STEM ACTIVITY: MATHEMATICS INSIDE THE HOLOGRAM PYRAMID[1]

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ABSTRACT

In this study, a STEM activity was designed and implemented for eighth-grade students. The activity was used with thirty students and involved the construction of various hologram pyramids. During the activity, the mathematical concepts used in designing the hologram pyramids were discussed and the resulting hologram images were compared considering reflection laws and characteristics of an image formed by a plane mirror. Moreover, the angle of the pyramid's surface in relation to the horizontal plane was discussed for the purpose of obtaining a clearer hologram image. It was revealed that the activity designed by the researchers is relevant for improving students' knowledge and skills related to STEM. Furthermore, the fact that the hologram pyramid was created by paper folding, as opposed to using the design methods reported in the literature, made the activity suitable for middle school students. Finally, suggestions for teachers who wish to implement the activity are offered.
Keywords: hologram pyramid, STEM, mathematics, science, design.

HOLOGRAM PİRAMİDİNDEKİ MATEMATİK: STEM ETKİNLİĞİ UYGULAMASI

ÖZ

Bu çalışmada sekizinci sınıf öğrencileri için bir STEM etkinliği tasarlanmış ve uygulanmıştır. Otuz öğrenci ile gerçekleştirilen bu etkinlikte yatay düzlemle yaptıkları açılar farklı olan çeşitli hologram piramitleri elde edilmiştir. Etkinlikte hologram piramidi tasarım aşamalarında kullanılan matematiksel fikirler sorgulanmış, elde edilen farklı piramitlerdeki hologram görüntüleri yansıma kanunları ve düz aynada görüntü oluşumu konusu ile ilişkilendirilerek karşılaştırılmıştır. Ayrıca, daha net hologram görüntüsü elde etmek için hologram piramidinin yüzeyinin yatay düzlemle yaptığı açı ölçüsü tartışılmıştır. Bu çalışmada, araştırmacılar tarafından tasarlanan hologram etkinliğinin öğrencilerin STEM ile ilgili bilgi ve becerilerini geliştirmek için uygulanabilir bir etkinlik olduğu görülmüştür. Ayrıca, alanyazında karşılaşılan hologram piramidi tasarım yöntemlerinden farklı olarak bu çalışmada hologram piramidinin katlama yoluyla elde edilmiş olması, etkinliği ortaokul öğrencilerinin seviyesine uygun hale getirmiştir. Çalışmada etkinliğin uygulanması sırasında ortaya çıkan öğrenci düşünceleri açıklanmıştır. Son olarak, etkinliği uygulamak isteyen öğretmenler için çeşitli önerilerde bulunulmuştur.
Anahtar kelimeler: hologram piramidi, STEM, matematik, fen bilimleri, tasarım.

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INTRODUCTION

Mathematics is often perceived as a challenging field by individuals (Fritz et al., 2019), and this perception frequently leads students to disengage from the subject (Li & Schoenfeld, 2019). Learning mathematical concepts in relevant contexts aids students in developing their beliefs about the importance of mathematics (Kilpatrick et al., 2001). Mathematics is acknowledged as the foundation for various disciplines, such as technology, engineering, and science (Li & Schoenfeld, 2019). The concept of STEM, which integrates these disciplines, has become one of the most popular topics in recent years and provides effective contexts for students to learn mathematics. STEM-related practices and instilling students with STEM knowledge at an early age enhance students' motivation to enroll in extra science and mathematics courses in later grades (Harackiewicz et al., 2012). It is also stated that students' beliefs about their competencies and interests begin to consolidate in middle school years (Simpkins et al., 2006). Therefore, middle school years are regarded as a crucial time period for developing students' interest in STEM and fostering their self-efficacy in mathematics and science courses (Blotnicky et al., 2018).

Various approaches are followed in integrating STEM topics. One of these approaches is STEM studies that emphasize engineering (Bryan et al., 2015; Shaughnessy, 2013). Shaughnessy (2013, p.324) defines STEM education as "...solving problems that draw on concepts and procedures from mathematics and science while incorporating the teamwork and design methodology of engineering and using appropriate technology." Bryan et al. (2015) state that STEM education should be based on engineering practices and its aim should be to teach science and mathematics concepts. However, there is limited research in the literature that explains how to integrate STEM based on this approach (Lawrenz et al., 2017). In STEM education, knowledge about science and mathematics can be gained by integrating various disciplines and using engineering and technology fields together. Based on this idea, in this study, a hologram pyramid was designed in which STEM disciplines were integrated, and this activity aimed to provide students with various skills related to all areas in the STEM discipline. Many studies emphasize the role of teachers in developing students' ability to apply their existing knowledge during activities involving different disciplines, such as STEM activities (e.g., Bryan et al., 2015; Honey et al., 2014; Moore et al., 2014; Nathan et al., 2013). However, it has been revealed that teachers in different fields also lack the necessary pedagogical knowledge to effectively implement integrated STEM activities (Peterman et al., 2017). It is thought that this study will make significant contributions to the literature, both because it is a resource that teachers can use to increase students' awareness and because it reveals an effective teaching practice in which STEM disciplines are integrated.

In this activity, the focus was on the mathematical dimension of STEM when creating a three-dimensional hologram pyramid by folding a paper and examining the angles of the hologram pyramid in relation to the horizontal plane while it was on the science dimension of STEM when explaining the hologram image with the help of reflection. Furthermore, attention was directed toward the engineering dimension of STEM during the analysis of images from various hologram pyramids with differing angles in relation to the horizontal plane and during obtaining the clearest hologram image. Lastly, when modeling the hologram pyramid, which is expected to generate the clearest image, and determining the angle measurements of the Hologram with the help of the GeoGebra software the focus was on the technological dimension of STEM.

This activity includes the following contents related to the science curriculum: principles of light reflection (F.5.5.2.2), image formation in flat mirrors (F.7.5.2.2), and designing a viewing apparatus using a mirror (F.7.5.3.5) (Ministry of National Education [MoNE], 2018a). Furthermore, the activity includes the following contents related to mathematics curriculum: triangles and quadrilaterals (M.5.2.2.3), lines and angles (M.7.3.1.2), polygons (M.7.3.2.2 and M.7.3.2.3), congruence and similarity (M.8.3.3.1 and M.8.3.3.2), transformation geometry (M.8.3.2.2), and right pyramid (M.8.3.4.5) (MEB, 2018b).

IMPLEMENTATION OF THE ACTIVITY

Activity

In this study, a STEM activity for designing various hologram pyramids was developed and implemented. As seen in Photograph 1, the hologram pyramid is composed of four identical isosceles trapezoids. The hologram pyramid design methods found in the literature typically entail drawing four equal isosceles trapezoids with specified side lengths on a transparent material, cutting out the drawn trapezoids using a utility knife or a cutting tool, and appropriately joining the equal-length sides of the trapezoids together with tape (Orcos & Magreñán, 2018). Shortly, this process involves cutting and attaching all edges of the four isosceles trapezoids with tape.

Photograph 1. A Hologram Pyramid

Middle school students can complete the process of drawing, cutting, and gluing isosceles trapezoids with the given dimensions properly only with the help of an adult. Moreover, this process requires significant work, effort, and time even from adults. Within the scope of this study, a hologram pyramid construction method that can be easily followed and applied by anyone who knows how to measure and construct angles was proposed.

In this study, a two-stage activity (see Appendix 1) was designed and implemented to help middle school students query the mathematical ideas used in designing hologram pyramids, compare the images obtained from the various hologram pyramids created, and relate the images with the reflection of light and image formation. In the first stage of the activity, the hologram pyramid was created using A4 papers and mathematical investigations were conducted. Then, hologram pyramids were created using transparent acetate papers and hologram images were examined with the help of smartphones.

Participants

The participants of the study were 30 eighth-grade students, 19 of them were girls and 11 were boys. They voluntarily participated in the study. The activity was implemented in a public school at the end of the spring semester of the academic year 2021–2022.

Implementation

The implementation of the activity lasted for 3 class hours. During this activity, students worked in groups of three. The implementation process of the activity was recorded with a video camera. In addition to the video recordings, the student worksheets presented in Appendix 3, and the observation notes created by the researchers during the implementation were also used to explain and evaluate the implementation process. Before the implementation of the activity, the meaning of "hologram" was asked to the students. It was observed that none of them had heard the word "hologram" before. To motivate the students to the activity, a hologram pyramid that was already created by the authors was brought to the classroom. Using a smartphone, a three-dimensional image was generated inside the hologram pyramid and the students were given an opportunity to examine the hologram image (Photograph 2).

