

The Effectiveness of Didactic Designs for Solutions to Learning- Obstacle Problems for Prospective Mathematics Teacher Students: Case Studies on Higher-Level Derivative Concepts

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Abstract: Various studies have concluded that many students have difficulty understanding concepts related to function derivatives. One of the concepts in function derivatives is higher-order derivatives, primarily the n^{th} derivative pattern. This study aims to: 1) identify various types of learning obstacles experienced by prospective mathematics teacher students on high-level derivative topics; 2) design alternative didactic designs based on the findings of learning obstacles; 3) identify the effectiveness of the didactic design developed for the learning obstacle solution. This study used a qualitative method with a Didactic Design Research (DDR). Involves 41 students who have received Differential Calculus courses and 43 second-semester students at one of the universities in Indonesia. Data from test results, interviews, and document studies were analyzed through identification, clarification, reduction, and verification techniques and presented narratively. The results showed that: 1) students were still experiencing epistemological, didactical, and ontological types of conceptual and instrumental learning obstacles; 2) the development of a didactic design based on learning obstacle findings refers to the Theory of Didactical Situation combined with the Socratic Questioning technique; 3) the alternative didactic design is quite effective in overcoming the initial learning obstacle, but a new obstacle appears, namely the ontological psychological type obstacle. The results of this study indicate that the phenomenon of learning obstacle findings and the developed didactic design can be used to improve the quality of learning on an ongoing basis.

Keywords: higher-level derivatives, didactic design, learning obstacles, prospective teachers

INTRODUCTION

There are problems in mathematics, physics, and many other branches of science that cannot be solved by ordinary geometry and algebra, so calculus is needed to solve them (Rohde et al., 2012). In addition, derivatives are one of the mathematical concepts in college needed to learn other concepts, subjects, or applications to solve real-world problems (Tall, 2012; Pepper et al., 2012; Tarmizi, 2010).

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In the Differential Calculus course, the derivative is an essential prerequisite for several other concepts. In the Study Program that produces prospective mathematics teachers at one of the universities in Indonesia, Differential Calculus is a subject that is a prerequisite for several other courses. Puspita et al. (2020), in their research, concluded that there was a significant correlation between the achievement of the Differential Calculus course and the student's cumulative achievement index.

Various things related to learning Differential Calculus have become interesting studies for researchers. Many researchers investigate various causes of difficulties and topics in calculus that are considered problematic by students. For example, most undergraduate students still think derivatives are difficult to learn (Willcox & Bounova, 2004; Tarmizi, 2010; Pepper et al., 2012; and Tall, 2012). These findings strengthen the results of research that focus on the difficulties experienced by students in several sub-topics of derivative functions. For example, students still have difficulty in determining the derivative of rational functions and derivatives of functions using chain rules (Tokgöz, 2012), determining the extreme values of functions (Fatimah & Yerizon, 2019). In addition, students are still experiencing difficulties learning the concept of limit, a prerequisite for function derivatives (Kim et al., 2015; Wahyuni, 2017; Fatimah & Yerizon, 2019; Arnal-Palacián & Claros-Mellado, 2022). The lack of conceptual understanding (Tall, 2012; Denbel, 2015; Orton, 1983b, Dahlia et al., 2018), there is a relationship between difficulties with mathematical thinking processes and the complexity of mathematical objects (Quezada, 2020), and the level of student ability (Gray & Tall, 1991) are suspected to be the cause of the difficulties experienced.

The difficulties experienced by students, as revealed from the various studies above, are also expected to be experienced by prospective mathematics teacher students. Therefore, studying the difficulties experienced by prospective mathematics teachers is necessary. In turn, they will spearhead the success of a learning process later when they become teachers. In this study, learning constraints will be a guide in identifying students' learning difficulties; This means that the impact of didactic design on the knowledge construction of prospective mathematics teacher students will be the focus of the study. Furthermore, knowing the learning obstacles of prospective mathematics teachers will assist lecturers in developing a hypothetical didactic design to overcome learning obstacles.

