Students' Mathematics Achievement Based on Performance Assessment through Problem Solving-Posing and Metacognition Level

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Abstract: Performance assessment through problem-solving or problem-posing provides benefits in learning mathematics. This study aims to obtain empirical evidence about the effects of performance assessment and metacognition on senior high school students' mathematics achievement. A quasi-experimental method was employed to engage 163 students in four classes selected through cluster random sampling. In addition, data were collected via an achievement test and a metacognition scale and analyzed using a two-way analysis of variance. The study results indicated a statistically significant discrepancy in the mathematics scores of students who were given performance assessments utilizing problem-solving and problem posing. Depending on the students' levels of metacognition, performance assessment had varying effects on the students' mathematical achievement. Students with a high level of metacognition and performance assessments through problem-solving had more effective mathematics achievement than those with performance assessments through problem-posing. In contrast, in students with medium and low metacognition, the performance assessment through problem-solving and problem-posing did not differ significantly and were classified as having low scores. This study suggested that using performance assessment and considering the level of metacognition support further efforts to enhance students' mathematics achievement.

Keywords: mathematics achievement, metacognition, performance assessment, problem-solving, problem-posing

INTRODUCTION

Performance assessment has a prominent role in assessing the students' progress in learning mathematics. Meanwhile, problem-solving and posing tasks are integral parts of learning and serve as the core of the performance assessment. The tasks help students develop mathematical thinking skills, such as modeling, pattern recognition, building logical arguments, studying, and developing creative thinking. One of the most important aspects of applying successful problem-solving and posing tasks is developing students' metacognition of the tasks.

The term "metacognition" has been defined in numerous ways, but its primary components are knowledge and the control of cognition (Boekaerts, 1997; Fernandez-Duque et al., 2000; Flavell, 1979; Sperling et al., 2004). In addition, metacognition refers to the degree to which students are self-aware in terms of their own memory, cognitive monitoring, and the learning processes themselves. The term "regulation of cognition" is used to describe to what degree students have command over their own mental processes during the learning process. For instance, matching to goal setting, executing strategies, and being aware of the problem they face. Metacognitive activities include planning how to learn something, checking to see if you understand it, and judging your progress toward finishing a task.

Several studies reported that there was a significant correlation between academic achievement and learning-related metacognition. Learners with high metacognitive skills perform much better than those with low metacognitive skills in mathematics classes (Boekaerts, 1997; Jaafar & Ayub, 2010). Also Özsoy (2011) shows that there is a significant and positive relationship ($r = .648$, $p < .01$) between metacognition and mathematics achievement. Furthermore, research results showed that 42% of total variance of mathematics achievement could be explained with metacognitive knowledge and skills.

Mathematics achievement is still relatively low and has shown a decreasing trend in the last ten years. The Program for International Student Assessment (PISA) in 2018 showed that students' mathematics abilities in Indonesia rank 72 out of 78 countries (OECD, 2019). The PISA test divided students' mathematical abilities into 6 levels, where level 1 was the lowest and level 6 was the highest in the higher-order thinking ability. The ability to answer the level 5 and 6 tests of Indonesian students is still low. In addition, the low achievement levels suggest fundamental problems in the process of mathematical learning at school.

An alternative to enhance the students' mathematics achievement is to use performance assessment for teaching, an approach based on problem-solving and posing tasks. Performance in mathematics can be evaluated based on various stages and the quality of students' problem-posing ability towards a problem. According to Nitko and Brookhart (2014), a performance assessment is an authentic procedure in which students are tasked to obtain information on how well they have learned. The rubric comprises two distinct components, namely the assignment and the criteria for assessment of the students' performance. In addition, a performance task is given that aims to show the learning target. The rubric for scoring is a set of guidelines that are used to assess the quality of student performance.

This study proposes that the current low achievement levels are related to the lack of opportunities to explore problem-solving and posing in learning. Therefore, learning mathematics should encourage students to apply solving problems. It is believed that students should develop the capability to attain novel mathematical understanding through the process of problem-solving. This requires them to effectively employ, modify, and adapt a variety of strategies, as well as continuously evaluate and reflect upon their problem-solving process in mathematics.

A multitude of scholars in the field of mathematics education have examined the teaching and learning of mathematics utilizing a problem-solving approach. Their research endeavors have primarily focused on the elaboration of novel instructional techniques and the design of innovative assessment instruments. These studies have made substantial contributions to the advancement of our knowledge on the most effective methods for supporting students' mathematical learning through problem-solving (Charles et al., 1987; Fuchs et al., 1999; Malloy & Jones, 1998; Schoenfeld, 1992). In addition, some concepts were applied to frameworks for evaluating a student's comprehension of problem-posing tasks (Chen et al., 2011; Leung & Silver, 1997; Yuan & Sriraman, 2011).

Brown and Walter (2005) confirmed that problem-posing involves two cognitive aspects such as accepting and challenging. Accepting is an activity where the students obtain the task or problem. Meanwhile, challenging is an activity that questions or tests the task through the formulation of problems. Silver (1994) asserted that challenging is channeled through generating new problems and questions aimed at exploring a given situation and the reformulation during the process of solving it. These situations require students to formulate questions like: (1) proposing solvable math questions within the existing context without providing additional information on core tasks, and (2) formulating math questions that are solvable by creatively adding new information.

This study offers the chance to investigate the effect of performance assessment in the problem-solving and problem-posing approach on the mathematical achievement of students by involving them in the classroom. Integrating students' level of metacognition in performance assessments could improve students' mathematical achievement. Through tasks and rubrics as the core of performance assessment, students solve various mathematical problems and simultaneously assess the quality of processes and results.

Performance assessment as a learning intervention or teaching model is a good empirical study with little related research. Problem-solving and problem-posing studies have not used performance assessment as a learning intervention.

Research Questions

This study uses a quasi-experimental approach to investigate the impact of performance assessment through 'problem-solving and problem-posing' considering the level of metacognitive awareness and its impact on students' mathematical performance. A Split Plot Design was used to test the role of students' metacognition level in performance assessment interventions through 'problem-solving and problem-posing.' The main research questions provided below.

1. Is there a difference in mathematics achievement scores between students who receive performance assessments through problem-solving as opposed to those who receive problem-posing?
2. Is there an effect of performance assessment on mathematics achievement for each level of students' metacognition?

Theoretical Framework

Metacognition and Mathematics Achievement

According to the National Council of Teachers of Mathematics (NCTM, 2000), the mathematical disposition of students can be described as exhibiting confidence in the application of mathematical concepts, having elevated self-expectations, displaying attentiveness during lectures, demonstrating persistence in resolving mathematical problems, possessing a keen sense of curiosity, exhibiting the ability and willingness to articulate mathematical ideas to others, and exhibiting a strong awareness of their own thought processes. Kadir and Sappaile (2019) stated that students' metacognition on a mathematics assignment is a state in which they begin thinking and using what they know and applying the knowledge prior to beginning the assignment itself. In terms of metacognition, it is important to note that possessing a significant amount of knowledge and skills is not sufficient without the ability to make informed decisions, manage, and regulate what has been learned effectively, and apply them to solve mathematical problems. Thus, the capacity for metacognition encompasses the development of executive, managerial, and self-regulatory skills, as they pertain to the acquisition and application of mathematical knowledge.

