

Developing reflective practice among teachers of mathematics

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ABSTRACT

This qualitative case study of four middle school and secondary school mathematics teachers examines how teachers' reflective practice supported their selection of strategies that align with the needs of current mathematics education according to the 10 dimensions of mathematics education. The findings suggest that teachers' reflection practice, through collaboration with other teachers and creating a positive safe environment within their classroom for students' feedback, impacts their design of classroom practices. They prioritize planning and preparing for the dimensions that are being discussed in the meetings with other teachers. Another major finding is that high level reflective teachers' attitudes toward teaching mathematics is consistent with the needs of current mathematics education. Lastly, meeting individual needs of students is the dimension that all teachers in this study, regardless of their level of reflection, were struggling to reflect on and plan for. Overall, each teacher reported reflective practices that appeared to align with the attitudes and practices for teaching mathematics survey.

Keywords: critical reflection, self-reflection, meeting individual needs, mathematics education, learning environment, collaboration

INTRODUCTION

Mathematics education has always been the center of attention among researchers, educators, as well as the general public. However, teaching mathematics does not simply require the content knowledge of teacher and delivering the subject by lecturing it, because the objective of learning mathematics is far more than knowing the content. Developing critical thinking, connecting mathematics to other disciplines, applying it in real-life situations, and preparing students for their future careers are the goals of teaching and learning mathematics. To reach these objectives and to provide a structured framework for teaching mathematics, the principals to actions (National Council of Teachers of Mathematics, 2014) outline guiding principles, which focus on the need for well-prepared and well-supported teachers and administrators to engage students in meaningful learning and to provide them with a high quality of mathematics. Implementation of principles in the teaching and learning calls for practitioners who reflect on each component and plan accordingly. It has been commonly discussed that teachers who regularly question their own practice are more likely to make use of reflection in ways that benefit both their professional practice and students' learning (Caro-Bruce & Klehr, 2007). They see reflection as a way of knowing themselves in their career.

There is ample evidence for the benefits of reflection as a pedagogical tool, such as advantages in leadership and management, healthcare, and nursing, and other careers. However, reflective thinking or inquiry, as a standard toward which all teachers and students must strive, has been identified in the past decades (Rodgers, 2002). The emphasis on reflectivity in teacher education programs occurred in response to attacks on the adequacy of teacher preparation (Etscheidt et al., 2012), when teacher education was characterized as being technocratic, routine, and recipe-oriented (Price, 1989). Over time, further research papers have been written on the proposed standards and on various approaches and levels for developing reflective practice and its role on teachers' performance. Attending students' voices and the impact of descriptive feedback on learning and teaching (Rodgers, 2006), creating a meaningful learning environment by reflection (White & Guthrie, 2015), and teacher self-assessment as a mechanism for facilitating professional growth (Ross & Bruce, 2007) are some of the research studies that presented the importance of encouraging reflection for teachers and elaborated on the improvement on instructional practices.

In identifying reflection practices, the importance of the higher order of reflection is highlighted (Larrivee, 2008). Reflection at the highest level—critical reflection—is known as the most powerful level of reflection that leads to fundamental changes in education. Because at higher levels of reflection, teachers examine the ethical, social and political consequences of their teaching, they can grapple with the ultimate purposes of schooling (Larrivee, 2008). Self-reflection at this level involves deep examination of values and beliefs embodied in the assumptions teachers make and the expectations they have of students (Larrivee, 2008).

Regardless of the extensive research and debates that have taken place regarding reflective practice, and its levels of practice, one point remains consistent: reflection practices hold a significant place in the practice of teaching and are contributing factors in selecting classroom assignments and tasks.

Since reflection practices hold such a vital role in both teachers' attitudes and practices toward teaching mathematics and students' learning of mathematics, it is important to understand and learn about the various ways and levels of reflective practice and how it contributes to choosing instructional approaches that meet the needs of current mathematics education. This study attempts to answer some of these questions by looking at how teachers make decisions as well as how teachers select and design classroom activities and tasks in mathematics upon reflecting on their teaching practice. The major components and classroom practice that teachers are encouraged to learn to facilitate a students' achieving mathematical success are identified by the 10 dimensions of mathematics education (McDougall, 2004). These dimensions will guide this study to develop reflective practice focusing on the needs of mathematics education.

This paper focuses on the following research questions:

1. How does the reflective level of teachers of mathematics, according to Larrivee's (2008) reflective practice criteria, relate to their selection of classroom practices?
2. How does the teachers' level of reflection, according to Larrivee's (2008) reflection levels, relate to their attitudes and beliefs about mathematics?
3. What are some of the possible reflective strategies that contribute to how teachers set and identify classroom practices?