Photograph 2. An Example of a Hologram Image

Presenting a hologram-related problem situation can be an alternative introductory activity for future implementations. An example of such a problem situation is as follows: "A firm wants to manufacture hologram designs with different images for their customers. To help this firm, design different hologram pyramids". This problem can motivate students to design hologram pyramids and engage them in problem-solving processes.

After the examination of the hologram images, the pyramid was shown to the students and they were asked what this shape was like. Some of the students answered as “pyramid”. The other students were asked if they agreed with their friends’ answers and a brief discussion was held. It was concluded that this shape resembled a vertical square pyramid, but since it did not have an apex, it did not meet all the features of the pyramid. Consequently, the teacher explained to the students that the shape was a truncated square pyramid. Additionally, they were informed that this shape also refers to a hologram pyramid. The students were informed that each group would obtain a hologram pyramid at the end of the activity. This enabled them to be aware of the purpose of the activity and to feel motivated to the lesson.

The activity was continued by following the hologram pyramid construction steps presented in Appendix 1. These construction steps were prepared for the implementer. At the beginning of the activity, the Hologram Worksheet in Appendix 3 was distributed to the students. To assist the implementer, the questions in the worksheet in Appendix 3 were integrated into the relevant sections of Appendix 1. As mentioned before, the students worked in groups and the worksheet (Appendix 3) completed by each group was used in evaluating the activity.

The teacher had the students complete Steps 1–4 in Appendix 1 by using an A4 sheet of paper. The students formed the angle A in the center of the A4 paper (Appendix 1; Step 4). Then, the teacher asked the students to determine the measure of angle A in the first question of the student worksheet in Appendix 3 without using a protractor. Most of the students answered this question as 270° in a very short time. The following dialogue took place between the teacher and the students:

> Teacher: How did you arrive at the answer of 270°?
> Student 3: Initially, there was a complete angle at point A.
> Student 7: Yes, at first, there was an angle of 360°.
> Teacher: What about afterward?
> Student 3: Then, we created an angle of 90° and separated it from the paper by cutting.
> Teacher: Well, where is angle A?
> Student 3: Actually, angle A is the remaining angle after cutting out the 90-degree angle.
> Student 4: Yes, the remaining angle. So, it is 270.
> Teacher: Yes. We obtain 270° by subtracting 90° from 360°.

Then, the teacher had the students perform Steps 5 and 6 in Appendix 1, respectively. The students folded the angle A two times in succession and named the newly formed angle as angle B (Step 6). Next, the teacher asked the students to find the measure of the angle B (Question 2, Appendix 3).

> Teacher: What is the measure of angle B?
> Student 6: 135°.
> Teacher: Can you explain how you arrived at that?
> Student 6: Angle A is 270°. Since we folded angle A, its half is 135°.
> Teacher: Yes, in fact, we did divide the 270° into equal parts by folding it, but into how many parts?
> A group of students: 4
> A few students: 2
> Teacher: Well, how many layers of paper does angle B have? Can you count the layers?
> Students: 4 layers.
> Teacher: Now, let’s unfold the paper and see how many equal parts angle A is divided into. (Students unfolded the paper, saw the fold lines, and decided that the angle was divided into 4 equal parts)
> Student 3: Yes, it is divided into four. Therefore, angle B is 67.5°.
> Teacher: Correct. Now, please write 67.5° to the vertex where angle B exists.

Later, the teacher guided the students in carrying out Steps 7 –10 given in Appendix 1. In these steps, the students identified two points on the arms of angle B and connected the opposing points with line segments (see Appendix 1, Step 9). After completing this process, the teacher proceeded with the activity by asking the students the third and fourth questions given in Appendix 3. Namely, the teacher asked the students to tell the geometric shapes that were formed on the paper after they drew the line segments CD and EF. (Appendix 3; Question 3) Responses from the students included ‘triangle’ and ‘trapezoid’.

> Teacher: Well, which points form the corners of the triangle? Can you name the triangle you noticed?
> Student 7: Triangle BCD.
> Teacher: Are there any other triangles?

Student 8: There is a larger triangle, triangle BEF.

Teacher: You previously determined the measure of angle B as 67.5°. Can you find the measures of other interior angles in these triangles? (Appendix 3; Question 4)

Student 9: Actually, triangle BCD is an isosceles triangle, meaning angles C and D are equal.

Teacher: So, what are the measures of angles C and D?

Student 9: Both are 56.25°.

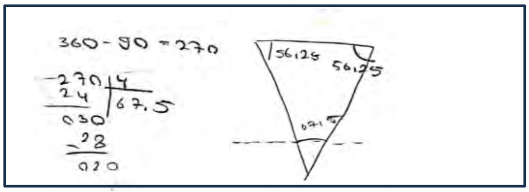
Teacher: Well, what about triangle BEF?

Student 9: It is also an isosceles triangle, so angles E and F are equal, they are 56.25°.

Teacher: Yes, you can note the angle measures. You mentioned it was skewed, where is it? Can you name it?

Student 8: The trapezoid is formed by points C, D, E, and F.

An example of the calculations performed by the students in Question 4 in the worksheet is displayed in Photograph 3.



Photograph 3. A Student Worksheet

The teacher asked the students to state which properties a quadrilateral must possess in order for it to be a trapezoid. Next, a short in-class discussion took place and it was concluded that at least one pair of sides must be parallel. In the middle school mathematics curriculum, students are expected to work on trapezoids when they are in grade 7. Therefore, it is not surprising that the participating students were able to identify the trapezoid shape formed on the paper and articulate this crucial characteristic of the trapezoid.

Teacher: Considering the definition of a trapezoid, is quadrilateral CDEF a trapezoid?

Students: Yes.

Teacher: You stated that at least one pair of sides must be parallel, correct?

Student 8: Yes, CD and EF are parallel.

Teacher: Why? How did you determine that?

Student 8: They are parallel because angles C and E are equal.

Teacher: Is that sufficient?

Student 8: Because C and E are corresponding angles. They are on the same line. That is, if CD and EF are parallel, angles C and E will be equal.

As can be understood from the dialogue, Student 8 demonstrated that line segments CD and EF are parallel by using the knowledge of the angles formed by two parallel lines (i.e., CD and EF) and a transversal (CE). Additionally, since triangles BCD and BEF are similar, it can be concluded that line segments CD and EF are parallel. More precisely, triangles BCD and BEF are similar because the ratios |BC|/|BE| and |BD|/|BF| are equal to 1.5/10. Therefore, line segments CD and EF are parallel. Consequently, the students concluded that the quadrilateral CDEF is a trapezoid after revealing that the two opposite sides are parallel.

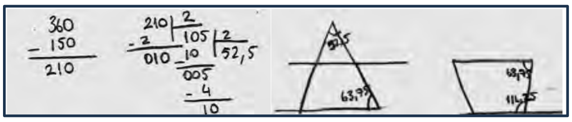
Next, the teacher asked the students to re-examine the angle measures and side lengths of the trapezoid CDEF. The students explained that CDEF is an isosceles trapezoid, based on the fact that the side lengths CD and DF and the measures of the angles E and F are equal in the trapezoid. Afterward, the teacher asked the students to name the angles that are supplements of angles C and D as angles G and H, respectively, and to determine the angle measures without using a protractor (Appendix 1; Step 10). Since the students had previously determined angles C and D to be 56.25°, they calculated the measurements of angles G and H to be 123.75°. The students recorded the angle measures on their papers. Next, the papers were cut using scissors along the line segments CD and EF (Appendix 1; Step 11). The activity continued by following Steps 12 and 13 given in Appendix 1.

Upon unfolding the four-layer paper, four identical isosceles trapezoids were obtained (Appendix 1; Step 12). The rightmost and leftmost edges were aligned so that they coincided with each other, next they were attached with tape and a three-dimensional shape (i.e., the hologram pyramid) was created (Appendix 1; Steps 13 and 14). Later, two different hologram pyramids were created with acetate paper by following the instructions in Appendix 2. The hologram pyramids differ because pieces of varying angle sizes were cut from the initial acetate papers (90° and 150°).

Students were asked to answer Questions 5–7 on the worksheet (Appendix 3), which involved comparing the two hologram pyramids they created. Sufficient time was provided to the students to complete the worksheets with their groupmates. Then, a whole class discussion was conducted and these questions were discussed as presented below.