Students with developed metacognitive skills possess the ability to identify limitations and weaknesses in their thinking process, which includes recognizing others' perspectives, monitoring their progress, and making distinctions between comprehended and misunderstood information. According to Marzano et al. (1988), metacognition is a competency that can be broken down into several different categories. These categories encompass the following: (1) The cultivation of self-regulatory capacities, including the demonstration of a persistent dedication to academic tasks, the adoption of a positive student mindset, and the regulation of one's attentional focus in response to the requirements of academic tasks; (2) the integration of various forms of knowledge, including factual knowledge, step-by-step knowledge, and knowledge based on conditions; and (3) the application of executive control abilities, such as the formation of plans, ongoing progress monitoring, and the systematic evaluation of procedures.

Schoenfeld (1992) proposed that the process of finding a solution to the issue required highly developed organizational skills, as well as control and monitoring mechanisms. It is imperative for educators to highlight the significance of these processes within the pedagogical framework that adopts a problem-solving approach. It is of utmost importance to integrate these processes into the educational curriculum, given their paramount significance in fostering the development of metacognitive abilities. The definition of metacognition encompasses concepts such as self-regulation, monitoring, and controlling of one's own cognitive processes. According to Cohors-Fresenborg et al. (2010), it is crucial to regulate the use of appropriate mathematical tools in the field of school algebra. To correctly measure metacognitive development, it is crucial to monitor its utilization. Numerous scientific investigations have established that monitoring procedures are indicative of achievement. However, there was no connection between monitoring reports and actual behavior or outcomes. Therefore, efforts in learning mathematics must be focused on monitoring changes in student behavior.

Several studies (Hasbullah, 2015; Listiani et al., 2014; Smith, 2007; Suriyon et al., 2013), have indicated that practices or learning activities based on metacognitive strategies have a significant impact on students' learning and mathematics learning achievement through the application of several approaches introduced as part of 21st-century learning. Furthermore, Veenman et al. (2006), several studies reveal that there is a correlation between mathematical performance and metacognitive skills. These studies place metacognition as an independent variable for intellectual ability. Menz and Xin (2016) indicated that students' mastery and proficiency in mathematics are linked to their metacognitive skills. When metacognitive abilities are robust, their performance in mathematics will be outstanding.

Recently, there has been extensive research on the pivotal role of self-regulation in academic achievement. For instance, Zee and de Bree (2017) Studies have revealed positive correlations between self-regulation and academic achievement in mathematics and reading among elementary students in the Netherlands. Kaur et al. (2018) showed that Punjabi secondary school students' academic success was positively impacted by metacognition and self-regulation. A study conducted by Dradeka (2018) in Saudi Arabia revealed a substantial disparity in student self-regulation, with students demonstrating higher academic achievement displaying a greater degree of self-regulation. The results of the study indicated that male students displayed a greater propensity towards academic self-regulation when compared to their female counterparts. The results also showed that male students, on average, reported higher levels of academic self-regulation compared to female students. In addition, Annalakshmi (2019) posits that adolescent girls from low-income rural families in Tamil Nadu who engage in self-regulation demonstrate higher levels of resilience and academic success. Zhou and Wang (2019) also found evidence supporting a positive correlation between academic achievement, self-regulation, and learning motivation, aligning with the findings of the present study.

Performance Assessment

The Principles and Standards for School Mathematics published by the National Council of Teachers of Mathematics (NCTM, 2000) encourage teachers to make use of real-world mathematical problems in the classroom to make the learning process more interactive and engaging. According to Nitko and Brookhart (2014), performance assessment is a kind of alternative assessment or authentic assessment. A performance assessment is a procedure in which students are tasked to obtain information on how well a student has learned. Unlike multiple-choice questions, performance assessment tasks require students to demonstrate mastery of a learning target by integrating knowledge and skills from a variety of subject areas.

According to Nitko and Brookhart (2014) a performance assessment is made up of two parts: the assignment itself, and the rubric that will be used to evaluate the students' work. Performance tasks contain student activity that aims to show the performance of a learning target. Some examples of performance assessment tasks in the study of mathematics are mathematical writing, the practical use of three-dimensional space props, a research project, measuring the height of an object, 'problem-solving, problem posing, and mathematical modeling.' A rubric for scoring is a set of guidelines that are used to assess the quality of student performance. The rubric, therefore, is used to provide guidelines for assessors to ensure consistency of assessment results. The grading rubric should be coherent in order to evaluate the performance quality of students. The rubric may take the form of a rating scale or a check list, and it addresses two aspects of student performance: achievement and processes.

According to Kulm (1994) a holistic rubric can be used to evaluate both problem-solving and problem-posing abilities. This hybrid of analytic and holistic approaches to grading includes criteria for evaluating students' grasp of key concepts and skills in application. The Anaholistics Rubric is a great way to evaluate a student's progress in mathematics because it provides a holistic score across multiple areas of study. Thus, the implementation of performance assessment referred to in this research is a measurement and assessment activity that requires students to display their performance through the task of problem-solving and problem-posing in writing related to the processes and outcomes of learning mathematics.

Problem-Solving Task

The study of mathematics education at schools raises two main questions: (1) How to instruct students in how to solve problems, and (2) how to evaluate the performance of students based on their ability to solve problems. Problem-solving activities should be used in the classroom as a means for teachers to evaluate the students' complex thought processes. These activities should require students to comprehend, create, implement, and evaluate their plans based on some theoretical arguments related to problem-solving strategies (NCTM, 2000; Pólya & Conway, 2004). Pólya and Conway (2004) developed a set of questions to be asked at each stage of the process in order to guide the students through it and check the results of problem resolution. Through the use of the following questions, this procedure can be carried out at each stage of the problem-solving process.

First stage: Understanding the problem (it is crucial to comprehend the issue), it is essential to first understand its nature. This requires consideration of the following questions: (1) What is the unknown? (2) Where can the data be located? (3) Can you clearly define the problem at hand? (4) Can the condition be fulfilled? (5) Does the condition fully determine the unknown, or is it inadequate, redundant, or incompatible with other information? (6) Can you create a diagram and use appropriate symbols to represent the information? (7) Can you break down the different parts of the condition? (8) Have you thought about documenting the information and conditions?

