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# Review of the mathematical thinking levels of gifted and non-gifted students 

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#### Abstract

This study aimed to determine the mathematical thinking levels of gifted and non-gifted students and to compare them according to some variables. In this study, screening model was used from quantitative research methods. The study group of the study consists of 194 gifted students and 168 non-gifted students studying in grades $5-8$ in a province in the southern region of Turkey in the 2022 to 2023 academic year. According to the results of the research, it was found that the mathematical thinking levels of gifted students were higher than their non-gifted peers. In addition, the mathematical thinking levels of gifted students showed a significant difference compared to grade level. This does not apply to non-gifted students. It was found that the mathematical thinking levels of female students were higher than the mathematical thinking levels of male students in gifted and non-gifted student groups, but this difference was not statistically significant. In addition, it was seen that the success grade of the mathematics course made a significant difference on the mathematical thinking level of both the gifted and non-gifted student groups.


Key words: Gifted students, mathematical thinking, non-gifted students.

## INTRODUCTION

Today, with the rapid pace of change, the education systems of the countries make it an important goal to educate students in a way that they dominate technology and contribute to their country by directing the change in the world. Education systems shaped in this direction will be successful when they can raise individuals who can offer different and qualified answers to original problems in daily life by using current opportunities in the most effective and efficient way. For this reason, curricula take into account the development of 21st century basic skills
and attach importance to learning processes that can provide this. Mathematical thinking is one of the basic skills that the nations want to gain in the qualified educational environments targeted by the nations and that the nations integrate into their curricula. The mathematics curriculum, which was revised by the Ministry of National Education in Turkey, started to be implemented in all grade levels of primary and secondary schools as of the 2018-2019 academic year. It is stated that instead of a structure that only conveys information

[^0]in the new curriculum, a simple structure that takes into account individual differences, focuses on gaining values and skills is aimed. In addition, it is stated that it is aimed to raise individuals with knowledge, skills and behaviors integrated with values and competencies (MEB, 2018). This study focused on examining the mathematical thinking levels of gifted and non-gifted student groups in terms of different variables. Thus, it was thought that it would be important to reveal the reflection of the achievements of mathematical thinking skills in the mathematics teaching curriculum to the students.

## Mathematical thinking

Mathematics has its own language and way of thinking. Since mathematics is a systematic structure, mathematical thinking has an important place in this system. Mathematical thinking skills can enable people to know themselves and the universe as a common thinking tool. Individuals who have acquired mathematical thinking skills can be successful in solving problems related to themselves and their environment (Tall, 1991). The importance of mathematical thinking emerges at this point, as appropriate responses to problems will be achieved with a sound and functional thinking skill (Yenilmez, 2007). Mathematical thinking helps individuals to realize the place and importance of mathematics in the world, to be constructive, reflective and sensitive citizens and to make sound decisions (PISA, 2015).In fact, mathematics, beyond all this, is often considered as life itself. Learning mathematics requires learning basic concepts and skills, as well as thinking mathematically, solving problems, and understanding that mathematics is important in real life.

The development of the concept of mathematical thinking in education and the importance of highlighting this skill in the learning process began in the 1980s, and Egan (1975), Freudenthal (1981), Krutetskii (1976), Burton (1984) and Schoenfeld (1992) studies reveal how important mathematical thinking is in mathematics education. Although there is no common definition and set of components for the concept of mathematical thinking in the field literature, it can be seen that mathematical thinking is a high-level thinking process that requires the management of processes that will solve the problem situation other than finding only the answer to a problem (Polya, 1945). According to Henderson et al. (2003), mathematical thinking is the direct or indirect application of mathematical processes to solving problems. Burton (1984) stated that mathematical thinking is not thinking about the subject of mathematics, but a way of thinking that is a function of known mathematical dynamics, processes and certain processes. Liu (2003) defined mathematical thinking as "the combination of estimating, induction, generalization, sampling, deduction, description, verification, formal and
non-formal reasoning, and similar processes." Yeşildere (2006) stated that mathematical thinking occurs if highlevel thinking skills such as customization, generalization, prediction, hypothesis generation, checking the accuracy of a hypothesis are required in the solution of a problem. When defining mathematical thinking, some of its elements are mentioned. Mason et al. (1985) stated that mathematical thinking consists of the processes of customization, generalization, hypothetical, and verification-persuasion, while Tall (2002) states that it consists of abstraction, synthesis, generalization, modeling, and proof. Alkan and Güzel (2005) also stated that mathematical thinking occurs by using the individual's previous knowledge of mathematics and abstraction, prediction, generalization, hypothesis and testing, reasoning and synthesis, etc.
Although mathematical thinking is perceived as a process used to solve mathematical problems, it is a process that can be used in every problem to be encountered in daily life. It is important to learn and teach mathematical thinking (Demirtaş, 2018) and to have a good mathematical thinking. In fact, mathematical thinking is one of the elements that form the basis of problem-solving skills. Similarly, Tuncay (2015) emphasized the importance and impact of mathematical thinking on the problem-solving process.

