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Epistemological Obstacle in 3D Geometry Thinking: Representation, Spatial Structuring, and Measurement

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Abstract

3D geometry is an essential concept in mathematics because it relates to real-world experiences and numerous mathematical topics. However, some students still have difficulty understanding this geometry. Therefore, this study aims to identify and describe students' epistemological obstacles to the dimensions of representation, spatial structure, and measurement of 3D geometry in junior high school. This study was carried out using an exploratory case study design with the purposive sampling method used to obtain data from26 students from three junior high schools in Indramayu Regency, Indonesia, through tests and interviews. The data collected were analyzed, synthesized, and described based on the framework that had been prepared. The result showed that students experience epistemological obstacles in performing a series of 3D geometric thinking tasks. They experience difficulties translating the representative model from 2D to 3D and determining the number of unit cubes of the 3D shape. Meanwhile, in the measurement dimensions, students experience difficulties in calculating surface area and comparing volumes of 3D geometric shapes. Therefore, to minimize the epistemological obstacle experienced by students, this study suggests teachers or prospective teachers pay attention to the characteristics of the material to be taught to students.

Key words: epistemological obstacle, 3D geometry, construction of thinking representation, spatial structuring, measurement.

Introduction

Geometry is one of the oldest areas of mathematics study (Mammarella, Giofrè, & Caviola, 2017) that plays an important role in education and school's curriculum (Herbst, Fujita, Halverscheid, & Weiss, 2017; Marchis, 2012; Serin, 2018). This is because it has a long history in the transmission process of human cultural civilization (Herbst et al., 2017). Furthermore, geometric physical objects that are visually, aesthetically, and intuitively attractive (Jones, 2002a) can be used to produce cultural relics (Herbst et al., 2017; Marchis, 2012; Serin, 2018) which are essential components of various aspects of human life (Jones, 2002a). These characteristics consider geometry as a science that bridges mathematical concepts and real-world experinces (Clements & Sarama, 2011).

Although this concept is an essential mathematical study taught in schools (Jones, 2002b; The International Commission on Mathematical Instruction, 1995; Zuya & Kwalat, 2015), many students experience difficulties in representing (Hwang, Su, Huang, & Dong, 2009; Panaoura, 2014; Parzysz, 1988, 1991; Pittalis & Christou, 2013; Kuzu, 2020), translating spatial structures (Battista, 1999; Battista & Clements, 1996; Ben-Haim, Lappan, & Houang, 1985; Pruden, Levine, & Huttenlocher, 2011; Tan-sisman & Aksu, 2016) and measuring (Battista, 1999; Hong & Runnalls, 2020; Huang & Wu, 2019; Özerem, 2012a; Tan-sisman & Aksu, 2016) 3D geometry objects. Therefore, students' obstacles in representing 3D geometric objects are divided into 2 dimensions, namely coding and decoding (Pittalis & Christou, 2013). Coding is the construction of plane representations and 3D shape nets, while decoding is the interpretation of structural elements and properties of 3D geometric shapes (Pittalis & Christou, 2013). Students' inabilities to translate spatial structures are closely related to visual-spatial abilities (Ben-Haim et al., 1985). Therefore, they need to study and understand the parts of objects to transform them from 3D to 2D during the representation of 3D geometric nets (Pittalis, Mousoulides, & Christou, 2010). Identifying the structure of 3D objects is not easy because it requires combining the internal representation with the visual structure of 3-dimensional objects (Cooper, 1990). Furthermore, misconceptions in measuring area and calculating volume from 3D geometry are caused by background knowledge, lack of reasoning, and basic operating measurement errors (Özerem, 2012b). The ability to determine the area and volume of objects is essential for students to be successful in both math and science (Vasilyeva et al., 2012).

Although several studies have been conducted on students' epistemological obstacles in mathematical concepts (Brown, 2008; Job & Schneider, 2014; Mesquita, 1998; Moru, 2014;

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Sunariah & Mulyana, 2020), none have been widely carried out on 3D geometric thinking processes. Epistemological obstacle occurs because of the nature of the mathematical concept itself (Brousseau, 1997). Epistemological obstacles will appear when students make mistakes when responding to assignments or questions made by the teacher (Brousseau, 1997). This is also emphasized by (Suryadi, 2013) who explains that the epistemological obstacle is essentially a person's knowledge which is only limited to a certain context. If the person is faced with a different context, then the knowledge he has becomes unusable or he has difficulty using it (Suryadi, 2013). In addition, in planning to teach or designing teaching materials, it is very important to determine possible obstacles, especially epistemological obstacles (Cornu, 2012). Therefore, in the context of this study, the study of the epistemological obstacle aims to identify possible obstacles that occur when students perform a series of cognitive actions in 3D geometric thinking. The formulation of the research problem is as follows.