Second stage: Devising a plan[1] which consist of: (1) Have you encountered a similar problem before or in a slightly different form? (2) Are you aware of a similar issue or a potentially helpful theorem? (3) Take into account the unknown and compare it to a familiar problem with a similar or identical unknown. (4) Here's a previously solved problem that is similar to yours, could it be helpful in your situation? (5) Can you utilize the outcome of the similar problem? (6) Can you apply the technique used in the similar problem to your situation? (7) Do you need to add any additional elements to make it work? (8) Could you rephrase the problem? (9) Is it possible to

[1] Note that a plan may identify the relationship between the data and the unidentified. If an immediate connection cannot be found, you may be required to consider auxiliary issues. You should eventually obtain a solution plan.

restate the issue in a different way? Revisit the definitions and try solving a related problem if you can't solve the original one. (10) Can you visualize a simpler related problem? A broader or a more specific problem? A similar problem? (11) Is it possible to solve a part of the issue? By retaining only a part of the conditions and eliminating the rest, to what extent is the unknown then determined and how can it change? (12) Are there any useful insights that can be gained from the available data? (13) Can you think of any additional data that would be helpful in finding the unknown? (14) Would it be possible to modify either the unknown or the data, or both, to bring the unknown and data closer together? (15) Have you used all the available data? (16) Have you considered all the conditions? (17) Have you taken into account all the important concepts related to the problem?

Note that a plan may identify the relationship between the data and the unidentified. If an immediate connection cannot be found, you may be required to consider auxiliary issues. You should eventually obtain a solution plan.

Third stage: Carrying out the plan which comprises of: (1) While putting your plan for the solution into action, make sure to check each step. (2) Can you confidently affirm that the step taken is appropriate? (3) Can you provide evidence that it is accurate?

Fourth stage: Looking back (test the result obtained), such as: (1) Can you confirm the result? (2) Would it be achievable the argument? (3) Is there another way you can arrive at the solution? (4) Is it obvious to you at first glance? (5) Is it possible for you to apply the solution or the method to an additional issue?

Problem Posing Task

According to Silver (1994), problem posing involves two cognitive aspects, namely accepting and challenging. Accepting stage, namely a situation such as pictures, manipulation of kids' tools, game, theorem, or concept, equipment, problem, or solution of a problem. Challenging is done by coming up with new problems and questions that aim to learn more about a given situation, or by rewriting a problem as you try to solve it. Silver and Cai (1996) conducted a study with a large group of sixth and seventh grade students and developed a problem-posing scheme based on the type and complexity of student responses, as shown in Figure 1.

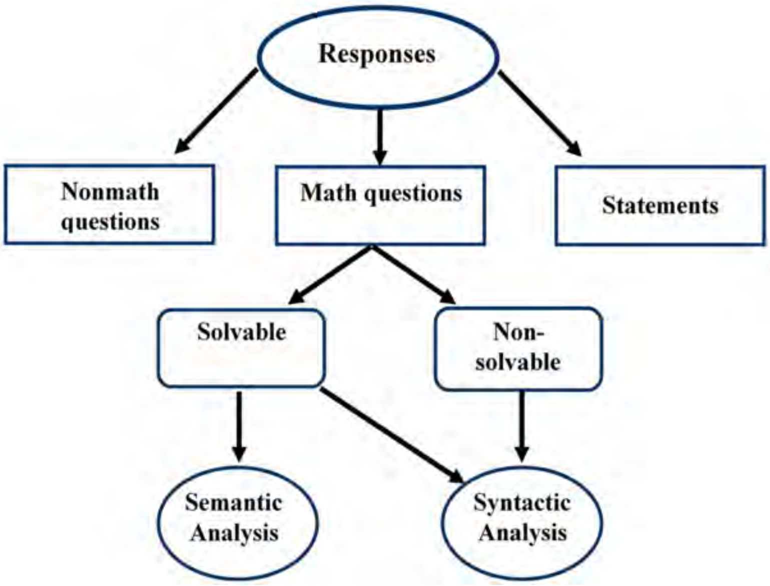


Figure 1: Problem Posing Response (Silver and Cai, 1996)

In Figure 1, the student-formulated responses were classified into three categories: (1) mathematical questions, (2) non-mathematical questions, and (3) statements. Mathematical questions refer to those that involve mathematical problems. These questions are further divided into two types: (1) solvable mathematical questions and (2) non-solvable mathematical questions. Solvable mathematical questions are those that contain sufficient information or conditions from the task. Solvable questions can be further classified into two subcategories: (1) questions that only include information provided in the task, and (2) questions that introduce new information beyond the task. Non-solvable questions are those that lack sufficient information from the task to be solved. Non-mathematical questions, on the other hand, do not involve mathematical problems and are unrelated to the task.

Furthermore, the complexity of the problem generated by the students can be classified into two types: (1) complexity related to the language structure (syntax), and (2) complexity related to the mathematical structure (semantics). The level of syntactic complexity is represented in the form of propositions, such as assignment, relationship, and hypothetical statements. Meanwhile, the level of semantic complexity includes categories such as transforming, grouping, comparing, and stating.

METHOD

Participants

This was a randomized study with a quasi-experimental approach, and class X (tenth) students in the Senior High Schools in Jakarta Indonesia, which consisted of 12 classes with similar characteristics (XA, XB, XC, XD, XE, XF, XG, XH, XJ, XK, XL). Furthermore, four (4) out of 12 classes (XA, XH, XL, XG) were selected at random using the cluster random sampling technique. The study involved 163 students, 99 (60.7%) women, and 64 (39.3%) men, with the following details: class XA 40 students 23 women (57.5%) and men 17 (42.5%), XH 42 students 27 women (64.3%) and men 15 (35.7%), XL 40 students 24 women (60.0%) and 16 men (40.0%), and XG 41 students 25 women (61.0%) and 16 (39.0%).

Measures

The independent and dependent variable used was performance assessment (treatments, namely problem-solving and problem-posing tasks), and mathematics achievement was measured by using a test. The validity of the contents was determined by the "Quantification of Content Validity" method from (Gregory, 2014), involving 10 expert raters, and obtained 47 valid-content items with ranges (0.700 to 0.930) and inter-rater reliability of 0.901. Additionally, the empirical results yielded 40 valid items with a validity range of (0.342 to 0.720), and the reliability coefficient was determined to be 0.917. The test materials include rational exponent and root forms, quadratic equations, inequalities, comparisons and trigonometric functions, logarithms, rules of 'sine and cosine, and the area of triangles.'

Furthermore, another independent variable includes metacognition as a moderator (categorical) variable that was measured using a scale and had been performed before the treatment was conducted. Measurement of student metacognition uses a scale developed by Kadir and Sappaile (2019), and the results was validated through consultation employed a panel of specialists, with 'the inter-rater reliability coefficient' among the panelists having been determined to be 0.830. Empirical test results of the scale consisted of 46 items with a validity range (of 0.197 to 0.804), and the construct reliability coefficient is about 0.938. The Confirmatory Factor Analysis (CFA) reached construct reliability scores of 0.990 for self-regulation skills, 0.980 for type of knowledge, and 0.982 for executive control skills.

