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Research Article

Problem-solving and posing skills in middle school students: The impact of gender, school type and grade level

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This paper examined the problem-solving and problem-posing skills of middle school students in terms of whether the factors of gender and school type have an impact, as well as to illustrate these skills with respect to grade levels. A total of 461 students from different middle school levels of both private and public schools participated in the study. The instrument and framework reported by Cai (2003) were applied in the data collection and analysis phases of the study. The results revealed no significant differences between private and public schools in terms of problem-solving and problem-posing skills. In terms of gender, similar characteristics were found in terms of problem-solving skills, while differences in problem-posing skills were noted. In addition, some differences were observed in terms of problem-solving and problem-posing skills according to grade level. These results are discussed in the context of the existing literature.

Keywords: Middle school students; Problem solving; Problem posing; Gender; Public schools; Private schools

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1. Introduction

The *mathematics education for all* trend that began in the 1950s has brought about significant changes in the mathematics instruction paradigm worldwide (Davis, 2001). Rather than the outdated approaches of the past, which aimed at equipping students with computational proficiency through rote procedures (Morris, 1999), contemporary perspectives support the development of various skills such as critical thinking, problem solving and problem posing, all of which center on conceptual understanding (Aydın-Güç & Daltaban, 2021; Baki, 2018). This study focused on the problem-solving and problem-posing skills of middle school students that have come to the fore in mathematics curricula in more recent years.

Problem solving, in this regard, has been characterized by researchers as "at the heart of mathematics" (Cockcroft, 1982, p. 249; Ellerton, 2013, p. 87). Wilson et al. (1993) draw attention to its significance with their list of five reasons that explain the importance of problem solving in mathematics: ① Problem solving is a major part of mathematics and is the core of this discipline;

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reducing it [mathematics] to a series of exercises leads to both misrepresenting mathematics as a discipline and shortchanging the students. ② Mathematics, which has many fields of application, is intertwined with other disciplines, and it has to work with other disciplines in solving the problems encountered in order to understand and communicate with them. ③ Mathematical problem solving involves intrinsic motivation, and school mathematics has the potential to stimulate student interests and enthusiasms. ④ Most of us deal with math problems for recreation because problem solving can be fun. ⑤ Problem solving has an artistic side, and this side is so important to the understanding and appreciation of mathematics that it should be an educational goal. According to Reiss and Törner (2007), problem solving assumes that there is a starting point and a target that cannot be immediately achieved by procedures identified by the problem solver. In other words, problem solving is more than a simple calculation, such as the sum of two digits, but requires the individual to go beyond what is already known. In this sense, as Schoenfeld (1985) noted, a problem is a task that an individual wants to achieve but without an obvious and specific method for finding a solution.

Problem posing, moreover, is an equally important skill in mathematics education, although it has received more limited attention than problem solving. Problem posing, referred to by Silver (1994) as both the "generation of new problems and … the re-formulation of given problems" (p. 19), has been described by Ellerton (2013) as being as fundamental as problem solving in mathematics instruction, because one cannot solve a problem unless it has been posed first. Einstein and Infeld (1938, as cited in Xie & Masingila, 2017) further assert that posing an interesting problem is more important than solving it. Dickerson (1999) emphasizes this point, indicating that posing problem sentences allows students to use their own language and grammatical structures, as well as syntax and scenarios. Researchers such as Isik and Kar (2012) point out that problem-solving and problem-posing skills are closely related, while Gonzales (1998) identifies problem posing as the fifth step of Polya's problem-solving process.

Within the scope of the present study, the problem-solving and problem-posing skills of middle school students at different grade levels were examined in terms of whether these skills differ with respect to gender, type of school (public or private) and middle school grade level.

