REACHING PYTHAGORIAN THEOREM BY FOLDING PATTY PAPER[1]

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ABSTRACT

In this study, a paper folding activity task, which involved reaching the Pythagorean Theorem with a series of steps was designed. The aim of the task is to reach the Pythagorean Theorem with folding activities by deductive reasoning and logical inference. The study aimed to examine the effectiveness of the task and to share the patty paper folding task with the teachers. The activity was used with 20 ninth grade students studying in a public school. The patty paper folding activity task was carried out for a total of 6 lesson hours for three weeks. The students did not have difficulty while folding; they generally had difficulties in performing algebraic operations and expressing the concepts mathematically. The students participated in the activity willingly and the patty paper folding task helped students understand why the theorem is true and contributed to meaningful learning in terms of being a explanatory proof.
Keywords: patty paper folding, Pythagorian Theorem, teaching proof, teaching mathematics.

YAĞLI KÂĞIT KATLAMA İLE PİSAGOR TEOREMİNE ULAŞMA

ÖZ

Bu çalışmada bir dizi adımla Pisagor Teoremi'ne ulaşmayı içeren yağlı kâğıt katlama etkinlik görevi tasarlanmıştır. Çalışmanın başlıca amacı, öğrencilerin tümdengelimsel akıl yürütme ve mantıksal çıkarımın rolüne ilişkin anlayış kazanarak Pisagor Teoremi'ne yağlı kâğıt katlama etkinlikleri ile ulaşmalarıdır. Bunun yanısıra, üç farklı etkinlik içeren yağlı kâğıt katlama görevinin yararlı olup olmadığının gözlenmesi ve öğretmenlerin yağlı kâğıt katlama görevini yakından tanımaları amaçlanmıştır. Etkinlik bir devlet okulunda öğrenim gören 20 dokuzuncu sınıf öğrencisi ile yürütülmüştür. Yağlı kâğıt katlama etkinlik görevi öğrencilerle üç hafta boyunca toplam altı ders saatinde gerçekleştirilmiştir. Öğrenciler yağlı kâğıt katlama görevinde, çalışma kağıtlarında bulunan adımları uygularken katlama aşamasında zorluk yaşamamışlar; fakat genel olarak cebirsel işlemleri yapmada ve bildikleri kavramları matematiksel olarak ifade etmeye çalışırken zorlanmışlardır. Öğrencilerin etkinliği uygulama sürecinde istekli ve keyif alarak etkinliğe katıldıkları gözlenmiştir. Yağlı kâğıt katlama görevinin teoremin niçin doğru olduğunu anlamalarını sağladığı ve açıklayıcı bir ispat niteliğinde olması açısından anlamlı öğrenmeyi destekleyici katkısı olduğu görülmüştür.
Anahtar kelimeler: yağlı kâğıt katlama, Pisagor Teoremi, ispat öğretimi, matematik öğretimi.

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INTRODUCTION

In the 300s BC, geometry became a scientific phenomenon established by the logical rules of a deductive science and axiomatic system, with the work called "Elements" written by Euclid, which is considered one of the peaks of success reached by human intelligence. As a logical system, two basic results emerged with the work "Elements" of the Geometry-Euclid period. One of these is the Pythagorean Theorem, and the other is that the sum of the interior angle measurements of a triangle in the plane is 180° (Argün et al., 2014). Although the Pythagorean Theorem is one of the first two results of the work called "Elements", it is undoubtedly the most famous theorem of geometry and has more than 300 known proofs (Benjamin, 2019). In this study, instead of directly introducing the Pythagorean Theorem to students as a rule, activities were developed on two proofs, and how to structure these proofs. This is because to use proof applications in school mathematics is considered to be useful. The Principles and Standards for School Mathematics consider the reasoning and proof standard as one of the parallel standards of mathematical knowledge that students should acquire and use, as one of the integral components of mathematics teaching and learning in determining all processes and methods of doing mathematics (National Council of Teachers of Mathematics [NCTM], 2000; van de Walle, 2013). Students need to acquire the habit of providing logical justification for answers to questions and curiosities, and it is extremely important to learn the value of proving their ideas through logical evidence. Therefore, all students from kindergarten through 12th grade should view reasoning and proof as fundamental components of mathematics; they must be able to create and examine mathematical hypotheses, develop and evaluate mathematical claims and proofs, and choose and use various forms of proof methods and reasoning (NCTM, 2000; van de Walle, 2013). However, in classrooms, the proof is presented to students as a series of symbolic manipulations that lead to a result prepared for them to accept, which students will have difficulty following in a purely deductive and narrow framework due to the formal nature of formal proof, and thus the nature of the rules and formulas is not understood by the students. Therefore, the proof skills of students become weak (Hanna & Jahnke, 1993, Tall, 1989). Hsieh et al. (2021) stated in their study that students were able to write formal proofs with the help of paper folding, even though they were not advanced students and had not learned the concept of proof in a formal way before. Therefore, it seems possible for students to progress to formal proof with the help of paper folding. While the sharp definitions produced by certain mathematicians for proof do not include proofs made by folding paper, in recent years, with the inclusion of technology in the proof processes, alternative proof methods have become visible and have begun to be accepted among mathematicians (Boz Yaman, 2020). However, concepts such as reasoning and justification are not synonymous with proof, so it is necessary to distinguish between reasoning and proof. Concepts such as reasoning, justification, and generalization are concepts closely related to proof. Although proof is a form of argument formation and justification, not all arguments and justifications are proof. In addition, formal proof may not always be a sufficient justification and explanation of mathematical ideas. What is important in geometry education is that students examine why a mathematical expression is true or false (Güler, 2020). In this way, students can structure the proof with correct reasoning. Different solutions have been suggested to improve students' proof skills.