1. What is the description of the epistemological obstacles of students in solving problems in 3D geometry material?
2. What is the description of the epistemological obstacles in the dimensions of the representation of 3D geometric shapes?
3. What is the description of the epistemological obstacles in the dimensions of the spatial structure of 3D geometric shapes?
4. What is the description of the epistemological obstacles in the measurement dimensions of 3D geometric shapes?

Method

Context

In Indonesia, 3D geometric thinking is studied at the elementary and junior high school levels. Table 1 shows the description and distribution of 3D geometry material in schools.

Furthermore, the study of mathematics curriculum in Indonesia at the elementary school level contain activities such as the elements and properties of 3D geometry. Meanwhile, at the secondary school level, it is associated with the measurement of surface area and volume of 3D geometric shapes. Furthermore, the overall concept of 3D geometry is taught in a structured manner, starting from recognizing, identifying, and understanding the properties, such as cubes, cuboids, prisms, pyramids, cubes, cones, and spheres. This is further associated with student activities in drawing, constructing nets, and determining the surface area and volume of cubes, cuboids, prisms, pyramids, cubes, cones, and spheres.

Design and Participants

The exploratory case study design is used to identify and describe the epistemological obstacles faced by students in carrying out a series of 3D geometric thinking processes. This case study was used due to its ability to explore situations where the intervention being evaluated does not have a clear set of outcomes (Yin, 2018). Practically, the research starts by selecting 3 secondary schools in Indramayu Regency, Indonesia, to be used as study sites. These schools were chosen due to their outstanding performance at the national exam on geometry material. Before the study was carried out at these schools, permission was obtained from the 3 principals, followed by the assistance of mathematics and assistant teachers to find students leaving close to the schools and were given permission by their parents to participate in this study. A total of 9, 11 and 7 students at the VIII grade levels in schools A, B, and C voluntarily participate in this study. Table 2 shows the characteristics of these students.

Initially, 26 students were invited to explain and discuss the technical study through a zoom meeting and afterwards, the number was reduced after seeking their parents' consent to minimize misunderstandings.

Table 1:*Distribution of 3D Geometry in Schools*

No	*Level*	*Description*
1	Elementary School (Level 1-6)	Recognize 3D and 2D geometry using a variety of concrete objects. Describes 3D and 2D geometry based on their characteristics. Explain and determine the area and volume of 3D geometry in non-standard units using concrete objects. Explain and find simple 3D geometric webs, such as cube and cuboid. Describes and determines volumes of 3D geometries using volume units, such as cubes and the cube-root relationship. Compare prisms, cylinders, pyramids, cones, and spheres. Explain 3D geometry, which is a combination of several geometric shapes and their surface area and volume.
2	Junior High School Level 8(13-14 years old)	Distinguish and determine the surface area and volume of 3D geometry, such as cube, cuboid, prism, and pyramid.
3	Junior High School Level (14-15 years old)	Generalize the surface area and volume of 3D geometry, such as tubes, cones, and spheres.

Table 2: *Characteristics of students*

No	*School*	*Amount*	*Level*	*Gender*	*Age*
1.	School A	9	VIII	3 male students, 6 female students	12-14 Years old
2.	School B	11	VIII	4 male students, 7 female students	12- 14 Years old
3.	School C	6	VIII	2 male students, 4 female students	13-14 Years old

Data collection

In this study, students' 3D geometric thinking ability test results were used as a reference to explore their epistemological obstacles. In addition, before the test questions are used, we have ensured that the test instruments used have met the validity and reliability. The test questions include 6 3D geometric thinking skills indicators developed by (Pittalis & Christou, 2010). Each indicator represents one question, and the first investigates students' ability to identify the properties of 3D geometry. The second test question is related to identifying and creating the associated net shapes. For example, students were asked to construct nets from cuboids with each side marked. The third question is related to students' abilities to translate 2D from 3D representations by drawing 2D shapes from the front, top, and side views of the 3D. The fourth test question relates to students' abilities to interpret the structural elements of 3D shapes, such as counting the number of cubes from the arrangement of unit cubes that form a 3D geometric figure. The fifth test question relates to students' ability to calculate the surface area and volume of the cuboids that make up certain 3D geometric shapes. The last question relates to their abilities to compare 3D geometric shapes based on their properties. Data were also collected from interviews to enrich the students' thinking process profiles and determine their difficulties identifying 3D geometry objects.