2. Literature Review

In the following section, the literature will be reviewed in terms of relationships among the variables of gender and school type in problem solving and problem posing. Research on mathematics education has frequently focused on the factor of gender. The studies that addressed the impact of gender on the performance of mathematics produced different conclusions. Some studies concluded that males are better than females with respect to mathematics achievement (Ferretti & Giberti, 2021; Zhao et al., 2022). On the other hand, numerous studies have been conducted that do not confirm this claim. For instance, in their meta-analysis study, Hyde et al. (1990) found male mathematics achievement to be higher at a negligible effect size ($d = 0.15$); however, a negligible effect size ($d = -0.05$) was calculated, indicating that females performed better in samples from the general population (excluding selective samples). In a separate study, Duffy et al. (1997) investigated the role of gender in twelve-year-old learners and found no difference in their mathematical problem-solving achievement. In another study conducted on college-level students, Ajayan and Panwar (2021) similarly found no difference between males and females with respect to performance on problem solving. In a more recent meta-analysis, moreover, Lindberg et al. (2010) found that males and females perform statistically similarly in mathematics, while further studies examining the sources of differences in gender problem solving, albeit small, revealed that the characteristics of the selected samples (e.g. comparison of low achiever males and females, outcome measure (easy problems versus difficult problems) (Innabi & Dodeen, 2018) sometimes favored females and sometimes males. From a global point of view, the results of TIMMS 2019 survey indicated that in the 8th grade level there was considerable gender equality in average mathematics achievement (Mullis et al., 2020). In detail, in seven countries girls outperformed boys significantly whereas the opposite was true for 6 countries, and in 26 countries there was no significant difference in terms of gender factor (Mullis et al., 2020). In the case of Turkey, although the average score of girls was higher the difference was not significant (Mullis et al., 2020).

Nosek et al (2009) argued that the different findings with respect to gender factor on mathematics achievement across different samples indicate that gender differences on mathematics achievement might stem from stereotypes which emphasize the belief that man are naturally more talented towards math and science topics. Continuous exposure to the situations which signal that doing mathematics or science is a man-oriented job may result in a misbelief on the part of women and consequently raise an anxiety towards mathematics. For instance, in the study of Murphy et al. (2007) each student watched a math, science and engineering conference video depicted either a 1:1 ratio of men to women or a 3:1 ratio of men to women. The researchers found that female students anticipated a lower sense of belonging and less desire to participate for the unbalanced version compared to balanced version.

Considering the literature, it seems that the gender difference in mathematical performance may be more likely to depend on a psychological ground than a biological ground, and the stereotypes may be the main reason if differences exist. In this study, we compared the performances of male and female students and regarded the outcome of this comparison as a clue for the existence or non-existence of possible stereotypes. While this factor is also explored in the present case, the reason for including it in the study is not to take the belief, as explained in the introduction section, that gender impacts problem solving, but to reveal the extent to which the stereotypes (as detailed in Nosek et al., 2009) that hold females to be less proficient in this regard reflect reality and to discuss gender equalities (Guiso et al., 2008) in light of the existing literature. In this respect, mathematics has been frequently typecast as male discipline, not only by learners and their parents, but even by teachers (Lindberg et al., 2010). On the other hand, numerous studies have been conducted that do not confirm this claim. For instance, in their meta-analysis study, Hyde et al. (1990) found male mathematics achievement to be higher at a negligible effect size ($d = 0.15$); however, a negligible effect size ($d = -0.05$) was calculated, indicating that females performed better in samples from the general population (excluding selective samples). In a separate study, Duffy et al. (1997) investigated the role of gender in twelve-year-old learners and found no difference in their mathematical problem-solving achievement. In a more recent meta-analysis, moreover, Lindberg et al. (2010) found that males and females perform statistically similarly in mathematics, while further studies examining the sources of differences in gender problem solving, albeit small, revealed that the characteristics of the selected samples (e.g. comparison of low achiever males and females, outcome measure (easy problems versus difficult problems) (Innabi & Dodeen, 2018) sometimes favored females and sometimes males. Similar results were also obtained regarding problem-posing skills. For instance, a study conducted by Bunar (2011) concluded that even though some differences existed in terms of problem posing in certain concepts, middle school students' skills were similar in general.

In terms of school types, on the other hand, it is generally believed that public schools offer lower quality education, while the private schools adopt a more market-oriented approach and provide more resources, and therefore, result in a higher-quality education (Mahuteau & Mavromaras, 2014). In studies that have evaluated this belief in terms of student performance, some researchers have reached mixed conclusions. One the one hand, for example, Aydın et al. (2017) examined the problem-solving skills of middle school students relating to fractions and found that student achievement in private schools was statistically significant compared to public schools. On the other hand, in the Australian context, Mahuteau and Mavromaras (2014) revealed that the type of school alone did not have a significant effect on mathematical problem-solving success in their multilevel regression study using PISA 2009 data. Looking at the Turkish context, Sulku and Abdioglu (2015) examined the factors affecting mathematics achievement through TIMSS 2011 relating to eighth graders (the focus group of the current study) and found that the average mathematics score of students in private schools were 80 points higher than those in public schools. In this sense, although there is limited support for the belief that student achievement varies between private schools and public schools, in Turkey, this gap seems more evident. The current study aims to shed further light on this issue by investigating whether school type is a significant variable in terms of problem-solving and problem-posing skills.