NCTM (2014) emphasized the necessity of three effective mathematics teaching practices as a solution to the problem of weak proof skills of students. These are a) Including activities that require mathematical reasoning and proof, b) Using and relating mathematical representations, c) Supporting productive challenge in mathematics classes (Zeybek Şimşek, 2020). Therefore, it was thought that this activity, which requires students to prove the most important theorem of geometry by using mathematical notations and making connections, as recommended, would be useful. Thus, it is aimed for students to gain an understanding of the role of deductive reasoning and logical inference by using patty paper material, and at the same time to encourage them to better understand mathematical concepts and propositions (Hanna, 2018). In this way, it is thought that the mathematics teaching practices mentioned by NCTM (2014) will be achieved. Patty paper folding activity design requires mathematical reasoning in the folding steps; it involves algebraic calculations and mathematical representations between folding steps, and since the steps after folding can be clearly observed, it includes learning through discovery and thus supports efficient forcing. Therefore, this activity design is considered to be an effective mathematics teaching practice. The use of paper folding in geometry allows us to visually present some concepts, properties and relationships without technology or measuring tools such as squares, or rulers (Duatepe-Paksu, 2016). This method, which supports conceptual learning on appropriate subjects, also increases the motivation of students towards the subject and even the course (Robichaux & Rodrigue, 2003). Paper folding is an effective material and method that can be used to prove theorems, and examining these applications is a useful method for teachers and students to develop a different perspective on proof and geometry. (Boz Yaman, 2020). The reason why the patty paper folding method, is used in this activity is the transparent structure of patty paper. In this way, it allows what is written and drawn on it to be easily seen and to make geometric discoveries with ease of folding. Patty papers are waxed square papers used by fast food restaurants to separate hamburger patties (Serra, 1994). Patty papers sold in markets, also known as baking papers, serve the same function when cut to the appropriate size (usually 15*15cm, but any suitable size can be used). It can be easily written on patty papers with pencils and felt-tip pens, and it is important for patty papers to be transparent in proof activities, to easily observe the folded side (fold marks) when folded and written on with a pencil, and to be able to observe relationships in visualization and justification, allowing reasoning. This enables students to work on proof tasks. Patty paper tasks thus differ from other paper folding tasks.

Patty paper geometry allows the discovery of any feature that can be discovered by using compasses, rulers and protractors. For example, since the patty paper is considered as a square based on the initial assumption, in the activities it is assumed that all its edges are equal, and the corner angle measurements are 90˚. Thus, in applications, length measurements can be made by using the edges of the patty paper, assuming that each side of the patty paper is equal, or whether a triangle is an acute or wide angle triangle can be determined by using the corner angles and side lengths of the square and making measurements, and special triangles such as right triangles, equilateral triangles can be created, different quadrilaterals can be created. The relationships between them can be determined using these measurements. In addition, comparing angle, area and length measurements by making copies using the transparency feature of patty paper is one of the useful discoveries of patty paper tasks. In a classroom where patty paper geometry activity is carried out, the cooperative learning model comes to the fore. A double-shared group structure is recommended for patty paper geometry. In this structure, students working in groups of four are divided into two pairs. One student from each pair reads the instructions to the partner while the partner does the folding. Each pair compares their results with the results of the other pair in the group. The group then forms its hypothesis and both pairs and roles change for the next investigation. This cooperative group structure allows them to learn by discovery. Everyone has a role in paired sharing and helps reduce children's mathematics anxiety (Serra, 1994). In this study, the mentioned double-shared group structure was adopted. In this study, it was aimed to not only reveal the Pythagorean theorem with the patty paper folding method, but also to observe whether this activity is useful or not, to reveal the students' reactions to the activity, and to familiarize teachers with the patty paper folding tasks. Students often have difficulty memorizing the formulas that are mainly used in geometry lessons. For this reason, it is considered important to create such environments where the reasons for rules and formulas can be investigated.

Preparing Students for the Activity Process: Readiness Activities

Readiness activities include folds that contain the preliminary information that students will need while doing the folds in the relevant proof activities. These include basic activities such as creating squares and triangles, measuring angles and lengths, and finding common points; It also consists of applications such as finding the midpoint of a line, discovering the shortest distance from a point to a line, reducing the height of a scalene or obtuse angle triangle, and creating a line parallel to a line. For example, the first activity task involves reaching the

Pythagorean Theorem by calculating the areas of triangles and quadrilaterals. For this purpose, it is necessary to measure the corner angles with the help of patty paper, knowing that the value of the corner angle measurement of patty paper is 90º. This is based on the initial assumptions of patty paper. The second activity requires knowing the shortest distance of a point to a line and folding a line parallel to a line with the help of patty paper to make vertical folds. In the last activity, whether the three line segments can form a right triangle requires the use of the transparency feature of patty papers. The fold marks formed when folding the patty paper represented the line segment and the edges of the shape desired to be created. The combination of folds formed angles, and their intersections were treated as points.