Data analysis

The data analysis process in this study is in accordance with the research carried out by (Tan-sisman & Aksu, 2016) in analyzing students' misconceptions and errors in spatial measurements such as length, area and volume of 3D geometric shapes. It consists of 7 stages, namely (1) making an assessment key, (2) developing a theory-based framework, (3) identifying student errors in answering questions, (4) linking students' difficulties with existing conceptions, (5) explaining the reasons for the conception identified as an epistemological obstacle, (6) conducting discussions using 3D geometry thinking theories and (7) descriptive analysis.

In the first stage, this study arranges questions to measure students thinking ability regarding 3D geometry and prepares the answer key. Points 1 and 0 are given for correct and wrong answer/explanation, respectively. The next stage is to create a framework based on the 3D geometry thinking ability indicator in accordance with the research carried out by (Pittalis & Christou, 2010), which is contained in the article entitled "Types of reasoning in 3D geometry thinking and their relations with spatial ability". In the third stage, the students' answers are transformed into Ms. Word and categorized sequentially by considering the framework in the second stage. This is followed by the fourth stage, which links students' errors with a theory-based framework solving 6 questions based on 3D geometric thinking skills indicators. The fifth stage provides the reasons for the existence of an epistemological obstacle, while the sixth discusses the results of identifying students' epistemological obstacles. In the last stage, the results of student errors were analyzed descriptively, as shown in Figure 1.

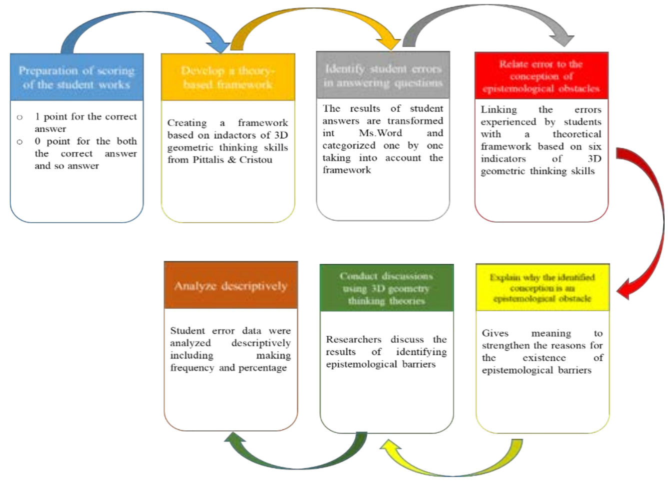

Fig. 1: Data Analysis Framework

Findings

The 3D geometry thinking ability test was examined on students in each school. Empirically, the analysis results of answers to 26 students are shown in Table 3.

In this study, the maximum score obtained by students from each question is 10. Based on the results, it was found that the average score of students in the first and second questions was 4.85 and 4.58, which indicates that the majority have difficulties in identifying and constructing nets from 3D geometric objects. Meanwhile, the average score on the representation dimension is 4.71, which also explains that students experience obstacles in representing 3D geometric objects. Furthermore, for the third and fourth questions, the students' average scores were 3.71 and 2.73, which indicate that they experience difficulties in translating 2D geometric shapes from 3D. They also experience obstacles in determining the number of unit cubes in 3D geometric shapes. Meanwhile, in the aspect of spatial structure dimensions, the average score is 3.25, which means that students experience obstacles in this aspect. Furthermore, for the fifth and sixth questions, the average scores were 4.96 and 4.35, which indicates that they experience difficulties in determining the surface area and volume of 3D geometric objects. Meanwhile, an average score of 4.65 was obtained in the aspect of measurement dimensions, which means that they experience obstacles in the measurement dimension.

These findings are also reinforced by the percentage of students that cannot answer the questions correctly, as shown in Table 4.

Table 4 shows that most students provided wrong answers to the questions. The table shows that in the first question, only 11 out of 26 students answered correctly, while the remaining 15, or 57.7%, answered incorrectly. Only 12 out of 26 students answered correctly for the second question, while the remaining 14 or 53.9%, answered incorrectly. Furthermore, only 10 students provided correct answers for the third question, while the remaining 16 or 61.5%, gave incorrect answers. Furthermore, only 6 out of 26 students answered correctly for the fourth question, and the remaining 20 or 76.9%, answered incorrectly. In the sixth question, out of 26 students, only 11 answered correctly, and the remaining 15 or 57.7%, answered incorrectly. For the fifth question, the number of students that answered correctly and incorrectly was the same. Overall the number of students that provided incorrect answers indicates that students experience epistemological obstacles in carrying out a series of 3D geometry thinking tasks such as in representing geometric objects, determining the spatial structure, and measuring the area and volume.