METHODS

This article explores relation between the selection of teaching approaches and reflection practices of mathematics teachers by talking to the teachers directly and hearing about the activities in their classrooms. A qualitative approach was selected for this study to explore how educators reflect on their practice as well as how they change their classroom activities accordingly. According to Taylor and Wallace (2007), qualitative inquiry provides a vehicle within which researchers may study human experience in a holistic way. It opens a new field of possibilities in science and mathematics education research and new tools with which to recognize the importance of relationships in education. In addition, a collective case study focusing on secondary and middle school teachers was chosen in order to conduct a cross-case analysis to describe the findings. As this study revolves around teachers' reflections and how it affects their selection of pedagogical approaches, a constructivist approach throughout the research is employed. The constructivist paradigm allows one to build up a pattern through the lived experiences of teachers.

To select the participants of the study, convenience sampling method was used. The participants of this study included secondary and middle school teachers who were experienced and novice teachers: Margaret, Panita, Sam, and Sarah. All four participants had been teaching mathematics during the previous months.

In addition to administering the attitudes and practices for teaching mathematics survey (McDougall, 2004) two semi-structured interviews were conducted with each participant. All of the semi-structured interview sessions were recorded and then transcribed. Interviewing teachers allowed the researcher to interpret how teachers select strategies that support students learning and determined how this is connected to their reflection in practice. The goal was to provide a comprehensive description of each case to better understand how a teacher makes decisions in the classroom regarding strategy selection and implementation. The results of surveys along with identifying the reflection level of teachers by analyzing semi-structured interviews supported this study to answer the research question about exploring the impact of the level of reflection on identifying teaching approaches.

The first interview included background questions and visions of success of teachers. It included degree of specialization, the grades and subjects the teachers teach, experience, their goals in education and visions of success, mathematics classroom practices such as the approaches they take to meet the individual needs, communication of mathematical concepts, the students' tasks, classroom environment, technology, and manipulatives that teachers use.

The second interview included questions about school culture and environment, self-reflection, and teachers' professional development. The questions are, as follows:

A. School culture and environment

1. How do you create an environment, which supports success in mathematics?
2. What challenges have you faced in trying to create a culture that supports student achievement in mathematics?
3. How do you work with staff and administration to develop the goals/vision of the school to develop mathematics improvement?

B. Self-reflection

1. How do you think the student's voice is important in teaching practice?
2. What instructional strategies would you use to successfully overcome student's difficulties in a mathematics lesson?
3. What are some factors that you think teachers should look back after teaching a mathematics lesson?
4. Please describe some methods that teachers can reflect on their practices.
5. Have you used any above tools or approaches? Please explain.

6. Do you think the self-reflection of teachers can be considered as an important factor in teaching mathematics? In what ways?
 7. What real impact does reflective teaching have on student learning?
 8. Overall, what are the factors that support success in a mathematics classroom?
 9. What area of teaching mathematics do you like to examine/improve?
 10. How do you know about your student's success or failure in understanding a lesson?
 11. How would you modify a lesson that did not go well?
 12. How do you consider what needs to be changed in a future lesson?
- C. Professional development
1. Does your school encourage teachers in participating in professional development programs by considering compensation or other motivative approaches?
 2. On what type of professional development schemes, you were involved or are being involved (if there is any)?
 3. How do you work with staff and administration to develop the goals/vision of the school to develop mathematics improvement?
 4. Do you have a mathematics department team at your school? If so, what is their role and what do they do?
 5. Do you think the student's interest is more important than a teacher's skills in students' participation and involvement in the class?
 6. How do your approaches in teaching meet an individual's needs in a mathematics class?
 7. Do you have any plans for students who need specific attention academically or mentally?
 8. Which of the 10 dimensions have you selected for your professional development plan? Why did you select those dimensions?

RESULTS

The main purpose of this research study was to investigate and analyze the way in which the reflective practice level affected classroom practices, with respect to particular dimensions of the 10 dimensions of mathematics education (McDougall, 2004), which include meeting individual needs, learning environment, students tasks, manipulative and technology, students' mathematical communication, and teacher's attitude and comfort with mathematics. The best position to take control of decision-making in order to preauthorize efficiency and expediency of standards of performances of students, according to Larrivee (2008), is "to develop the habit of engaging in systematic reflection about their work" (p. 341). Each individual case study was analyzed to examine the relation between the results from the attitudes and practices for teaching mathematics survey and the reflection level interpretations according to conversations with teachers.

Reflection Levels

Margaret is adoptive to students' responses and needs. She builds relationships with her students in order to hear their voice and to consider their needs in teaching plans. Margaret consistently searches for various resources in teaching mathematics to see if there is a lack of teaching materials in her individual work and mathematics team at her school. When school improvement plans were discussed during the interview, Margaret mentioned the unstructured decisions that were made by the school administrative team regarding the lack of numeracy and literacy goals in school plans. However, she and her colleagues addressed the issue by creating small informal group meetings and embedding the numeracy and literacy learning objectives in their lesson plans.

According to Brookfield (1995), engaging in respectful and critically precise conversations with peers who have the same challenges offers an ideal chance for critical reflection on practices. Margaret explained that she uses peer conversations and encourages co-planning sessions. She was involved in a professional development plan that supported her to reflect on teaching strategies and made her think about moving beyond the curriculum expectations. When it comes to the stage of implementing classroom practices, Margaret's focus is on demands of written curriculum.