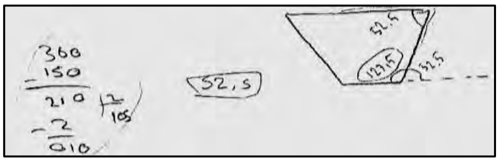
> Teacher: What are the differences between these two holograms?
> Student 23: One of the holograms is big and the other is small.
> Teacher: In what way is it bigger?
> Student 23: Well, it is wider.
> Teacher: Where is the wider one?
> Student 23: The angle there (indicating angle G in the hologram in their hand, Appendix 1; Step 10).
> Teacher: [Turning to the class] We labeled this angle as angle G on paper. What are the angles corresponding to G in both holograms?
> Student 9: The first hologram we made (referring to the hologram created by subtracting 90°) is exactly the same as the paper hologram. Since the angle G of the paper hologram has a measure of 123.75°, this angle is the same.
> Teacher: Then, what is the measure of the angle G in the second hologram?

At this point, the teacher allocated some time to the students to calculate the angle measure corresponding to angle G in the second hologram. The teacher monitored the students' work during this period and observed that most students found the correct answer, 116.25°, using the method they applied to the paper hologram. In Photograph 4, an example of a student solution is presented.



Photograph 4: An Example of a Student Solution

One of the groups calculated the angle measure incorrectly (Photograph 5). When two examples in Photograph 4 and Photograph 5 are compared, it can be seen that both groups calculated the angle measure 52.5° correctly, but the group whose solution is depicted in Photograph 5 arrived at an incorrect answer due to placing the angle with a measure of 52.5° in the wrong corner of the shape.



Photograph 5: An Example of an Incorrect Student Solution

The classroom discussion about the measure of angle G in the second hologram took place as follows:

> Teacher: How did you determine angle G in the second hologram?
> Student 12: We subtracted the 150-degree angle to create the hologram. We divided the remaining 210° by 2 and then again by 2 and found 52.5 degrees. This angle formed one angle of the trapezoid.
> Teacher: Can you indicate which angle is 52.5°?
> Student 12: The angle there (pointing to angle F on her own paper as shown in Appendix 1; Step 10).

At this point, the teacher took a hologram paper that she folded as in Appendix 1; Step 10 and continued the discussion by marking the angle shown by the student.

> Teacher: What do you think about your friend's answer?
> Student 8: The angle we found to be 52.5° is not located correctly.
> Teacher: Well, where should it be located?
> Student 8: The corner that we folded twice should be 52.5°. So, the apex angle of the triangle before cutting the paper.
> Student 12: Teacher, we calculated angle G incorrectly because we wrote 52.5 degrees in the wrong place.
> Teacher: Ok then. Now find the measure of angle G. …
> Student 12: Angle G is 116.25°.

After all groups correctly calculated the measure of angle G, the teacher continued the classroom discussion by asking them to explain how the angle measures of the pieces removed from the paper affected the hologram pyramid.

> Teacher: Well, we cut out the 90-degree angle when creating the first hologram, and the 150-degree angle when creating the second hologram. In our previous discussion, your friend (Student 23 in the dialogue above) emphasized the magnitude of the angle measure. How did these angles affect our holograms?

Student 7: In the hologram that we cut out a 150-degree angle, angle G of the trapezoid became 116.25°. The lateral surfaces of this hologram appear steeper.

Teacher: Well, how about the other hologram?

Student 12: Angle G of the trapezoid that was obtained by cutting out a 90-degree angle became 123.75°. The lateral surfaces of that hologram are closer to the ground.

Student 9: So, its slope is less.

To help the reader compare the two holograms more effectively, the two holograms nested together are presented in Step 14 in Appendix 2. Since the angle between the lateral surfaces of the outer hologram and the horizontal plane is greater than that of the inner hologram, it can be observed that the inner hologram remains higher and does not fully fit into the other.

The investigations conducted thus far aimed at creating a hologram and examining its mathematical properties. Next, investigations on science concepts (i.e., reflection and formation of images in a plane mirror) were conducted. For this reason, initially, the students were asked to compare the images formed in the holograms, identify the differences between the images, and explain the possible reasons for these differences (Appendix 3; Question 8). Students were asked to open a hologram video from an online video-sharing platform on their smartphones and examine the images created in their holograms (Photograph 6).

Photograph 6: A Snapshot of Students' Hologram Image Examination

Teacher: How are the images inside the holograms are formed? What do you think?

Student 27: It is formed by the reflection of the lights on the smartphone.

Teacher: Yes, it occurs by reflection. Well, what is reflection?

Student 22: It is traveling back of the light at the same angle as it came up.

Teacher: Where does the light coming out of the phone screen go?

Student 22: It goes to the hologram. Namely, it goes to its lateral surface.

Teacher: Well, how do we determine the angle of the incidence for the light hitting the hologram?

Student 25: By drawing the normal to the surface.

Teacher: Well, where will you draw the normal?

Student 25: The light coming out of the phone screen hits the lateral surface of the hologram and reflects. We should draw the normal to the point where it hits.

Teacher: Then, can you show how the image is formed on the graph paper? Now, let's think about this.

Here, the teacher aimed to have the students apply the laws of reflection on the hologram. Students drew on their worksheets the hologram models and the reflections that occurred in these holograms. Eighth-grade students did not have much difficulty explaining the reflection and formation of images on the hologram surface. The fifth-grade learning objectives in the science curriculum regarding the relationship between incident rays, reflected rays, and the normal to the surface and the seventh-grade learning objectives regarding the formation of images in plane mirrors helped students make these explanations with less difficulty.

Here, the activity aimed to help students ponder on the optimal degree of angle between the hologram pyramid and the horizontal plane to ensure that the clearest hologram image is obtained that it is of equal size to the image on the smartphone. To achieve this aim, the teacher asked the students to compare the images formed in two different holograms.

Teacher: What did you notice about the images formed in the two holograms? Are they the same or different?

Student 17: The image appeared more faded in the hologram that we cut out 150-degree angle. The image appeared clearer in the hologram that we cut out a 90-degree angle.

Teacher: Well, is the image obtained in the hologram by cutting out a 90-degree angle the clearest image? How do we obtain the clearest image?

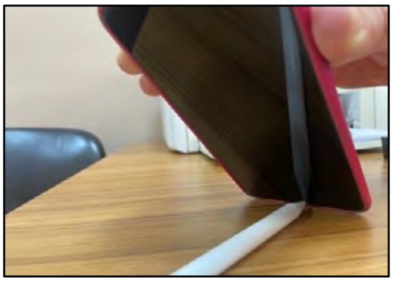
Student 17: Actually, the image that stands horizontally on the phone screen becomes an image that stands vertically inside the hologram. The best image is obtained when we create a fully vertical image.

> Teacher: Then, can we say this... In our hands, there is a plane mirror. We want to see an object in a horizontal position to stand vertically. Then, how should we use the plane mirror? Let's think about this.

Meanwhile, the teacher opened on the smart board the GeoGebra file that she had previously prepared. (https://www.geogebra.org/m/h8fxcvsn). The screenshot of the GeoGebra file is presented in Appendix 5(a). In this screenshot; an object, a reflection line (mirror/hologram surface), and its reflected image along this line can be seen. The teacher obtained different images by altering the slope of the reflection line in the classroom.

With the help of GeoGebra, it can be easily seen that the reflection of a horizontal line segment along a 45° inclined line is a vertical line segment. The teacher explained to the students with the help of GeoGebra that the angle of inclination of the hologram surface should be 45° by making an association with the 8th-grade learning objective "Construct the reflected image of points, line segments and polygons along a line". The research conducted on obtaining images in the hologram pyramid indicated that the angle inclination of the surface of the hologram pyramid with the horizontal plane should be 45° (Thap et al., 2018). The readers are advised to search "Pepper's Ghost Pyramid" for more detailed information on this topic.

During the implementation, the explanations provided by Student 17 in the aforementioned dialogue helped to move forward in the discussion. However, in future implementations where such an answer is not received from the students, teachers may use an additional example to implement the activity. For instance, the image of a pen, positioned horizontally on a desk, might be examined using the screen of a smartphone in the "off" mode (i.e., a black screen) as a reflective surface (see Photograph 7). By slowly tilting the smartphone until it is at an angle of approximately 45 degrees from the vertical position, it can be observed how the image of the pen on the black screen becomes perpendicular to the desk.