Epistemological obstacles for students in identifying the elements and properties of 3D geometry

Students must be able to represent the field of the colored unit cube to solve the first question. Furthermore, they need to identify and count the number of edges of a unit cube that represents the length, width, and height of the large cube. The analysis results of students' obstacles in answering the first question (see Table 5) showed that only 42.3% provided correct answers. In comparison, the remaining 57.7% experienced difficulties identifying the elements and properties of 3D geometry (Table 5).

There are 4 types of epistemological obstacles found in the first question. The first type shows that 26.9% of students experience difficulties in visually representing the unit cube field because it is not directly visible. Meanwhile, the second shows that 11.5% experience difficulties in identifying and counting the number of edges in the length, width, and height of a large cube. This obstacle causes students to be unable to calculate and add up the remaining unit cubes in the 3D form in each row. The point of view of the 3D shape in the image causes not all unit cubes to be seen directly due to obstruction. These obstacles prevent students from calculating unit cubes that are not visually visible. In the third type of obstacle, 7.7% of students were unable to calculate the rest of the unit cube in the middle because they were not able to visually represent

Table 3: 3D Geometry Thinking Ability Test Results

	Representation Dimension			*Spatial Structure Dimension*		
	Question 1	*Question 2*	*Question 3*	*Question 4*	*Question 5*	*Question 5*
N	*26*	*26*	*26*	*26*	*26*	*26*
Average	4,85	4,58	3,77	2,73	4,96	4,35
	N	52	**N**	**52**	N	**52**
	Average	**4,71**	Average	**3,25**	Average	**4,65**

Table 4: *The percentage of students unable to provide correct answers*

			Percentage of Students that cannot answer correctly	
No	*Question Indicator*	*Total Students*	Total	(%)
1	Identify the elements and properties of 3D geometry.	26	15	57,7
2	Construct nets from 3D geometries.	26	14	53,9
3	Translate 2D geometric shapes formed from 3D geometries.	26	16	61,5
4	Determine the number of unit cubes of a 3D geometric shape.	26	20	76,9
5	Calculates surface area and volume of 3D geometry.	26	13	50
6	Compares volumes of 3D geometries based on their properties.	26	15	57,7

that the three-unit cubes in the middle have colored sides at the bottom. Therefore, it can be concluded that students are only able to represent the visible side of the unit cube visually. The fourth type of obstacle showed that 11.5% of students did not understand the meaning of the question, such as the terms "remaining unit cube", "colored side", "uncolored unit cube remainder." Furthermore, they also assume that the "remaining unit cube" does not appear in the picture. The "colored side" is only on the front, right, left side, and top, while the bottom is not colored. Mistakes in interpreting terms make them unable to represent all parts of the unit cube that are located at the bottom, middle and top.

Epistemological obstacles in constructing 3D geometric nets

Students need to be able to identify the planes and edges of the cuboids that have been marked and their sizes to answer the second question. Afterwards, they must pay attention to the structure of the edges and planes of the 3D geometric object. The analysis results of students' obstacles in answering the second question are shown in Table 6, with only 46.1% able to answer the second question correctly. Meanwhile, the remaining 53.9% experienced difficulties in constructing webs from 3D geometric shapes.

Hree types of epistemological obstacles are found in the second question, with 26.9% caused by the fact that students were only able to construct one form of marked cuboid nets. Misinterpretations cause students' inability to construct at least 2 different cuboid shapes. Furthermore, the second type of obstacle is caused because they are not right in constructing the shape of the cuboid nets, as shown in Figure 2.

The figure shows that students are not able to arrange the parts of the cuboid plane into nets perfectly. This type of difficulty is called joining the disc and the lateral surface along a line [34]. The third type of difficulty is that students do not understand that the cuboid planes consisting of the front, back, right side, left side, base, and top are parts of the cuboid nets.

Epistemological obstacles in translating parts of 2D geometry formed from 3D geometry

Students need to ensure that the shapes from the top and sides are in accordance with their point of view to complete the third question. However, based on the analysis results shown in Table 7, only 38.6% were able to correctly answer the third question, while the remaining 61.5% had difficulties in translating the 2D formed from 3D geometric shapes.

