this effect developed through the success in the mathematics course. For this reason, mathematics teachers' use of metacognition in designing in-class activities can improve students' problem-posing performances. Additionally, as mathematics, the native language course also has a mediating effect on problem posing. Accordingly, it is recommended to include activities that stimulate students' metacognition in native language courses, especially in the learning goals related to understanding and writing a text. Thus, the metacognition of the students will be effective on their problem-posing performance as well as their academic success in the native language course. The greatest mediating role in the effect of metacognition on problem-posing performance is formed by considering mathematics and native language course success together, respectively. In this regard, it is recommended to add real-time interdisciplinary learning goals and activities related to problem posing to the mathematics lesson curriculum and the native language lesson curriculum. It is recommended to examine the problem-posing practices the students perform through metacognitive activities within the mathematics lesson in terms of language and expression within the scope of the native language lesson. Researches can be carried out for this purpose. Students' language skills are relevant in problem-posing performance. In the literature, it is stated that reading a book increases the ability to use a language and is a valuable tool for abstract thinking and self-expression skills (Akçamete, 1990; Odabaş et al., 2008). In this context, these skills can be developed by presenting students with reading goals in native language and mathematics lessons. Mathematics-themed novels, stories etc. can be suggested especially in mathematics lessons. There are only a few studies in the literature that examine the relationship between metacognition and problem-posing performance. Through the examination of this relationship with different variables, mediator and regulatory models can be revealed. Considering the determined serial mediation effect model, it becomes evident that interdisciplinary studies should be conducted regarding problem posing within the scope of the mathematics lesson and the native language lesson. In this respect, this study offers guidance in terms of designing and organising learning environments. Accordingly, it is believed that the current success in the national and international exams (the Trends in International Mathematics and Science Study [TIMSS] and the Program for International Student Assessment [PISA]), in which students' mathematical thinking skills are evaluated, will also increase.

This study was conducted under certain limitations. The first of these limitations is related to the data collection. The end of the year grade averages of the students were taken into consideration for the Native language and mathematics courses’ academic success scores, which were considered as mediator variables in the study. Using data from secondary sources is a factor that jeopardises the reliability of original studies. For this reason, tests that measure the academic success in the native language and mathematics courses comprehensively can be developed and used in future studies on the subject. Another limitation of the study is related to the sample. The sample of the study consists of 121 students. The sample of the study was limited to 121 students due to the fact that the time required from students for the data collection process of the study was too long (one hour outside of school hours) and only students who participated voluntarily were selected. Future researchers may reach more reliable results by working with larger samples.

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