Photograph 7. An Examination of the Pen's Image via a Smartphone

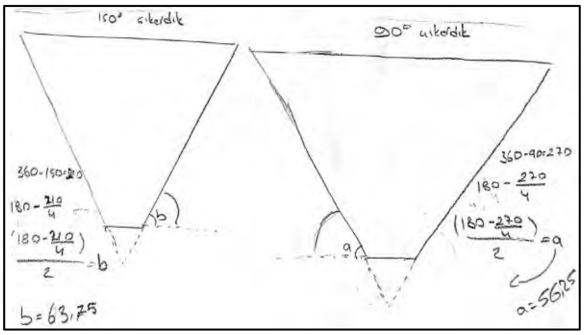
At this point, the measure of angle G in the hologram when the inclination angle of the hologram pyramid's surface is 45° should be questioned. Given that these two angles are on distinct planes, the calculation of angle G requires knowledge of trigonometry. Therefore, 8th-grade students will not be able to calculate the measure of angle G. Consequently, the authors modeled a hologram pyramid in GeoGebra and the angles were examined on the smartboard with the aid of this model (Appendix 5(b), https://www.geogebra.org/m/gwxk7qut). In GeoGebra, the inclination angle of the hologram pyramid's lateral surface can be changed by dragging point A, and the measure of angle G, which changes accordingly, is displayed on the smartboard.

Upon determining via GeoGebra that the measure of angle G is 125.26° when the inclination angle of the hologram pyramid's lateral surface is 45°, the section concerning reflection and image formation in the hologram was completed, and mathematical examinations were re-started. The teacher asked the students the following question: "What is the measure of the angle that we must cut out of the paper in order to obtain a hologram pyramid with an angle G of 125.26°?" Given that the students had performed similar calculations several times since the beginning of the activity, they readily determined that an angle with a measure of 78° should be cut out of the paper for the new hologram design.

Students performed calculations concerning three distinct hologram pyramids since the beginning of the activity. Finally, the teacher asked the students to develop a mathematical equation that is used for calculating angle measures (i.e., the measure of angle G in the hologram pyramid and the measure of the angle that needs to be cut out of the paper) when designing a hologram pyramid. The purpose of this final activity is to help students arrive at an algebraic expression that involves a generalization.

Teacher: When constructing a hologram pyramid, how do we formulate an algebraic expression to find the measure of angle G?
Student 12: We subtracted the cut out angle from 360 and divided the result by 4. In the first pyramid, we cut out 90°, leaving 270°. We determined the angle by dividing 270° by 4.
Teacher: Well, does this yield the measure of angle G?
Student 12: No (examining the paper in her hand). This provided the apex angle.
Teacher: Whose apex angle?
Student 7: The apex angle of the triangle we obtained in the first step.
Teacher: So, what did we do next?
Student: We subtracted the result we obtained from 180 and divided it by 2.
Teacher: Why did we perform this operation?
Student 7: To find the base angles of the triangle.
Teacher: Write this as a single mathematical operation on your papers.

At this point, the students initially struggled to formulate an algebraic expression that involved a generalization. The teacher aimed to help students identify the changing values by going over the mathematical operations written by the students. Students wrote two equations for the holograms formed when 90-degree angle and 150-degree angle were cut out of the papers. (Photograph 8).

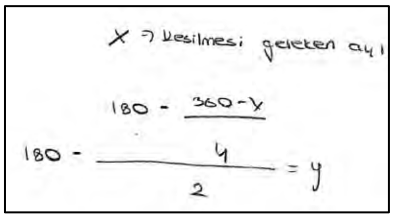


Photograph 8. Equations Formulated by the Students

After writing these mathematical operations, the teacher had students examine the constant and changing values in the written equations.

Teacher: What do you notice when you examine the written equations?
Student: We write the same operations; only the angles that we cut out differ. For this reason, the values that we divide by 4 are different.
Teacher: Suppose that the value we cut out is unknown. Then, how might we represent it in the equation?
Student: We can say "x" for the value that we do not know.
Teacher: Then, how can we reformulate our equation?
Student: 180 minus x is divided by 4.
Teacher: What does "x" represent in this equation?
Student: It represents the remaining value when we cut out the angle.
Teacher: Well, if we define x as the value of the angle we cut out instead of the remaining value, how do we write the expression?
Student: Can we express it as 360 minus x?

After expressing the values given to the angles with variables exactly, the students formulated the general expression. The teacher formed a table on the board while writing the algebraic expression. In this part of the activity, arithmetic operations were generalized and students were asked to write an algebraic equation by using changing values and constant values. The mathematical operations written on the board are summarized in Appendix 6 to provide an additional resource for future implementers. One of the algebraic equations written by students can be seen in Photograph 9.



Photograph 9. An Example of an Algebraic Equation Formulated by Students

In the expression documented in Photograph 9, the variable denoted with y corresponds to the measure of angle G, while the variable denoted with x corresponds to the measure of the angle that must be cut out of the paper at the beginning of the hologram creation.

At this point, the teacher asked the students the following questions about the mathematics and science concepts covered in the hologram activity in order to summarize and evaluate the activity: Which geometric shapes did we use when designing the hologram? Which angle calculations did we perform through these geometric shapes? Which geometric properties did we use during these calculations? How is image formation achieved in a hologram? What are the variables that influence image formation in a hologram? What modifications do these variables necessitate in our design?

The activity introduced in this study was evaluated through whole class discussion. During the final discussion, the students correctly answered the questions posed. However, an assessment form (see Appendix 4), was developed for educators who are interested in implementing this activity in their classrooms. This form allows for the individual assessment of knowledge of mathematics and science gained when designing a hologram. Furthermore, future implementers can extend the activity by spending an additional two lesson hours and adding the activity "A company wants to introduce its hologram designs to its customers in detail. Prepare the presentations/posters/videos required for this company." Implementers can use this added activity not only to improve students' process skills such as communication and discussion and but also to assess their knowledge of mathematics and science concepts.

CONCLUSION and RECOMMENDATIONS

In this study, a STEM activity was designed and implemented for eighth-grade students. An analysis of the video recordings of the implementation process, the students' worksheets, and the observation notes of the implementer showed that the students were able to use the targeted science and mathematics concepts during the activity and it was concluded that the activity was appropriate for eighth-grade students. Research has shown that teaching mathematical concepts in specific contexts supports students' conceptual learning of mathematics and develops their beliefs about the importance of mathematics (Kilpatrick et al., 2001). Furthermore, Gijsbers et al. (2020) emphasized that students should be provided with different contexts to support their learning of mathematics at an early age. Thus, it can be concluded that the hologram design can be a context for teaching both mathematics and science concepts through the implementation of the STEM activity designed in this study.

During the development of this activity, some support was received from science teachers and academicians working in this field. However, for future implementations, it can be suggested that mathematics and science teachers should work together in order to have richer discussions in terms of science related concepts. Indeed, Li et al. (2019) stated that in order to effectively link mathematics with other disciplines in STEM, teaching should be shaped around student ideas and the development of mathematical thinking. Therefore, it is suggested that implementers in different disciplines should work together to link mathematics with other disciplines and develop students' mathematical thinking more effectively.

This activity was carried out with eighth-grade students. However, it has great potential for future implementations at lower and higher school levels. Knowledge of trigonometry is needed to calculate the measure of the angle G when the angle of inclination of the surface of the hologram pyramid is 45°. As eighth graders cannot make this calculation, in this activity the measure of angle G was found by creating a hologram pyramid in GeoGebra. Therefore, this activity can be adapted to high school students as a trigonometry activity. The GeoGebra files used in this activity were prepared by the authors of the current study. As a follow-up to this activity, a new activity can be designed in GeoGebra to investigate the hologram pyramid and the implementation process can be carried out over a longer period of time. Li and Schoenfeld (2019) stated that considering mathematics not only as a tool but also as an empirical discipline will strengthen its relationship with STEM. The activity introduced in this study discussed how mathematics impacts the hologram design process. Besides, the additional activities suggested by the authors of the current study may extend the empirical features of the implemented activity. Therefore, it is considered that this study would contribute much to the perception of mathematics not only as a tool but also as an empirical discipline.