The research will focus on prospective mathematics teacher students, particularly student learning obstacles on high-level derivative topics. This topic is important to study because students must master many concepts, including function derivatives, derivatives of the quotient of two functions, sequence patterns, factorial ideas, and exponential properties. Furthermore, Students must master the concepts they have acquired before high-level derivative concepts to make it easier to solve high-level derivative problems, especially the n^{th} derivative pattern. The objectives of this study are 1) to identify various types of learning obstacles experienced by prospective mathematics teacher students on high-level derivative topics; 2) to design alternative didactic designs based on the findings of learning obstacles; 3) to identify the effectiveness of the didactic design developed for the learning obstacle solution.

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THEORETICAL FRAMEWORK

If you pay attention, the causes of various student learning difficulties revealed in previous research could come from internal or external factors. Difficulties caused by external factors are referred to as obstacles; if they are associated with learning, then the difficulties are called learning obstacles. According to the Theory of Didactical Situation (TDS), an obstacle is a learning obstacle caused by external factors (Brousseau, 2005). Strictly speaking, Suryadi (2019) said that the external factor that could cause obstacles was didactic design. According to the source, there are three types of learning obstacles, namely: ontological obstacles, didactical obstacles, and epistemological obstacles (Brousseau, 2002).

According to the Theory of Didactical Situations (TDS), learning has a tiered flow that allows students to construct knowledge. The learning flow starts from an action situation, a formulation situation, a validation situation, and ends with an institutionalization situation. This learning flow is in line with the results of Obreque & Andalon's (2020) research, which concluded that most teachers understand mathematics as a priori knowledge, which requires action to discover, interpret, and formalize it. Knowledge can be constructed using the Socratic Questioning Technique at each stage of the TDS. Learning is guided by questions posed to promote students' independent thinking. The Socratic Questioning technique has six categories of questions in learning, which allows students to know the importance of questions, clarify, investigate assumptions, investigate reasons, investigate alternative solutions, and investigate answer implications (Paul, 1990).

Several studies concluded that the Socratic Questioning Technique could facilitate teachers and students to get the best results. For example, Cojocariu & Butnaru (2014) concluded that using the Socratic Questioning technique in learning can facilitate teachers to provoke students to be directly involved in learning. Higher-order thinking skills occur when students think, discuss, debate, evaluate, and analyze concepts through their thinking and those around them (Elder & Paul, 2007; Roger D. Jensen Jr., 2015).

METHOD

The qualitative method chosen in this study was the Didactical Design Research (DDR) design. DDR started to develop in 2010 (Suryadi, 2019), which explores the characteristics of learning design and its impact on the development of students' thinking processes (Fuadiah et al., 2019). At the same time, Sidik et al. (2021) said that DDR is a form of educational innovation. The paradigm used in qualitative research is the interpretive paradigm (Suryadi, 2019; Denzin & Lincoln, 2018; Creswell, 2014). The interpretive paradigm underlies researchers in understanding didactic design problems of high-level derivative topics in textbooks, which are references for Differential Calculus courses. The research data is sourced from the Respondent's Ability Test (RAT), the results of interviews, and document studies. The research data is sourced from the Respondent's Ability Test (RAT), the results of interviews, and document studies. The data from the RAT on learning obstacles was carried out qualitatively. The analysis was carried out simultaneously

through data reduction techniques and presented narratively. In addition, interviews and document studies were conducted to strengthen the analysis of RAT results.

The learning obstacle findings are then used to design alternative didactic designs on high-level derivative topics. The critical paradigm is the basis for designing the intended alternative didactic design. The didactic design developed is based on the Didactic Situation Theory with four stages of situations, each step guided by the Socratic Questioning Technique. Prospective mathematics teachers became the target of a didactic design trial developed based on the findings of the learning obstacle. Finally, based on the trial results data, the effectiveness of the didactic design will be seen in overcoming the findings of the identified learning obstacles.