With respect to the effect of grade level on students' performances on problem solving and problem posing Cai (2003) investigated fourth, fifth and sixth grade students mathematical thinking in problem solving and posing. The findings of the study showed that the majority of the students in all grades were able to implement strategies in the solution of the problems and choose appropriate representations to explain their solution processes. In addition to this, the number of correct answers to the problems increased as the grade level ascends, and this difference was statistically significant between the fourth and fifth grades. As to the problem posing tasks, the difficulty level of the problems posed by students increased in line with the grade level, however, this difference did not reach a statistically significant level across grade levels.

3. Method

3.1. Participants

The participants in this study were middle school students from public and private schools who were enrolled in the sixth, seventh and eighth grades. They were selected from schools in a city in northeastern Turkey with a population of less than one million. Since there are no private schools in the rural areas outside the city, the participants were chosen from schools in or close to the city center. Therefore, it can be said that a purposeful sampling method was employed. Schools with varying levels of student achievement were chosen by considering the results of the student selection exam for high schools in the case of both public and private schools. A description of the participants is presented in Table 1.

Table 1
Characteristics of participants (N=461)

Category		*Sample (%)*	*Number of participants*
School type			
	Public	40.1	185
	Private	59.9	276
Gender			
	Male	54.9	253
	Female	45.1	208
Grade			
	6th grade	38.4	177
	7th grade	20.1	93
	8th grade	41.5	191

3.2. Data Collection Tool

An instrument consisting of four tasks developed by Cai (2003) was used as the data collection tool (see Appendix). The instrument was originally developed to measure the mathematical thinking of students in the fourth, fifth and sixth grades in Singapore from different perspectives. However, both classroom teachers and academicians who are experts in problem solving concurred that the tasks in the instrument were more suitable for sixth, seventh and eighth grade middle school students in terms of the objectives in the Turkish curriculum. This view was considered in the selection of the grade levels of the participants. The first task (the Hats Average) aimed to measure students' understanding of the mathematical concept of average. An average value was given, and the participants were asked to find the missing data in the data set. This question was also presented visually. On the other hand, the second task (Pizza Ratio) examined the students' skills in comparing fractions through dividing a pizza into portions, which is an example frequently used in teaching fractions. In order to solve the problem, a ratio needed to be established for the number of persons and pizzas, and comparisons were to be made through visual or numerical representations. In the third task (odd number pattern), the students' generalization skills were measured. This question, presented as a party scenario, asked students to determine the relationship between a ring and the number of guests and use it to find a value that they could not find by writing one by one. The final task involved problem posing, asking students to pose problems at various difficulty levels, taking into account the figures provided to them.

3.3. Data Analysis

Each of the first three tasks related to problem solving according to different premises. For example, the first problem consisted of a single item, the second problem consisted of two items, and the third problem consisted of three items. In scoring the students' responses, each correct item was given a score of 1 point. The criterion for correct answers to the problems involved a response that explained or illustrated how the student found the answer. Thus, on the first task, for instance, "10" answers were given a score of 1 only if the participants demonstrated how they found the response (e.g., formula of average). Accordingly, the maximum score students could obtain on the problem-solving questions was 6, while the minimum score was zero.

In analyzing the problem-posing task, Cai's (2003) classification was taken into consideration, and the posed problems were coded in three categories: mathematical problems, nonmathematical or irrelevant problems, and no responses given. The mathematical problems were then further classified as extension or non-extension problems. According to Cai , extension problems are those that require a higher level of cognitive processing beyond three given figures (e.g., "How many black dots are there in the 50th figure?"). Non-extension problems, on the other hand, focus on the given figures in a pattern (e.g., "How many white dots are there in the third figure?"). However, some of Cai's criteria were simplified in this case in order to clarify the scoring after the classification of the problems. Therefore:

a. The students posed less than two mathematical problems received 0 points on this task.