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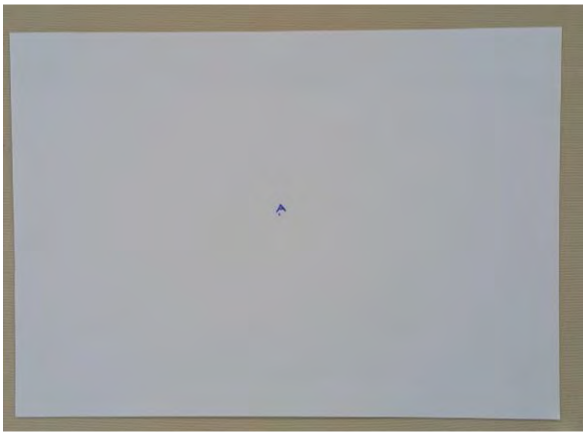
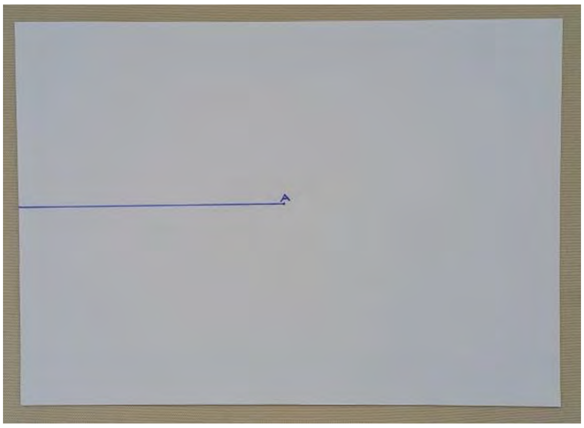
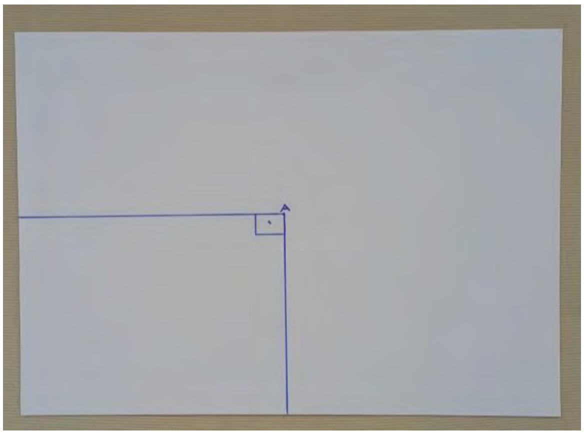
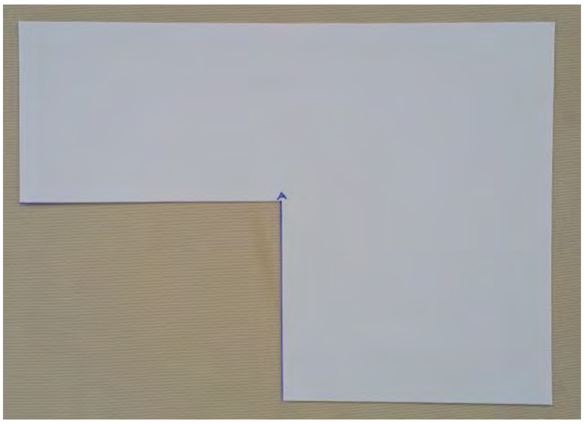
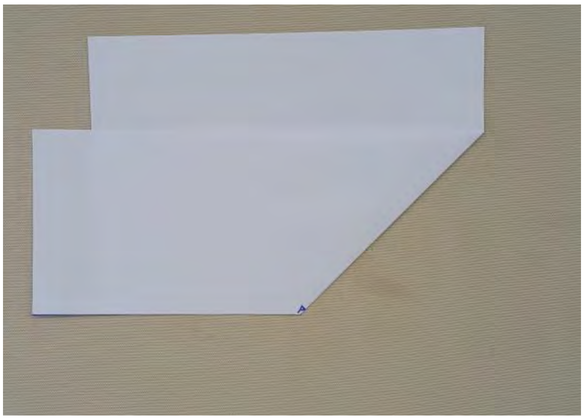
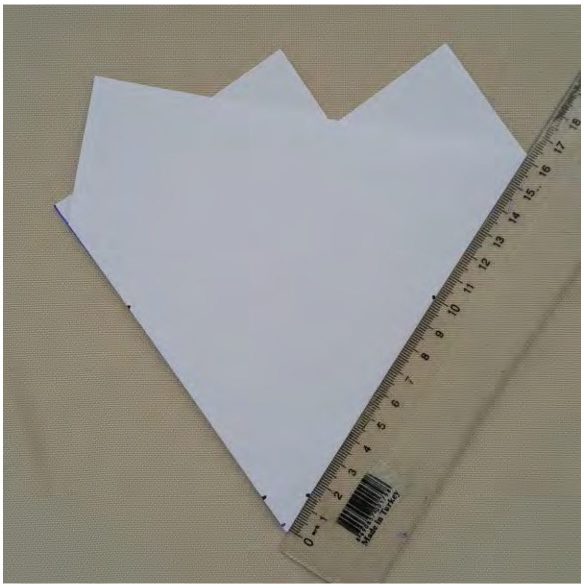
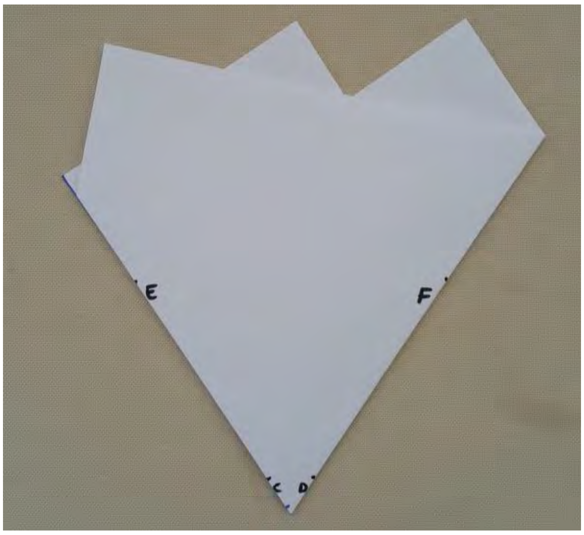
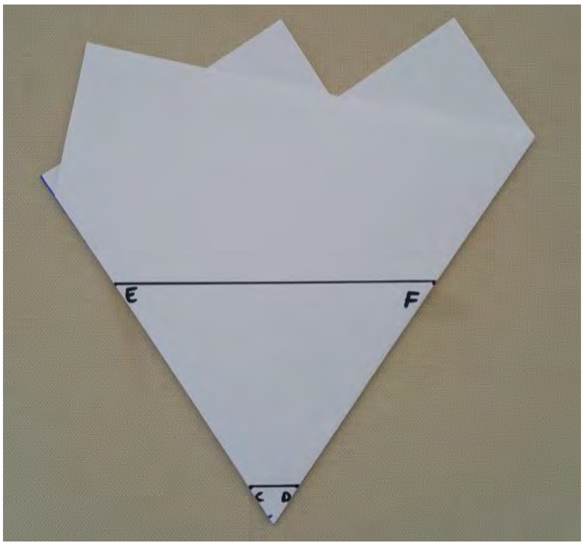
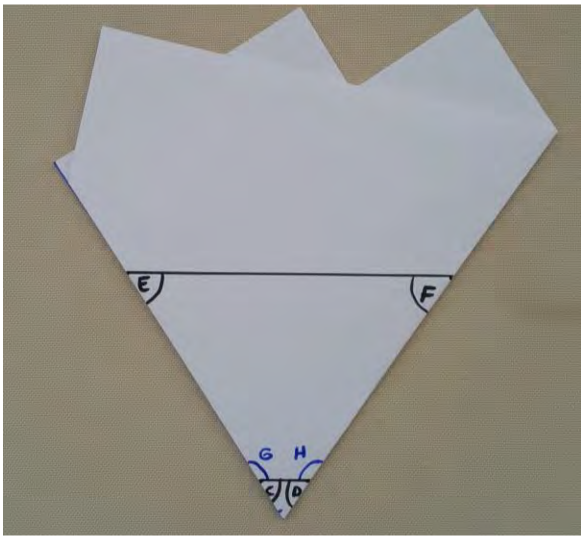
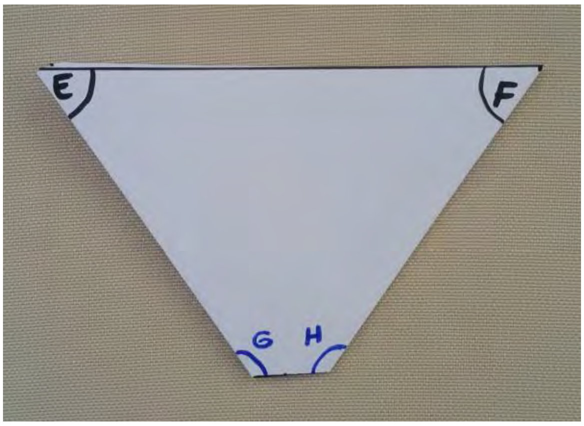
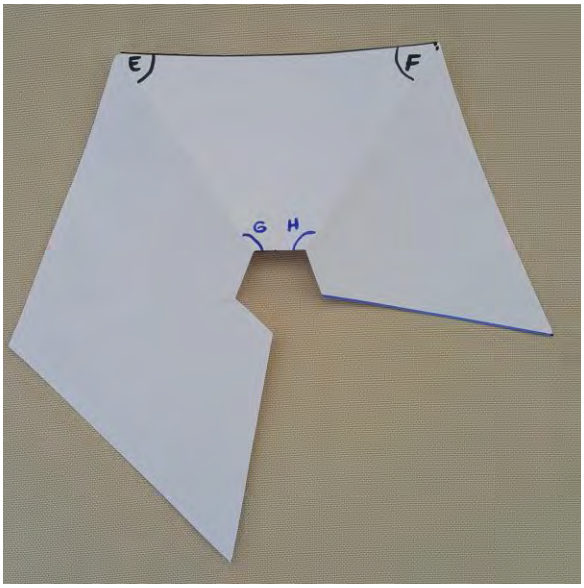
Appendix 1