During the interview, meeting individual needs (dimension 2) and how participants reflect on this dimension were discussed. Margaret talked about assessment as her dominant tool to reflect on how she met her student's needs, however, she described it as a non-reliable tool for a teacher to reflect on student's learning. The reason that she relied on evaluations as an indicator of reflection on teaching was because time was a big concern in her teaching, especially when she taught mathematics classrooms.

Margaret explained that, by looking at products, everyone can realize whether the individual needs are met or not, or at least this can be determined to a high degree. When asked about clarifying the products, she mentioned assessments as part of the product, which she considers as a non-reliable criterion for assessing students, especially if we want to know about individual students' learning. She said:

Unfortunately, assessments are the only way to see if we meet the individual needs, because there is no time for (doing) anything else in order to evaluate learning. There are always kids that surprise me by doing perfect work on a regular basis in the classroom but are not good performers in the assessment setting (Margaret, interview, January 2021).

Margaret explained that, although she is in favour of group work as an activity that fulfills students' individual needs to a high degree, she feels the lack of time and the numerous curriculum expectations makes it difficult to use structured group activities in her classes.

Referring to use of manipulatives (dimension 7), Margaret mentioned the difficulty of using manipulatives in her classroom was due to the challenge of organizing various activities in such limited time. In describing students' tasks (dimension 4), she emphasized embedding open-ended questions in evaluations and some class activities regardless of time constraints. Margaret stated that, based on her experience, students prepare differently when they know there are going to be open-ended questions in assignments and that is, where the quality of thinking arises. According to her statements, Margaret's reflection in practice falls under the surface level, but this does not mean that she has not reached the next levels of reflection; as explained by Larrivee (2008), "teachers may reflect at various levels simultaneously, interweaving various levels" (p. 344). Her attempt at developing the resources and her visions, seeing teaching practices as remaining open for further investigation and development, her commitment to continuous learning and improvement practices, and considering student's perspectives in classroom practices placed her reflection level in the pedagogical level. Overall, Margaret showed that she does some surface reflecting, some pedagogical reflecting, and rarely at critical reflecting.

Panita's belief about the significant role of students' individual characteristics, emotional differences, cultural backgrounds on their learning process and pace, encourages her to dedicate a time for communication and group work in almost all of her class activities. She views teaching practice within the broader sociological and cultural contexts. In describing student's tasks (dimension 4), Panita explained that conversations that arise during group work, either in collaboration with group members or her speaking directly to students, is the best way that she can find out about her students' different ethnicities and backgrounds. She added that this practice also supports her in identifying the level of expectations for her students.

Although Panita acknowledges the difficulties of promoting the student's mathematical communications (dimension 8) in her classes and finds it a big challenge because of student's reluctance to explain mathematical problems orally or in written format, she manages to highlight the value and benefits of mathematics talks and communications in group activities. This way, she challenges assumptions about her students and sets elevated expectations in practice. According to practice indicators of Larrivee (2008), "critical reflective teachers are aware of incongruence between beliefs and actions and takes action to rectify" (p. 352).

Panita uses technology as a means of integration of subjects and connecting mathematics to scientific real-life examples. Her visions of success and beliefs about education align with the strategies that she employs in her teaching. This practice is consistent with the pedagogical level of reflection. Panita is certain that she has achieved the class objectives when the students can incorporate their learning in real-life situations, and they can think critically about the situations apart from the content learning. Panita talked about many similar activities that involved real-life examples of mathematics. She described an activity that she called graph of the week in which she integrated social topics such as smoking and health, teen pregnancy and sport and nutrition in mathematical graphs lesson. Panita talked about students' reluctance at the beginning weeks of doing this activity and she passionately reported how inserting social themes changed the position of students:

When I was teaching graphs, students liked it only as a theoretical lesson, but then when I taught it incorporating graphs using real-life examples the story became completely different. I could clearly see that more discussions were happening. In addition, better teamwork skills could be observed. Overall, the quality of our task significantly improved when I started using these types of examples with my students, and all of a sudden, their interest was heightened (Panita, teacher interview, July 2020).

Panita continuously collaborates with her colleagues and searches for ways to develop practical ways of teaching mathematics for her students and applies the explored and designed activities in her classrooms. On average, according to reflection indicators, Panita's reflection level is at pedagogical and critical reflection, but most of her reflecting tends to be at the critical level. Similar to Panita, Sam's belief about teaching mathematics is in accordance with the pedagogical reflection level. In contrast, the reflection indicators show that his reflection in practice is not in the category of pedagogical level. This was evident in the inconsistency of his teaching approaches and visions and goals for designing activities. Although he believes in student-centered teaching and inserts various thoughtfully designed group activities into his teaching assignments and exercises, his goal is to engage students and fulfil the school and parents' expectations. In a typical classroom setting, Sam applies teaching strategies by knowing what works best for his students and the expectations of parents.