There are 3 types of epistemological obstacles experienced by students in answering the third question. In the first obstacle, 23.1% of them were wrong in drawing 2D shapes from a 3D view. They simply drew the arrangement of the unit cubes by rotating the position, as shown in Figure 3.

Figure 3 show that students do not understand the meaning of the third question, which is associated with drawing an image in 2D form. This indicates that they do not

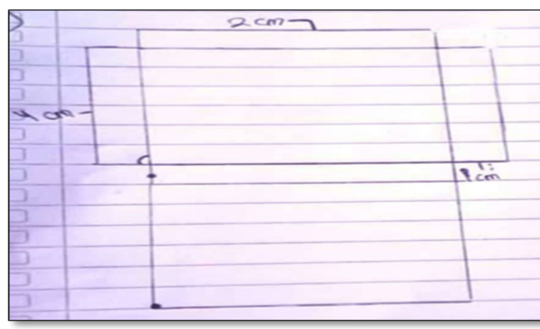

Fig. 2: One of the answers to the second question

Table 5: Epistemological obstacles in the first question

Task	*Answers*	*f*	*%*
Identify the elements and properties of 3D geometry	Incorrect answers	15	57,7
	Epistemological obstacles students		
	Cannot count and add up the remaining unit cubes in the 3D drawing in each row	3	11,5
	Unable to represent colored and uncolored unit cube plane	7	23,1
	Unable to determine the number of remaining cubes in the middle	2	7,7
	Incorrect in interpreting the question command	3	11,5

Table 6: Epistemological obstacles in the second question

Task	Answers	f	%
Constructing nets from 3D geometries.	Cannot answer correctly	14	53,9
	Epistemological obstacles experienced by students		
	Just draw one shape of cuboids nets	5	19,2
	Unable to draw the shapes of cuboid nets	7	26,9
	Other obstacles	2	7,7

Table 7: Epistemological obstacles to the third question

Task	*Answers*	*f*	*%*
Draw 2D geometry sections formed from 3D geometry	Cannot answer correctly	16	61,5
	Epistemological obstacles experienced by students		
	Improper drawing of the arrangement of unit cubes in 3 directions of view	6	23,1
	Difficulty in representing 2D images on the side view and top view.	8	30,7
	Other difficulties	2	7,7

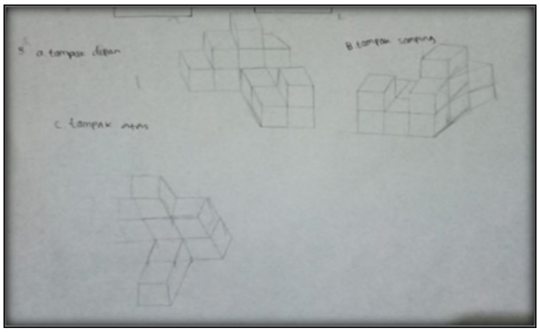

Figure 3. *a student's answer*

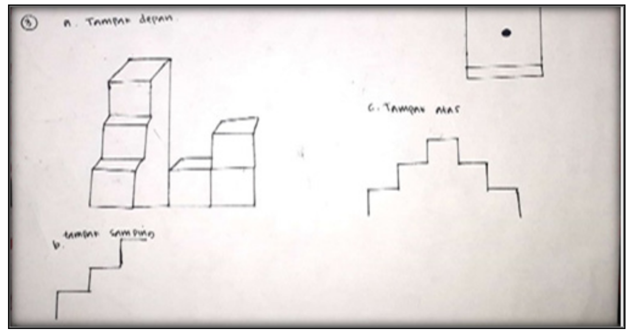

Fig. 4: A student's answer

Table 8: *The epistemological obstacle in the fourth question*

Task	*Answers*	*f*	*%*
Determine the number of unit cubes from the specified 3D geometric figure	Cannot answer correctly	20	76,9
	Epistemological obstacles experienced by students		
	Cannot represent 3D object shape	7	19,2
	Improper calculation of the number of levels.	10	19,2
	Not answering questions	3	7,7

have adequate knowledge regarding the techniques needed to draw projections from 3D geometric shapes. Furthermore, in the second obstacle, 30.7% of the students had difficulty representing 2D images from the side and top views, as shown in Figure 4. They think that drawing a side view means arranging it in the form of a ladder. Meanwhile, on the top view, students assumed that the picture was in the form of a stack of unit cubes because the majority did not understand the concept of drawing with perspective. In the third obstacle, 7.7% of students described the front, side, and top views separately without putting the pictures together.