The grouping of research participants are: 1) to find out the initial learning obstacle, RAT was given to 41 prospective mathematics teacher students who had received a differential calculus course after this, referred to as group one, and 2) after the didactic design was developed based on the findings of the leaning obstacle, alternative didactic designs were tested on students who were attending lectures Differential Calculus has as many as 43 people after this, referred to as group two. The trial aims to see the effectiveness of the didactic design in overcoming learning obstacles. The analysis will focus on seeing which learning obstacles can be overcome, which learning obstacles still arise, and the possibility of new learning obstacles emerging. Things that need to be known further were conducted by interviewing several students as an effort to clarify and carried out after the researchers gave tests related to high-level derivative topics. In simple terms, the stages of DDR are presented in Figure 1 below:

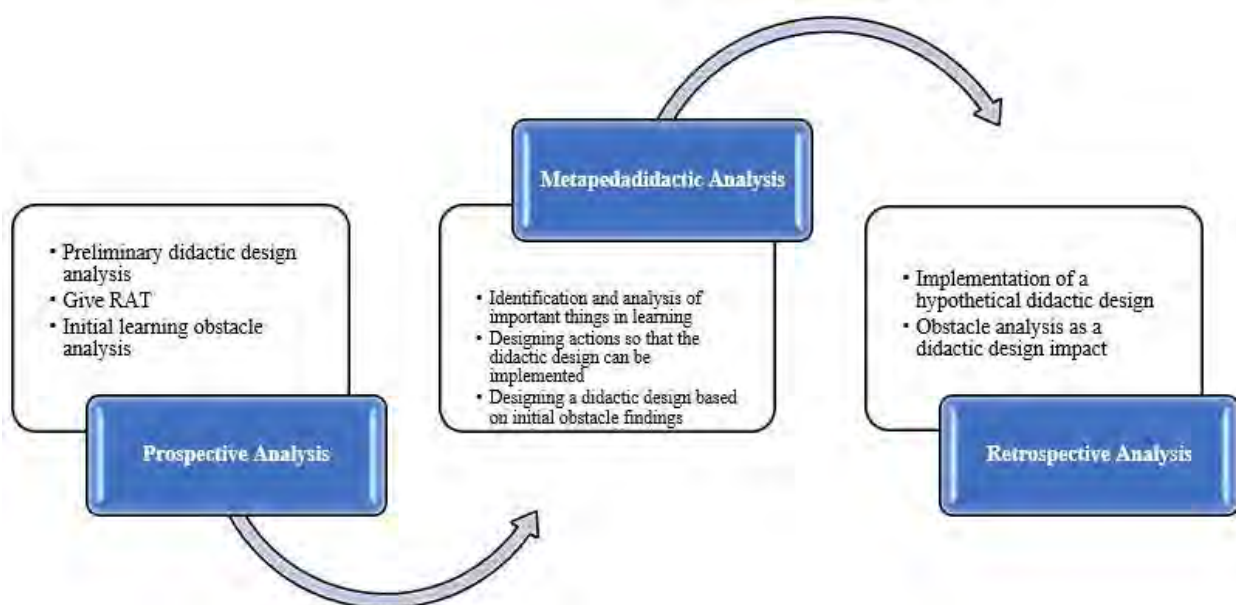


Figure 1: Stages of DDR

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RESULTS

The ability of students to determine the higher-order derivative of a function depends on their understanding of various rules for finding the first derivative of a function. Students often have problems determining a higher-level derivative because they do not master the rules for finding derivatives, especially the derivative of the quotient of two functions. The reference book presents the concept of higher-order derivatives after the derivation search rules for different types of functions. In the sourcebook, examples and problems related to higher-order derivatives are generally for simple functions, one of which is a polynomial function. When high-level derivatives are associated with other concepts, for example, the derivatives of rational functions, exponential properties, sequence patterns, or others, students often experience difficulties. Based on these problems, the authors designed a test called the Responsive Ability Test (RAT) has the intention of identifying various learning barriers experienced by students in high-level derivative concepts, particularly determining the n^{th} derivative pattern of rational functions. The problem on the RAT is "Find the n^{th} derivative of $y = \frac{1}{x^2}$, present it in the simplest form!".

From the findings of the learning obstacle, a didactic design was developed to overcome these problems. Furthermore, the implementation of the didactic design aims to see its effectiveness in solving the problem of learning obstacles. Therefore, in addition to analyzing the solution to the problems found by the initial obstacle, it will also look at the possibility of the emergence of new obstacles, which will be the basis for further revision of the didactic design. The following sections describe the analysis of research findings for each research objective.

Findings of early learning obstacles

Group one students generally can determine the 1st, 2nd,, n^{th} derivatives of the given function. However, students still have difficulty determining the n^{th} derivative formula in the simplest form. Students can already use exponential properties from the answers given, so the power rule determines the function's derivative for various levels. Although some students use the derivative of the quotient without using the exponential property, the mistakes made are more than those using the exponential property.