In readiness activities, students worked with a double-sharing group structure as suggested by Serra (1994). The teacher informed the students about this group structure in the preparation activities. Students are grouped in a group of four people, two people in a row, front and back. In each row, this group of four is divided into two pairs. While one student from each pair practiced the activity, the other did the common folding and read the instructions. Afterwards, the couples shared their ideas with the other couple in the group where they had problems; in the next activity, they performed the doublings by changing the order, that is, the roles, between the couples. In this way, they had the opportunity to share their thoughts and reasoning with each other and had the opportunity to turn their reasoning into hypotheses.

As can be seen in Table 1, in the first activity, students were first asked to fold and unfold the patty paper in any way they wanted. They were then asked to fold and unfold a second line. What will happen after repeating this process several times and at how many points two lines can intersect each other have been asked and discussed. In the second activity, students were asked to fold a line on patty paper and mark a point that was not on this line, and to find the shortest distance of the point to the line. They were expected to discover that the shortest distance of the point to the line is the line perpendicular to it. For this purpose, first of all, the students were asked to mark any point on the patty paper and then fold a line outside the point. Afterwards, they were asked to fold lines of different lengths from this point to this line, and it was stated that they could fold by selecting more than one point on the line they first folded and that they could go over the fold marks with a pencil. The students were asked to determine the lengths of the lines they folded in pairs. Students were asked to find the shortest line, and they were expected to justify why it was "the shortest" according to the answers given. Students were expected to discover that the shortest distance is the line that forms a right angle. After the discovery, they were expected to justify the reason for forming a right angle. Students were expected to notice that, due to the transparency of patty paper, as they folded, the shorter lines overlapped with the folds; it created complete overlap when folded on themselves, that is, it turns into a right-angle measurement of 180 degrees/ 2 = 90 degrees since it forms half of the straight angle (Figure 1).

Table 1. Readiness Activities

Activity No/Content	Aim
1/ Folding and discussions on how many points two lines can intersect each other	It is aimed to enable students to have an idea about the intersection points of the lines they fold and their positions relative to each other. For example, it is aimed to provide students with an intuitive understanding of the positions of the lines relative to each other while creating triangular and quadrilateral areas formed by the intersection points of the folded lines or discovering whether the line segments will combine to form a triangle, as in the last activity. The transparency feature of patty paper provided benefits such as making fold marks visible and making it easier to place the lines end to end in Worksheet 3.
2/ Folding and discussions on finding the shortest distance of a point from a line	In Worksheet 2, for right triangle folds, it is necessary to know how to fold the shortest distance of a point to a line and that this distance is also the height. Thanks to the transparency feature of patty paper, the height drawn from the point to the line causes the lines to overlap when folded on top of each other.
3/ Folding a line parallel to another line	In Worksheets 1 and 2, to observe that the opposite sides of the square are parallel to each other; knowing that another line that folds into a line and is perpendicular to a line is parallel to the first line gives students an insight into the edges of the square and the positions of the lines relative to each other.

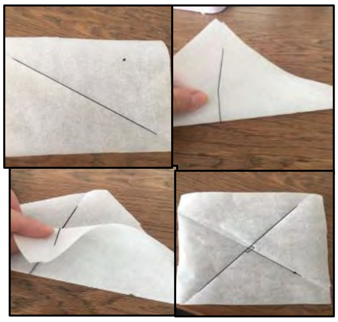


Figure 1. Finding the Shortest Distance of a Point from a Line

It was aimed for the students to realize that the shortest line they created by folding was formed by overlapping the first line they folded and dividing the straight angle in half. In other words, it was aimed for students to say the expression "when a straight angle overlaps, it divides the region into two". The double-sharing group structure helped students when they could not explain it. So much so that the students tried to persuade each other, and when they could not, they argued with the other pair in the other group. If they were still not convinced, an opportunity was created to discuss their ideas with other groups in the class.

After finding the shortest distance of a point to a line, the students were asked to find a line parallel to the line on the same patty paper. Students were expected to discover how to draw parallel lines (a hint that they could use perpendicular if necessary). Students were expected to justify their reasoning and to realize and express that when the height is drawn again on a perpendicular line, parallelism is achieved.

Pilot Study

Worksheets and patty papers were distributed to the students, one for each group. A section was left for them to draw the folds they made with patty paper on the worksheet. The students first made the folds by trial and error, then drew the folded image on paper, and finally checked whether they made the folds correctly in the powerpoint presentation. In the pilot study, it was determined that students' drawing on paper caused a waste of time, and it was observed that they felt the need to use a ruler to draw properly. For this reason, the worksheets used in the pilot application were updated. Students achieved the expected goals through trial and error. However, although they found the shortest distance of a point to a line by folding it and discovered that the resulting line formed a right angle, they could not explain the reason for this.