b. If a non-extension or a non-mathematical problem following an extension problem was expressed by a student as a more difficult problem, the student was given a score of 0.

c. If two or more posed problems were non-extension problems formulated in consideration of the figures provided, the following approaches were taken with respect to the difficulty levels of the problems: 1) Problems that compared the points in the figures were assigned greater difficulty than questions that focused only on the number of points in a figure. 2) Questions that combined the dots in the figures were more difficult than questions that focused on the number of dots in the figures only. If two separate questions with similar characteristics were formulated, only one of them was included in the scoring. For example, the question "How many black dots are there in Figure 1?" posed by a student was considered as easy, while the question "How many white dots are there in Figure 1?" was considered as moderately difficult one. Since no hierarchical classification was made, only one question was included in the scoring. Each non-extension question was scored as 1 point, as with the sample student responses mentioned here.

d. Since an extension problem is more complicated and more difficult than a non-extension problem, each extension problem was scored as 2 points.

- If a student provided a non-extension and two extension problems for the A, B, and C sections of the problem posing item respectively then received 5 points.
- If a student provided two non-extension and one extension problem for the A, B, and C sections of the problem posing item respectively then s/he received either 3 or 4 points based on the criteria explained in the "c." section.

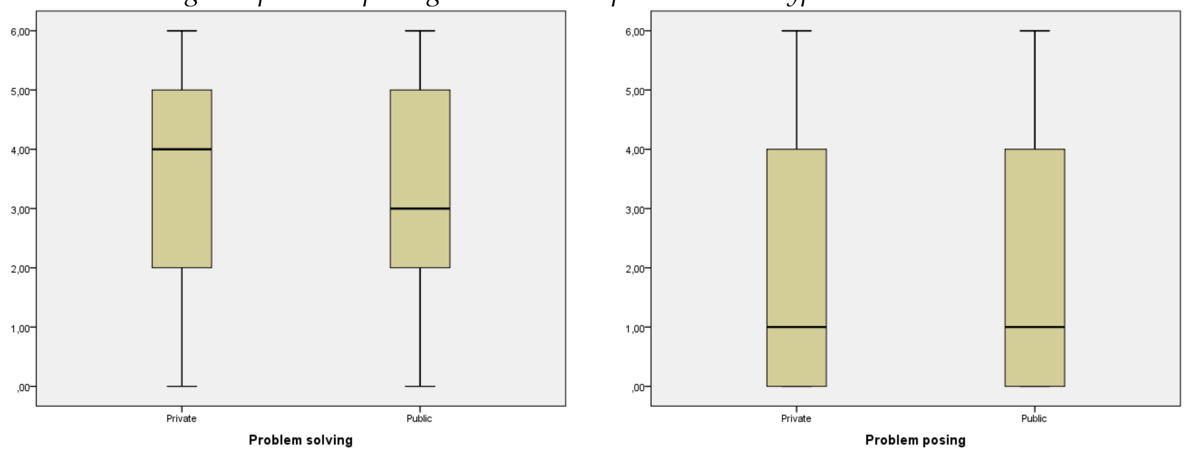
Within the scope of the study, the relationships between the variables were also examined. Since the normality test indicated that the data sets were not normally distributed, a Mann-Whitney U-test was performed for the variables of gender and school type, and a Kruskal-Wallis test for the variable of grade level. As a final note, to ensure reliability, the researchers carried out the coding of the data twice, on two separate occasions, two months apart. The inter-codes were in agreement at a rate of 95% for the problem-solving tasks and at 80% for the problem-posing task.

4. Results

4.1. School Type

The descriptive data revealed that the problem-solving skills of students in private schools (n=276 and mean = 3.47) was slightly higher in comparison to public school students (n = 185 and mean =3.34). In other words, the students in private schools outperformed those in public schools. In terms of problem-posing skills, on the other hand, it was determined that the mean scores of students in public schools (mean = 2.16) were higher than those in private schools (mean = 1.95). The box plot obtained from these data is shown in Figure 1.

Figure 1
Problem-solving and problem-posing skills with respect to school types



To determine whether the data were suitable for conducting a normality test, the Shapiro-Wilk test was carried out. Since the normality test results revealed that the data sets for both the problem-solving and the problem-posing skills were not suitable for parametric tests, Mann-Whitney non-parametric tests were performed to examine whether statistically significant differences were present between private and public schools (see Table 2).