Sam acknowledges the significant role of student's mathematical communication (dimension 8) and use of manipulatives and technology (dimension 7) in learning mathematics. Regardless of his student's hesitancy to use communication, Sam strives to employ various methods to foster this dimension in his classrooms. Sam talked about the four wall board activity in which students can use the walls that are covered in paper as a board to write the steps for solving a problem. The steps can be shown in many ways, such as bullet points, a mind map, or their own creative methods. In addition, Sam talked about another activity, which he named staring confer. In this activity, students are given a very difficult set of problems, which are often wordy and based on a real-life question. He explained some parts of the activity:

I give the instructions and then I get myself out of the situation and I tell them that I am not the resource here, the dictionary is there, computers are on your desk. You can ask questions if you want me to look at your computer and confirm something. This way, I see that communication starts in groups quickly. They start talking about if they all understood the question; they try to convince each other and to listen carefully, and sometimes they search for symbols and signs that are new to them. I think the more teachers spend time planning such activities before class, the more communication skills are built up among kids (Sam, teacher interview, July 2020).

Sam normally adjusts his practice to the situation and his beliefs are shaped and supported based on his experience and evidence from past classrooms. In sum, the reflection level of Sam in practice can be considered at surface reflection level and the pedagogical reflection level. Sarah was constantly concerned about time when she taught mathematics. She believes that the high volume of curriculum expectations and the teaching material was a barrier for her to employ teaching practices that allow her students to explore and connect mathematical concepts and to use real-life situation problems. However, considering herself new to her teaching career, Sarah anticipates that, as she gains experience, she will be able to manage her time for implying effective teaching approaches such as posing more open-ended problems in teaching, devoting more time to mathematical communications, and using more of technology and manipulatives in her classes. Sarah is confident in use of technology in her classes, and she analyzed the relationship between this practice and her students' learning in dimension 7 (manipulatives and technology). There was also an unpredicted outcome in analyzing interviews: teacher's content knowledge in the subject of teaching supports selection of teaching practices and increases the competency of teachers in practice. This was evident in Sarah's case study that her major in computer science guided and supported her in the usage of technology.

Sarah believes that experienced teachers can manage and master all classroom practices at a time, and she hopes that she will also be more confident in using alternate practices for teaching over time. She connects her abilities and skills in practice to the lack of experience. Sarah emphasizes the connection between mathematics and real-life situations throughout both interviews and her attempt to embed real-life problems in her lesson plans. In describing the meeting individual needs (dimension 2), she discussed her accommodation for English as second language students. She provides some differentiated instruction to address student's needs. She provided more explanation of her practice for grade 10 mathematics course when she spoke about this dimension:

Because there is a heavy emphasis on quadratics on this course, we do a lot of work such as transformations, characteristics of quadratics and etc. What I have done to make it more tangible is that at the beginning of the school year, I gathered information on students' personal hobbies and activities. Then I projected those activities in graphs. More specific examples are when I showed them emotions in throwing a ball and reaching to the highest point and hitting the ground in different shapes of quadratics (Sarah, teacher interview, July 2020).

Sarah explained that success for her students is mainly determined by their future goals, such as moving from one grade to another, but in lower grades, the focus should be on teaching them to connect mathematics to the world around them. Sarah's overall level of reflection meets the surface level criteria. Although her vision of success and belief systems is aligned with the pedagogical reflection, her level of reflection in practice is in the surface reflection category.

Overall, Panita was the only teacher that met most of the criteria for being at the pedagogical and critical reflection level. Margaret and Sam were at the intersection of the surface level and pedagogical level of reflection. Sarah's reflection level about classroom practices remains at the surface level of reflection.

Reflection level & selection of classroom practices

According to the findings, there is a pattern between the level of reflection of teachers and selection of their strategies and classroom practices including the students' tasks (dimension 4), use of manipulatives and technology (dimension 7), and focus on mathematical communication (dimension 8). High level reflective teachers focused on improving and incorporating the classroom practices that support student learning. These teachers' assumptions about their lack of adequate knowledge and skills on these dimensions, as well as about their students' absence of background knowledge, were not a barrier to implementing effective instructional approaches.

Panita was at the highest levels of reflection: pedagogical and critical. Although her students were resistant to using manipulatives and technology in mathematics classrooms, she incorporated technology in her lessons because she was aware of the significant role of technology in learning mathematics. Panita designed integrated mathematics and science activities through collaboration with the mathematics department. She explained that she was successful in capturing the student's attention and interest over time. Although her score was five out of six in this dimension, Panita stated that she considers the technology and manipulatives among the dimensions that she wants to improve on because she believes that technology is growing faster than her learning process in this regard.

This practice was not observed in Sam's and Margaret's practice, two instructors who were at the intersection of the surface and pedagogical levels of reflection. Although Margaret believes that deeper understanding of mathematics can be nurtured though using technology and manipulative, she does not often employ technology or use manipulative-based activities. This is because she thinks the design of manipulatives that exist at their school is mostly for lower grades, and also because she is not very organized in using them. Like Margaret, Sam did not generally use technology because his students do not see it as being interesting and they think it is a childish tool.