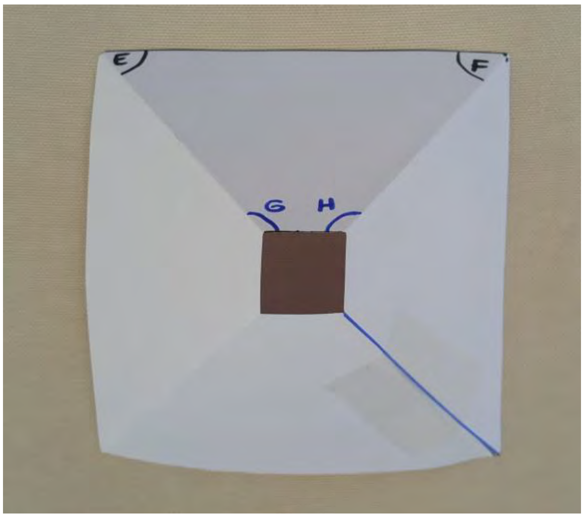
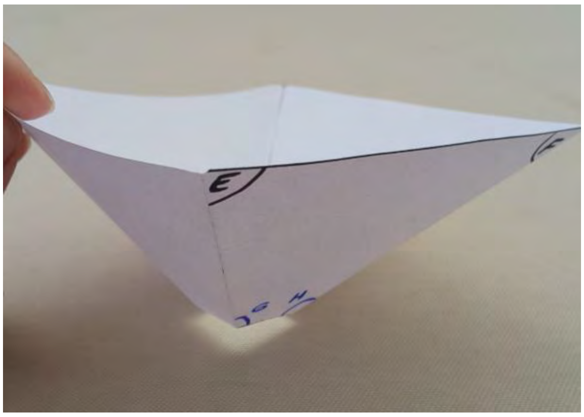
Hologram Pyramid Construction Steps (A4 paper)

Step	Explanation	Image
1	Choose a point in the center of the A4 paper and name it as Point A.	
2	Draw a line segment from Point A to the short side of the paper.	
3	Draw a 90-degree angle so that point A is the vertex of the angle and the line segment you drew in the second step is an arm of the angle. Extend the arm of the angle to the edge of the paper (use a protractor and ruler).	

4	Cut the 90-degree angle you have drawn and separate it from the rectangular piece of paper. Name the remaining angle as angle A. Ask your students to find the measure of angle A without using a protractor.	
5	Fold angle A in half so that the arms of the angle coincide and the fold line passes through point A.	
6	Fold the new angle in half so that the fold line passes through point A again and the arms coincide. Press the edges so that the fold is smooth and the fold lines are well-defined. Name the new angle you have obtained as angle B. Ask your students to find the measure of angle B.	

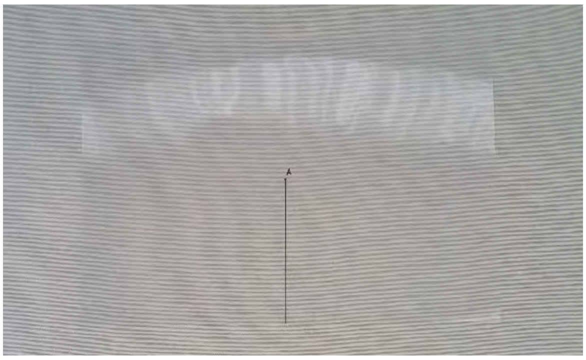
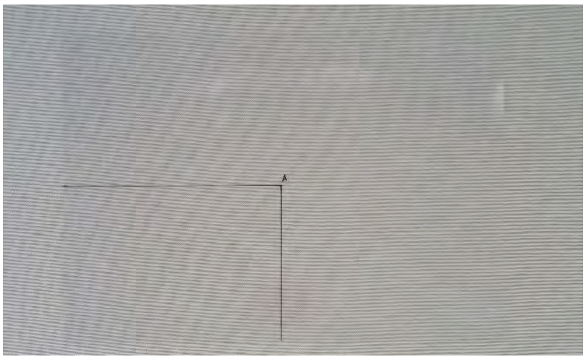
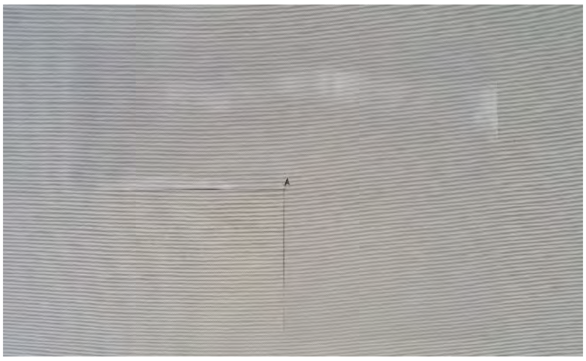
7	Identify the points on both arms of angle B that are 1.5 cm and 10 cm from the vertex of angle B. Mark these four points with a pencil.	
8	Name the two points 1.5 cm from the vertex of the angle as C and D. Name the two points 10 cm from the vertex of the angle as E and F.	
9	Connect points C and D by drawing a line segment using a ruler. Connect points E and F by drawing a line segment using a ruler. Ask your students to identify the geometric shapes formed by the points they have determined and the two line segments they have drawn. Ask your students to identify the properties of these shapes and to justify and explain their ideas.	

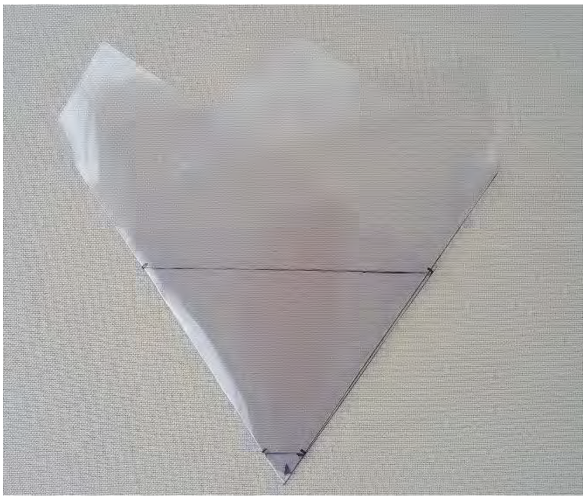
10	Form angles C, D, E and F as shown on the right. Ask your students to name the supplementary angles of C and D as angles G and H respectively. Ask your students to find the angle measures of the geometric shape without using a protractor and to explain their answers by giving reasons.	
11	Cut the folded paper from the line segments you drew in Step 9.	
12	Unfold the paper you have obtained. Notice that the paper consists of four identical geometric shapes.	