The following figure presents some student answers regarding the derivative of a function and the general form of the n^{th} derivative:

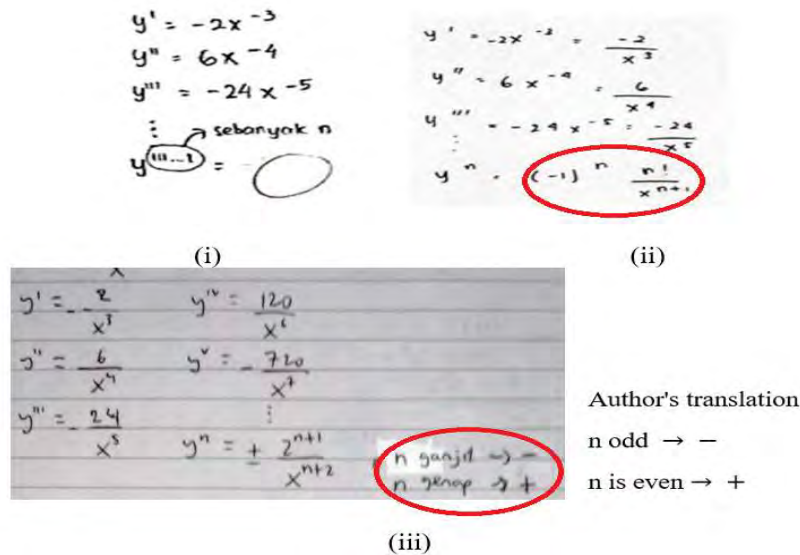


Figure 2: Student answers regarding high-level derivatives

From the answers given, some of the students' difficulties in determining the identified n^{th} term can be grouped as follows: 1) students cannot write the n^{th} term pattern (Figure 2. part (i)); 2) students can write the pattern of changing signs well, but an error occurs in determining the pattern by utilizing the factorial concept (Figure 2. part (ii)); 3) students cannot determine the pattern of changing signs appropriately, only writing positive or negative signs when n is odd or even (Figure 2 part (iii)).

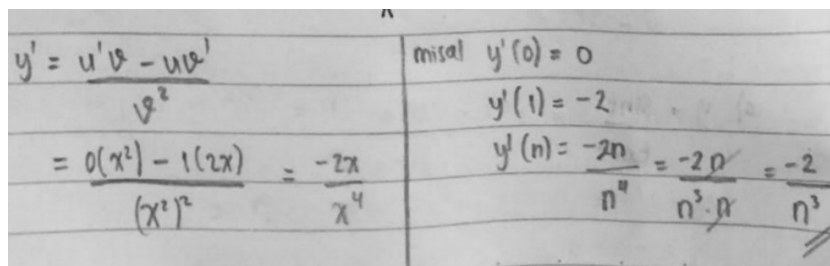
The difficulties experienced by these students can be categorized as obstacles, with the following details:

- Students experience an epistemological type obstacle because of the limited understanding and mastery of high-level derived concepts associated with their habits. For example, students are used to high-level derivatives for simple functions and never determine the pattern of the n^{th} derivative in a simple form; existing lecture notes and sourcebooks reveal these problems.
- Students experience didactic-type obstacles caused by the existing didactic designs that are not following the continuity of students' thinking processes. In the sourcebook, it only asks students to determine the 1st, 2nd, ... and not asked to determine the general form of the n^{th} derivative. The functions given are generally simple functions, for example, polynomial functions. More complex functions, one of which is a rational function, are rarely used for the concept of higher-order derivatives.
- Students experience the ontological obstacle of the instrument type; students cannot solve the problem entirely because they do not master technical matters. The technical thing in question is to determine the pattern of the sequence using the factorial concept.

To strengthen the findings of the obstacle, the authors conducted semi-structured interviews with one of the first group participants. The results of interviews with students reinforce the findings of

these obstacles. The results of the interviews provide the following information: 1) Students cannot see the relationship between the 1st, 2nd, 3rd, and so on with the resulting exponents of -3, -5, -7, etc.; 2) Students cannot see patterns 2, 6, 24, and so on as regular patterns when associated with the factorial concept; 3) Students have difficulty presenting the pattern of alternating signs -, +, -, +, ... as a result of the 1st, 2nd, 3rd, etc.