Sarah's comfort with the use of technology cannot be completely linked to her level of reflection. Sarah's second teaching subject was computer science, and this can be considered as an influential factor that served her in the use technology in her classes very often. During the interviews, Sarah generally talked about her difficulty with managing time for various tasks with the exception of adopting technology-based activities in her classroom practices. She explained that she is very comfortable using technology in her mathematics classes. Sarah stated that, unlike other strategies that she normally attempts to use in a mathematics class, she was not concerned about time for fulfilling the curriculum expectations. She explained that technology encourages student participation and brings joy to her classes. But she added that it becomes difficult to manage classroom when she integrates technology-based activities in her classrooms. Sarah did not use manipulatives in her practice very often, because she feels that she lacks enough skills to use them, and her students do not understand lesson well when manipulatives were used.

Students' tasks and selection of practices that promote mathematical communication were the other practices that were aligned with the reflection level of participants of this study. Panita discussed the challenge of encouraging her students to communicate their thoughts and presented a complete description of solutions in written and oral form. Nevertheless, she provided a variety of choices for her students within group work such as having them in peer observation, reflecting on peers' answers, and sharing their solutions in groups to provoke communicating mathematics.

Sam, being at one level lower than Panita at the surface level, acknowledges the importance of mathematical communication in mathematics, but he rarely challenges those students who are not willing to communicate mathematical thinking. Sam accepts the approach of his students who are at a high level of mathematics and can reach the final answer without being able to communicate it, but he occasionally considers activities that promote communication. Margaret, at the same level of reflection as Sam, sees mathematical communication as the key part of learning. She uses limited strategies to foster communication such as focusing on discussing provincial large-scale assessment test samples and posing open-ended mathematics problems to discuss different solutions. Margaret explained that the student task dimension is the dimension that she feels a deep discomfort. Similar to Margaret, Sarah believes that there should be a greater emphasis on mathematical communications and students' tasks, but she also had limited numbers of approaches in practice, due to her concern about time. Sarah develops students' mathematical communication by occasionally embedding problems in group activities in which students do not necessarily solve a problem, but they need to talk about it, analyze and discuss it in their groups.

Teachers' beliefs about effectiveness of teaching strategies aligned with the in-practice approaches if they were at a higher level of reflection. All of the teachers recognized the importance of technology and manipulatives in mathematics learning as well as the value of designing tasks that encourage mathematical communication. However, the implementation of tasks was highlighted and focused on the practice of high-level reflective teachers.

Attitudes & Beliefs about Mathematics in Relation to Reflection

The findings revealed a meaningful link between the reflection level and the attitudes and beliefs of each teacher on the 10 dimensions of mathematics education. Three patterns were observed during the study. The first observation was that teachers who had a higher overall score in the attitudes survey were situated at the highest level of reflection. Panita was the only teacher whose reflection level was in the category of pedagogical and critical reflection, and her overall score was highest among all other teachers at 5.03. Next, Margaret and Sam, with an overall score of 5.02 and 4.83, respectively, fell in between surface and pedagogical reflection. Lastly, Sarah, being at between pre-reflection and surface levels, scored an average of 4.35. These findings encourage teachers of mathematics to be critical reflective teachers in order to better understand the needs of mathematics education and identify goals for their improvement. Being reflective at lower levels, limits a teacher's perceptions and thoughts for planning tasks that align with mathematics classroom demands.

The second observation is that almost all the teachers were aware of the gaps and strengths in implementing classroom practices. This represented the self-awareness of teachers and their beliefs about their abilities in teaching. Meeting individual needs of students was the dimension that Panita, Margaret, and Sarah stated as being a goal that they wanted to reflect on and to improve. With the exception of Sam, other teachers scored lowest on this dimension in the attitudes and beliefs survey. Sam scored 4.6 on meeting individual needs, which was his second lowest score. Unlike the other cases, Sam did not mention meeting individual needs as a dimension that he wants to improve on. However, his approach for improving on the dimension he chose to advance, students tasks, was to assign tasks in a way that he can meet his students' individual needs. Sam stated that he struggles in the students' tasks (dimension 4) and his aim is to find ways to grow on it. The consistency between Sam's goal to progress and his lowest scored dimension, could be seen in his score in the students' tasks, 3.5 out of six.

Lastly, the interpretations of interviews and the surveys showed that all the teachers scored highest in the teacher's attitude and comfort with mathematics (dimension 10). Panita and Margaret described multiple factors that enhanced their attitudes toward teaching mathematics and being comfortable with teaching subject. Panita mentioned her studies at universities—majoring in mathematics, physics, and education—supported her comfort with the subject and her passion for teaching is reflected in the care and time she devoted to teaching to deliver the lesson as best as she could. Margaret highlighted her ongoing passion for seeking resources available for exploring the most effective educational approaches. She stated that the passion and comfort for teaching is the dimension that she is very thoughtful about. Both Panita and Margaret scored highest on this dimension, 5.4 out of six.

Sam and Sarah did not have a consistency between the most frequent reflected dimension and the score on the dimension, as they presented other dimensions as their focus of attention, learning environment and mathematics communication, respectively. However, both teachers' second highest score was on the teacher's attitude and comfort with mathematics (dimension 10). Sam considered his complex journey in studying different subjects in his university programs made him to be open to the design of class activities and to be critical of his approaches in assigning open-ended tasks. Sarah reflected on being careful about the words she and her students use of positive words, allowing her to show her enthusiasm and positive attitude about mathematics to her students. Overall, the results represent the consistency of the teacher's self-reflection and their attitudes and beliefs about teaching mathematics.

Reflection Strategies for Teaching Mathematics

Teachers in this study reflect on their instructional approaches depending on their school environment, desire to improve, and their attitudes toward teaching mathematics. The major reflection methods that teachers used in this study include collaboration and creating a safe environment for reflection in classroom.

Collaboration

Teachers' communication and collaboration with other teachers is considered as one of the most effective strategies for developing reflective practice. According to Pultorak (2010), "defining reflection as only activity to be pursued by an individual greatly limits its potential for personal growth" (p. 141). Reflection through collaboration, which is considered a strategy that promotes critical reflection, can be seen in the practice of those teachers who were at higher levels of reflection in this study. In addition, the results concluded that teachers focused on reflecting on the dimensions, which were outlined in schools' co-planning meetings and in the teacher's journal writings.

As a leader of a professional development program, Margaret described how co-planning sessions brought great opportunities for teachers to reflect on their previous experiences and to deliver the lessons in a new way. They used observations and note-taking in their co-planning sessions to plan for certain classes before they went into their own classrooms again and started teaching. In particular, she mentioned the important objective of discussions in these sessions, which revolved more around addressing students' needs than rather than considering curriculum objectives. Margaret called this experience the most valuable practice during her teaching time because she benefited from the observations by implementing the results in planning for her students' tasks.

Panita and Sam both admired the supportive collaborative team that they had at their schools. In Panita's school, teachers share their successful and non-successful teaching experiences in mathematics with the group. They reflect on the deficits and the patterns that exist among different classes and analyze the experiences to design and construct new ways of teaching mathematics. The administrative team addresses the needs of the teachers and students with regards to preparing the material and resources. The assistance of principal and administrative team encourages teachers to create new activities and design collaborative projects according to the reflection results. Panita mentioned that, because students, teachers, and administration participated in planning, this was a great way to move on from description to knowledge and construction. This was shown in Manouchehri's (2002) study that collaboration and discussions facilitated a shift in the level of reflection, moving from describing explaining, to a theorizing and restricting phase.

In agreement with Panita, Sam appreciates the professional team that he was collaborating with in his school. In Sam's school, they have two collaborative meetings, one type involves individual meetings with their small subject department, where they independently write their interpretations and understandings of implemented activities, while sharing ideas in group sessions with other departments. He explained that their regular meetings with the mathematics department, which involved reflecting on past classes, creating new lesson plans, unit plans, and even in planning small projects, guided him in his teaching practice. Sam pointed out that the value and effort that his team as well as the school board facilitated an inclusive environment and learning atmosphere. Therefore, it became a habit for him to be attentive to this dimension in teaching. In the interview, he expressed that creating a safe learning environment is the dimension that he reflects on more often than other dimensions.

Because Sarah was in different schools in her five years of teaching experience, she did not have an opportunity to benefit from regular meetings at a certain school for a lengthy period. However, she explained that, in the professional development sessions at her last school, they discussed and interpreted the Education Quality and Accountability Office evaluation results and after writing their thoughts, they shared their individual ideas about different types of question with the group. This resulted in realizing the gaps in students learning, in particular, the topic of mathematics that was the connection between real-life situation and mathematics.

She explained that the results of discussions encouraged the mathematics department to collaborate with the science department. They collaboratively created activities to promote students' problem-solving skills in real-life situations. Sarah mentioned that she realized that the implementation of group activities that they created in collaboration with other teachers impacted her student's learning, as she could see more engagement of her students in class. She reflected on the tasks she implemented. She stated that she should have devoted more time to class activities and discussions, because she realized that she had been continuously worrying about the time needed for implementing activities.

Creating a safe environment for reflection in action

Teachers in this study identified that, by observing their own students as a part of their teaching performance and dialogue with individuals, they were able to reflect on their teaching practice. According to the literature, reflective dialogue can take place individually or in small groups (Tsang, 2007). All four participants of this study employed the notion of creating a safe environment, with varied emphasis, as a means of preparing the site to provoke conversation and allowing for teacher reflection in action.