## Gifted students

Not every individual is the same in educational settings. There are individuals who think at different speeds and have different individual characteristics. One of these student groups is gifted students. Gifted students have cognitive skills that will enable them to be successful in the fields of science and art, and they constitute approximately $2 \%$ of the society (Ataman, 2004). For this reason, gifted individuals need to be well trained and guided through education prepared in accordance with their characteristics (Davis, 2006). There are different approaches to define giftedness in the field literature.

Renzulli (1978), called the three-ring model, expresses giftedness as the intersection of above-average ability, task delivery, and creativity. Gifted individuals will form the intellectual segment in the society if their potential is evaluated correctly by giving appropriate trainings to them (Bakar et al., 2018). With appropriate trainings, it is inevitable that gifted individuals will contribute positively to the development of both their immediate environment and society (Sontay et al., 2014). In Turkey, gifted people are diagnosed in 5 stages.

## Stage 1: Announcement phase

Stage 2: Filling out observation forms by classroom teachers
Stage 3: Evaluation of forms
Stage 4: Group screening

## Stage 5: Individual review

The basic stages in which the diagnosis of gifted are 4th and 5th stages, in which students who succeed in the group screening are evaluated individually. In this individual evaluation, the intelligence test preferred by the Ministry of National Education is applied. In 2015, WNV, or Wechsler intelligence test, was applied to 2nd, 3rd and 4th grade students. In 2016, while Kbit or Kaufman short intelligence test was applied to the first and second grades, the WNV test was applied to the third and fourth grades. In 2017, this time, ASIS, namely SAK Intelligence Scale, was applied to first and second year students in SAC auditions, while WNV test was applied to third year students. In SAC intelligence tests, a new regulation is made every year and it is preferred to change the tests. In this context, it is not possible to talk about a certain intelligence test every year, and individuals with an IQ score of 130 and above in the tests are defined as gifted individuals. In addition to the formal education activity in their schools, individuals who are diagnosed as gifted receive education according to the talent area (General Ability, Painting, Music) in SAC. In the diagnosis, there is no process for whether individuals are gifted in the field of mathematics (Aygün et al., 2020). In addition to the (formal) mathematics teaching curriculum in their schools, gifted students also experience different teaching processes in the Science and Art Center. Since mathematical thinking is a high-level thinking activity, it is expected that gifted individuals will emerge in problemsolving processes. In this case, it is important to determine whether the mathematical thinking skills of students diagnosed as gifted differ from their mathematical thinking levels compared to their peers.