Epistemological obstacles in calculating many unit cubes from 3D geometric shapes

Students need to be able to identify the edges of the unit cube in the length, width, and height of the 3D geometric shape to be able to answer the fourth question. However, based on the analysis results shown in Table 8, only 23.1% were able to answer the fourth question correctly. In comparison, the remaining 76.1% experienced difficulties in determining the number of unit cubes from 3D geometric shapes.

Students' experienced 2 types of epistemological obstacles while answering the fourth question. The first is their inability to represent the number of levels of 3D shapes. The students need to represent the topmost arrangement as the level in the tallest building therefore the number of existing levels is 3. The second obstacle is that they are not precise in calculating the number of rooms at each level due to their inability to identify the various unit cubes (room).

These obstacles are related to students' inability to visualize unit cubes that are not visible at each level, such as determining the rib structure on the base. Students' incompetency can be minimized, assuming they are able to understand the rib structure for each section. The rib structure on the front, side, and top are called the length, width and height of the 3D geometry, respectively. Students' abilities to count the number of edges that appear front, side, and top, means that they are can determine the number of unit cubes as a whole.

Epistemological in calculating surface area and volume of 3D geometry

Students' abilities to identify 3D geometric shapes are needed to answer the fifth question. Furthermore, they need to identify the size of each rib and determine the parts that belong to the figure's surface. However, based on the analysis results of students' answers shown in Table 9, only 50% provided

Table 9: The epistemological obstacle in the fifth question

Task	*Answers*	*f*	*%*
Calculating the surface area and volume of a flat-sided shape.	Cannot answer correctly	13	50
	Epistemological obstacles experienced by students		
	Unable to use 3D geometry surface area and volume formulas	8	30,8
	Unable to define surface shape from 3D geometry	5	19,2

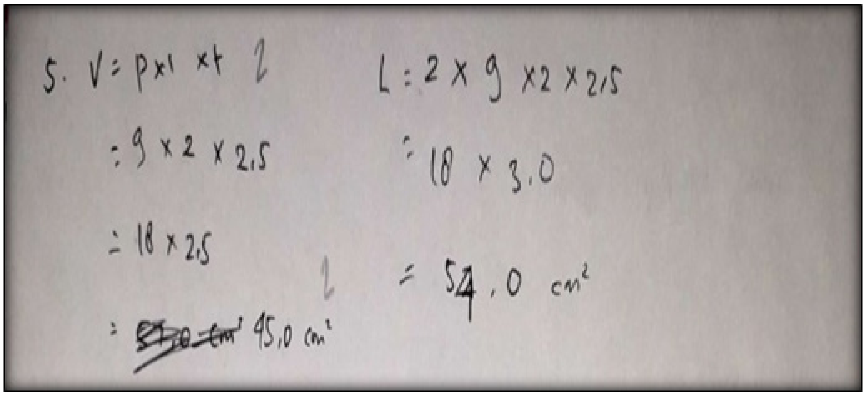

Fig. 5: A Student's Answer

Table 10: Epistemological Obstacle in the Sixth Question

Task	*Answers*	*f*	*%*
Comparing the volumes of flat-sided figures based on their properties.	Cannot answer correctly	15	57,7
	Epistemological obstacles experienced by students		
	Difficulty in determining 3D geometric volume formulas	6	23,1
	Difficulty in determining the volume of the prism	3	11,5
	Incorrect answers in comparing the volume calculation results of each 3D geometry	2	7,7
	Other difficulties	2	7,7

correct responses. Meanwhile, the rest experienced obstacles in calculating the surface area and volume of 3D geometry.

There are two types of obstacles experienced by students. First, 30.8% are unable to use the formula to correctly calculate the surface area and volume of the cuboid. The error was because they answered by interchanging the volume and surface area formulas or using other spatial formulas, as shown in Figure 7. Other causes include their inability to know the formula for a cuboid's surface area and volume, not working carefully, and inadequate understanding of the formula.

The second type of obstacle is that students do not determine the surface shape of 3D geometric objects, rather they assume it is only on the sides, back and front, thereby leading to a misconception. Some students also determined the surface area by dividing 3D geometric shapes into several blocks, and after obtaining the pieces in the form of cuboids, the students added up all the surfaces of the blocks. However, they made mistakes in identifying the length of the side of the cuboid, edges and surface area of the 3D geometric shape.