Margaret focused on developing a growth mindset as her major approach to make the environment inclusive and safe in order to promote dialogues. She explained the two main approaches that enhanced conversation and communication in her classes: First, her welcoming manner and appreciation of students' comments, regardless of being wrong or right in class activities. Secondly, journaling, which included having students record their thoughts, descriptions of mathematical concepts, and exploring and note-taking of what they learned, was also helpful. She mentioned that the creation of a safe environment, one that students can trust and become engaged in, can be developed over time and it is not something that can be created in one session.

Margaret sometimes uses students' friendly conversations in a group to reflect on certain topics taught in mathematics. Similar to Margaret, Sam expressed their school emphasized the creation of a safe and inclusive environment. Thus, he designed various activities for his students to become more interactive and more engaged with one another. He stated that, when he observed joy, involvement, and happiness among students, he was sure that learning was in progress.

Panita, as a teacher and a mother of a son in grade 12, sees the importance of building relationships with students and how it can make the environment safe for learning. She described the fluctuation of her son's attitude toward mathematics with having

good or non-pleasant relationships with teachers and how it impacted his learning. Panita referred to this experience as a tangible example of the role of the relationship of a student with a teacher within a safe environment for learning. She explained that, when students make a good relationship with her, they are willing to disclose their lack of understanding than hiding it.

The correlation between the interpersonal relationships with students and reflective teachers was presented in the results of Korthagen and Wubbels' (1995) study. They concluded that "reflective teachers are capable of not only adequately analyzing their interpersonal relationships with students, but also succeed in making those relationships better" (p. 64). Panita expressed that, by assigning small groups of students, she allows students to feel safe to ask their questions, and with a good relationship with students, those who have difficulty use the opportunity of lunch time and talk to her about their misunderstandings in friendly conversations.

When reflection on individual needs was discussed, Margaret mentioned that assessment was the only tool they could use to evaluate students' learning, but she was not in favour of the written assessment as a reliable tool. Her approach followed Rodger's (2006) two ways of knowing about students learning: "to observe their learning as it is happening, including (but not limited to) the products (projects, papers, self-assessments, even some tests) that result from learning" (p. 231), secondly, to ask students about their learning. Margaret reflects on her students' learning using the results of evaluations of group projects and tests. She explained that, because tests are not always a perfect indicator of students' learning, she creates small groups and encourages students to communicate their understandings of the lesson. This method provides her with a good opportunity to reflect on how the lesson is being delivered. Margaret described the observation as her most effective way of reflecting on teaching and learning.

Sarah was the teacher with the least experience, and she discussed her difficulty in managing time for group activities. Similar to the three other teachers, Sarah also highlighted the importance of relationships with students and spoke about the small groups that she assigns for her classes. She sets certain norms at the beginning of each school year and speaks about having an inclusive, respectful, and valuable learning atmosphere. Sarah, a teacher who was shy as a child, expressed her background experiences of not being able to present herself and communicate in mathematics classrooms.

Unlike other participants, Sarah focused on dimension four, student's tasks, to reflect on her students learning. She assigns tasks that connect students' previous knowledge to the current lesson and incorporates open-ended problems to promote conversation. She believes that, through student's discussions in groups, sometimes she can better understand students' learning than giving them giant evaluations. Although Sara's focus for reflecting on student's learning was different from others, she used observation during the task implementation in order to understand about their learning. She focused on making a friendly and safe environment through these assignments to promote communication and to hear students' voices about their learning.

Three of the participants of this study create a safe environment as a means of preparing the class for reflection. Margaret considers the number of questions that her students ask in group works and forms the conclusion as a reflection piece of the learning process. Sam and Panita look at the joy and involvement of students to reflect on their practice. All of the teachers in this study emphasized the importance of relationships with students to promote discussion about the concepts of each lesson.

CONCLUSIONS

This research study examined the reflection practices of four middle and secondary school mathematics teachers and how their reflection level served them in selecting their teaching approaches. The study showed what strategies support the higher levels of reflection and how reflective practice is linked to dominance on the implementing the 10 dimensions of mathematics education (McDougall, 2004). The major findings of multiple case studies are summarized, as follows:

1. The teachers reflect on their practice of teaching through collaboration with colleagues, creating a safe environment for students' feedback, and enhancing communication through assigning various students' tasks.
2. Establishing collaborative sessions with peer teachers contributes to higher order of reflection and influences teachers' attitude and practices toward teaching mathematics.
3. For self-reflective teaches, lack of adequate skills and knowledge were not barriers for implementing the tasks that align with the needs of mathematics education. They employed various ways to improve instructional skills for effective teaching.
4. Being at higher level of reflection can have an influence on one's attitude towards mathematics, as seen with all the teacher participants from this study who are more consistent with current mathematics education reform ideas.
5. Meeting individual needs is the dimension that all teachers in this study want to improve in their practice.
6. Building relationship with student for promoting a safe environment and communication was important to all the teachers in this study. However, self-reflective teachers succeeded in making interpersonal relationships.
7. Due to the lack of adequate time to cover a high level of mathematics content, teachers limit the number of activities that contribute to creating effective learning environments. This was highlighted in the practices of the less experienced teachers.