## Research questions

Examining the field literature, it is observed that there are limited number of studies that focus on gifted talent specific to the field of mathematics, and limited number of studies showing that the mathematical problem-solving processes of gifted students (Kim et al., 2004; Yıldız et al., 2012) and their mathematical creativity (Haavold, 2013; Taşkın Can, 2013) differ from those of non-gifted students. There are studies on revealing the mathematical thinking processes of the students (Aygün et al., 2020; Baltaci, 2017; Kamarulzaman et al., 2022). In addition, teacher awareness studies on mathematical thinking (Baki and Işık, 2018; Baş, 2013; Coskun et al., 2021; Fisher et al., 2018; Krupa et al., 2017; Lee and Francis, 2018; Nickerson et al., 2017; Superfine et al., 2018; Türker and Yetkin, 2021) and review studies (Akdoğan, 2021) was made. As a result, determining the level of mathematical thinking of gifted students is considered an important issue in terms of revealing the factors behind the success of these students as well as reflecting the
reasons for their failures on the contrary. In this study, unlike other studies, the mathematical thinking levels of gifted and non-gifted students have been discussed comparatively. In addition, the scores of both student groups from the Mathematical Thinking Scale (MTS) will be compared in terms of gender, grade level and mathematics achievement grade variables, and it is thought that the research findings will contribute to the field literature. In this context, this study aimed to determine the mathematical thinking levels of gifted and non-gifted students and to compare them according to some variables (gender, grade level and mathematical achievement). For this purpose, the following research questions were formed:

1. What are the levels of mathematical thinking of gifted and non-gifted students?
2. Do gifted students differ in mathematical thinking levels based on gender, grade level and mathematics achievement?
3. Do the levels of mathematical thinking of non-gifted students differ according to gender, grade level and mathematics achievement?

## METHOD

## Research pattern

In this study, which aims to compare the Mathematical Thinking (MT) levels of gifted (G) and non-gifted (NG) students from 5-8 grade secondary school students in terms of different variables, a screening model from quantitative research methods was used. Screening models are research approaches that aim to depict a past or current situation as it exists (Karasar, 2009).

## Study group

The research was carried out with 194 G students attending SAC from 5-6-7-8 grade students of secondary school in a province located in the southern region of Turkey and 168 NG students studying at the $5-8$ th grade level of a randomly selected public school from the same province. The fact that students attend SAC means that they are diagnosed as G. The Gstudent group is limited to students who are nominated by their teachers with the observation forms sent to the schools by SAC and who continue to SAC by successfully completing the diagnostic processes. These students, who study at SAC in addition to their formal education, are students whose IQ score is determined to be 130 and above by a standardized test conducted by the Ministry of National Education. Descriptive statistics about the study group of the study are presented in Table 1.
While G students constitute $53.59 \%$ of the sample, NG students constitute $46.40 \%$. NG students at all grade levels ( 43 fifth graders, 40 sixth graders, 50 seventh graders, 35 eighth graders) are evenly distributed. In the distribution of G students ( 50 fifth graders, 82 sixth graders, 36 seventh graders, 26 eighth graders), there is a decrease in the number of students with the increase in grade level. It can be seen that female students ( 105 girls, 63 boys) are higher than female students ( 105 girls, 63 boys) according to gender, while the ratio of male students ( 85 girls, 109 boys) is higher in the distribution of $G$ students. It is seen that the majority of the students

Table 1. Descriptive statistics.

| Variable | G |  | NG |  |
| :--- | :---: | :---: | :---: | :---: |
|  | N | \% | N | \% |
| Gender |  |  |  |  |
| Female | 85 | 43.8 | 105 | 62.5 |
| Male | 109 | 56.2 | 63 | 37.5 |
| Grade Level |  |  |  |  |
| 5 | 50 | 25.8 | 43 | 26.5 |
| 6 | 82 | 42.3 | 40 | 23.8 |
| 7 | 36 | 18.6 | 50 | 29.8 |
| 8 | 26 | 13.4 | 35 | 20.8 |
| Mathematics course grade |  |  |  |  |
| Low level (3 and lower) | 33 | 17.0 | 38 | 22.6 |
| Moderate level (4) | 42 | 21.6 | 40 | 23.8 |
| Good level (5) | 119 | 61.3 | 90 | 53.6 |
| Total | 194 | 100 | 168 | 100 |

have a mathematics report card grade at the intermediate level and above.

## Data collection tool and analysis of data

In the study, the Mathematical Thinking Scale (MTS) developed by Er et al. (2023) was used to determine the MT levels of the participants. The sub-factors of the MTS include "Inductive and deductive thinking ( 7 items)", "Utilitarian thinking (3 items)", "Planned thinking (3 items)", and "Problem-solving based thinking ( 3 items)". The internal consistency coefficient of the 16 -item MTS used in this study was found to be 0.87 . The obtained reliability coefficient shows that the scores related to the scale are reliable and can be used for research purposes (Tavşancıl, 2002).