Table 2
Comparison of skills across school types

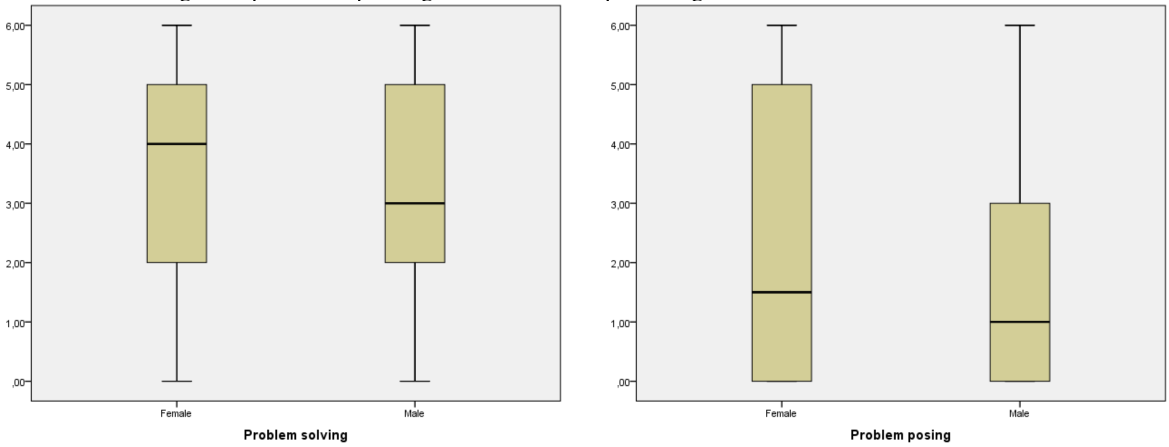
	Private schools (n=276)		*Public schools (n=185)*			
	Mean rank	Sum of rank	Mean rank	Sum of rank	*U*	*p*
Problem solving	235.04	64870.5	224.98	41620.5	24415.5	.42
Problem posing	227.70	62844.5	235.93	43646.5	24618.5	.49

As can be seen in Table 2, school type was not a significant factor in the problem-solving or problem-posing skills of the participants. In other words, the students in both groups had similar characteristics in terms of these skills.

4.2. Gender

When the skills of the middle school students were examined in terms of gender, it was revealed that their performance in problem solving was similar, and that the average scores of females (mean=3.53) was slightly higher than males (mean=3.33). In the context of problem-posing skills, it was likewise determined that the mean scores of the females (mean = 2.37) were higher than those of the males (mean = 1.76). Figure 2 shows the boxplots regarding the related data.

Figure 2
Problem-solving and problem-posing skills with respect to gender



The Shapiro-Wilk test result showed that the data were not normally distributed and therefore unsuitable for parametric testing; therefore, non-parametric testing was carried out. Table 3 shows the results of the Mann-Whitney test conducted on whether gender was a significant variable in relation to problem solving and problem posing.

As indicated in Table 3, the participants' problem-solving skills did not differ statistically in terms of gender. On the other hand, their problem-posing skills differed significantly in favor of females.

Table 3
Comparison of skills across gender

	Male (n=253)		*Female (n=208)*			
	Mean rank	Sum of rank	Mean rank	Sum of rank	*U*	*p*
Problem solving	224.15	56711.00	239.33	49780.00	24580.00	.22
Problem posing	219.22	55463.00	245.33	51028.00	23332.00	.03*

Note. $* p < .05$

4.3. Grade Levels

The final variable examined in the study was grade level, with the aim of determining the extent to which the targeted skills had been achieved by the participants and how these skills differed in terms of grade levels. With respect to problem solving, there was a slight decrease in the transition from the 6th grade (N=177; mean=3.28) to the 7th grade (N=93; mean=3.15). On the other hand, it was observed that the average performance of the 8th graders increased considerably (N=191; mean=3.68). A similar picture was observed for the change in problem-posing skills. While the 6th grade students (mean=1.90) performed better than the 7th graders (mean=1.60), the highest score was obtained by the 8th graders (mean=2.38). The related box plots are presented below, in Figure 3.

The normality test indicated that the data were not normally distributed, and that therefore, a non-parametric analysis should be performed. The results of the Kruskall-Wallis one-way analysis of variance indicated that the grade level significantly affected students' problem-solving skills, $\chi^2 = 8.59, p = 0.02$. Pairwise comparisons were conducted to uncover the source of the difference among groups by using the Mann–Whitney U-tests for each pair, as shown in Table 4.