ACTIVITY IMPLEMENTATION

This activity was held in six lesson hours in the 2022-2023 academic year with 20 ninth grade students studying in a public school in Honaz district of Denizli province, in groups of two. The aim is to eliminate the difficulties experienced by the students who learned the Pythagorean Theorem in the previous year, if any, towards these achievements, to ensure that they develop a positive attitude towards proof by folding patty paper, and to test the effectiveness of this activity in generating the Pythagorean Theorem. Thus, it was aimed to observe whether students' conceptual understanding could be reinforced.

In addition to patty paper, rulers, pencils and scissors were used in the activity. The pencil was used so that the students could draw on patty papers after folding them, and the scissors were used to make the appropriate cuts. The ruler was not used for measuring purposes, but to go over the fold marks properly. This activity consists of two parts throughout the first two activities called “Patty Paper Folding Activity Task” (Table 2).

Table 2. Patty Paper Folding Activity Task

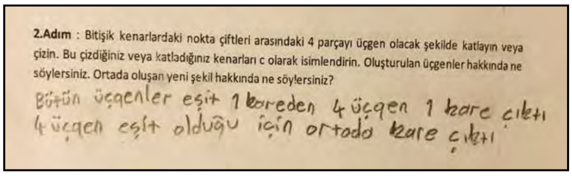
Part One: Patty Paper Folding Activities	Patty Paper Folding Activity Worksheet 1
	Patty Paper Folding Activity Worksheet 2
	Patty Paper Folding Activity Worksheet 1 (Control Activity)
Part Two: Folding Images	Folding Visuals with PowerPoint Presentation

The first and second parts were conducted simultaneously. In the first part, Patty Paper Folding Activity Worksheet 1", and “Patty Paper Folding Activity Worksheet 2", which contained instructions were distributed to the students, respectively. These worksheets consist of two different activities that reach the Pythagorean Theorem in two different ways. After the application, a post-application control activity (“Patty Paper Folding Activity Worksheet 3”) was carried out, which included discoveries about whether the three side lengths that make the Pythagorean Theorem true would form a right triangle. In the first two Patty Paper Folding Activity Task worksheet applications; students followed the worksheet activities (first part), and at the end of the step, they checked whether they had folded correctly by watching the relevant visuals (with a PowerPoint presentation) (second part, Folding Visuals). These two parts were carried out simultaneously (Table 2). In "Patty Paper Folding Activity Worksheet 3", which includes the control activity, students were given three side length measurements of a right triangle that meets the formula $a^2 + b^2 = c^2$ and were asked whether they could form a right triangle using these sides. After the pilot study, the first prepared working papers were revised (Appendix 1).

Patty Paper Folding Activity Worksheet 1

Along with the "Patty Paper Folding Activity Worksheet 1", 15x15 sized patty papers were distributed, one to each group consisting of two people. The folding visuals previously prepared by the teacher were made ready by opening the powerpoint presentation. In "Patty Paper Folding Activity Worksheet 1", students are expected to reach the Pythagorean Theorem by reasoning with the help of area calculation using congruence-similarity in triangles (Appendix 1).

Students started the activity with the instructions specified in the first step. Students made the markings on patty paper using other patty paper of the same size. After naming the edges of the patty paper, the students were asked: "How can we express an edge of your patty paper in terms of a and b?" When asked, the majority of the class, except for a few students, answered "*ab*". After the students were given an example with numbers instead of a and b, the whole class realized that the answer was "*a*+*b*" Since the students could not make the algebraic extension of $(a + b)^2$ the teacher drew a square on the board in order to enable them to reach this identity and gain experience in the steps. This situation was not a part of the patty paper folding activity but was a situation that developed during the activity. The teacher aimed for the students to reach this algebraic extension from the shape instead of giving it directly. All of the students reached the identity $(a + b)^2$, and all of the students completed their folding correctly in the second step of Worksheet 1. The students correctly identified that there were four triangles and a quadrilateral formed as a result of the folds they marked on the patty paper. However, they used the expression "equality" when talking about the "congruence" of triangles. Below is an example of a workgroup. A student's statement is as follows: "All triangles are equal. 4 triangles and 1 square came out of 1 square. Since 4 triangles are equal, a square appeared in the middle" (Figure 2).


2.Adım : Bitişik kenarlardaki nokta çiftleri arasındaki 4 parçayı üçgen olacak şekilde katlayın veya çizin. Bu çizdiğiniz veya katladığınız kenarları c olarak isimlendirin. Oluşturulan üçgenler hakkında ne söylersiniz. Ortada oluşan yeni şekil hakkında ne söylersiniz?

Bütün üçgenler eşit 1 kareden 4 üçgen 1 kare çıktı 4 üçgen eşit olduğu için ortada kare çıktı

Figure 2. Example of a Student Group's Response in Step Two

In the third step, students were asked to name the angles of the triangles formed on the patty paper they folded. In this way, they were expected to realize that the third sides of triangles whose two sides and one angle were equal were also equal. All of the students noticed that the angles in right triangles were supplementary angles and stated this. In the second step, students were expected to express with a mathematical basis why the shape formed in the middle was a square. What is expected from the students was to realize that these triangles were congruent because their two sides were congruent and the angles formed by these two sides were congruent. All groups named the angles in the triangle as shown in Figure 3. While implementing these steps, they also had to fill in the sections on the activity sheet according to their task sharing within the group (Photograph 1).

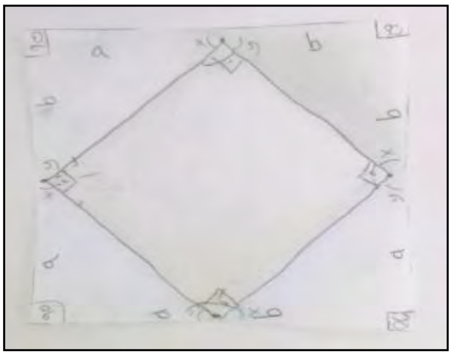
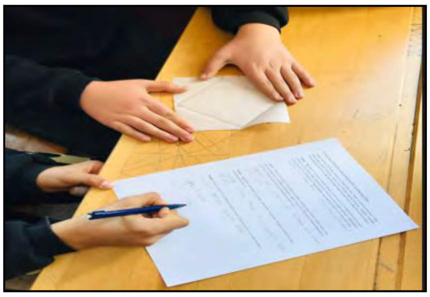


Figure 3. Labeling Example Made in the Third Step



Photograph 1. Example of Students Filling in the Worksheet

Since the students worked with a double-sharing group structure in the activities, they took turns on three worksheets and shared their folding and reasoning with each other. For example, the student who folded in Worksheet 1 filled in the sections in Worksheet 2, and the other student made folds in Worksheet 2. Since the students acted with the same group structure in the preparation activities, they were prepared for group work in advance. Most of the students (except one group) stated that one corner of the patty paper was 90° and found that the sum of the angles they named in the triangle was 90°, by taking advantage of the transparency of patty paper. First of all, they placed another patty paper on the patty paper they were working on, marked (scribbled) the angle a on the patty paper, and placed it on the acute angle of the other right triangle, which they thought was the same, and determined that it was the same size. Thus, for triangles whose one angle measure was equal, the other acute angle measures (y angle) were also equal. Since it is known that the corner angle of a patty paper is 90°; the students placed the acute angle measurements (x and y) side by side on the patty paper and checked them on a corner angle of the other patty paper (Figure 4).

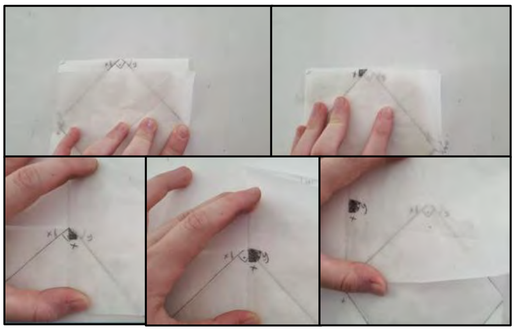


Figure 4. Students Reaching the Right-Angle Measurement by Taking Advantage of the Transparency of Patty Paper

Thus, they said that the sum of the x and y angle measurements was 90^o and one angle of the new quadrilateral formed in the middle was 90^o Students who stated in the previous step that the shape was a square due to its appearance were able to express their thoughts on a mathematical basis. For example, a group of students stated as follows an then expressed this with equality (Figure 5): "I know that one side is 90˚ because the sum of the interior angles of the triangle is 180 degrees."

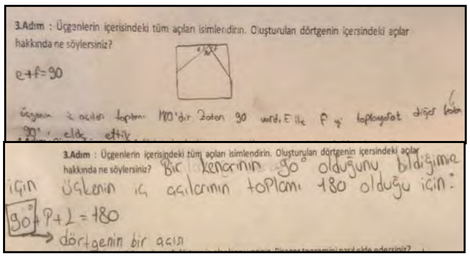


Figure 5. Example of a Group's Response for the Third Step

In the example of another student group, students made correct justifications based on the sum of the interior angles of the triangle. For example, one student expressed it as e+f=90, drew on a square and continued as follows:

> The sum of the interior angles of a triangle is 180. There were already 90. By adding E and F, we got the remaining 90.
> Since we know that one side is 90º, the sum of the measures of the interior angles of the triangle is 180. (Figure 5).

In the fourth step, only four students in two groups realized that they could reach the desired theorem with the help of the area. For this reason, all groups were reminded of their first-step practices. In this way, all groups reached the Pythagorean Theorem with the help of the area. The response of one group is given in Figure 6. (Next to the algebraic calculations, the student expression is "area of the square in the middle". While applying the steps in this activity, which lasted 2 class hours, it was observed that students generally had difficulty performing algebraic operations and had difficulty expressing the concepts they knew mathematically. When students were asked why the shape was a square, they used expressions such as "All sides are equal." and "It has four sides." instead of investigating and justifying that the reason why the side lengths of the squares are equal could be due to identical triangles.

Patty Paper Folding Activity Worksheet 2

"Patty Paper Folding Activity Worksheet 2" was directed to the students in the second week of the study. It is an activity created by dividing wax paper into four triangles and a square, and the Pythagorean Theorem is achieved with the help of algebra. The difference from the first activity is that in this activity, students will be able to reach the square formed in the middle with the help of triangles that can only be formed by folding perpendiculars to the line drawn at the corner point of the square. The activity started by distributing patty papers to the groups (Appendix 1). What is expected from the students is to reach the Pythagorean Theorem by using the congruence-similarity rules and area measurement in triangles.

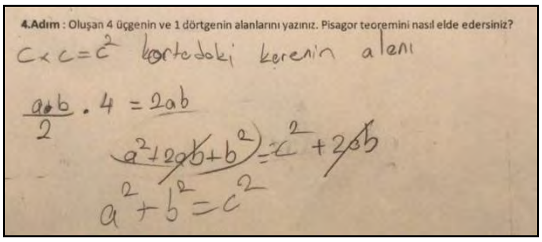


Figure 6. Example of a Student's Work in Step Three

The students easily completed the triangular fold in the first step and then the triangular fold perpendicular to the first fold. In particular, its task in the second step is to fold a perpendicular line (draw a perpendicular) from a point in one corner of the square to the hypotenuse of the triangle folded in the first step. In this step, which was thought to be difficult for the students, the students completed the folding easily. It is thought that the reason for this is that they have previously experienced the task of "finding the shortest distance from a point outside the line to the line" in the readiness activity. In addition, the transparency feature of patty paper shows that the fold coincides with the line, and thus the division of the right angle into two equal parts can be easily observed. In this way, all of the students successfully completed four folds, resulting in four right triangles (Photograph 2).

Photograph 2. A Group Making the Desired Fold

What is expected in the third step is that they realize that the four triangles formed are equal to each other and that they discover through mathematical inference that the quadrilateral formed in the middle is a square. One student used the expression "Because we folded it perpendicularly" as the reason. Other students stated that all the edges of the patty paper were equal and that there was one of these edges in each triangle. Thus, they stated that the triangles they formed were equal. Students in a group explained, "We prove that the angles are equal by naming them." While explaining why the quadrilateral formed in the middle was a square, most of the students wrote an explanation, but they did not use the idea of similarity as a reason. All of the students were able to see that all angle measurements of the shape formed in the middle as a result of vertical folding were 90 degrees, but they did not give a reason why these sides were equal in length. The answer of one group is given in Figure 7: "Since its sides are equal, its corners are also perpendicular. (Because we folded it vertically). It is a square because its corners are equal" (Figure 7).

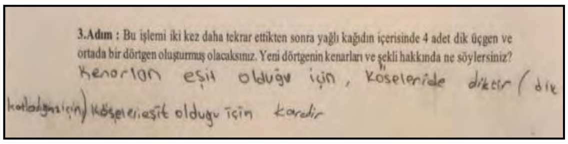


Figure 7. One Group's Answer in Step Three

The students responded by being aware of the vertical folds, but they could not base the idea that the quadrilateral formed in the middle was a square on the idea of similarity of triangles. In particular, the statement "It is a square because its corners are equal" showed that the students made the case that a shape is a square by focusing only on the angle measurements, and that they did not need to explain why all the sides were equal. In the 4th step, students were asked to name the edges and right angles of the shapes they formed on patty paper and to cut out the resulting triangles and squares. The students carried out the instructions requested in the fourth step, as shown in Figure 8.

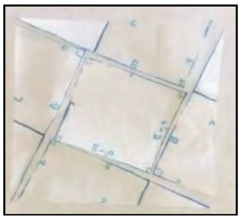


Figure 8. Example of the Cutting Made in the Fourth Step

In the last step, the students were expected to reach the Pythagorean Theorem with the help of the areas of the shapes they cut from patty paper. Since students had previously gained experience with algebraic equations using area calculus in Worksheet 1, they did not have much difficulty in this step. Only some of the students made a notation mistake while writing the expression $(b-a)^2$ to find the area of the newly formed square with side $(b-a)$. For this reason, one of the students in the groups who made calculation error was asked to perform the operations on the blackboard. The procedures were discussed with the whole class and errors were identified. Then, the whole class performed the relevant algebraic operations and arrived at the Pythagorean Theorem. In this activity, which lasted two sessions, students had less difficulty than the first activity. However, since the algebra steps in this activity were more detailed than the first activity, they had more difficulty in algebraic operations in this activity. In these two proof approaches, it is seen that the shapes can be changed without changing the areas, and this will not affect the result. In both activities, at the end of the step, the relevant visual was simultaneously shown to the students via a powerpoint presentation. In this way, students had the chance to check and change the accuracy of their operations and steps and proceed to the next step.

Patty Paper Folding Activity Worksheet 3

The control activity was applied in the third week after the patty paper folding activities, which were carried out one week apart. Before starting the activity, the students were asked:

> Now you know that if the side lengths of a right triangle are a and b and the length of the hypotenuse is c, these three lengths prove the formula $a^2 + b^2 = c^2$. So, is the opposite of the Pythagorean Theorem also true? If you have three segments whose lengths correspond to the Pythagorean formula, should these three lengths form a right triangle?

The answers given by the students were discussed by the class. What is expected from students in this activity is to discover that three-length measures corresponding to the Pythagorean Theorem will form a right triangle (Appendix 1).

In the first and second steps of the activity, students were asked to choose one of the Pythagorean triples in the given worksheet and check whether they could meet the Pythagorean Theorem. Students drew the triples they controlled on patty paper with the help of a ruler, and by placing another sheet of patty paper on top of the drawn patty paper, they drew a triangle with each side being one of these three lengths. Here, the lengths of the sides forming the triangle were given ready-made to the students. What is expected from the students were to know whether the triangle formed was a right triangle or not. The transparency of patty papers made it possible to easily see whether the given line segments formed a triangle or not. Students stated that they knew that the corner of patty paper was 90˚ (the initial assumption of patty paper folding activities). Students made statements as follows: "I realized that the inside of the triangle I drew by drawing a line at a ninety-degree angle on the vertical corner of the paper was 90°"; "I found it using the corner of patty paper" (Figure 9).

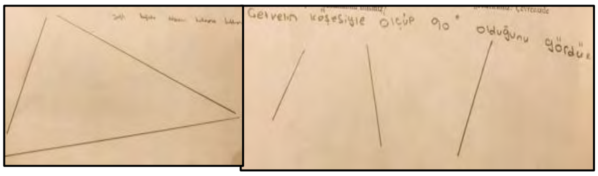


Figure 9. Examples of Forming a Right Triangle

Students were able to form their triangles easily. Some students expressed that they showed with a ruler that one angle of the triangles they formed was 90°. In the last example in Figure 9, it can also be seen in the statement "We measured it with the corner of the ruler and saw that it was 90˚." Students acted with the assumption that the corner of the ruler was a right angle. In general, the entire class participated in the activity willingly.

CONCLUSION and RECCOMENDATIONS

In this study, it was aimed to reach the Pythagorean Theorem by students using the paper folding method, as well as to observe whether the patty paper folding activity is useful or not, and to reveal the students' reactions to the activity. It is thought that the study achieved its purpose. The patty paper folding activity, designed to achieve the Pythagorean Theorem, contains both mathematical and geometric concepts. Students reached the Pythagorean Theorem, which they learned in the eighth grade, by establishing relationships and reasoning between the areas of geometric shapes they formed by folding. Instead of a classroom environment where the Pythagorean theorem is given and then sample questions about it are solved, students were presented with an activity that increases their social interactions in a cooperative learning environment and allows them to reason by considering the information and rules they have already grasped. During the implementation process of the activity, it was observed that the students participated in the activity willingly and with pleasure. It has been observed that the patty paper folding task helps students understand why the theorem is true and that it supports meaningful learning in terms of being a visual proof. It was observed that even students who exhibited negative attitudes and behaviors towards mathematics lessons participated in the activity caressingly and willingly. Thanks to this activity, it was aimed for students to gain an understanding of the role of deductive reasoning and logical inference by using patty paper material and at the same time to encourage a better understanding of mathematical concepts and propositions (Hanna, 2018). It was observed that students exhibited reasoning and logical inference behaviors with this activity and were motivated by these behaviors.

Although the students did not have difficulty in the folding steps, they had difficulty in expressing mathematical concepts. It can be said that the reason for this is due to the generally low academic achievement levels of the students. In addition, students encountered mathematics courses in which they were accustomed to solving test questions within the general academic system. The reason for this may be that they have not been in environments where they can demonstrate both algebraic and cognitive reasoning skills. However, after the implementation of "Patty Paper Folding Activity Worksheet 1", the students had less difficulty in expressing the mathematical concepts in "Patty Paper Folding Activity Worksheet 2". In this case, it can be said that this activity has the potential to facilitate understanding and provide permanent learning. This activity was carried out with high school students. It is thought that the activity can be easily carried out with eight grade school students to teach the Pythagorean Theorem conceptually. This study consists of two proof activities and control activities of the Pythagorean Theorem. There are many proofs of the Pythagorean Theorem and different proof activities are suitable for application by folding patty paper (Serra, 1994).

In mathematics classes, patty paper folding can be used in teaching mathematical concepts, in reaching rules and theorems, discovering patterns and many other areas. In introducing the basic properties and definitions of geometry, many properties such as the situations of intersecting lines and angles, finding the shortest distance of the point to the line, types of angles and folding the properties of parallel lines can be discovered with the help of folding and congruence properties. Many properties such as the properties of angle bisectors and medians, reaching the rules related to them, finding the measurements of the interior angles of the triangle, finding the height and midpoint can be discovered with the help of patty paper geometry. Not only the properties of triangles; many concepts related to the properties of polygons, such as the sum of interior and exterior angle measurements of polygons, and the properties of certain families of quadrilaterals such as parallelograms, trapezoids, deltoids, and even transformation geometry, can be taught by the patty paper folding method (Serra, 1994).

REFERENCES

Argün, Z., & Arıkan, A., Bulut, S., & Halıcıoğlu, S. (2014). *Temel matematik kavramların künyesi [The list of basic mathematical concepts]*. Gazi Kitabevi.

Benjamin, A. (2019). *Matematiğin sihirli dünyası [The magic world of mathematics].* Nika Yayınevi.

Boz Yaman, B. (2020). Kağıt katlama (origami) ve ispat [Paper folding (origami) and proof]. In I. Uğurel (Ed.), *Matematiksel ispat ve öğretimi [Mathematical proof and teaching]* (pp. 333-362). Anı Yayıncılık.