In addition to these obstacles, students are also still experiencing conceptual type ontological obstacles. For example, students do not understand the concept of the quotient rule and cannot distinguish the nth derivative from the value of the derivative of a function at time n. This is revealed from student answers as follows:



The image shows handwritten mathematical work on lined paper. On the left, the quotient rule is applied to $y = \frac{0(x^2) - 1(2x)}{(x^2)^2}$, resulting in $y' = \frac{-2x}{x^4}$. On the right, a pattern for the derivative is shown: $y'(0) = 0$, $y'(1) = -2$, and $y'(n) = \frac{-2n}{n^4} = \frac{-2n}{n^3 \cdot n} = \frac{-2}{n^3}$.

Figure 3 : Examples of conceptual type ontological obstacles

Based on the results of the analysis of various identified learning obstacles indicates the need for an alternative didactic design designed to overcome these learning obstacles.

Didactic design based on learning obstacle findings

The author designs a didactic design to overcome student learning obstacles. The didactic design developed on high-level derivative topics allows students to construct knowledge of the concept. The didactical design enables students to determine the nth derivative and facilitates them to build the nth derivative pattern. In addition, the stages in didactic design encourage students to relate the concepts they have learned to other concepts, such as the concept of sequences, exponential properties, and factorial concepts. The didactic design developed includes 1) topics and sub-topics, 2) predictions of student responses, 3) didactic and pedagogical anticipation, and 4) developed mathematical objects and abilities.

Topic and sub-topic components contain concepts that will be discussed and will become students' learning objects. Finally, the predictive element of student responses has various estimates of student responses that will appear during the learning process; this will be useful for lecturers in preparing various anticipations. Various student response predictions that the author identified are: being able to determine the nth derivative for all types of functions; can determine the nth derivative of a polynomial function but experiencing constraints for rational functions; experiencing problems in determining the nth derivative and the general form of the nth derivative of trigonometric functions or simple rational functions; be able to determine the nth derivative of trigonometric functions or simple rational functions but experience problems when determining

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the general form of the n th derivative of these functions; can apply the concept of high-level derivatives in solving a given problem.

The didactic and pedagogical anticipation components contain the stages of learning that follow the TDS stage. The TDS stages in question are: 1) action situations, in the form of presenting problems so that it helps students determine high-level derivatives, understand the meaning of function derivatives, and determine the general form of the n th derivative of several types of functions; 2) formulation situations, students are directed to the formation of an understanding of high-level derivative concepts; 3) validation situations, if several students have different formulations or even come up with erroneous constructions, then a validation process is needed that allows improvements or reinforcement of the concept to be made; and 4) the institutionalization situation, at the end of the TDS design stage closed with problems, aiming to see students' abilities in applying high-level derivative concepts to other problems in different contexts. Through the Socratic Questioning Technique, students are guided in understanding problems, formulating, and validating derivatives for various levels of rational and trigonometric functions. The stages continue until students can determine the general pattern of the n th derivative of the given function.

The didactic design developed by the researcher based on the findings of the initial learning obstacle contains four components. The topic components discussed are high-level derivatives; the student response prediction component contains various student response predictions, as explained in the second paragraph of this section. The other two components, namely the didactic and pedagogical anticipation components, contain various stages of TDS, while the components of the mathematical objects developed are high-order derivatives (including polynomial functions, trigonometric functions, and rational functions), general forms of high-order derivatives, and high-order derivative applications. The didactic anticipation and pedagogical design components contain situation stages in TDS, including (a) action situations in the form of problem presentations related to high-level derived notations and meanings that stimulate participants to think; (b) the formulation situation in the form of efforts made by researchers to lead to the formation of an understanding of high-level derivative concepts; (c) the validation situation contains case examples in the form of a rational function that encourages participants to carry out validation at various stages of determining the derivative and its n th derivative pattern; (d) the institutionalization situation in the form of presenting various problems that encourage participants to apply high-level derivative concepts.

The Effectiveness of Alternative Didactic Designs on Learning Obstacle Solutions

After the implementation of the didactic design in group 2 learning, the didactic design's effectiveness will be analyzed. For example, will it be investigated which early learning obstacles are overcome, which learning barriers are still emerging, or are new learning barriers emerging?