Figure 3

Problem-solving and problem-posing skills with respect to grades

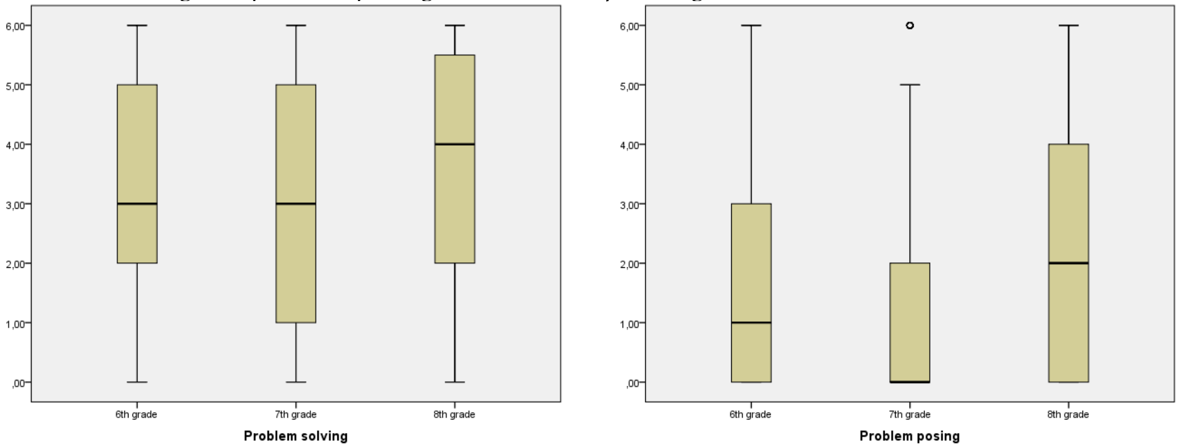


Table 4

Comparison of skills across grade levels

Grade	*Mean rank*	*Sum of ranks*	*U*	*p*
Problem solving				
6	137.52	24341.50	7872.50	.55
7	131.65	12243.50		
6	172.98	30617.00	14864.00	.04*
8	195.18	37279.00		
7	127.50	11857.50	7486.50	.03*
8	149.80	28612.50		
Problem posing				
6	140.74	24910.50	7303.50	.11
7	125.53	11674.50		
6	175.01	30977.50	15224.50	.09
8	193.29	36918.50		
7	124.18	11549.00	7178.00	.006**
8	151.42	28921.00		

Note. $* p < .05$; $** p < .01$

As the statistics in Table 4 indicate, there was no significant difference between the 6th and 7th grades in terms of problem solving, while a significant difference was found between the 8th grade and the other two grades. In terms of problem posing, due to the high performance of the 6th graders, the only difference was found between the 7th- and 8th-grade students in favor of the 8th graders.

5. Discussion and Conclusion

The development of mathematical problem-solving skills is among the main goals of school mathematics, with the aim not only of supporting academic success, but also of providing individuals with the ability to solve the problems that they will encounter in everyday life (Çelik & Güler, 2013). Problem-posing skills, on the other hand, have been historically neglected in mathematics curricula (Schwartz, 1992), although its importance has been increasingly recognized over the last two decades (Cai et al., 2016; Polat & Özkaya, 2023; Xu et al., 2020). Due to the importance of both skills, the current study examined the problem-solving and problem-posing skills of middle school students in terms of specified variables using a descriptive approach.

Private schools are generally smaller than public schools; and because they typically offer smaller class sizes, there is a common belief that students are more successful (Benveniste et al., 2003). However, contradictory results can be found in the literature. For instance, using TIMSS data, Rutkowski and Rutkowski (2009) revealed that the effect of private schools on learner success varies in different countries. In the case of the United States, for example, by adding the effect according to school-type and between-school variance, the school type effect was found to be 1%. In Chile, on the other hand, the same effect was 60%. In the case of Turkey, moreover, inferences have been made in the literature from the results of the PISA exam in the existing literature. Alacacı and Erbaş (2010), for example, found a large variance between schools in mathematics performance using PISA scores. However, the schools included in the PISA exam in Turkey are high schools of varying types (e.g., vocational, science, health, etc.), and as such, generalizability of the results with respect to middle school students is limited.