Design and Procedure

Based on the selected class sample, XL, XA, XG, and XH, two classes (XA and XH) were randomly assigned to performance assessment through problem-solving while two classes (XL and XG) were randomly assigned through problem-posing. The experimental design used was Group within Treatment (GWT) design and is considered the heterogeneity of the treatment groups/classes. Through the proper grouping of classes, this design can reduce treatment errors (Kadir, 2022). Furthermore, mathematics lessons were delivered for all classes involved for a full semester (5 months) following the national curriculum of Indonesia. There were 2 meetings each week which consisted of 2 X 45 minutes, and at the end of the semester, mathematics achievement was recorded for each participating student.

Student metacognition data were obtained before treatment. Each item of the metacognition scale consists of a Likert scale from 1 to 5. For 46 items, the total metacognition scores for each range from 46 – 230. The higher the scores, the better the metacognition. The results of the metacognition level scale in each class were divided into 3 major categories, namely the high (70th quantile and higher), medium (between 30th and 70th quantiles), and low level (less than or equal to 30th quantile). The metacognition classifications of the high, medium, and low-level students were not conducted in the treatment conditions, but the classification was needed for data analysis. The study design is presented in Figure 2.

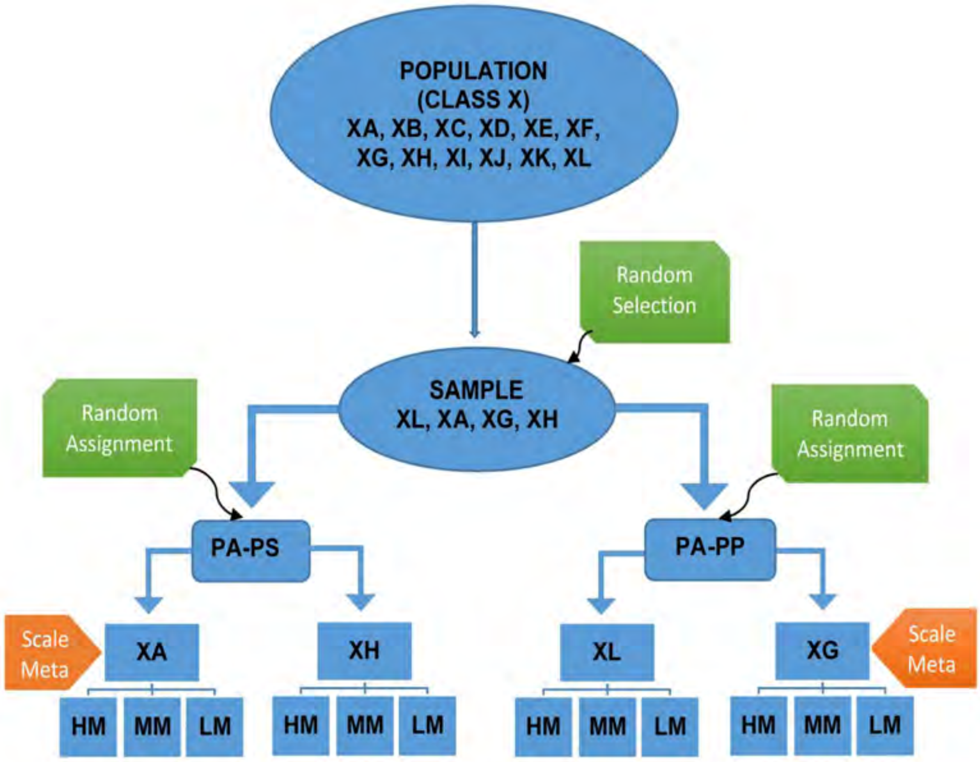


Figure 2: Research Design

Notes:
PA-PS = Performance Assessment-Problem Solving
PA-PP = Performance Assessment-Problem Posing
HM = High Metacognition; MM = Medium Metacognition; LM = Low Metacognition
XA = Class XA 40 Students, HM = 13; MM = 15; LM = 12
XH = Class XH 42 Students, HM = 13; MM = 17; LM = 12
XL = Class XL 40 Students, HM = 13; MM = 15; LM = 12
XG = Class XG 41 Students, HM = 13; MM = 16; LM = 12

Experiment 1: Performance Assessment through Problem-solving

Treatment through performance assessment with problem-solving was conducted in some stages of the task as presented in Table 1.

Stages	Activities
Understanding the problem	Students were given 'a problem-solving task' Students 'were guided to understand the tasks' (e.g., to know what was the known fact and what was asked) Students were guided to see the data fulfillment of the tasks
Devising a plan	Students were guided to identify the problem of the tasks. Students changed the task into more simple language Students were guided to memorize the similar tasks Students were guided to connect mathematical concepts to the tasks Students developed a strategic design or solution method
Carrying out the plan	Students were guided to apply the plan in appropriate steps Students checked the technique of solving problems
Looking back	Students checked the correct process of solving problems Students checked the correct results of solving problems

Table 1: Guide to doing problem-solving tasks

The rubric for assessing students' levels of success in solving problems is in Table 2.

Stages	Score	Descriptors
Understanding the problem	: **2**	Understanding the problems correctly
	: **1**	Misinterpreting in partial/neglecting the condition of the task.
	: **0**	Misinterpretation to all.
Devising a plan	: **4**	Select the procedure which leads to the correct solution
	: **3**	Select some strategies but incomplete
	: **2**	Select a strategy but unsuccessful/not trying another technique
	: **1**	Select a plan which undoable to be implemented
	: **0**	Select a plan which irrelevant/no strategy at all
Carrying out the plan	: **4**	Doing the procedure correctly and there is a correct solution
	: **3**	Using correct strategy but less incorrect calculation
	: **2**	Doing correct procedure which may be giving a correct answer but incorrect in structuring and calculating
	: **1**	Using a part of procedures that is correct but leads to the incorrect answer
	: **0**	Using inappropriate plan and pause/cannot use plan or correct algorithm
Looking back	: **2**	Checking is conducted to the results and the process
	: **1**	There is checking but incomplete
	: **0**	There is no check given or no check at all

Table 2: Problem-solving Rubric

Experiment 2: Performance Assessment through Problem Posing

Treatment of performance assessment problem-posing was conducted in some stages of the task in Table 3.

Stages	Activities
Accepting	▪ Gave the students problem-posing tasks ▪ Students were guided to be familiar with the tasks (observing, writing the data of the tasks) ▪ Students were guided to think about the concepts, formulas, patterns, or samples which connected to the tasks
Challenging	▪ Students were guided to make solvable math questions based on the situation of the tasks ▪ Students made Math questions based on the tasks by adding new information from the main tasks ▪ Students made conditional questions to enrich the context of the main tasks ▪ Students combined other situations with the main tasks ▪ Students were guided to change the previous tasks ▪ Students were guided to make Math questions with new data and new context out of the main tasks.

Table 3: Guide to doing problem-posing task

Rubric for assessing students' response and proposition type in posing, in Table 4.