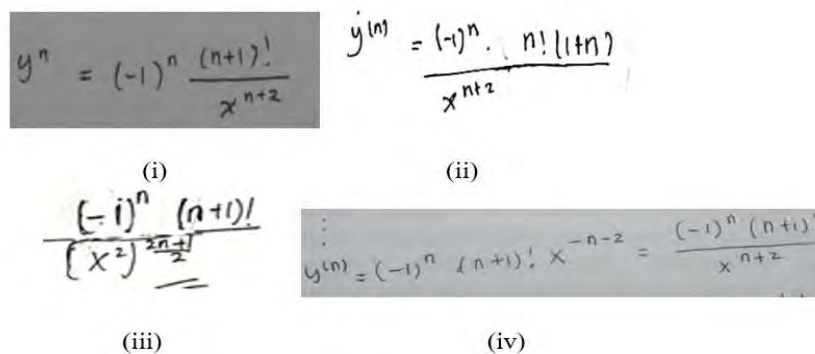
After implementing the alternative didactic design, it was found that most of the learning obstacles could be overcome properly. However, the findings indicate that there are still unresolved obstacles. The initial constraint findings still emerge after the implementation of the didactic

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design, although in a slightly different form. These problems will be considered in making improvements to the didactic design. This process continues as an effort to improve the quality of learning on an ongoing basis. In detail, the analysis results after implementing the developed didactic design are presented below.

The developed didactic design has overcome various obstacles in group one. Students' answers in group two corroborate these findings. For example, some of the results related to determining the simplest form of the n^{th} derivative are:



(i) $y^n = (-1)^n \frac{(n+1)!}{x^{n+2}}$

(ii) $y^{(n)} = (-1)^n \cdot \frac{n!(1+n)}{x^{n+2}}$

(iii) $\frac{(-1)^n (n+1)!}{(x^2)^{n+1}}$

(iv) $y^{(n)} = (-1)^n (n+1)! x^{-n-2} = \frac{(-1)^n (n+1)!}{x^{n+2}}$

Figure 4: Student answers after the implementation of the didactical design

Associated with problems related to the initial obstacles indicated, an analysis will be carried out on the various answers given after the implementation of the didactic design. From various student answers, the authors conclude that 1) Students can determine patterns by utilizing the factorial concept in the general form, namely $(n + 1)!$, even in a different form (flexibility) by presenting it in the form of $(n + 1)n!$; 2) Students already understand the concept of a sign change sequence and present it in a general form, and can even give it in a different condition.

Students can generally determine the available form of the n^{th} derivative of the given problem. This is reinforced by the answer of one of the students who not only correctly determined the general form of the n^{th} derivative but provided detailed processing steps. This phenomenon illustrates that students completely master the concepts of quotient derivatives, power derivatives, factorial concepts, and the concept of sign pronouns. The following picture presents student answers regarding the problem in question:

cara polanya.

1. Pola untuk Pembilang suang Selang (negatif, positif, negatif, dst) $(-1)^n$
2. Pola untuk 2, 6, 24 pada Pembilang adalah $(n+1)!$
3. Pola untuk x^3, x^4, x^5 (pada penyebut) adalah x^{n+2}

maka Turunan ke n adalah

$$y^{(n)} = \frac{(-1)^n \cdot (n+1)!}{x^{n+2}}$$

Author's translation:

How to determine the pattern:

1. The pattern for alternating numerators (-, +, -, +) is presented in the form $(-1)^n$
2. The pattern for 2, 6, and 24 in the numerator is $(n + 1)!$
3. The pattern for $x^3, x^4, x^5 \dots$ (in the numerator) is x^{n+2}
4. Then the nth derivative is $y^{(n)} = \frac{(-1)^n (n+1)!}{x^{n+2}}$

Figure 5: Evidence of complete mastery of derivative concepts

Although the didactic design has overcome most obstacles, some students still have difficulty determining the pattern of alternation of signs in a general form. For example, some students only associate the change of sign with the type of the n^{th} derivative; the n^{th} derivative will be negative when n is odd and positive when n is even. In addition, some students only mention the sign of the n^{th} derivative, which will alternate negative and positive. The following student answers reveal the problem:

$$\frac{y^n = (n+1)!}{x^{n+2}} \Rightarrow \begin{cases} n \text{ ganjil, maka } y^n < 0 \\ n \text{ genap, maka } y^n > 0 \end{cases}$$