On the other hand, an exam known as Monitoring and Evaluation of Academic Skills, which was carried out with 8th grade students by the Ministry of National Education as domestic version of the PISA, revealed that problem-solving skills differ in favor of private schools in terms of problem types when compared to students in public schools (MoNE, 2019). However, the data presented in the report released by the ministry are pertain to Turkey in general. On the other hand, some localized studies have reported that students in private schools have significantly greater problem-solving skills than public school students. In this regard, Aydın et al (2017) found a significant difference in favor of private schools in Istanbul in terms of solving fraction problems. However, contrary to those results, no significant difference was found in the current study between students in public schools and students in private schools in terms of either problem-solving or problem-posing skills. This situation can be regarded from numerous perspectives, including school size, gender of students, and grade level.

With regard to school size, when student performance on the high school entrance exam is examined through data from 2019, it can be seen that provinces with an average size population (around 500k - 1.5 m) are more successful than larger cosmopolitan cities (MoNE, 2020). In this respect, a study conducted by Temli-Durmus (2014) in three metropolitan public schools in Turkey revealed that one of the main reasons that teachers were not able to apply constructivist approaches was crowded classrooms. Another interesting finding in the same studies was that teachers in public schools believe that a constructivist approach can be best applied in private schools. On the other hand, it is known that families of high socioeconomic status living in large cities tend to send their children to private schools. From this viewpoint, it can be asserted that it is not possible to evaluate the achievement between private and public schools based solely on school type, and other variables, such as the population of the school, class size and socioeconomic status of the students should also be considered. With this in mind, the fact that the current study was conducted in a city with a much smaller population compared to cosmopolitan cities, as well as smaller class sizes, is considered to be the main factor in the similarities in problem-solving and problem-posing skills between private and public schools.

From another perspective, the role of gender in mathematical problem solving has been emphasized by researchers for decades, while few studies have addressed the gender variable in terms of problem posing (e.g. Dickerson, 1999). Although there is an extensive reporting in the older literature that males are more successful in mathematical problem solving (e.g. Fennema & Carpenter, 1981; Maccoby & Jacklin, 1974), some relatively new studies have revealed the opposite in terms of mathematics achievement (Priya, 2017; Voyer & Voyer, 2014), while others show no significant difference (Caplan, 2005). Moreover, other researchers have pointed out that it is difficult to reduce mathematical problem solving to gender, because it can be affected by many psychological characteristics underlying gender roles (Zhu, 2007). Still other researchers have reported that the type of question used may cause gender differences in both problem solving (Duffy et al., 1997) and problem posing (Dickerson, 1999). However, in the current study, gender was not found to have a significant impact in terms of problem-solving skills, similar to the results of results PISA Turkey (Gevrek & Seiberlich, 2014). Although the current study is not aimed at generalization, it was observed that the gender gap in favor of males found in OECD countries in mathematics (Batyra, 2017) was not the case in this instance. Furthermore, the results of the current study were significant in favor of females in terms of problem-posing skills. Although only a single question relating to problem posing was included in the instrument, females posed more extension problems, whereas males posed more non-extension problems or gave non-mathematical responses. In this sense, the question type may have had an impact; namely, the result mentioned above and the fact that the structure of the problem was complex or computational might differ in terms of gender in problem-solving skills, and may be valid for problem posing. In this context, for future studies, students can be presented with different types of scenarios in which they can pose different types of problems in order to test their responses.

Finally, in examining performance in terms of grade level, the significant difference in both the problem-solving and problem-posing skills of eighth-grade students was found to be noteworthy. There may be two main reasons for this result. First, due to the spiral nature of the curriculum, as grade level increases, the complexity of the objectives taught increases, and the skills to be developed are enriched. Second, Turkish students take a national exam for high school selection in the 8th grade, and they typically exhibit extra effort in preparing for this exam, because it is believed that a high level of readiness for the exam is important. Moreover, surprisingly, it was found that the performance of 6th grade students in problem solving and problem posing was better than the 7th graders. Moreover, no statistically significant differences in problem-posing skills were found between the 6th grades and the other two grades. As a final remark, comparing the results of the current study with Cai's (2003) work, which was conducted with Singaporean students using the same instrument, it was noted that mean scores of Turkish students were considerably lower in all items of problem solving. Similar results were also found for problem-posing performance.