Response Type	Code	Score	Proposition Type
Statement	(Q0)	: 0	
Non-math questions	(Q1)	: 0	
Math questions non- solvable	(Q2)	: 1	A/R/S
Math questions solvable:			
(a) Without new information	(Q3)	: 2	A/R/S
(b) With new information	(Q4)	: 3	A/R/S

Table 4: Problem Posing Rubric

Note:
Q0 = Statement did not contain math question;
Q1 = Question had no math problem and was unrelated to tasks;
Q2 = Math question had no adequate information to be solved;
Q3 = Math question based on the information provided on the tasks;
Q4 = Math question using additional information out of the main tasks;
A = Assignment (task proposition, namely the question must be solved);
R = Relationship (Relationship proposition, namely the question to compare):
S = Supposition (Conditional proposition, namely the question using a conditional sentence)

Data Analysis

Metacognition is a moderating variable integrated into this study to form a factorial design and was classified into three categories, namely high-level, medium-level, and low-level students. In reality, the design was a split plot with a completely randomized setting. The analysis technique used was a Two Way Analysis of Variance (ANOVA) with a split-plot (Montgomery, 2013). Furthermore, the main and subplot were the class and metacognition levels respectively. Analysis for the split-plot design and the corresponding contrasts were conducted using IBM SPSS Statistics 23.

RESULTS AND DISCUSSION

Students' Metacognition

Baseline information on students' metacognition for each class is presented in Table 5.

Metacognition	Class/Group				Total
	XA (N=40)	XH (N=42)	XL (N=40)	XG (N=41)	(N=163)
Mean	166.9	139.3	144.2	150.3	150.0
Median	166.5	145.0	145.0	152.0	152.0
Mode	158.0	120.0	132.0	154.0	148.0
Std. Deviation	9.97	17.64	11.22	11.12	16.49
Range	152.0 - 191.0	110.0 - 166.0	128.0 - 169.0	130.0 - 176.0	110.0 - 191.0
Q1-Q3	158.0 - 174.0	120.0 - 155.0	133.0 - 151.0	140.0 - 157.0	138.0 - 161.0

Table 5: Baseline metacognition for all students in 4 different classes

Table 5 shows that the median (range) of students' metacognition level was 166.5 (152-191), 145 (110-166), 145 (128-169), and 152 (110-191) for class XA, XH, X, L, and XG respectively. Furthermore, the mean and the mode indicate that the classes XA (166.9 > 158.0), XH (139.3 > 120.0), XL (144.2 > 132.0), XG (150.3 > 154.0,) and X total (150.0 > 148.0). When the theoretical average score is set at 138 (46x3), then the mean metacognition before being given the intervention with performance assessments has exceeded the average theoretical and empirical scores above the mode of each class. Therefore, students' average metacognition before being given a performance assessment is in a good category.

Students' Mathematics Achievement

The mathematics achievement average scores are expressed in percentages unless otherwise specified. The scores are classified by treatment group (problem-solving and problem-posing) and according to students' metacognition level which is presented in Table 6.

Performance Assessment	Metacognition Level	Class	Mean	Std. Deviation	N
Problem-Solving	High	XA	78.62	5.867	13
		XH	78.85	8.143	13
		Total	78.73	6.954	26
	Medium	XA	73.20	3.895	15
		XH	68.24	9.890	17
		Total	70.56	7.980	32
	Low	XA	67.42	8.174	12
		XH	68.00	9.863	12
		Total	67.71	8.864	24
	Total	XA	73.23	7.413	40
		XH	71.45	10.430	42
		Total	72.32	9.073	82
Problem Posing	High	XL	68.31	9.313	13
		XG	70.85	10.431	13
		Total	69.58	9.774	26
	Medium	XL	70.33	6.705	15
		XG	69.38	6.752	16
		Total	69.84	6.634	31
	Low	XL	67.75	8.864	12
		XG	68.08	9.080	12
		Total	67.92	8.777	24
	Total	XL	68.90	8.142	40
		XG	69.46	8.579	41
		Total	69.19	8.319	81

Table 6: Students' mathematics achievement

Table 6 showed that the average scores for students receiving performance assessment through problem-solving were higher (73.32 and std. deviation 9.07) than those based on problem posing (69.19 and std. deviation 8.32). Furthermore, looking at a simple effect of treatment within each level of metacognition, shows that in students with high metacognition, the average mathematics achievement score based on problem-solving was higher than problem-posing (78.73 vs. 69.58 with a standard deviation of 6.95 and 9.77). However, this is not the case for those in the low and medium metacognition, where the mathematics achievement was relatively similar (70.56 vs 69.84 and 67.71 vs. 67.92).

This descriptive finding showed that students with a high level of metacognition given performance assessments both using problem-solving and problem-posing could achieve relatively good mathematics scores. These findings are consistent with the results of Abdellah (2015), that positive correlation was identified between the Metacognitive Awareness Inventory (MAI), academic achievement, and teaching performance. Moreover, the deployment of metacognitive skills was found to have a marked and positive influence on both academic achievement and teaching performance.

Performance of problem-solving

The description of the students' performance of problem-solving according to various stages is in Table 7.

Stages	Max Score	Mean	%
Understanding the problem	20	12.290	61.45
Devising a plan	30	21.060	70.20
Carrying out the plan	30	22.485	74.95
Looking back	20	13.540	67.70
Total	100	69.375	69.38

Table 7: Problem-solving scores according to the stages (N = 82)

Table 7 showed that the average score for problem-solving skills was 69.40%. This value was an average from various stages of problem-solving; conducting the plan contributes to the highest percentages (75.00%) relative to the rest of the stages (understanding the problem (61.50%), devising a plan (70.20%), and looking back (67.70%). Assessment of stages of problem-solving showed that under a maximum score of 100%, students' score was 69.40%. Therefore, the students have shown fairly good problem-solving performance in every step of Polya. This finding aligns with the results of the study conducted by Lee and Chen (2015) that the geometric reasoning performance of students that received Polya question-based learning was superior to those that received direct presentations. In addition, students that received instruction based on Polya's questions expressed a stronger sense of participation than in direct presentations. The recent finding resemble with the results of Aguilar and Telese (2018) on a sample of elementary pre-service teachers, who reported significant advancements in their procedural fluency, conceptual understanding, and strategic problem-solving competencies following their participation in a series of non-routine problem-solving tasks. The participants, as prospective teachers, demonstrated an improvement in their proficiency in employing procedures and exhibited a sufficient level of conceptual knowledge in relation to problem-solving.

Performance assessment problem-solving in teaching-learning started with giving mathematical tasks to students. An example of a problem-solving task is shown in Figure 3.

<table>
<tr><td colspan="2">Tugas pemecahan masalah
“ Siswa kelas X mengikuti penggalangan dana untuk mengumpulkan sejumlah Rp960.000,- kegiatan lomba seni antar kelas. Setiap siswa harus menyumbang jumlah uang yang sama, tetapi empat siswa tidak dapat membayar. Untuk mengatasi kekurangan itu, siswa lainnya harus menambah uang masing-masing sebesar Rp20.000,-. Berapa banyak siswa yang mengumpulkan dana?”</td></tr>
<tr><td>Tahap</td><td>Contoh kinerja siswa</td></tr>
<tr><td>Memahami masalah</td><td>Apa yang diketahui ?
(i) Jumlah dana yg akan dikumpulkan = Rp 960.000,-
(ii) Jumlah siswa yg tidak dapat membayar : 4 orang.
(iii) Uang tambahan yg harus ditanggungulangi setiap siswa : Rp 20.000,-
Apa yang ditanyakan?
Jumlah siswa yg berpartisipasi dalam pengumpulan dana.</td></tr>
<tr><td>Menyusun rencana</td><td>(i) Misalkan jumlah siswa pengumpulan dana : x maka setiap siswa membayar : (Rp 960.000 / x)
(ii) 4 siswa tidak membayar, jadi pembayar (x−4).
(iii) Total dana yg terkumpul dengan 4 siswa tak bayar : (x−4) (Rp 960.000 / x)
(iv) Jumlah dana tambahan untuk mengatasi 4 siswa tidak bisa bayar (x−4). (Rp 20.000)
(v) Jumlah dana yg terkumpul, dinyatakan dengan model matematika (x−4) (960.000 / x) + (x−4) (20.000) = 960.000</td></tr>
<tr><td>Melaksanakan rencana</td><td>Dari persamaan : (x−4) (960.000 / x) + (x−4) (20.000) = 960.000 ... (i)
$\Leftrightarrow (x-4)\left(\frac{48}{x}\right) + (x-4) = 48$ (kedua ruas bagi 20.000)
$\Leftrightarrow 48 - \frac{192}{x} + x - 4 - 48 = 0 \Leftrightarrow x - \frac{192}{x} - 4 = 0$
$\Leftrightarrow x^2 - 4x - 192 = 0 \Leftrightarrow (x-16)(x+12) = 0$
$\Leftrightarrow x = 16$ atau $x = -12$</td></tr>
<tr><td>Memeriksa proses dan hasil</td><td>Nilai x = 16 (memenuhi) dan x = −12 (tidak memenuhi)
Jadi jumlah siswa yg membayar sebanyak 16.
Total dana terkumpul = jumlah dengan 4 siswa tidak membayar + total dana tambahan.
⇔ (16−4) (Rp 960.000 / 16) + (16−4). Rp 20.000 = Rp 960.000
⇔ $\frac{12}{16}$ (Rp 960.000) + 12. Rp 20.000 = Rp 960.000
⇔ Rp 720.000 + Rp 240.000 = Rp 960.000 (benar).</td></tr>
</table>

<table>
<tr><td colspan="2">Translate in English:
Problem-solving task
“Grade X students participate in fundraising to collect an amount of Rp960.000,- for the inter-classes arts competition. Each student should contribute an equal amount, but there are four students who could not pay. To overcome the shortness, the rest of the students must add an amount of Rp20.000,- each. How many students who pay/participate in collecting the budget?”</td></tr>
<tr><td>Stage</td><td>Example of the students’ performance</td></tr>
<tr><td>Understanding the problem</td><td>What is known?
(i) The amount of fund to be collected: Rp960.000,-
(ii) The number of students who could not pay are 4 students.
(iii) Additional money of each student must pay is Rp20.000,-
What is asked?
“The number of students who participate in the fundraising”</td></tr>
<tr><td>Devising a plan</td><td>Develop mathematical model
(i) Suppose the number of students who raise funds is x, then each student must pay: $\frac{\text{Rp}960.000}{x}$
(ii) There are four students who do not pay, so the number of payer only: $(x - 4)$.
(iii) The total funds collected taking into account four non-payer students: $(x-4)\left(\frac{\text{Rp}960.000}{x}\right)$
(iv) The amount of additional funds to cope with four non-payer students: (x - 4). Rp20.000,-
(v) All funds collected expressed by the equation:
$(x-4)\left(\frac{\text{Rp}960.000}{x}\right) + (x-4)(\text{Rp}20.000) = \text{Rp}960.000$</td></tr>
<tr><td>Carrying out the plan</td><td>From $(x-4)\left(\frac{\text{Rp}960.000}{x}\right) + (x-4)(\text{Rp}20.000) = \text{Rp}960.000$
$\Leftrightarrow (x-4)\left(\frac{48}{x}\right) + (x-4) = 48$
$\Leftrightarrow 48 - \left(\frac{192}{x}\right) + x - 52 = 0$
$\Leftrightarrow x - \left(\frac{192}{x}\right) - 4 = 0 \Leftrightarrow x^2 - 4x - 192 = 0$
$\Leftrightarrow (x-16)(x+12) = 0 \Leftrightarrow x = 16 \text{ or } x = -12$</td></tr>
<tr><td>Looking back</td><td>Retrieve: x = 16 (satisfied) or x = -12 (is not satisfied). So the number of payers is 16 students. Total funds collected = the amount of fund + 4 non-payers + the total of additional fund:
$\Leftrightarrow (16-4)\left(\frac{\text{Rp}960.000}{16}\right) + (16-4)(\text{Rp}20.000) = \text{Rp}960.000$
$\Leftrightarrow (12)\left(\frac{\text{Rp}960.000}{16}\right) + (12)(\text{Rp}20.000) = \text{Rp}960.000$
$\Leftrightarrow$ Rp720.000, - + Rp240.000, - = Rp960.000,- (correct)</td></tr>
</table>

Figure 3: Performance of Problem-Solving

Performance of problem posing

The description of the students' problem-posing ability according to the response and proposition after receiving a performance assessment is presented in Table 8.

Response Type	Score	%	Proposition Type	Score	%
Statement or Non-math questions (Q0/Q1)	0	0	Assignment (A)	0	0
Math questions non-solvable (Q2)	44	0.94	Assignment (A)	3393	45.60
Math questions solvable (Q3)					
a. Without new information	5639	73.54	Relationship (R)	2259	27.27
b. With new information	2029	26.46	Supposition (S)	2019	27.13
Total	7668	100	Total	7441	100