Results of this study also revealed that lack of adequate skills and knowledge were not barriers for self-reflective teachers. They could implement the tasks that align with the needs of mathematics education. Self-reflective teachers employed various ways to improve instructional skills for effective teaching.

According to this research, being at a higher level of reflection can have an influence on a teacher's attitude towards mathematics, as seen with all the teacher participants from this study who are more consistent with current mathematics education reform ideas.

Teachers in this study attempted to meet their students' individual needs with similar methods. They used mathematical communication within group work as a major tool to create a safe environment and to provide space for reflection. This approach is suggested by McDougall (2004) in describing the integration of the 10 dimensions. McDougall (2004) explained that it is important for teachers to acknowledge that the dimensions of mathematics education are not discrete: "for example, an appropriate learning environment can help with meeting individual needs, and rich student tasks help students constructing knowledge" (p. 16).

The teachers in this study expressed that they experience challenges in meeting individual needs and in finding the best approaches to reflect and to improve their skills on this dimension. A longitudinal research study is needed to investigate the methods that teachers can apply in order to reflect on their practice for meeting individual needs, and to identify effective teaching strategies that support teachers fulfilling needs of students individually.

This study has provided a wealth of information about how the higher levels of reflective practice can support the selection of strategies that contribute to the needs of current mathematics education. Further research in this area will shed more light on how to best identify reflective practices for meeting individual needs, emphasizing the use of different presentations and improving learning outcomes. Through conducting broader research, the approaches and considerations that teachers should examine to promote the link between reflection and action in the practice of teaching can be further investigated.

The findings of this study indicate that reflective practice among teachers of mathematics is facilitated through collaboration with colleagues, creating a safe environment for student feedback, and enhancing communication through diverse student tasks. Collaborative sessions with peer teachers contribute to higher levels of reflection and influence teachers' attitudes and practices towards teaching mathematics. The research also reveals that a lack of skills and knowledge does not hinder self-reflective teachers from implementing tasks aligned with the needs of mathematics education. These teachers employ various strategies to enhance their instructional skills, demonstrating their commitment to effective teaching. Moreover, the study suggests that a higher level of reflection influences teachers' attitudes towards mathematics, as evidenced by the alignment of teacher participants with current mathematics education reform ideas.

However, time constraints often limit the number of activities teachers can incorporate to create effective learning environments, particularly among less experienced teachers. Meeting individual needs emerged as a dimension that all teachers aimed to improve. Building relationships with students, promoting a safe environment, and fostering effective communication were identified as crucial elements of mathematics instruction. Self-reflective teachers excelled in establishing interpersonal relationships with their students. They employed mathematical communication within group work as a major tool to create a safe environment and foster reflection. It is suggested that further research is needed to explore effective methods for self-reflection and to develop strategies that address individual needs, considering the integration of the 10 dimensions proposed by McDougall (2004). Additionally, investigating how administrators and principals can facilitate a collaborative school environment that encourages teacher reflection, and the sharing of experiences would be beneficial.

This study provides valuable insights into the importance of reflective practice in mathematics education and its impact on teachers' attitudes and instructional practices. It highlights the challenges faced by teachers in meeting individual needs and the potential for further research to identify effective reflective practices and promote a stronger link between reflection and action in teaching.

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Ethical statement: The authors stated that the University of Toronto Ethics Office approved this study on May 28, 2020. Prior to the start of the study, the ethical review process was completed by filling out the Human Participant Ethics Protocol form and submitting the required documents to the University of Toronto's Research Ethical Approval Department. This study is considered low risk, as it involves teacher participants who are adults and willingly participate in interviews. The authors further stated that an email was sent to middle school and secondary school teachers of Mathematics who were teaching at school or had taught mathematics within a few months of that time. Those who agreed to be part of this study signed a formal consent letter to confirm their participation. In addition, a description of the proposed research and the nature of participation was via email sent to the prospective participants. In the consent letter, participants were informed about the level of their involvement in more detail. Participants' names or anything else was not used so that might identify them in the written words, oral presentations, or publications. The authors ensured that the information remained confidential during the study and will remain confidential after the completion of the research study. Participants were free to change their minds at any time and to withdraw even after they had consented to participate. They had the opportunity to decline to answer any specific questions. The authors stated that they destroyed the tape recording after the research has been presented and/or published which may take up to five years after the data has been collected. There are no known risks or benefits to participants in assisting in the project, and the authors shared with them a copy of the notes to ensure accuracy. Additionally, the authors stated that, to ensure confidentiality, participants were reminded that pseudonyms for teachers, as well as schools, were used in this study. Teacher participants were given the opportunity to review the data derived from their participation in the interviews and surveys in order to verify the accuracy of data and interpretations of the interviews. The participants were completely aware of their rights regarding the possibility of withdrawing from the study at any time. It was mentioned to participants that, at all times, they could review collected data and findings to ensure accuracy and reliability.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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