6. Limitations and Future Studies

Although the current study is important with respect to examining middle school students' problem-solving and problem-posing skills in terms of certain variables, there are two key limitations. The first is the assumption that different school types have similar characteristics. Although sampling diversity was taken into account, the role of schools' characteristics in this regard was ignored. Regression studies taking these variables into consideration are needed in future research. Second, the region where the study was conducted displays a homogeneous structure in terms of socioeconomic levels. Making similar comparisons between regions may offer richer content in terms of examining equal opportunities in education.

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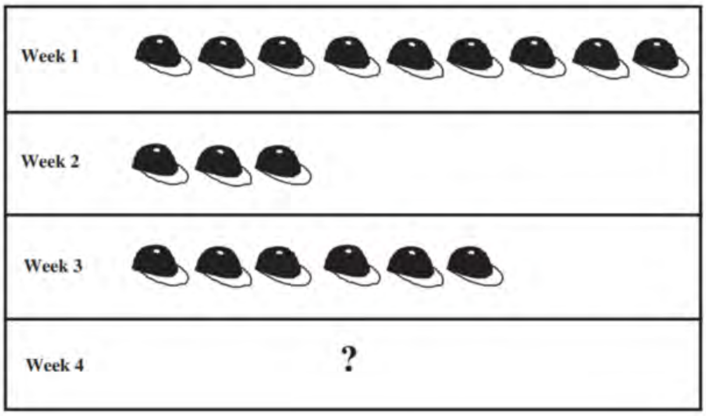
Appendix. Four tasks (Adopted from Cai, 2003, p.734-736)

1. The hats averaging problem

Angela is selling hats for the Mathematics Club. This picture shows the number of hats Angela sold during the first three weeks.

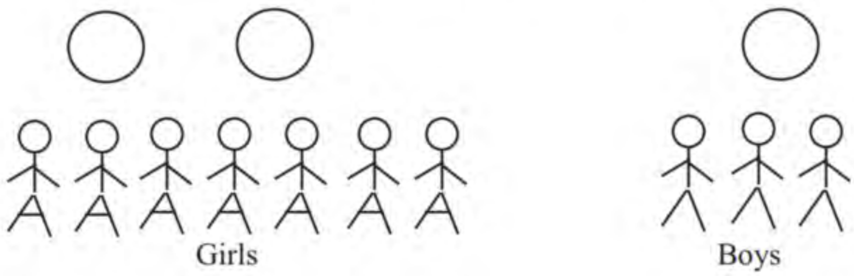
How many hats must Angela sell in Week 4 so that the average number of hats sold is 7?

Show how you found your answer.



2. The Pizza Ratio Problem

Here are some children and pizzas. 7 girls share 2 pizzas equally and 3 boys share 1 pizza equally.



A. Does each girl get the same amount as each boy? Explain or show how you found your answer.

B. If each girl does not get the same amount as each boy, who gets more? Explain or show how you found your answer.

3. The Odd Number Pattern Problem

Sally is having a party.

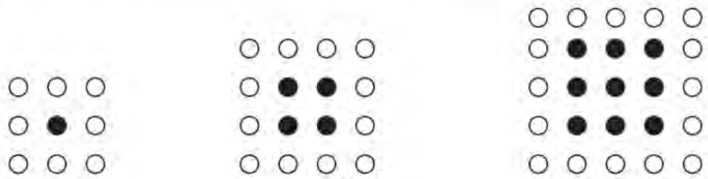
The first time the doorbell rings, 1 guest enters.
The second time the doorbell rings, 3 guests enter.
The third time the doorbell rings, 5 guests enter.
The fourth time the doorbell rings, 7 guests enter.

Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 10th ring? Explain or show how you found your answer.
B. In the space below, write a rule or describe in words how to find the number of guests that entered on each ring.
C. 99 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.

4. Problem Posing Task

Mr. Su drew the following figures in a pattern, as shown below.



(Figure 1) (Figure 2) (Figure 3)

For his student's homework, he wanted to make up three problems BASED ON THE ABOVE SITUATION: an easy problem, a moderate problem, and a difficult problem. These problems can be solved using the information in the situation. Help Mr. Su make up three problems and write these problems in the space below.

A. The easy problem:

B. The moderately difficult problem:

C. The difficult problem: