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Original Research

Ticks and crosses in primary mathematics assessments: What purpose do they serve?

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Dates:

Received: 25 Aug. 2021 Accepted: 28 Apr. 2022 Published: 11 Oct. 2023

How to cite this article:

Chihodzi, B., Mwakapenda, W., & Ngulube, B. (2023). Ticks and crosses in primary mathematics assessments: What purpose do they serve? *Pythagoras*, *44*(1), a647. https://doi.org/10.4102/ pythagoras.v44i1.647

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Scan this QR code with your smart phone or mobile device to read online. Ticks and crosses (TCs) are a common aspect of teachers' classroom practice in relation to assessment in many learning areas including mathematics. Putting TCs in learners' written work is a strategy of feedback. Even though these TCs are frequently used in different types of mathematics assessments, there is limited research in relation to what they actually stand for and what functions they are designed for and especially what purpose they eventually serve in practice. This article emerged from a broader study that aimed at exploring classroom formative assessment practices of Grades 4-6 mathematics teachers, a learning goals and documentary analysis. Since this study was qualitative in nature, we used qualitative, non-probability sampling to recruit respondents according to pre-selected criteria relevant to our research questions. The study participants were 43 qualified and experienced Intermediate Phase mathematics teachers and 95 Grades 4-6 learners from the Tshwane South district, where a phenomenon of low achievement was of great concern. We engaged in document analysis of all the 95 learners' mathematics workbooks. Questionnaires were administered to the 43 teachers. We report on an analysis of teachers' assessment practices of Grades 4-6 learners' mathematics work. We narrate the extent of the use of TCs among teachers from selected schools in Tshwane South district in Gauteng, South Africa. Our analysis shows that while there is prevalent use of TCs among teachers, there are critical gaps in relation to knowledge of TCs in assessing mathematics. We present a qualitative and quantitative data analysis to illustrate how these were used in connection with assessment of learners' mathematics work linked to the concepts of numerical, geometric, and graphical relationships. We use our analysis of the vignettes to explore and argue that teachers use TCs without adequate understanding of what these actually mean in relation to assessment broadly and assessment intended at collecting and clarifying goals for mathematical learning specifically. Despite teachers having mathematical qualifications and a repertoire of experience for teaching, the majority of teachers grappled with understanding mathematical concepts as evidence in how they marked learners' mathematics work. The study also found that teachers' understandings of assessment of mathematics were diverse and largely inconsistent with the formal definitions of mathematics.

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Contribution: This study indicated that there are critical gaps in relation to knowledge of TCs in assessing mathematics. A clear-cut marking policy will guide teachers to provide effective marking using TCs.

Keywords: ticks; crosses; assessment; primary mathematics; classroom practices; feedback; primary school learners; low achievement.

Introduction

The use of ticks and crosses (TCs) in general is a common practice in education and they are commonly used by teachers for the purpose of ascertaining the correctness or incorrectness of written responses from learners to given questions. Teachers use TCs as key ways of communicating or interacting with learners in relation to achievement of learning goals. However, we noted the different ways teachers use TCs when assessing learners' mathematics work. Hence, our interest was driven by the need to understand the teachers' knowledge on the purpose of TCs in classroom formative assessments in mathematics. In this study, we explored teachers' understanding of TCs in their daily assessments which was guided by the research question: *What knowledge do Intermediate Phase mathematics teachers have in the effective use of TCs in classroom formative assessment practices to accomplish learning goals*?

We use our analysis of teachers' data to illustrate and argue that teachers appear to use TCs with limited understanding of what they actually mean in relation to assessment, generally, and more specifically in relation to classroom formative assessment in mathematics as they are intended at collecting evidence of clarifying goals for mathematical learning.

In the next sections of the article, we provide definitions of assessment and what assessment means in terms of a broader study that aimed at exploring classroom formative assessment practices of mathematics teachers of Grades 4–6. We further discuss the evidence of assessment in learners' work and what constitutes marking in terms of TCs in primary mathematics assessment. We use the evidence from the analysis to respond to the question about teachers' understanding of assessment with respect to the use of TCs. The question is: what purposes do these TCs serve, according to teachers, and what are teachers' understanding of these?

The conceptual framework

The conceptual framework for the study was grounded in the pedagogies of formative assessment, an aspect identified as instrumental and paramount in improving teaching and learning (Jones & Heritage, 2012; Wiliam, 2007). The theory of constructivism and socio-constructivism underpinned this study and constituted the point of reference from which explorations and investigations were executed and interpreted. The conceptual framework represents the interactions of several important constructs that facilitate learning (Figure 1). All these constructs are hinged on the teacher as the principal 'driver' of formative assessment because of the teachers' deep knowledge of mathematics. The conceptual framework of this study was adopted from the concept of activity theory. According to activity theory:

[*T*]he unity of analysis is motivated by activity directed at an object [*goal*]. It includes cultural and technical mediation of human activity, artifacts in use (and not in isolation). Activities consist of goal-directed actions that are conscious. (Vygotsky et al., 1920, in Engestrom, Miettinen & Punamaki, 1999, p. 1)

Activity theory's main emphasis is on producing an outcome (goal). In this study, it refers to the achievement of learning goals. Therefore, the conceptual framework was based on several constructs that work together to establish a goal (Vygotsky et al., 1999). Thus, the conceptual framework was

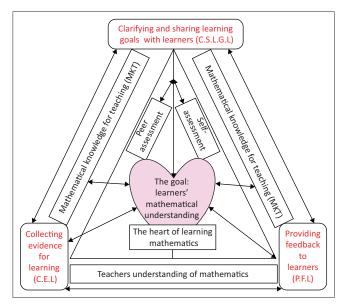


FIGURE 1: Conceptual framework of the study.

built upon the attributes that are presented as constructs in the triangle (Figure 1). These constructs interact with each other to bring about learning of mathematics which is housed in the 'heart' or the centre of the triangle. The constructs include Learning Goal (learners' mathematical understanding) in the centre or 'heart' and hinges to the three sides of the triangle which are: Mathematical Knowledge for Teaching (MKT), Clarifying and Sharing Learning Goals with Learners (CSLGL), Collecting Evidence of Learning (CEL), Providing Feedback to Learners (PFL), Learner Self-Assessment (LSA), and Peer Assessment (PA). This is denoted by: Learning Goal = (MKT) + (CSLGL) + (CEL) + (PFL) + (LSA) + (PA).The most significant part and the central issue is that the learning goal is achieved because there has been increasing concern about continuous underperformance bv mathematics learners for the past two decades. If these constructs are not aligned together, poor achievement is likely to persist. The constructs are dependent on the teacher as the principal 'driver' of this alignment. Hence, our interest was driven by the need to understand the teachers' knowledge on the purpose of TCs in classroom formative assessments in mathematics to achieve learning goals.

Mathematical knowledge for teaching

Maree (2004) concludes that the common and dominant perspectives in the teaching and learning of mathematics through the 20th century developed from expository teaching and rote learning, realistic contextualisation, and problemsolving in related learning areas. Teachers' understanding of mathematics is crucial in that it is what we may call a 'bridge of knowledge'. The teacher passes on mathematics knowledge they clearly understand to the learners. That means they bring their clarity and connectedness of their understanding to the learners. One of the fundamental roles in classroom interaction and the teacher's role as pedagogical and assessor (Bauersfeld, 1994) was the primary focus of the study: to understand the teachers' knowledge of the purpose of TCs in classroom formative assessments in mathematics. It requires teachers who have the ability or skill to assess formatively, and flexibly adapt effective teaching that facilitates the restructuring of learners' mental state as a prerequisite. In other words, the teachers' ability to engage learners in a mathematical activity improves learning. Thus, using TCs and providing effective feedback moves learners forward.

Gatt and Vella (2003) say that learning involves mental processing, interpreting, and making sense of experiences and information. It is argued that teachers should view the causes of mathematical errors as 'fertile ground' to 'plant' mathematics knowledge by modification or 'refinement and reorganization' (Smith et al., 1993, p. 116). For example, the first stages of learning are often characterised by errors due to the nature of existing knowledge structures and the inappropriate context in which they are applied.

However, understanding that learners' misconceptions are a normal part of the process of internalising new information in existing mental structures is vital (Olivier, 1992; Smith et al., 1993); these misconceptions must be replaced through passing on mathematical knowledge that learners are required to have (Leu & Wu, 2005; Santagata, 2005). When teachers deal with the misconceptions of learners' everyday experiences (gaps) sufficiently, learners' mental structures are facilitated (Smith et al., 1993).

Clarifying and sharing learning goals with learners

Moss and Brookhart (2009) point out that sharing and clarifying learning goals with learners is only the first step in developing their understanding of what they are to learn. It has been concluded that formative assessment is most effective when teachers actively engage learners in negotiating criteria for success so that learners have clear understanding of what teachers expect of them (Cauley & McMillan, 2010; Clark, 2008; Wiliam, 2007). Thus, learners should be familiar with the instructional goals, the standards they are required to meet, and how the achievements are to be assessed using TCs. Sharing the learning goals with learners enables learners to clearly understand what they are to learn or what the objective was in a particular lesson (Heritage, 2010; Moss & Brookhart, 2009) so that placing of TCs in their mathematics work becomes effective.

The bottom line is: CSLGL became the 'compass' that guides both the teacher and the learners to know what needs to be done, and by what means, to achieve the set goals (Moss & Brookhart, 2009).

Collecting evidence of learning from learners

Sound classroom formative assessment practices adhere to gather information on evidence of learning to move the learner forward (Wiliam, 2007) through the use of TCs. Elicited information helps to detect learning gaps and then design ways to address them. Thus, analysis and interpretation of evidence of learning takes centre stage. Inaccurate analysis and interpretation lead to learners' mathematical 'deficiencies' which result in failing to close the gaps. In the process of eliciting evidence of learning, teachers should examine the evidence from the learners' viewpoints of their conceptions, misconceptions, skills, and knowledge by means of placing TCs. Thus, teachers' domain mathematical knowledge for teaching influences their interpretation of learners' mathematical understanding (Heritage, 2007). Interpreting and analysing learners' work requires the teacher's mathematical content knowledge. Hamre and Pianta (2005) argue that an average teacher perpetuates mathematics learning. For example, teachers' incorrect marking exacerbates mathematical deficiencies in learners.

It must be understood that learners' misconceptions (gaps) are a normal part of the process of internalising new information in existing mental structures (Smith et al., 1993), which must be replaced through instruction (Leu & Wu, 2005; Santagata, 2005). Understanding why an error has been made is of higher importance and pedagogical value than merely categorising a learner's contribution as correct or erroneous (Smith et al., 1993) by simply putting TCs. When teachers deal with learners' errors or misconceptions (gaps) sufficiently and adapt effective teaching accordingly, restructuring of learners' mental structures is facilitated (Smith et al., 1993).

Quality feedback for adjusting teaching to close the gap

Feedback is viewed as a formative assessment strategy or tool (Black & Wiliam, 2009; Hattie & Timperley, 2007), which influences learning processes. Feedback is one of the key components of formative assessment that teachers use to give learners advice on oral or written work. In other words, feedback focuses on correcting learners' misunderstandings and errors and specifies ways in which learners' work could be improved. Feedback aims at closing the gap between what learners know and do not know (Wiliam, 2011). The policy also clearly states that the teacher should provide learners with feedback that should be developmental (Basic Education, National Protocol for Assessment Grades R - 12, 2012). Sadler (1989) made it clear that information itself is not feedback, but only becomes feedback when it is actively used 'to alter the gap' (p. 121). However, the clarification of learning goals with the learners takes precedence. Teachers clarify learning goals to align their assessments to ensure that learners focus on tasks they need to know and also to guide effective feedback.

Written feedback is the most effective way teachers dialogue with each learner, particularly when they pay attention to the learner's responses to the feedback (Wiliam, 2011). If learners are provided with 'rich' information (feedback) they are likely to make substantial progress in future tasks (Wiliam, 2011). It is argued that the lack of written feedback is 'poverty of practice' (Black & Wiliam, 2001, p. 4) which is beset with problems and shortcomings.

Learner self-assessment and peer assessment

Self-assessment evolved from Black and Wiliam's (1998, 2009) work; it plays an essential role in classroom formative assessment practices. It empowers the learner to do the learning themselves through self-reflection (Noon & Duncan, 2005). Learners make decisions, reflect on their work, and identify their misconceptions (McMillan & Hearn, 2008). Self-reflection is enhanced when learners have a clear knowledge of what they need to learn. Noon and Duncan (2005) argue that learners can judge their performance and make decisions about themselves and their abilities. All this takes place with the use of TCs.

Gielen (2007) describes PA as:

[*A*] strong vehicle of 'assessment for learning' because it actively involves learners in evaluating their learning and allows them to participate in a collaborative appraisal using multiple perspectives when incorporating viewpoints from different learners. (pp. 102–103)

As a result of its socio-constructivist thrust, Leu and Wu (2005) and Santagata (2005) claim that PA is instrumental in

improving learning. Thus, learners can make comments about themselves and others as a way of pointing out their weaknesses and strengths to bring forth 'fruition' in learning mathematical concepts. Learners assume a teacher's role to the other learners when they use TCs.

However, research findings argue that teachers' perceptions of assessment for learning have a great impact on their assessment practices which also depends on how often teachers use PA (Panadero & Brown, 2017). PA is regarded as a strategy for formative assessment that focuses on other learners from the same class assessing each other. Heritage (2010) notes that:

[*T*]he feedback students provide to each other can also be an element of formative assessment for teachers. What students say or write about each other's work can be good evidence of how well they understand the learning goals and success criteria, and the depth of their thinking about the task at hand. (p. 14)

However, this can only be applied if there is a conducive environment. Besides, it could pose a challenge for learners to provide feedback to each other effectively. Not every learner can provide feedback because of different levels of understanding, and the poor performers will always rely on the top achievers.

What is assessment?

Assessment in general is a judgement about something that has been thought about in a specific way with the purpose of achieving desired outcomes. For example, in education, assessment is about determining whether or not the set goals of education have been achieved. It involves various methods of collecting data by teachers or interested parties, say the Department of Basic Education, to evaluate learners' progress, in terms of skill acquisition and knowledge and make decisions on what learning gaps need to be closed. More so, assessment is a broad term that constitutes four different types of assessments: baseline assessment, diagnostic assessment, formative assessment and summative assessment. However, we do not discuss them but we only focus on classroom formative assessment which takes an aspect of each one of them in some way 'to effectively address the mathematical "deficiencies"' (Chihodzi, 2020, p. 76) in learners. Hence, education goals are met.

Definition of assessment

According to CAPS (Department of Education, 2011):

Assessment is a continuous planned process of identifying, gathering, and interpreting information regarding the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement, evaluating this evidence, recording the findings and using this information to understand and thereby assist the learner's development to improve the process of learning and teaching. (p. 293)

This definition of assessment points out four key processes, namely: (1) generating and collecting evidence of learners'

achievement, (2) evaluating evidence of learning, (3) recording findings, and (4) providing constant feedback to advance learning and teaching. For all these processes to take place TCs are used as a first step. Thus, the teachers identify gaps, collect the evidence, evaluate it, record it and then provide feedback to move learners forward.

Chihodzi (2020) views assessment as:

[A] language that makes use of symbols and notations to describe the collected evidence of learners' mastery of numerical, geometric, graphical relationships and the provision of feedback to learners. It is an activity that involves gathering evidence of learners' abilities to observe, represent, and investigate patterns and quantitative relationships in life processes and between mathematical objects themselves. An assessment helps in developmental processes that enhance reasoning, analyzing, and problem-solving that will contribute to decision-making. (pp. 8–9)

The main aspect of Chihodzi's (2020) definition of assessment is to recognise its strong connection to mathematics. The aspect is the language of symbols that are used to communicate the collected evidence of learners' mastery of mathematical concepts, namely numerical, geometric, and graphical relationships, to mention but a few. More so, it identifies the teacher as the sole interpreter of the evidence who must provide quality feedback to the learners. The feedback should be 'pregnant' with information that informs and supports instruction (Dell & Dell, 2016). Hence, the instruction is designed to close the identified gaps. In this article, TCs are used to dialogue the elicited evidence of learning to both the teacher and the learners about the mastery of learnt concepts and address gaps accordingly.

Formative assessment (assessment for learning)

Dell and Dell (2016) say that formative assessment 'is a collection of formal and informal processes used to gather evidence to improve student learning—provides teachers and students with continuous, real-time information that informs and supports instruction' (p. 6).

According to the National Council of Teachers of English (NCTE) (2013) formative assessment is:

[*T*]he lived, daily embodiment of a teacher's desire to refine practice based on a keener understanding of current levels of student performance, undergirded by the teacher's knowledge of possible paths of student development within the discipline and of pedagogies that support such development. At its essence, true formative assessment is an assessment that is informing teachers, students, and families. (p. 2)

These definitions point out that formative assessment is a key process in the collection of evidence of day-to-day learning activities (formal or informal assessments) and provision of feedback which is based on achieving an intended outcome or learning goal. These assessments are marked which is part of 'processes of measuring knowledge, behaviour, skill, attitudes, and beliefs according to explicit rules and benchmarks' (Mahmoodi-Shahrebabaki, 2014, p. 3). We argue that assessment involves a number of different types; a test is a 'tool' that can be used in any of the four types of assessments to measure skills, attitudes, knowledge or beliefs. Interestingly, when teachers are assessing the skills acquisition and knowledge, they mark using TCs to gather evidence on learners' achievement.

Evidence of assessing learners' work

Feedback is known as one of the key components that are instrumental to classroom formative assessment only when collection of evidence is completed. It guides and helps learners to master mathematical misconceptions identified by putting TCs. There should be some evidence of interaction or dialogue or tracks of the teacher's markings, which include TCs and feedback comments to show that the learner's work has been assessed. Thus, the evidence of symbols like TCs, comments, signatures, totals and dated work should be noticeable in the learners' work. It must be understood that TCs are also forms of feedback. However, putting TCs only without written feedback comments may not actually serve any purpose. The reason is that teachers have failed to provide the guidelines on what necessary steps ought to be taken by the learners to address the mathematical misconceptions identified in the assessed work. Therefore, the written feedback comments in the learners' work become the evidence of assessment practices that assist the learners to address the identified challenges (Department of Basic Education, 2011). Hence, learning takes place. Signatures and dated or stamped work are important in that they serve as proof of accountability and responsibility of teachers' actions and decisions they make in learners' marked work as officials from the Department monitor the effectiveness and efficiency of teachers. The presence of TCs, dated or stamped work and quality written feedback gives a broader picture about an overall impression of classroom formative assessment practices. Thus, it becomes 'empirical knowledge ... and understanding' (Bowen, 2009, p. 33) of what is happening in the classrooms. The non-existence of TCs, stamped work and quality feedback gives an impression of ineffective teachers in terms of influencing learning because there is lack of proper dialogue around the learning of the concept. Sceptics in Dabell (2018) believe that if learners' work is left unmarked, mistakes, misconceptions and misunderstandings will appear in subtler ways and it becomes a challenge to deal with them. Sceptics in Dabell (2018) 'believe children need specific, accurate and clear feedback on paper so they can refer to it, go back to it and use it for growth' (p. 3).

Dabell (2018) also argues:

Many believe that written feedback is necessary when assessing written maths because it allows us to correct work, make useful comments, ask questions, set targets, award grades, and provide advice and guidance. Focused marking that pinpoints misconceptions and addresses gaps can be a key part of our maths dialogue with children. Meaningful marking also feeds into the next lesson because any areas of confusion can be flagged up, discussed together and dissected as a class. (p. 4) Dabell (2018) understands that marking should show a deep understanding of the purpose of focused marking. Learners need to know what must be corrected using comments to guide them into understanding what is missing in their answers and not TCs only. Such focused marking communicates to the learners what gaps should be addressed. Hence, learning is facilitated. Dabell agrees with Wiliam (2016) on effective feedback, which he calls 'detective work' (Dabell, 2018, p. 3). Wiliam advises teachers to provide feedback as detective work when he says:

If we're going to provide feedback on 20 answered questions then rather than just marking a pupil's work, we can instead enable deeper learning by making them look further. For example, by saying: 'Five of these are incorrect. Find them and fix them.' (p. 4)

Doing so makes the learners think more deeply about their original work in a more logical and critical way. This can be used when learners are trying to identify areas for improvement.

Marking is considered an important component in assessment which must be meaningful to bring about 'conceptual growth' in learners. Such practices in marking ensure that learners, as much as the teachers who provide the feedback, become the 'analysts' or detectives (Wiliam, 2016) in their own pieces of work. Asking learners to review what they have written empowers them to do the learning themselves through self-reflection in mathematics (Noon & Duncan, 2005; Wiliam, 2016). More so, learners will be taking centre stage in assessing themselves or other learners on their learning (Wiliam, 2007). It does not end there: learners should come back to check with their teachers to acknowledge if they are heading in the right direction.

What is marking?

In reality, the purpose of marking is to provide feedback to learners on their performance in a task. Marking must communicate to the learner what needs to be done to improve understanding of concepts (Shute, 2008). Marking learners' work enables teachers to make accurate judgements or evaluations on what learners ought to understand (Elliot et al., 2016). This involves the use of TCs and effective feedback.

When marking, the teacher interprets learners' understanding. To a larger extent marking plays a formative role when it is applied to a task directly related to the preceding teaching and learning process. If marking has to be meaningful, the parameters of the task need to be clear and applicable to all learners depending on the task. Therefore, sharing and clarifying the learning goal with learners becomes a 'beacon' that guides learners into knowing what they need to learn. Marking is intended to point out learners' strengths and weaknesses and prescribe to them the necessary steps to take them forward in their learning. In other words, marking should be about making appropriate comments on good performance and the seriousness of errors in learners'

marked work. However, the marking should be consistent and fair, giving positive feedback through TCs and scoring marks. Therefore, such practices are the indicators of what 'treatment' is needed to deal with learners' 'mathematical deficiency' or 'mathematical kwashiorkor syndromes' (Chihodzi, 2020, p. 203). In the same way, Elliot et al. (2016) agree with Chihodzi that marking provides the teacher with a better view of each individual learner's strengths and weaknesses and ultimately points to what the learner needs to do to move forward. Hence, our interest was driven by the need to understand the teachers' knowledge on the purpose of TCs in classroom formative assessments. However, putting TCs should clearly connect with questions given.

It is argued that constructive comments are a more effective form of marking than simply putting TCs. Herbert, Oates, Sherriff and Walker (2018) point out that 'marking has a positive impact upon pupils' learning and progression by tackling misconceptions early and giving the pupil an opportunity to correct errors immediately' (p. 17).

The emphasis here is not just on providing correct answers, but on allowing learners to be detectives of their own errors or mistakes or misconceptions (Wiliam, 2016). In Scotland, the Scottish Qualification Authority (SQA, 2014) states the following marking symbols for both internal and external marking of Mathematics:

- ____X At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, i.e. a cross is only used where a mark is not awarded.
- ★A cross-tick should be used to indicate 'correct' working where a mark is awarded as a result of follow-through from an error.
- X A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been eased.
- A tilde should be used to indicate a minor error which is not being penalised, e.g. bad form. This should be used where a candidate is given the benefit of the doubt.
- A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working. (pp. 3–4).

This is a clear policy on ways symbols help teachers to maintain consistency in their marking both internally and externally. This also allows learners to be able to understand the meaning of the symbols in their marked work. The lack of indicating these symbols or markings in learners' work implies lack of proper guidance in their learning. It would also imply a poor work ethic which thwarts learning. Using crosses only may have a negative impact on moving learners forward. Thus, it may be discouraging to the learners to simply see crosses without indications on how they should close the gap. Hence, the desire to learn the subject is thwarted. In this study, it has been shown that in South African primary schools there is no clear-cut policy on marking guidelines in assessments. Hence, marking is inappropriately done. The assumption was that in all the 43 public schools that participated in the study there was no evidence of proper formative assessment practices. Hence, teachers are directionless and with little knowledge of the purposes of TCs in mathematics assessment. Hence, continuous underperformance by learners is exacerbated.

However, the role of marking in assessment, particularly the use of TCs needs further exploration.

In the following sections, we present data from a study that examined the concept of assessment and provide an analysis that addresses the question related to the purpose that TCs serve in primary mathematics assessment.

Data collection

The data being analysed for this article formed part of a study that investigated three interlinked questions:

- What is the connection and influence between the teachers' mathematical understanding and classroom formative assessment practices?
- How do teachers share and clarify learning goals with learners to monitor progress towards achieving learning goals?
- How do teachers collect evidence of learning from learners, interpret it, and provide feedback to improve learning?

The participants for this study were 43 Intermediate Phase mathematics teachers and 95 learners from Grades 4–6 that were drawn from 16 public primary schools with permission from the Department of Basic Education. The selection of the 95 learners was not by design. Their parents or guardians gave the learners permission and they were also willing to participate in the study. Of over 600 teachers in Tshwane South district, 100 teachers were invited to participate, and only 43 were willing. Purposive non-sampling and convenient sampling methods were used to select the research participants as declared by other researchers like Rule and John (2011) who say:

[*A*] researcher ... has to choose people who can shed most light, or different lights, on a case. This is known as purposive sampling where the people selected as research participants are deliberately chosen because of their suitability in advancing the purpose of the research. (pp. 63–64)

Thus, the suitability of the convenient sampling and purposive non-sampling were strategically considered for this study.

Data collected involved an open-ended questionnaire for teachers and document analysis of learners' mathematics daily activity workbooks. The collected data were inductively and deductively analysed. That is to say, collected data were analysed inductively based on the exact written responses by the participants to the questionnaire and observations of the learners' vignettes in order to come to a general understanding of teachers' knowledge of classroom formative assessment. Also, the collected data were analysed deductively, meaning that we used reasoning from the responses of the teachers to the questionnaire to reach our conclusions.

The work that was analysed was from Terms 1–3 of 2018. The purpose of this study was to explore the Grades 4–6 mathematics teachers' knowledge in the effective use of TCs in classroom formative assessment practices to accomplish learning goals. This is so because classroom formative assessment is a fundamental tool that has been advocated by Dabell (2018) and Wiliam (2011, 2016) in the way it addresses issues of poor performances in mathematics.

Schools were purposively sampled on the basis of low performance in the Tshwane South district of the Gauteng province. Learners were also selected on the criteria of highest, average, and lowest achievers in each class. The purpose was also to have a better understanding of how teachers marked learners' work to move them forward. We used vignettes from learners' daily activity workbooks (informal) from which the information was tabulated on uses of TCs, feedback comments, scoring marks, signatures, and dating work by teachers from the 43 classes that participated. Secondly, in our interpretation of the data we tried our best to be as objective as possible to minimise biases so as to avoid discrediting the outcome of the study. Therefore, a thorough investigation of the vignettes was objectively analysed and credibility preserved (Bowen, 2009; O'Leary, 2014). To reduce subjectivity in the use of vignettes we used all the photographs of the learners' mathematics written and marked work to avoid bias.

Data analysis and discussion

In the study, we collected evidence needed to establish whether learners' work had been assessed. Table 1 and

Table 2 show data related to evidence of work for 95 learners' (n = 95) work that was assessed by 43 Mathematics teachers. The evidence in Table 1 and Table 2 focuses on the frequency of use of TCs, comments, totals and scores, signatures and dated work by the teachers. Vignettes were used to corroborate this evidence as a way to understand the teachers' knowledge of the purpose of TCs in classroom formative assessments.

Table 1 displays who frequently marks the work, be it the teachers themselves, learners only or both. The table shows that:

- Most Grade 4 work (48%) is marked by the learners themselves.
- In Grade 5, marked work is fairly distributed, with 34.4% being marked by learners while work marked by teachers only and by both teachers and learners have each 31.3%.
- In Grade 6, most of the work (38.2%) was marked by learners themselves while 35.3% of the work is marked by both the teachers and learners.
- On average, most work (39.3%) was marked by learners across the grades.

Also, Table 1 shows the extent to which TCs were used to mark learners' work by either the teachers or learners. The uses of the TCs in Table 1 is characterised by the following:

- Most of the Grade 4 work (41.4%) was not marked using TCs.
- Most of the Grade 5 work (31.3%) has ticks only while the evidence of those that used TCs and those that did not use TCs have 25% each.
- Most of the Grade 6 work (32.4%) has no evidence of TCs while 29.4% has evidence of TCs.
- On average, and across the three grades, most of the learners' work (30.5%) has no evidence of TCs while 29.5% has both.

This was an unexpected practice in marking learners' work because it deviates from effective marking. This practice fails to provide the teacher with a better view of each individual learner's strength and weaknesses. Marking learners' work

TABLE 1: Frequency of use of ticks and crosses in learners' assessment work.

Grade	N	Work marked by						Marking learners using ticks and crosses								Scoring marked work			
		Teacher		Learner		Both		Evidence of ticks and crosses (V/x)		Evidence of ticks only (√)		Evidence of crosses only (x)		No evidence of ticks and crosses (V/x)		Indicated		Not indicated	
		n	%	n	%	п	%	п	%	n	%	n	%	n	%	n	%	п	%
4	29	6	20.7	14	48.3	9	31	10	34.5	6	20.7	1	3.4	12	41.4	13	44.9	16	55.1
5	32	10	31.3	11	34.4	11	31.3	8	25	10	31.3	8	25	6	18.8	12	37.5	20	62.5
6	34	9	26.5	12	35.3	13	38.2	10	29.4	6	17.6	7	20.6	11	32.4	13	38.2	21	61.8
Total	95	25	26.2	37	39.3	33	24.2	28	29.5	22	23.2	16	16.8	29	30.5	38	40	57	60

TABLE 2: Frequency of commentary notes in learners' assessment work

Grade	N		Evidence of	f comments			Evidence of	f signatures		Evidence of dated work				
		Yes	%	No	%	Yes	%	No	%	Yes	%	No	%	
4	29	2	6.9	27	93.1	10	34.5	19	65.5	7	24.1	22	75.9	
5	32	1	3.1	31	96.9	12	37.5	20	62.5	10	31.3	22	68.8	
6	34	5	14.7	29	85.3	16	47.1	18	52.9	15	44.1	19	55.9	
Total	95	8	3.2	87	91.6	38	40	57	60	30	33.7	63	66.3	

involves the teacher's expectations to make judgements about what learners ought to understand (Elliot et al., 2016). The CAPS (Department of Education, 2011) emphasises producing and gathering evidence of learners' mastery of concepts, evaluating evidence of learning, and providing constant feedback to improve the process of learning and teaching; these teachers are deviating from effective marking that improves learning. Therefore, it shows that such practice by both teachers and learners when using TCs is inappropriate.

Table 1 also demonstrates the extent to which scoring marked work was done. Most of the Grade 4–6 learners' work did not have mark totals (Grade 4 55.1%, Grade 5 62.5% and Grade 6 61.8%).

Overall, learners' work did not have totals or scores. Dabell (2018) argues:

Many believe that written feedback is necessary when assessing written maths because it allows us to correct work, make useful comments, ask questions, set targets, award grades, and provide advice and guidance. (p 4)

Our interpretation of this evidence was that there were no scores for learners to see how much they had achieved, and there was a lack of feedback comments that point out learners' weaknesses and strengths and ways to close gaps. Hence, the learners' written activities were largely not advancing learning.

Table 2 shows overwhelming evidence of teachers not giving appropriate comments as feedback to learners across the three grades (93.1% in Grade 4, 96.9% in Grade 5 and 85.3% in Grade 6). From the evidence displayed in Table 2, it is clear that the role of teachers in providing feedback comments about learners as a means to point out their weaknesses and strengths to bring forth 'fruition' in learning mathematical concepts was not understood. Dabell (2018) argues:

Focused marking that pinpoints misconceptions and addresses gaps can be a key part of our maths dialogue with children. Meaningful marking also feeds into the next lesson because any areas of confusion can be flagged up, discussed together and dissected as a class. (p. 4)

Wiliam (2016) and Black and Wiliam (2009) point out that feedback comments have profound influence on improving the process of learning and teaching provided that the elicited learning evidence is effectively used to give quality feedback.

Also, Table 2 shows significant evidence of 65.5%, 62.5% and 52.9% of Grade 4, 5, and 6 teachers not putting signatures to authenticate that they have seen and marked the work themselves or they have seen and approved the marking that was done by the learners.

Dated work by teachers or by an official in a document is proof of acknowledging and authenticating that the contents in a document were seen and verified on the date written down. As demonstrated in Table 2, Grade 4 (75.9%), Grade 5 (68.8%) and Grade 6 (55.9%) work was not dated at all to indicate that the work was assessed on particular date by the teachers.

On average across the three grades, the data show that:

- In the majority (91.6%) of learners' work teachers did not seem to appreciate the value of writing feedback comments on what the learners should do to address learning gaps.
- The majority (60%) of learners' work implies that the teachers might have been ignorant about the importance of putting signatures to authenticate the genuineness of the marking that had been done and that they have seen and have verified the work as a true record.
- The majority (68%) of learners' work was not dated by the teachers to show when the work was marked to prove that the work was checked as a means of assessing.

If learners' work fails to have any of the three aspects highlighted, the teachers were not effectively assessing and monitoring learners' work. Sceptics in Dabell (2018) understood that if learners' work was left unmarked then misunderstandings, mistakes and misconceptions would crop up in subtle ways and would become problematic to deal with. Learners should get written, accurate, specific information and specific activities to work on to improve their learning. When effective marking practices are followed, there is a likelihood of gathering relevant information that will be used to provide effective feedback. Thus, learning is enhanced (Wiliam, 2007). Elicited learning evidence helps to detect learning gaps and then design ways to address them accordingly. The teachers examine the evidence of learning and review it after a lesson or series of lessons as a way to deal with learners' errors or misconceptions. The argument is that if all this is not taking place learning is incapacitated. The simple reason is that teachers are not dialoguing effectively with learners whose needs they have identified and are not paying particular attention to the learners' needs (Wiliam, 2011). The failure to dialogue with each individual learner might also be hindered by high teacher-learner ratios.

Numerous studies have found that the time invested by teachers to provide personalised feedback aims at providing learners with 'rich' information to make substantial progress in future tasks (Wiliam, 2011). However, personalised feedback is effective when it is clearly understood by the learners and also if they have a willingness to act on it (Ryan, Gašević, & Henderson, 2019; Winstone, Nash, Parker, & Rowntree, 2017). Otherwise, the 'feedback process is thwarted', as noted by Henderson, Ajjawi, Boud and Molloy (2019, p. 21) because it is not user-friendly to the learners as it fails to make sense to them. Lim, Joksimovic, Gašević and Fudge's (2020) and Winstone et al.'s (2017) findings indicate that when a teacher provides useful information as personalised feedback it increases the learners' desire to learn, increases their motivation and keeps them on task.

Exemplars of vignettes

To corroborate the data shown in Table 1 and Table 2, here are four examples of vignettes that demonstrate ways teachers and learners mark mathematics work using TCs. This enabled us to understand the teachers' knowledge of the purpose of TCs in classroom formative assessments. The vignettes also show how teachers write feedback comments, put signatures and put dates in all work seen by them as evidence that the work has been assessed by them.

In Figure 2, there is evidence that learner V2b's work was assessed by the teacher themself. The following is evidence that shows us that the teacher marked the work using ticks only in red. The teacher ticked in all the blocks but left out five. The reason for not ticking the five blocks is unknown. More so, both the ticked and unticked blocks serve no purpose to the learner regarding the concept of diagrammatic representation of fractions since the teacher did not make an effort to give feedback comments. What it also means is that both the marked and unmarked blocks are mathematically incorrect because it was not explained that each small block represents $\frac{1}{100}$. Hence, our interpretation was that the marking served no purpose at all because one small square was considered $\frac{1}{100} = 0.01 = 1\%$, 4 small squares were $\frac{4}{100} = 0.04 = 4\%$, etc. This sounds correct, but the diagrammatic representations of fractions in the top first row were inaccurate because each small block or square does not represent $\frac{1}{100}$

In this context, the only suitable way to deal with diagrammatic representation was to use a 100-block grid. This suggests that the teacher did not understand the diagrammatic representation of common fractions. Thus, the ticks in this case were inappropriately used. Therefore, the teacher conveyed the wrong concept to the learner. In this case, crosses and feedback comments should be prevalent to point out to the learner what is wrong. Surprisingly, no comment was given to guide the learner since their work could be construed as a misconception. To

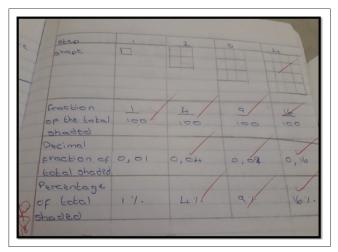


FIGURE 2: Evidence of teacher V2 assessing learner V2b's work.

make matters worse, at the bottom left of the vignette we see the teacher's signature and the date the work was marked, which officially signifies knowledge of correctness of answers, approval and acceptance that the teacher had seen and marked the work and that all was in order.