In general, students' obstacles in determining the surface area are due to their inability to identify the length of the edges in each 3D geometry plane. Students are not able to determine the surface area that appears on the front, back, side, top and bottom views, which makes it difficult for them to calculate their surface area and volume.

Comparing 3D geometric volumes based on their properties

Students' need to be able to identify the properties, size and calculate the volume of each 3D geometry. However, Table 10 shows that only 42.3% were able to compare the volumes, while the remaining 57.7% experienced difficulties.

Students experience 3 types of epistemological obstacles. The first is difficulty determining the exact volume formula, with the majority calculating the volume of each 3D geometric shape before comparing the results. Furthermore, some do not know the volume formula of a cuboid, triangular prism, or cube, while others made mistakes in performing the multiplication operation on the lengths of the edges. The second obstacle is the difficulty determining the volume of a triangular prism due to their inability to understand its nature by stating that the base and top planes have the same shape.

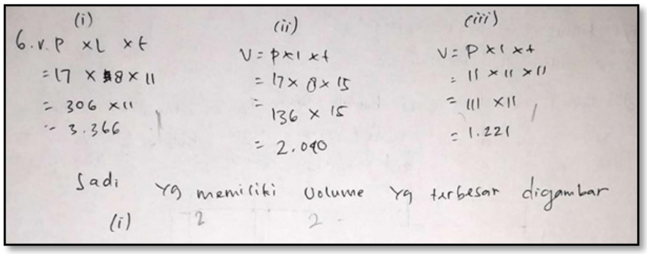

Fig. 6: A Student's Answer

The third obstacle is their inability to carefully calculates the volume of cubes, blocks, or triangular prisms.

Discussion

Epistemological obstacle on the representation dimension of 3D geometric thinking ability

The findings on the dimensions representation of 3D geometry thinking ability revealed that approximately 57.7% of students experienced difficulties in identifying the elements of 3D geometry. These findings explain that students do not have a set of spatial knowledge of the plane structure on the unit cube, which is located within the large cube, rather from one side. In theory, representing shapes from 3D geometry is a complex process involving the concept of 2D geometric objects and their properties (Mesquita, 1998). The process of representing 3D geometric shapes is closely related to two cognitive construction processes, namely coding and decoding (Mesquita, 1998). Coding construction is related to cognitive processes such as manipulating, building webs from 3D to 2D forms, and translating one mode of representation to another, while decoding is related to interpreting and recognizing elements of 3D objects such as vertices, edges, plane surfaces in various representation modes (Pittalis & Christou, 2013).

In terms of cognitive coding construction process, 53.9% of students experienced difficulties in constructing nets from 3D geometric shapes. These obstacles are due to students' inability to place marks on each 3D geometry field and arrange parts into nets perfectly. The construction process in making nets requires the ability to translate 3D objects and 2D nets by focusing and studying the parts of objects in both representation modes (Pittalis et al., 2010). Constructing nets from 2D geometric objects by folding and forming 3D geometries requires spatial ability (Cohen & Ben Gurion, 2003). This is because the transformation of a 3D object into its net is not a copy of the corresponding perception, rather a mental operation performed by manipulating the image (Cohen & Ben Gurion, 2003). Furthermore, the transition from the perception of 3D objects to their nets requires the activation of appropriate mental actions that coordinate the various perspectives of the object (Pittalis et al., 2010). (Mariotii, 1989) stated that the construction of the net requires coordination between the mental representation of the object as a whole and the decomposition of its parts.

Cohen & Ben Gurion (2003) identified 5 types of student errors when constructing nets from 2D to 3D objects, namely (1) confusion between the perspective view of the solid and its net due to confusion when visualizing 3D geometric shapes from various points of view to the construction of the nets. (2) Joining the disc and the lateral surface along a line relates to the strategies constructed from the various edges of the specified 2D plane. Students have a clear tendency to join parts of a 2D plane to the edges of a surface developed along a line and not just at a single point. (3) Wrong form of the edge to be joined during compilation in the formed field. Similar to the conical nets, the circular base needs to match the surface of the conical blanket. (4) Wrong placement of the parts due to students' failures in determining the net parts to be connected. (5) Other mistakes are usually associated with inadequate construction ideas. The five types of student errors are because the construction of nets from 3D shapes requires their ability to transform 3D objects into 2D nets by focusing on studying the parts of objects in both representation modes (Pittalis et al., 2010) and their ability to manipulate the image (Cohen & Ben Gurion, 2003).