(i)

faktorial dan selang selang positif negatif

x pangkat nya bertambah 1 dari sebelumnya

(ii)

Author's translation

<p>n is odd, then $y^n < 0$</p> <p>n is even, then $y^n > 0$</p> <p>(i)</p>	<p>factorial and sign alternating positive negative</p> <p>x is increased to the power of one from before</p> <p>(ii)</p>
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Figure 6: Examples of psychological-type ontological learning obstacles

The writer categorizes this difficulty as an ontological obstacle of psychological type because mentally, students are not yet ready to receive knowledge. The students' weak desire to determine the line pattern without considering the possible value of the variable x indicates the student's unpreparedness to obtain knowledge.

The findings reveal that research participants still experience various learning obstacles in high-level derivative concepts. The causes of these learning obstacles stem from external factors, namely mental readiness to receive knowledge, the applied didactic design, and the limited context

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that students have. The obstacle experienced by prospective mathematics teacher-students is focused on the concept of high-level derivatives, especially determining the pattern of the n^{th} derivative. A weak understanding of rational function derivatives, factorial concepts, exponential concepts, and the concept of a sign-changing sequence is allegedly the cause of the obstacle. The results of the study are in line with the results of the study: 1) Tokgoz (2012), students still have difficulty in the concepts of rational function derivatives and function derivatives using the chain theorem; 2) Tarmizi (2010), Tall (2012), Pepper (2012), Hashemi (2014), and Dahlia et al. (2018), understanding of derivative concepts is still weak and is a concept that is considered difficult by students; 3) Orton (1983), still found fundamental errors in the concept of derivatives from students majoring in mathematics.

The study also concluded that students experienced learning obstacles included in the Didactical obstacle category. The initial didactic design did not accommodate students in constructing the general form of the n^{th} derivative. The function given to the concept of higher order derivatives was generally simple (eg a polynomial function), and no examples asked students to determine the n^{th} sequence pattern. The author suspects the initial didactic design did not provide students sufficient experience constructing the n^{th} derivative pattern. Learning obstacles experienced by students can be a consideration for lecturers in designing didactic designs. The didactic design developed can facilitate students to construct knowledge without experiencing significant obstacles. This statement is in line with the opinion of 1) Amzat et al. (2021), who said that the task of educators is not only to educate but also to develop a curriculum, including preparing didactic designs; 2) Arnal-Palacian & Claros-Mellado (2022), who concluded that the teaching method was one of the causes of the difficulties experienced by students.

CONCLUSIONS

Some of the prospective mathematics teachers in group one still experience learning difficulties in high-level derivative concepts. The various types of learning obstacles found were: 1) epistemological type obstacles, due to limited understanding and mastery of high-level derived concepts associated with their habits; 2) didactic learning obstacles arise because the didactic design does not follow the flow of students' thinking processes, 3) the instrument-type ontological obstacle, the student cannot solve the problem entirely because they do not master technical matters related to the sequence pattern using the factorial concept. The findings of learning obstacles experienced by prospective mathematics teacher students in determining the n^{th} derivative pattern are caused by a weak understanding of the concept of rational function derivatives composite. Besides that, it also indicated that the learning barriers experienced are caused by the understanding of the concept of rational function derivatives which is still low, this is in line with the research of Tarmizi (2010), Tall (2012), Pepper (2012), Hashemi (2014), and Dahlia et al. (2018). After the implementation of the didactic design, which was developed based on the findings of the obstacles in group one, the results showed that most learning obstacles could be overcome properly. Therefore, the researcher concludes that the didactic design developed effectively overcame the learning obstacle problem. However, initial obstacles still arise after the implementation of the didactic design, although with a slightly different form. These problems will

be taken into consideration to make improvements to a better didactic design. The lecturer has to develop a didactic design that can overcome various student learning obstacles because the lecturer, apart from being an educator, also acts as a curriculum developer, one of which is developing a didactic design; this statement is in line with Amzat et al. (2021).

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