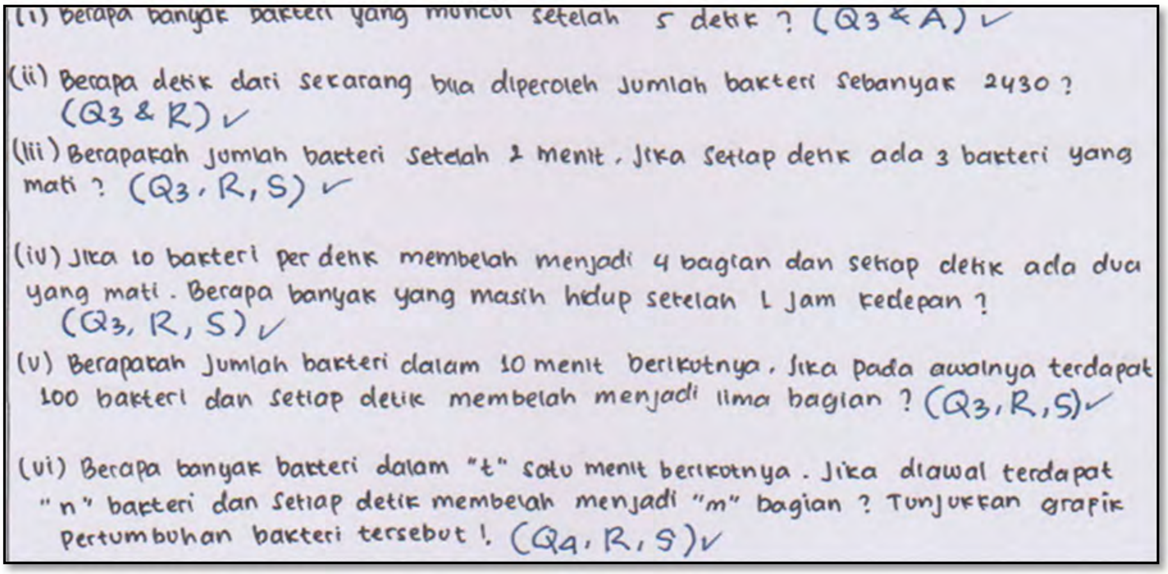
Table 8: Students' problem-posing ability by the response and proposition types (N = 81)

Table 8, showed that out of 7.668 problem-posing responses, all are mathematical questions, of which 0.94% are non-solvable, and 99% solvable mathematical questions (73.54% with new information and 26.46% without new information). The finding is slightly different from the report of Silver and Cai (1996) where out of 1.465 responses, more than 70% were mathematical questions, 20% were of the statement, and 10% were of non-mathematical questions. Currently, responses to statements, non-mathematical responses, and non-solvable mathematical questions were almost non-existent. Most responses are concentrated on solvable mathematical questions. This shows that applying performance assessment based on problem-posing to some extent, facilitated students' metacognition to control their learning activities in posing questions relevant to mathematics both with and without new information.

This finding is consistent with the results found by Cai et al. (2013) that the students generated many Mathematical problems that can be solved, including those that have a complex syntax and meaning. Almost half of 509 middle school students developed sets of related problems, and also eight fairly complex problems were solved. The nexus between problem-solving performance and problem-generating abilities revealed that individuals classified as "good" problem solvers generated more mathematical and intricate problems compared to those classified as "poor" problem solvers.

This finding is also similar to the features of problem-posing performance assessment task-based, which contained two specific stages: (1) accepting, where students are trained to establish their concept, explore prior knowledge, and connect to the problems given by the teachers and (2) challenging, where students are challenged to ask questions through changing the initial problems, obtaining data from the questions and at the end they can change the objectives and solve it to increase their higher-order thinking. This result can be interpreted as a performance assessment based on problem-posing meant for students with low metacognition in order to progress their mathematics learning.

Intervention performance assessment problems-posing in teaching-learning in the class started with giving mathematical tasks to students. An example of problem-posing tasks and student performance is in Figure 4.

Menerima (Problem-posing task)
“Sebuah penelitian tentang pembiakan koloni bakteri melaporkan bahwa satu bekteri setiap detiknya terbagi menjadi tiga begian. Pada awalnya, ada sepuluh bakteri di koloni tersebut”
Menantang


(i) Berapa banyak bakteri yang muncul setelah 5 detik ? (Q3 & A) ✓
(ii) Berapa detik dari sekarang bila diperoleh jumlah bakteri sebanyak 2430 ? (Q3 & R) ✓
(iii) Berapakah jumlah bakteri setelah 2 menit, Jika setiap detik ada 3 bakteri yang mati ? (Q3, R, S) ✓
(iv) Jika 10 bakteri per detik membelah menjadi 4 bagian dan setiap detik ada dua yang mati. Berapa banyak yang masih hidup setelah 1 jam kedepan ? (Q3, R, S) ✓
(v) Berapakah jumlah bakteri dalam 10 menit berikutnya, Jika pada awalnya terdapat 100 bakteri dan setiap detik membelah menjadi lima bagian ? (Q3, R, S) ✓
(vi) Berapa banyak bakteri dalam "t" satu menit berikutnya. Jika diawal terdapat "n" bakteri dan setiap detik membelah menjadi "m" bagian ? Tunjukkan grafik pertumbuhan bakteri tersebut ! (Q4, R, S) ✓

Translate in English:

Accepting (Problem-posing task)
“A research about breeding the colony of bacteria reported that a bacteria every second is divided become three parts. In the beginning, in the colony, there are ten bacteria”
Challenging Example of the students’ performance
(i) How many bacteria come after 5 seconds? (Q3 & A) (ii) How many seconds from now when the obtained amount of bacteria as much as in 2430? (Q3 & R) (iii) What is the number of bacteria after 2 minutes, if in every second there are 3 dead bacteria? (Q3, R & S) (iv) If the 10 bacteria per second splitting into 4 parts and every second there are two dead. How many bacteria still alive after 1 hour ahead? (Q3, R & S) (v) What is the amount of bacteria in the next 10 minutes, if at beginning, there are 100 bacteria and every second splitting into five parts? (Q3, R & S) (vi) How many bacteria "t" in the next minute, if at beginning there are "n" bacteria and every second splitting into "m" part? Show the growth of the bacteria graphically! (Q4, R & S).

Figure 4: Performance of Problem Posing

Hypothesis testing

The ANOVA GWT table according to the split-plot design is displayed in Table 9.

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	Hypothesis	804630.95	1	804630.95	24649.73	.000
	Error	68.717	2.105	32.643[a]		
Perform-Assess (A)	Hypothesis	423.055	1	423.055	12,960	.046
	Error	68.717	2.105	32.643[b]		
Metacognition (B)	Hypothesis	1034.124	2	517.062	7.702	.001
	Error	10405.896	155	67.135[c]		
Perform.Assess* Metcog (A*B)	Hypothesis	691.892	2	345.946	5.153	.007
	Error	10405.896	155	67.135[c]		
Class-G(A)	Hypothesis	64.426	2	32.213	.480	.620
	Error	10405.896	155	67.135[c]		

[a] .988 MS(G(A)) + .012 MS(Error); [b] .988 MS(G(A)) + .012 MS(Error); [c] MS(Error)

Table 9: The Summary of two-way ANOVA GWT Split-Plot on Students' Achievement

Table 9 showed that the interaction performance assessment and metacognition were significant on students' mathematics achievement (F = 5.15; *p*-value = 0.007). This result suggests that performance assessment has significant effect on students' mathematics achievement depending on the metacognition level. The simple effects, i.e., the treatment of Performance Assessment (Problem Solving versus Problem Posing) should then be further investigated within each metacognition level.

The study finding showed that there is an interaction effect between performance assessment and metacognition on students' mathematics achievement. These findings are also in line with the study conducted by Özcan and Erktin (2015), where they found a statistically significant difference in the mathematics scores of students who were assigned homework tasks that incorporated metacognitive questions, compared to those who were not. The findings differ from the study conducted by Gul and Shehzad (2012) which took the subject of public and private university students as a population. The study reported a moderate nexus among metacognition, goal orientation, as well as academic achievement. In contrast, a weak relationship was observed between metacognition and achievement.