All data of the study were collected by the researcher. The scale took approximately 25 minutes to implement. The mathematics achievements of the students are determined by the grades of the students' mathematics courses in the previous semester, which are registered in the system in their schools. After checking the assumptions of normality of the MTS scores of the students according to the variables and the equality of the variances, it was decided which analyzes would be performed. Kolmogorov Smirnov statistics analytical test values ( N and skewness-flatness values) were examined whether the scores obtained from the scale provided normality for each data group. The values of $\pm 1, \pm 1.5$, $\pm 1.96, \pm 2, \pm 3, \pm 3.29$ are proposed for the assumption of normality in the field literature. In this study, the $\pm 2$ approach (George and Mallery, 2019) was considered in the examination of skewness and flatness values.

Data Analysis of whether the MT levels of the G and NG students showed a statistically significant difference was examined by performing the t-test of unrelated samples. Whether the mean MTS scores of the G and NG students showed a significant difference according to the gender variable was examined by applying ANOVA test or not and whether there was a significant difference according to grade level and mathematics achievement grade variables was examined by applying a single-factor analysis of variance (Ford and Harris, 1992). Scheffetest, one of the complementary post-hoc analysis techniques, was applied to determine between which groups the significant difference found with single-factor ANOVA was found. In this process, $p=0.05$ was
accepted for the significance value in the process of analyzing all statistical analyzes.

## FINDINGS

## Comparison of MTS scores of G and NG students

First, it was examined by applying the unrelated groups' t-test, the total score obtained from the MTS scale and the scores obtained from the sub-factors. The results are given in Table 2. Examining Table 2 is examined, there is no statistically significant difference in the mean MTS total scores of $G$ and NG students $(\mathrm{t}(360)=1.825$, $\mathrm{p}>0.05$ ). MTS averages of NG students ( $X=61.52$ ), are lower than those of G students $(X=64.10)$. There is a statistically significant difference in the mean of factor 1 score, which is the sub-factor of the MTS scale of G and NG students. Accordingly, it can be said that the MT levels of $G$ students are higher than those of NG students, but this difference is not significant. In addition, considering that the maximum score that can be taken from the scale is 80 , it can be said that the MT levels of the students in both groups are high.

## Comparison of MTS scores of G and NG students by gender

The findings obtained as a result of the t-test are presented in Table 3. Examining Table 3, it was seen that the mean MTS total scores of the G and NG students did not show a statistically significant difference according to the gender variable $(\mathrm{t}(192)=.588, \mathrm{t}(166)=.276, \mathrm{p}>0.05)$. It was observed that average scores of female students in the G group from MTS ( $X=64.81$ ) were higher than

Table 2. Unrelated groups t-test results.

| Factors | Groups | N | A.O | Ss | T | Sd | P |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F1 | G | 194 | 29.38 | 7.10 | 2.199 | 360 | $0.029^{*}$ |
|  | NG | 168 | 27.91 | 5.33 |  |  |  |
| F2 | G | 194 | 11.74 | 3.66 | 1.444 | 360 | 0.150 |
|  | NG | 168 | 11.21 | 3.30 |  |  |  |
| F3 | G | 194 | 11.41 | 3.41 | 1.205 | 360 | 0.229 |
|  | NG | 168 | 11.00 | 3.02 |  |  |  |
| F4 | G | 194 | 11.56 | 3.61 | 0.463 | 360 | 0.644 |
|  | NG | 168 | 11.39 | 3.00 |  |  |  |
| Total | G | 194 | 64.10 | 14.69 | 1.825 | 360 | 0.069 |
|  | NG | 168 | 61.52 | 11.72 |  |  |  |

Table 3. Students' mathematics thinking scale scores by gender.

| Students | Gender | N | $X$ | Ss | T | Sd | P |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gifted | Female | 85 | 64.81 | 15.12 | 0.588 | 192 | 0.557 |
|  | Male | 109 | 63.55 | 14.40 |  |  |  |
| NG | Female | 105 | 61.72 | 11.64 | 0.276 | 166 | 0.783 |
|  | Male | 63 | 61.20 | 11.96 |  |  |  |