Duatepe-Paksu, A. (2016). Kâğıt katlama yöntemiyle dörtgenlerin incelenmesi [Examining quadrilaterals by paper folding]. *Journal of Inquiry Based Activities*, *6*(2), 80-88.

Hanna, G. (2018). Reflections on proof as explanation. In A. J. Stylianides & G. Harel (Eds.), *Advances in mathematics education research on proof and proving, ICME 13 Monographs* (pp. 3-18). Springer.

Hanna, G., & Jahnke, H. N. (1996). Proof and proving. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education Volume 4* (pp. 877-908). Kluwer Academic Publishers.

Hsieh, F. J., Horng, W. S., & Shy, H. Y. (2021). From exploration to proof production. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education* (pp. 279-326). Springer.

Güler, G. (2020). Akıl yürütme ve ispat ilişkisi [Relationship between reasoning and proof]. I. Uğurel (Ed.), *Matematiksel ispat ve öğretimi [Mathematical proof and teaching]* (pp. 89-108). Anı Yayıncılık.

Robichaux, R. R., & Rodrigue, P. R. (2003). Using origami to promote geometric communication. *Mathematics Teaching in the Middle School*, *9*(4), 222-229.

Serra, M. (1994). *Patty paper geometry.* Key Curriculum Press.

Tall, D. (1989). The nature of mathematical proof. *Mathematics Teaching*, 127, 28-32.

Zeybek Şimşek, Z. (2020). İspatın matematik öğretim programları ve uluslararası standartlardaki yeri ve önemi [The place and importance of proof in mathematics curriculum and international standards]. In I. Uğurel (Ed.), *Matematiksel ispat ve öğretimi [Mathematical proof and teaching]* (pp. 69-84). Anı yayıncılık.

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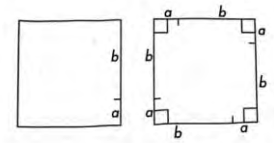
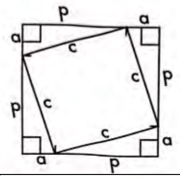
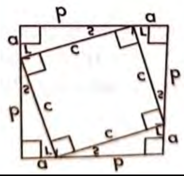
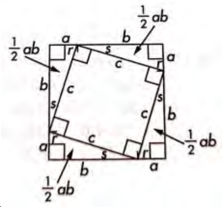
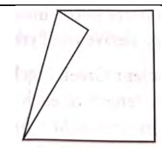
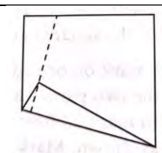
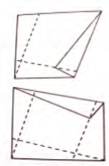
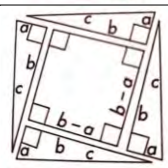
Appendix 1

Patty Paper Folding Task Worksheets

Part One: Patty Paper Folding Activities	**Part Two: Folding Images**
Patty Paper Folding Activity Worksheet 1	
Step 1: Make a mark on one side of the patty paper and divide it into 2 pieces and name these pieces as a and b. Mark the other 3 sides of the patty paper with the same dimensions.	
Step 2: Fold or draw the 4 pieces between pairs of dots on adjacent sides to form a triangle. Name the sides you drew or folded as c. What do you say about the triangles formed? What do you say about the new shape formed in the middle?	
Step 3: Name all of the angles inside the triangles. What would you say about the angles inside the quadrilateral**?**	
Step 4: Write the areas of the 4 triangles and 1 quadrilateral. How do you reach the Pythagorean theorem?	
Patty Paper Folding Activity Worksheet 1	
Step 1: Hold the patty paper by one corner, and fold it into a triangle so that one corner passes through the adjacent corner.	
Step 2: Fold another triangle perpendicular to the first fold and pass through a second corner of the square.	
Step 3: After repeating this process two more times, you will have created 4 right triangles and a rectangle in the middle in the patty paper. What would you say about the sides and shape of the new quadrilateral?	
Step 4: Mark the right angles and name the sides. After determining these triangles by going over 4 right triangles with a pencil, cut out the triangles and the square.	

Step 5: Reach the Pythagorean theorem with the data you have.	$4.\frac{a.b}{2} + (b-a)^2 = c^2$ $2ab + b^2 - 2ab + a^2 = c^2$ $b^2 + a^2 = c^2$ Images: (Serra, 1994, s.221, 222)
Patty Paper Folding Activity Worksheet 3	
Now you know that if a right triangle has side lengths a and b and hypotenuse length c, these three lengths satisfy the formula: $a^2 + b^2 = c^2$ So, is the converse of the Pythagorean theorem also true? If you have three pieces whose lengths satisfy the Pythagorean formula, should these three lengths form a right triangle?	
{3,4,5} {6,8,10} {9,12,15} {7,24,25} {8,15,17} {5,12,13}	
Step 1: Choose any of the Pythagorean triples in the list above.	
Step 2: Check that it satisfies the Pythagorean theorem.	
Step 3: Draw the triples you chose on your patty paper by measuring them with a ruler.	
Step 4: Put another patty paper on top of the patty paper on which you have drawn three pieces, and draw a triangle with each side being one of these three lengths.	
Step 5: How do you show whether one of the angles of your triangle is a right angle? Can you use the right angles you see around you?	