The difference in the mean score of mathematics achievement between the performance assessment treatments for each metacognition level was conducted using the t-test. The result (F = 1.358; *p*-value = 0.198) was obtained based on the homogeneity test of variance using Levene's test of Equality of Error Variances. Therefore, the variance distribution of data between treatment groups and the level of metacognition is assumed to be equal (homogeneous). The related contrasts (t-tests) of these simple effects assuming equal variances are presented in Table 10.

	F = 1.358 Sig. =0.198	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
Math Achiev	Assume equal variances	PA-PS*HM x PA-PP*HM	5.29[a]	1.457	3.632	151	.000
		PA-PS*MM x PA-PP*MM	1.62[a]	1.602	1.012	151	.313
		PA-PS*LM x PA-PP*LM	-.60[a]	1.400	-.427	151	.670
		PA-PS*HM x PA-PS*LM	10.30[a]	1.420	7.253	151	.000
		PA-PS*MM x PA-PS*LM	15.42[a]	1.502	10.27	151	.000
		PA-PP*HM x PA-PP*LM	4.41[a]	1.437	3.070	151	.003
		PA-PP*MM x PA-PP*LM	13.20[a]	1.507	8.759	151	.000

Table 10: Contrast Tests for Simple Effects ([a] The sum of the contrast coefficients is not zero)

The contrast tests in Table 10 showed the following: (1) for students with high metacognition level (HM), the mathematics achievement scores of those receiving performance assessment through problem-solving (PA-PS) were significantly higher than those receiving problem-posing (PA-PP) (t = 3.63; df = 151; *p*-value = 0.000), (2) for students with medium and low metacognition (MM & LM), there was no significant difference between mathematics achievement scores in those receiving performance assessment through problem-solving (PA-PS) and those receiving problem-posing (PA-PP) (t =1,01; df = 151; *p*-value = 0.31 & t = -0.43; df = 151; *p*-value = 0.67), (3) for students receiving performance assessment through problem-solving (PA-PS), mathematics achievement scores in those with high and medium metacognition (HM & MM) were significantly higher than those with low metacognition (LM) (t = 7.25; df = 151 *p*-value = 0.000 & t = 10.27; df =151; *p*-value = 0.000), and (4) for students receiving performance assessment through problem-posing (PA-PP), mathematics achievement scores in those with high and medium metacognition (HM & MM) were significantly higher than those with low metacognition (LM) (t = 3.07; df = 151; *p*-value = 0.003 & t = 8.76; df = 151; *p*-value = 0.000).

These results showed that students with high metacognition and performance assessments through problem problem-solving are better or more effective in increasing students' mathematics achievement compared to performance assessments through problem-posing. Meanwhile, for students with medium and low metacognition, the class given the performance assessment through problem-solving and problem-posing did not show a distinction in math scores and was classified as having low scores. These findings showed that students' level of metacognition in the performance assessment intervention determines their mathematics achievement.

The result is supported by students' metacognition, such as focusing and monitoring cognitive processes when analyzing and planning until an appropriate solution is achieved. Furthermore, students' skill in applying the mathematical concepts concisely in solving the problem is meant to train students and enhance their skill on the higher cognitive level which in turn may support improvement in mathematics achievement. This result is accordance with Chong et al. (2019), who reported that the senior high school learners in Brunei demonstrated positive attitudes and beliefs towards problem-solving in mathematics, which are not commonly observed in routine mathematics learning. The meaningful activities designed by teachers were found to facilitate the development of both cognitive-metacognitive abilities and student affect, which aligns with the findings of Tachie (2019) that the application of metacognitive skills and tactics was advantageous for solving mathematics problems. These elements encompass breaking down the task, planning, monitoring, reviewing, and reflecting, both individual and group monitoring capabilities, competency in reading and writing, and self-regulatory capacities.

Mathematics achievement scores in students with high metacognition were better than those with low metacognition. These findings indicate that performance assessment through problem-solving requires students to use metacognition skills optimally to understand problems, plan solution models, choose the right model, control, and provide solutions to problems through existing resources. In line with the study conducted by Jacinto and Carreira (2021), it was stated that the resources used during solving and disclosure activities affect the depth of the conceptual model developed in the progressive mathematical process.

Furthermore, the finding expresses that, for the students with high metacognition, mathematics achievement scores and those receiving performance assessment through problem-solving are higher than those with problem-posing. This implies that applying performance assessment through problem-solving for students with high metacognition can improve their mathematics achievement. In addition, this phenomenon was not observed in students with low metacognition, where the performance assessment through problem-solving and problem-posing did not improve students' mathematics achievement. This result is consistent with a prior study carried out by Du Toit and Du Toit (2013), on metacognition and achievement of students in class XI. Findings from the study shows that metacognitive behavior is consistent with the first three stages of Polya, problem recognition, strategy formulation, and implementation but not with the fourth stage (reflection). A similar study carried out at the elementary school level in Singapore by N. H. Lee et al. (2014) reported that the approach focused on metacognition had a significant impact on students' comprehension of the issue, their ability to plan solutions, their confidence, and their control over their actions and emotions during problem-solving.

Another finding is that in students with medium and low metacognition, the mathematics achievement score of those receiving performance assessments through problem-solving was not significantly different from those receiving problem-posing. This finding does not support the hypothesis that performance assessment through problem-posing will be a better option in students with medium and low metacognition. Therefore, students with medium and low metacognition struggle to produce math problems, understand, plan, and solve problems optimally. On the contrary, problem-posing requires students to use their metacognition skills to generate new and quality math questions by creatively adding new information to the task. This is consistent with Van Harpen and Sriraman (2013) which involved participants from one location in the USA and two locations in China. Nevertheless, according to their findings, high school students have difficulty formulating original and challenging mathematical problems.

CONCLUSION

The assessment of student performance through problem-solving and problem-posing, in conjunction with their level of metacognition, has a significant impact on their achievement in mathematics. Specifically, students who receive performance assessments through problem-solving tend to achieve higher scores compared to those who only receive problem-posing assessments. Teachers who adopt a teaching style that includes a diverse set of questions during problem-solving assessments facilitate opportunities for students to provide more accurate and comprehensive responses, given the provision of focused questions at each stage of problem-solving. Conversely, when students undergo performance assessments through problem-posing, they tend to generate a higher number of mathematical questions that they may not have sufficient time to answer. The influence of performance assessment on mathematics achievement is further moderated by the student's metacognition level. Students with high metacognition levels perform better on problem-solving assessments than those who receive problem-posing assessments. Metacognition plays a crucial role in students' ability to regulate their learning activities and ask pertinent questions during different stages of problem-solving. Conversely, for students with low and medium metacognition levels, the impact of performance assessment on their mathematics achievement is not significant.

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