Our conclusion in this case was that assessing and monitoring of learners by the teacher is ineffective which may be the result of lacking specialised content knowledge (Ball, Thames, & Phelps, 2008; Heritage, 2007; Stiggins, 2009). That is why the teacher was unable to detect the incorrect diagrammatic representation of fractions. Therefore, the purpose of using ticks is not clearly understood. A parent who has knowledge of mathematics could challenge the teacher for failing to impart the correct mathematical knowledge, because the teacher's signature authenticates and approves that the work was properly assessed by the teacher. Gill (2014, p. 1) says 'a signature is a mark or sign made by an individual on an instrument or document to signify knowledge, approval, acceptance, or obligation'. It is important that whenever teachers put signatures they must be sure of the accuracy of the information. We argue that this was futile marking. Hence, the teachers' marking may be interpreted as 'window dressing', meaning that their marking created an impression that they are marking effectively yet it served minimum purpose.

The teacher's pedagogical role as an assessor is questionable. How could one lead a way they do not know? Hence, the blind leading the blind, they will both fall into a ditch. This results in two conditions which Chihodzi (2020) described as:

- 'mathematical stagnation' which refers to a permanent hindrance to progressing to learning mathematics. Such learners drop or 'reject' the subject informally because they feel that mathematics is just too difficult for them (p. 220).
- 'perennial mathematical sterility', meaning that learners experienced a long period of continuation of inability to make sense of mathematics unless they meet a teacher whose subject knowledge is strong and his/her confidence levels to teach that subject is great (p. 230).

These scenarios imply that the reliability and validity of assessments are compromised. This raised questions about how classroom formative assessment practices were done in schools.

Figure 3 shows evidence that learner U1b's work was assessed by the teacher themself. What is seen in the vignette are many red ticks, two crosses and corrections that were written by the teacher. The work was not totalled. Instead, we see the teachers' signature and the date at the bottom left of the work to signify and to authenticate knowledge and approval that the work had been seen and marked by them. One of the essential components of marking that is missing in the vignette is feedback comments on the learner's work. Here, the teacher provides the correct answers as feedback without writing comments to guide the learner on how to

FIGURE 3: Evidence of teacher U1 assessing learner U1b's work.

close the gap by being 'detectives' (Wiliam, 2016) of their own work. In this case, the learner may still find it challenging to know how the answers were obtained.

Hence, teacher U1's marking practice is unethical and ineffective because it is not correct and acceptable. In addition, we do not see the addition and subtraction of the six-digit numbers in the topic clarified and shared with the learner so that learner can monitor their progress in light of the goals (Heritage, 2012). There is no connection between the topic and the procedure of arriving at the answers. More so, the topic is also not connected to the second part of the answers. In this case, this task cannot assist the learner to prepare for a formal assessment because it fails to demonstrate how the answers were obtained. If the teacher knew what assessment entails they could have started the task by sharing the learning goal with learners as the first step in developing their understanding of what they are to learn and then providing examples of what they are expected to produce. Nyquist's (2003) typology describes this marking as weaker feedback practices where learners are given only marks or grades known as 'knowledge of results', or only scores or grades with memoranda or answers often viewed as 'knowledge of correct results'. Dabell (2018) argues that teachers should correct work, make useful comments that provide advice and guidance to address the misconceptions.

Ideally, teachers give feedback comments to learners to help them better master content and improve their achievement (Fisher & Frey, 2009). Wiliam (2016) argues that:

[*I*]f a math teacher corrects a student's arithmetic errors, there's nothing left for the student to do but note how many of her calculations were incorrect. It's easy to see why such forms of feedback are unlikely to be effective. (p. 1)

It is arguable that merely giving feedback about current achievement produces minimal benefit, but when learners are engaged in mental activity, the effects are maximised.

In all instances of gaps between the learners' current learning and the desired instructional goals, the teacher identifies learners' emerging understanding or skills and modifies instruction to facilitate progress (Heritage, 2010). The analysis and interpretation of learning evidence are pivotal for the

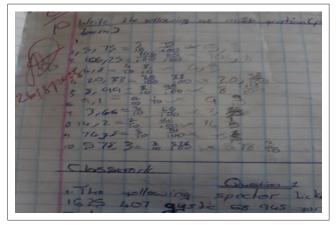


FIGURE 4: Evidence of learner or peer assessment of learner K5c's work.

effectiveness of classroom formative assessment to close the gaps (Wiliam & Thompson, 2007).

Figure 4 shows that the work was marked by the learner or by a peer as evidenced by pencil crosses. In this vignette, we see crosses the learner or peer used. Crosses are more prevalent, showing that there is a misconception of changing decimal fractions to common fractions. What is also surprising is that questions 1, 4, 5, 7, and 8 were marked incorrect yet they are all correct and, to make matters worse, the corrections given are incorrect. What we see here is the teacher's signature signifying knowledge and approval about correct marking (Quora, 2014).

The marking in Figure 4 does not help the learner for revision purposes because there are contradictions in the marking. Thus, correct answers (1, 4, 5, 7, and 8) are considered incorrect, and corrections provided are incorrect. What also lacks in this vignette is learning goals and exemplars of work or criteria that should have been given as guidance to what the learner was expected to learn. More interestingly, at the top left-hand corner of this activity, the teacher wrote the total, $\frac{0}{10}$ (grade), and signed and dated to authenticate knowledge and approval that the work has been seen and marked by the teacher. Provision of the correct answers makes learners think less because everything has been done for them (Wiliam, 2016). Spruce (2017) also argues that '... students need to know what is wrong and how they can correct it.' (p. 2) Studies have shown that when teachers analyse and interpret evidence it is pivotal for the effectiveness of classroom formative assessment that enhances closing gaps (Wiliam & Thompson, 2007).

Our interpretation of the evidence in Figure 4 is that the teacher does not use feedback comments to diagnose learners' strengths and weaknesses to guide instruction, a staple of teachers' instructional practices (Boston, 2002). According to past research feedback should be prescriptive, supportive, and specific in what the learners need to do (Shute, 2008) to close the gap. Sadler (1989) makes it clear that information itself is not feedback, but only becomes feedback when it is actively used 'to alter the gap' (p. 121). The lack of providing written feedback is described as

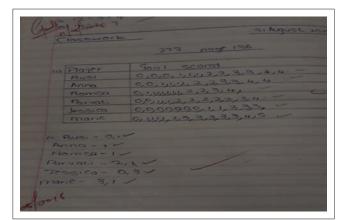


FIGURE 5: Evidence of assessing learner O2a's work.

'poverty of practice' (Black & Wiliam, 2001, p. 4) which is beset with problems and shortcomings that inhibit learning.

Figure 5 shows that this activity was marked by both the learner or peer and the teacher as evidenced by ticks in pencil and in red pen. The learner's work has evidence of ticks only from both the learner or peer and the teacher. The teacher's ticks, signature and date authenticate that they have seen the work and that it has been correctly marked by the learner or peer. What is missing in this activity is the total of what the learner achieved. Also in the vignette, we see the commonest feedback comments most teachers give. In this vignette, the teacher writes 'Good' at the top left-hand corner of the activity. This descriptive comment also motivates learners but it fails to provide the information the learner needs to use to move forward. In this case, the teacher could have commented, for example, 'Your answers are correct but there is something missing'. The learner would examine their work to see the missing piece of 'cake'.

The possible answer would be that the answers have not been written in full to convey the correct message to the learner for future reference. Our understanding is that the teacher is not using ticks effectively. They are mostly interested in answers. The teacher is not using the most effective way to dialogue with each learner and pay particular attention to the learners' responses (Wiliam, 2011). Sadler (1989) made it clear that information itself is not feedback, but only becomes feedback when it is actively used 'to alter the gap' (p. 121). We assume that the reason for teachers to provide effective feedback could be a result of high workloads, time scarcity and challenges with learners (Mahmoodi-Shahrebabaki, 2015).

Conclusion

In this article, we explored teachers' understanding of TCs in their daily assessments. The analysis in this study has shown that the teachers used TCs, feedback comments and signatures in assessing learners' work. However, the teachers have demonstrated a limited understanding of each one of the components of assessment. In most of the work that was marked, TCs were used inappropriately. The marking served no purpose and was misguiding to the learners towards achieving mathematical learning goals. Most teachers were mostly interested in the answers learners provided and not the procedures. In some cases, the work was unmarked and, thus, misunderstandings, mistakes and misconceptions would crop up in subtler ways and it would become problematic to deal with the learners' misconceptions. The way learners' work was being marked by the teachers cannot be expected from qualified mathematics teachers because they are deviating from effective marking practices. The reason is that it does not help the learners to move forward in learning mathematics. To a larger extent the teachers themselves lacked an understanding of each individual learner's strength and weaknesses. Teachers were simply running TCs across learners' written work for no clear purpose. In other words, learners were being taught by incompetent teachers. Hence, learners lacked mathematical proficiency.

On average, most work was marked by learners across Grades 4–6 (Cowie, Moreland, & Otrel-Cass, 2013; Gielen 2007; Heritage, 2007; Panadero & Alonso-Tapia, 2013; Panadero, Brown, & Strijbos, 2015). Research has identified peer and self-assessment as instrumental and paramount in assessment because they build a community of learners who are involved in evaluating and monitoring their learning. Gielen (2007) says 'peer assessment allows them to participate in a collaborative appraisal using multiple perspectives when incorporating viewpoints from different learners' (pp. 102–103). It means that peer and self-assessment enable learners to identify their own discrepancies between what they know and the desired achievement. Therefore, learning is enhanced.

In this study, it was also found that despite learners assessing themselves they received weaker feedback: only marks or grades, known as 'knowledge of results', or only scores or grades with memoranda or answers, often viewed as 'knowledge of correct results' (Nyquist, 2003) which lacked providing the learners with deep knowledge of how they could close the gaps. Teachers simply put signatures without feedback comments. This practice is not effective in advancing learning because learners are not fully guided on effective ways to examine their own mistakes and correct them. Rice (2014) says teachers should 'try to do more than just put ticks' to motivate the learners. The evidence we found in this study that teachers focus on answers was also deeply rooted in the learners. As long as their answers were correct all was well. This practice of marking may heavily affect learning mathematics.

Overall, learners' work did not have mark totals. In general, mark totals are two sides of a coin. To those learners who do well, mark totals may instil confidence and enthusiasm to learn while to those that do not, they pose discouragement to learning. However, the most effective way is to provide comments that focus on the learners' work rather than the learners (Nyquist, 2003). Metcalfe (2017) in her study describes this as corrective feedback that provides direct advantage to learners' improvement. Recent research has found that it is necessary to correct learners' work and then provide advice and guidance. Kornell, Klein, & Rawson (2015) investigated the effects of corrective feedback and they reached the same conclusions.

Across the three grades, we found that a significant amount of learners' work never received any written feedback comments from teachers. Possibly the teachers do not really understand the positive impact of comments on learners' work. It may also suggest a time factor due to the high teacher-learner ratio. Besides it may also require a lot of time to craft feedback comments. However, Dabell (2018) views feedback comments in mathematics teaching as pivotal in that they provide the directions that learners have to take to move forward. The study revealed that feedback to learners was mainly descriptive and focused on what the learner has achieved or improved. Yet, feedback comments are the most powerful and effective way to communicate with each learner and pay particular attention to the learners' needs and responses and help them to master the concepts and improve their learning (Boud & Molloy, 2013; Carless, 2015; Rowe, 2017). As a result of teachers' ineffective feedback to learners we assumed teacher burnout as they were likely to be overwhelmed with inadequate time and high teacherlearner ratio (Mahmoodi-Shahrebabaki, 2015).

The study findings show that some teachers marked incorrect answers in learners' mathematics work as correct. These teachers seemed to lack specialised content knowledge and were unable to detect learners' misconceptions. In addition, teachers put their signatures to signify knowledge, approval and acceptance of the marking as true records. Such practices could jeopardise one's profession if sued by parents who are knowledgeable about mathematics. It is important that whenever a teacher puts a signature they must be sure of the accuracy of the information they have acknowledged. To curb the misuse of TCs when marking there should be a clear-cut marking policy that guides teachers to provide effective marking that leads to achievement of learning goals.

Acknowledgements

Competing interests

The authors declare that they have no financial or personal relationships that may have inappropriately influenced them in writing this article.

Authors' contributions

B.C. implemented the research work and carried out the data analysis. B.N. and W.M. supervised the research work.

Ethical considerations

The Gauteng Department of Education approved the research study (8/4/4/1/2).

Funding informations

This research received no specific grant from any funding agency in the public, commercial or not-for-profit sectors.

Data availability

The data that support the findings of this study are available on request from the corresponding author, W.M.

Disclaimer

The views and opinions expressed in this article are those of the authors and do not reflect the official policy or position of any affiliated agency of the authors.

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