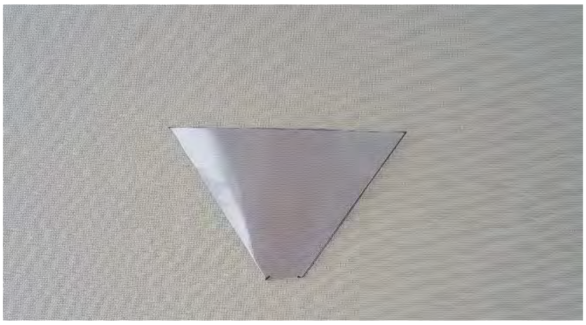
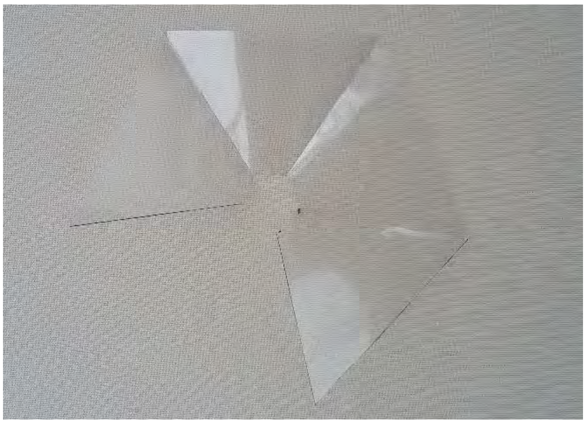
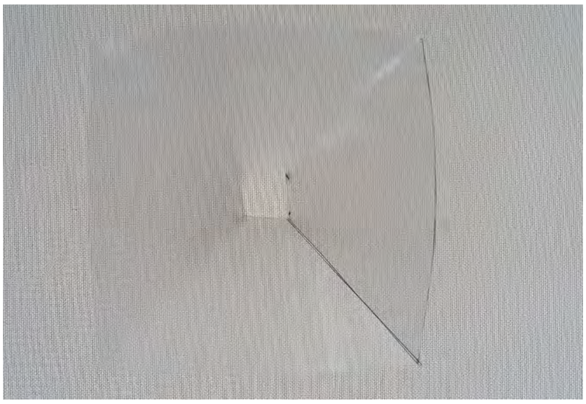
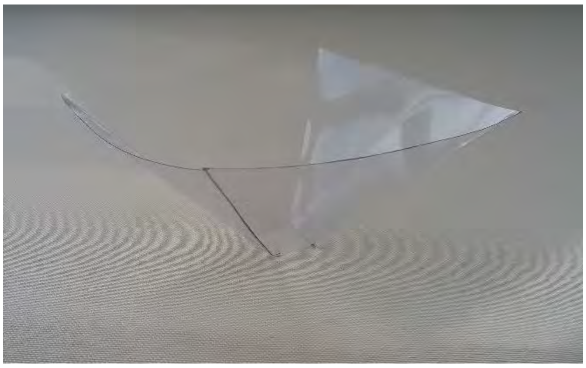
13	Align the rightmost and leftmost edges so that they coincide with each other and attach them with tape.	
14	Sharpen the fold lines properly so that the three-dimensional shape you have obtained forms the side faces of a truncated square pyramid. The resulting shape is a three-dimensional hologram pyramid.	

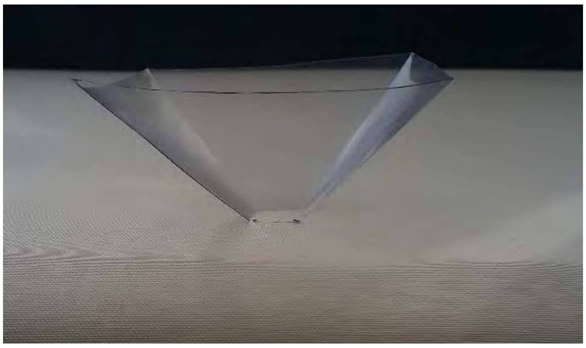
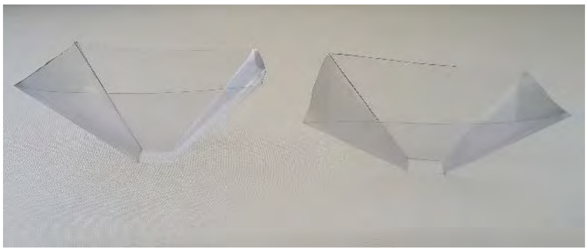
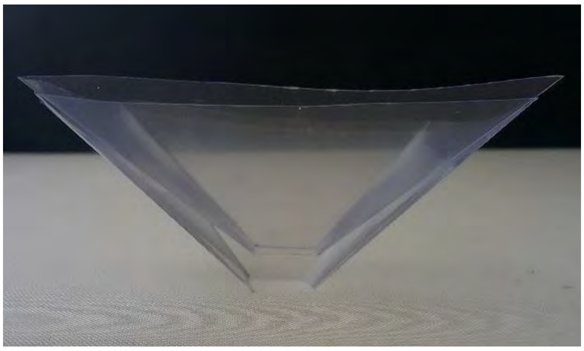
Appendix 2

Hologram Pyramid Construction Steps (Acetate Sheet)

Step	Explanation	Image
1	Choose a point in the center of the acetate sheet and name it as Point A. Draw a line segment from Point A to the long side of the acetate sheet.	
2	Draw a 90-degree angle so that point A is the vertex of the angle and the line segment you drew in the second step is an arm of the angle. Extend the arm of the angle to the edge of the paper (use a protractor and ruler).	
3	Cut the 90-degree angle you have drawn and separate it from the acetate sheet. Name the remaining angle as angle A.	
4	Fold angle A in half so that the arms of the angle coincide and the fold line passes through point A.	

5	Fold the new angle in half so that the fold line passes through point A again and the arms coincide. Press the edges so that the fold is smooth and the fold lines are well-defined. Name the new angle you have obtained as angle B. Ask your students to find the measure of angle B.	
6	Identify the points on both arms of angle B that are 1.5 cm and 10 cm from the vertex of angle B. Mark these four points with a pencil.	
7	Name the two points 1.5 cm from the vertex of the angle as C and D. Name the two points 10 cm from the vertex of the angle as E and F. Connect points C and D by drawing a line segment using a ruler. Connect points E and F by drawing a line segment using a ruler.	

8	Cut the folded paper from the line segments you drew in Step 7.	
9	Unfold the acetate sheet you have obtained. Notice that it consists of four identical geometric shapes.	
10	Align the rightmost and leftmost edges so that they coincide with each other and attach them with tape.	
11	Sharpen the fold lines properly so that the three-dimensional shape you have obtained forms the side faces of a truncated square pyramid.	

12	The resulting shape is a three-dimensional hologram pyramid. Open a hologram video on your smart device, place the hologram pyramid in the center of your device, and observe the image formed inside the hologram pyramid.	
13	Go back to Step 2 to obtain the second hologram pyramid. With a new sheet of acetate paper, create a hologram pyramid using the new measures. (Step 3: Cut a 150-degree angle from the paper).	
14	To compare the angles that the hologram surfaces make with the horizontal plane, a photograph of two holograms nested together is given on the right. Ask your students to compare the images formed in the two hologram pyramids, identify the differences, and explain the possible reasons for the differences.	

Appendix 3

Hologram Worksheet

1. Find the measure of angle A without using a protractor and explain how you found your answer.

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2. Find the measure of angle B without using a protractor and explain how you found your answer.

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3. Identify the geometric shape formed by the two line segments you have drawn. Identify the properties of this shape. Explain your answer with justification.

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4. Draw the geometric shape you obtained and determine the angle measurements of this shape without using a protractor. Explain your answer with justification.

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5. Compare the hologram pyramids you have obtained. Explain the different features, if any.

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6. Recall the construction steps of the two hologram pyramids and explain the differences.

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7. Explain how the differences in the hologram construction phase affect the hologram pyramids and give reasons.

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8. Compare the images produced by the two hologram pyramids. Identify the differences in the images and explain possible reasons for the differences.