Epistemological obstacle on spatial structure dimension of 3D geometric thinking ability

The spatial structure dimension of 3D geometry thinking skills revealed that 61.2% of students experienced difficulties in translating parts of 2D formed from 3D geometry, while 76.9% experienced difficulties in determining the rest of the unit cubes. These obstacles are related to the student's inability to visualize unit cubes that are not visible at each level, such as determining the rib structure of the unit cube located on the base. The rib structure in the front, side and top is called the length, width and height of the 3D geometry, respectively. Students' ability to count the number of edges on the front, side, and top, determine the number of unit cubes as a whole. Furthermore, their ability to arrange the 3D geometric objects is closely related to visual-spatiality. Students' ability to "read" the representation of 2D geometric objects from 3D is part of the spatial visualization ability (Ben-Haim et al., 1985). The recognition of 3D objects is related to the internal representation mode that combines the visual structure that arises from 2D geometric shapes (Cooper, 1990). This is understandable because the process of visualizing and conceptualizing 3D objects is a complex cognitive process, and both require the development of students' abilities to decode and encode spatial information (Markopoulos, Chaseling, Petta, Lake, & Boyd, 2015). Furthermore, it takes the ability to coordinate the line segments are in the cuboid and integrate them to build a coherent mental model of the line segments (Battista & Clements, 1996).

Epistemological obstacle on measurement dimension of 3D geometric thinking ability

In terms of students measurement dimensions of 3D geometry thinking ability, 50.0% and 57.7% experienced difficulties in calculating the surface area and comparing the volume of 3D shapes. Generally, their inability to determine surface area, volume, and comparing 3D geometric shapes are due to the difficulty to identify the length of the edges on each part of the geometry plane. Students are unable to determine the surface area on the front, back, and side trays. The inability to determine the surface area makes it difficult to calculate and compare the volume to real-world objects used in everyday life (Kara, Eames, Miller, Cullen, & Barrett, 2011) and many other mathematical topics (Hong & Runnalls, 2020). Battista & Clements (1996) identified 4 main strategies used by students in calculating the volume of geometric shapes. The strategy is to (1) conceptualize the unit cube as a rectangular array arranged in layers of 3D geometric shapes, (2) conceptualization a set of unit cubes as a space that fills the 3D shape volume without organizing it into layers, (3) conceptualize the set of unit cubes on the surface, and (4) using the volume formula. Students use the first two categories to demonstrate awareness of the spatial structure of 3D objects, including hidden parts (Vasilyeva et al., 2012). Furthermore, they use the third strategy to express a lack of understanding related to volume and fail to integrate different views of 3D objects instead of focusing on the parts that stand out from a particular perspective (Vasilyeva et al., 2012). Meanwhile, those that use the fourth strategy do not have a conceptual understanding of volume and only use the formula as a shortcut to mechanically understand the structure of the 3D arrangement (Vasilyeva et al., 2012).

Conclusion

In conclusion, students experience epistemological obstacles in carrying out a series of 3D geometric thinking processes on the dimensions of representation, spatial structure, and measurements. Of the 26 students, 15 (57.7%) and 14 (53.9%) experienced difficulties representing the elements and nets of 3D geometry, respectively. This is due to their inability to represent the structure of the field on a unit cube that is not directly visible. This causes students to be unable to determine the number of unit cubes that are indirectly visible and to arrange the parts of the 3D geometry into perfect nets. Furthermore, in the spatial structure dimensions, 16 (61%) students were unable to translate the 2D geometry part formed from 3D geometric shapes. Meanwhile, 20 (76.9%) students experienced obstacles in determining the number of unit cubes from 3D geometric shapes due to their inability to describe and visualize the representative mode of 2D shapes from 3D shapes and vice versa. Furthermore, another cause is the inability of students to visualize the rib structure found on the front, side, and top. The edge structure is named by the length, width, and height of the 3D geometric shape. Meanwhile, in the measurement dimensions, 13 students or 50%, experienced difficulties calculating the surface area and comparing 3D Therefore, we believe that this research can contribute to science related to the educational and pedagogical aspects of 3D geometry. In addition, to minimize the existence of epistemological obstacles experienced by students, this study suggests teachers or prospective teachers pay attention to the characteristics of the material to be taught to students.geometric shapes due to their inability to identify the front, back, side, and top. The inability to determine surface area causes students to be unable to calculate and compare volumes.

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