Table 4. Change of students' mts scores by grade level.

| Students | Grade | N | $X$ | Ss |  | Sum of squares | Sd | Squares Avg. | F | p |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| G | 5 | 50 | 67.28 | 14.24 | $G a$ | 8112.341 | 3 | 2704.114 | 15.295 | . 000 |
|  | 6 | 82 | 56.98 | 14.23 | Gi | 33592.386 | 190 | 176.802 | 0.753 | 0.522 |
|  | 7 | 36 | 68.47 | 13.81 | Total | 41704.727 | 193 |  |  |  |
|  | 8 | 26 | 74.42 | 4.74 |  |  |  |  |  |  |
| NG | 5 | 43 | 63.44 | 12.05 | $G a$ | 311.895 | 3 | 103.965 |  |  |
|  | 6 | 40 | 61.95 | 12.52 | Gi | 22653.956 | 164 | 138.134 |  |  |
|  | 7 | 50 | 60.88 | 11.54 | Total | 22965.851 | 167 |  |  |  |
|  | 8 | 35 | 59.62 | 10.71 |  |  |  |  |  |  |

Intergroup: Ga; In-group: Gi
those of male students ( $X=63.55$ ), and similarly that average scores of female students in the NG group from MTS ( $X=61.72$ ), were higher than those of male students ( $X=61.20$ ). Accordingly, it can be said that the MT levels of female students in both groups are higher than those of male students.

## Comparison of MTS scores of G and NG students according to grade levels

The findings obtained as a result of the ANOVA test are
presented in Table 4. According to Table 4, it was found that the mean scores of G students from MTS showed a significant difference according to their grade level ( F (3. 194) $=15.295, p<0.05)$. Total score averages of the G students participating in the study were as follows respectively from lowest to highest: sixth grade ( $X=56.98$ ), fifth grade ( $X=67.28$ ), seventh grade ( $X=68.47$ ) and eight grade ( $X=74.42$ ). The Scheffe test, one of the multiple comparison tests, was used to determine the direction of the significant difference. According to the analysis results the difference between fifth, seventh and eighth grade $G$ students is in favor of sixth grade students. In

Table 5. Change of students' mts scores according to mathematics achievement.

| Students | Mathematics <br> achievement | $\mathbf{N}$ | $\boldsymbol{X}$ | Ss |  | Sum of squares | Sd | Squares Avg. | F | p |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| G | Low | 30 | 49.60 | 18.50 | $G a$ | 12487.644 | 2 | 6243.822 | 40.818 | $0.000^{*}$ |  |
|  | Moderate | 35 | 56.14 | 15.46 | $G i$ | 29217.083 | 191 | 152.969 |  |  |  |
|  | Good | 129 | 69.64 | 9.33 | Total | 41704.727 | 193 |  |  |  |  |
|  |  |  |  |  |  |  |  | 1075.309 | 8.524 | $0.000^{*}$ |  |
| NG | Low | 38 | 56.26 | 15.14 | $G a$ | 2150.619 | 2 | 126.153 |  |  |  |
|  | Moderate | 40 | 59.37 | 9.93 | $G i$ | 20815.232 | 165 |  |  |  |  |
|  | Good | 90 | 64.71 | 9.75 | Total | 22965.851 | 167 |  |  |  |  |

addition, it was found that the mean scores of the NG students from MTS did not differ significantly according to the grade level ( $\mathrm{F}(3.168$ ) $=0.753, \mathrm{p}>.05$ ). Total score averages of the NG students participating in the study were as follows respectively from lowest to highest: eight grade ( $X=59.62$ ), seventh grade ( $X=60.88$ ) sixth grade ( $X=61.95$ ) and fifth grade ( $X=63.44$ ). Accordingly, it can be said that the level of MT of the G students increases as the grade level increases, while the MT levels of the NG students decrease slightly as the grade level increases.

## Comparison of MTS scores of G and NG students according to mathematics achievement

The findings obtained as a result of the ANOVA test are presented in Table 5. According to Table 5, it was found that the mean scores of G students from MTS showed a significant difference according to their mathematical achievement ( $F(3.194$ ) $=40.818, \mathrm{p}<.05$ ). Total score averages of the $G$ students participating in the study in MTS were as follows from lowest to highest: students with low mathematics achievement ( $X=49.60$ ), students with moderate mathematics achievement ( $X=56.64$ ) and students with good mathematics achievement ( $X=69.64$ ). The Scheffe test, one of the multiple comparison tests, was used to determine the direction of the significant difference. According to the analysis results the difference between G students with moderate mathematics achievement and good mathematics achievement is in favor of students with low mathematics achievement.
In addition, it was found that the mean scores of the NG students from MTS showed a significant difference according to their mathematical achievement ( $\mathrm{F}(2.168$ ) $=8.524, \mathrm{p}<.05$ ). Total score averages of the NG students participating in the study from MTS were as follows from lowest to highest: students with low mathematics achievement $(X=56.26)$, students with moderate mathematics achievement ( $X=59.37$ ) and students with good mathematics achievement ( $X=64.71$ ). The Scheffe test, one of the multiple comparison tests, was used to determine the direction of the significant difference.

According to the analysis results the difference between NG students with moderate mathematics achievement and good mathematics achievement is in favor of students with low mathematics achievement. Accordingly, it can be said that the MT levels of both G and NG students increase as their mathematical achievement levels increase.

## DISCUSSION

This study, which was conducted to determine whether the MT levels of G and NG students and the average scores they received from the MTS scale differed according to gender, grade level and mathematical achievements, was based on the data obtained from 362 students. In line with the first research question, the average scores of $G$ and NG students from MTS were compared. It was observed that there was a statistically significant difference in the mean scores from Factor 1, which is the sub-factor of the MTS scale, in favor of the G students, and as a result of the comparison of the average scores taken from the whole scale, it was seen that the G students had a higher average than the NG students, but this difference was not significant. Accordingly, it can be said that the MT levels of G students are higher than those of NG students. In addition, considering that the maximum score that can be taken from the scale is 80 , it was observed that that the MT levels of the students in both groups are high. Gagne (2004) expressed the concept of superior ability as the individual's intelligence age being above normal and performing at a high level in reasoning and abstract thinking skills. Gifted individuals have a high level of sense of duty and creative skills and above-average academic ability (Renzulli, 1978). Gifted individuals are by nature high-level performers in many areas compared to their peers. These research findings are similar to the findings of the field literature.

In line with the second and third research questions, the status of the MTS total score averages of the Gand NG students according to the variables (gender, grade level, mathematics course success) was examined. It
was observed that the mean MTS total score of both G students and NG students did not show a statistically significant difference according to gender variable. In addition, it was observed that the MT levels of female students in both groups were higher than male students. Akçakın and Kaya (2020) examined the variation of mathematical thinking styles according to gender in their study. Researchers stated that female students have more analytical thinking style than male students, and male students have more visual thinking style than female students. Gürtaş (2021) examined the mathematical thinking and problem solving skills of secondary school 7th grade students on rational numbers in his study, in which both qualitative and quantitative methods were used. As a result of the research, it was seen that the mathematical thinking and problem solving skills of the students were not at the desired level, and there were statistically significant differences when the participants were examined in terms of variables such as gender, parental education level, the schools they attended, and the reading time. In his study titled "A review on MT skills", Duran (2005) examined the power of some variables related to MT applied to 15-year-old students within the scope of PISA to predict the success of MT skills. As a result of the research, it was reported that male students were more successful than female students, and those who received pre-school education were more successful than those who did not receive pre-school education. In the study titled "Development of MT in Prospective Teachers" by Alkan and Güzel (2005), it was examined that the MT levels of prospective teachers did not constitute a significant difference in terms of gender. Although there is a significant difference in the two sub-factors, it can be said that parallel results have emerged with this research when the whole scale is considered. Him (2006) conducted research on the selfregulatory learning of gifted students in the field literature, and found that male and female students were similar in terms of average scores in terms of intrinsic motivation, test anxiety, cognitive strategy use, and self-regulation, and that the results did not differ significantly. Pajares and Graham (1999) also reached similar conclusions. Accordingly, it can be said that this research finding is similar to the field literature. It was found that the average scores of gifted students from MTS differed significantly according to grade level. However, this does not apply to NG students.

The difference between the average scores of gifted students from the fifth, seventh and eighth grade MTS is in favor of sixth grade students. It was observed that that the level of MT of the gifted students increases as the grade level increases, while the MT levels of the NG students decrease slightly as grade level increases. Baş (2019) investigated the relationship between secondary school students' attitudes towards mathematics, mathematical thinking, and problem solving. As a result
of the research, it was seen that the students' attitudes towards all three variables were above the medium level, and these attitudes did not differ according to gender, but differed according to grade level. In addition, it was seen that the average scores of both gifted students and NG students from MTS showed significant differences according to their mathematical success. Accordingly, it can be said that the MT levels of for both groups of students increase as their mathematical achievement levels increase. Tüzün and Cihangir (2020) determined the relationship between the MT stages of secondary school students and their mathematical self-efficacy. The researchers found that there was a positive moderate level relationship between the students' mathematics course final scores and their mathematical self-efficacy and MT stages, as well as between the MT stages and mathematics self-efficacy. In addition, in the field literature, Karakoca (2011) revealed a significant differentiation in the variable of mathematical achievement in MT situations in problem solving. In the study by Alkan and Güzel (2005), it was seen that there was a linear relationship between prospective mathematics teachers' scores from the analysis courses and their level of mathematical thinking. In addition, it was seen that there was a significant relationship between MT and mathematical success in Mubark (2005)'s study, there was a strong relationship between MT and mathematical achievements in Nepal (2016)'s study, and there was a positive significant relationship between MT and success in Kocaman (2017) study. Accordingly, it can be said that this research finding is similar to findings in the field literature.
Consequently, in this research, it has been observed that the MT levels of G students were significantly higher according to their grade level; however, this did not apply to NG students. In addition, it was found that the MT levels of female students were higher than the MT levels of male students in gifted and NG student groups, but this difference was not statistically significant. In addition, it was seen that the success grade of the mathematics course made a significant difference on the level of MT of both the gifted and NG student groups.

## CONCLUSION AND RECOMMENDATION

It is important for individuals to have this skill because they use MT skills in analyzing the events and phenomena they encounter at every stage of life. In his study titled "The effects of MT skills on the academic achievements, problem-solving skills and attitudes of primary school students in technology and science courses", Taşdemir (2008) examined the effects of a unit in technology and science course on attitude, academic achievement, and problem-solving skills of groups that continue constructivist learning and normal education
with teaching that includes constructivist learning-based MT activities. According to the results of the study, it was determined that constructivist-based teaching, which includes MT activities, has a significant effect on the development of students' academic achievements, attitudes and problem-solving skills and on ensuring their continuation. Ayllón et al. (2016) examined the relationship between the development of MT and creativity through mathematical problem posing and solving, and creativity in mathematical problem solving and creation. According to the results of the study, a significant relationship was found. In order for the MT levels of both gifted students and NG students to be at the desired level, it is recommended to design teaching environments that develop this skill and to examine the effectiveness. In addition, in this research, MT levels were discussed and compared with the MT scale. In addition, the mathematical processes of $G$ and NG students in the problem-solving process can be discussed together. In addition, it is recommended to conduct correlation studies in which different ways of thinking such as creative thinking and critical thinking are considered together. It is recommended to include more activities to develop mathematical thinking skills in the mathematics teaching curriculum. In addition, teachers can be given trainings to support the teaching process in gaining mathematical thinking skills. In addition, students', teachers' and pre-service teachers' mathematical thinking skills and processes can be examined.

## Limitations

This study, which was conducted to determine whether the MT levels of gifted and NG students and the average scores they received from the MTS scale differed according to gender, grade level and mathematical achievements, was limited with the data obtained from 362 students. It was assumed that the students sincerely responded to the scale items.

## CONFLICT OF INTERESTS

The author has not declared any conflict of interests.

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