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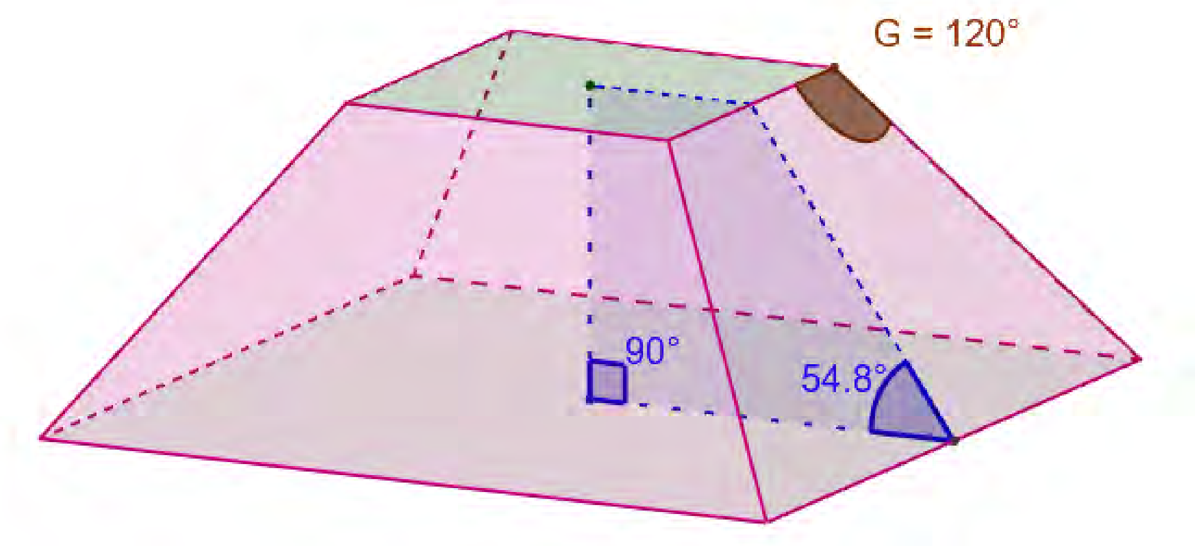
Appendix 4

Assessment Form

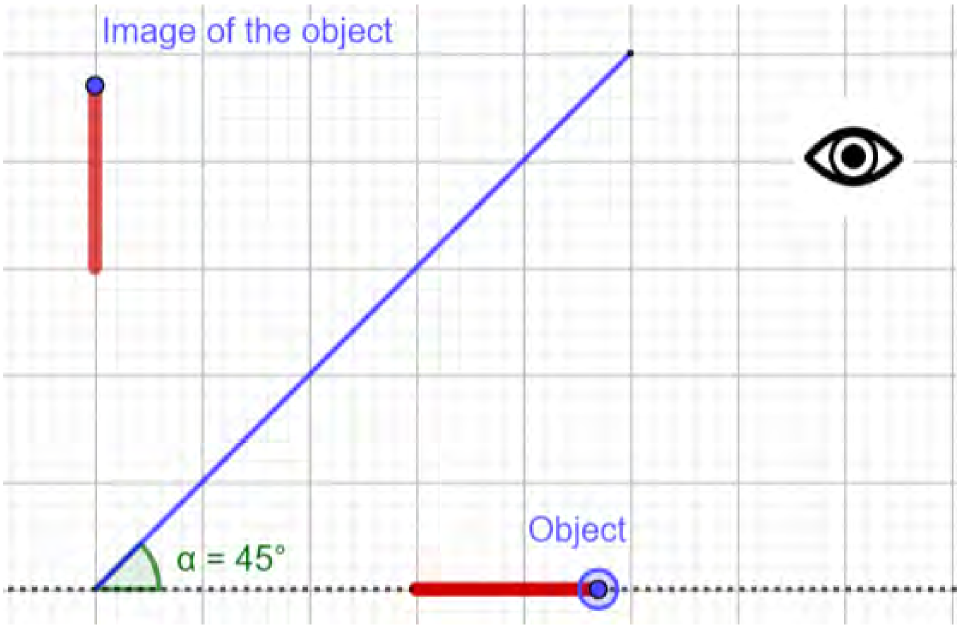


Construct the hologram pyramid shown above using acetate paper. Name the angles A, B, C and G as you named them in the activity and answer the questions below for the hologram pyramid you have constructed.

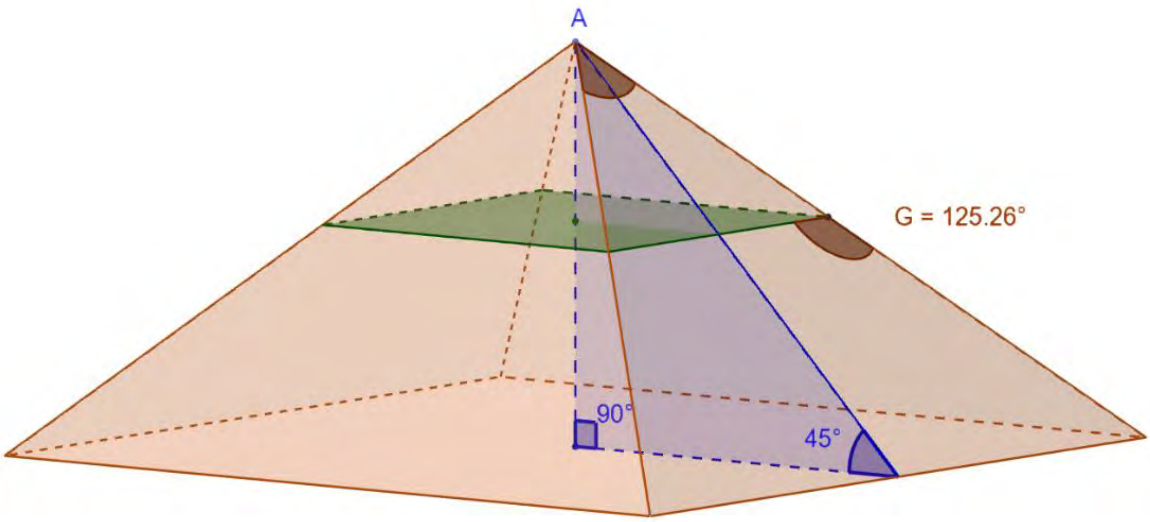
1. How many degrees of angle did you remove from the acetate paper? Explain your answer with justification.
2. What is the measure of angle A?
3. What is the measure of angle B?
4. What is the measure of angle C?
5. Write the algebraic expression that gives the relationship between the measure of the angle that must be subtracted from the acetate paper you determined in the first question and the measure of angle G.
6. Check your answer by using this algebraic expression.
7. What is the angle of the side faces of the hologram you have obtained in relation to the ground?

Appendix 5

GeoGebra Materials for Implementers



(a) https://www.geogebra.org/m/h8fxcvsn

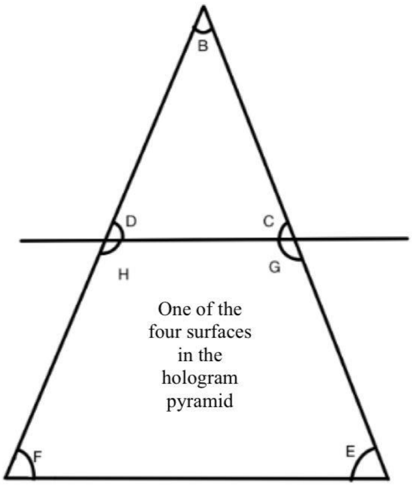


(b) https://www.geogebra.org/m/gwxk7qut

Appendix 6

Algebraic Generalization Process for Implementers

Problem: Formulate a rule to compute the angle G in the figure, considering the angle removed from the paper during the initial phase of the hologram pyramid construction. Note that the trapezoid illustrated below represents one of the four surfaces of the hologram pyramid.



	Angle B	**Angle C =>** $\frac{180-B}{2}$	**Angle G =>** $180-C$
When the 90° angle is cut out	$\frac{360-90}{4}$	$\frac{180-\frac{360-90}{4}}{2}=123{,}75$	$180-\frac{180-\frac{360-90}{4}}{2}=56{,}25$
When the 150° angle is cut out	$\frac{360-150}{4}$	$\frac{180-\frac{360-150}{4}}{2}=116{,}25$	$180-\frac{180-\frac{360-150}{4}}{2}=63{,}75$
When the unknown angle measure (x°) is cut out	$\frac{360-x}{4}$	$\frac{180-\frac{360-x}{4}}{2}=45+\frac{x}{8}$	$180-\frac{180-\frac{360-x}{4}}{2}=135-\frac{x}{8}$