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EXAMINATION OF DIAGRAMMATIC REPRESENTATION AND VERBAL PROBLEM-SOLVING REPRESENTATIONS OF PRIMARY SCHOOL STUDENTS

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Abstract

This study aimed to examine the diagrammatic representation skills and problem-solving performances of students according to their problem-solving representations. A cross-sectional survey design using quantitative methods was used in this study. The sample consisted of 31 second-grade and 41 third-grade students from a public primary school in Turkey. The Diagrammatic Representation Test and Mathematical Operations Test were used in this study. The data were analyzed with descriptive statistical analysis, the chi-square test, the independent samples t-test, discriminant analysis and logistic regression analysis. The findings indicated that while the preferred types of representations for solving verbal problems and problem-solving performance did not vary significantly based on grade level, scores obtained from the diagrammatic representation test exhibited significant differences. It was observed that students' problem-solving performance and diagrammatic skills could predict their preferred types of representations for solving verbal problems. Consequently, students who possess knowledge regarding effective representation preferences, as well as the ability to construct and utilize them, are more likely to generate appropriate and high-quality representations, leading to accurate problem-solving outcomes. This, in turn, enhances their performance in diagrammatic representation tasks.

Keywords: Diagrammatic representation, pictorial representation, schematic representation, symbolic representation, verbal problem solving.

INTRODUCTION

People use various mechanisms to understand reality, including representing their understanding of it in their mind or a non-mind environment. Representations are a means of simplifying, reducing, and making complex concepts more understandable. They provide opportunities to support and monitor one's thinking and help others understand how information is processed. In this context, representations become a significant structure to demonstrate what students think and know (Diezmann & Lowrie, 2009).

Representation is a crucial skill for students learning mathematics, as it is one of the mathematical processes defined in the primary school mathematics curriculum, along with problem-solving, reasoning and proof, and communication. Therefore, in mathematics lessons, teachers are expected to guide students to integrate meanings across verbal, visual-spatial, embodied, and symbolic representations (Tytler et al., 2023). Students should be encouraged to choose representations to solve problems, switch between representations, and use multiple representations to develop representation skills (Lowrie, 2020). Studies have shown that the types of representations used to solve a problem can affect problem-solving success and provide insights into the effectiveness of internal (imagery-based) and external (diagrammatic and analytical-based) representations (e.g., Blazhenkova & Kozhevnikov, 2009; Kozhevnikov et al., 2005). External representations are particularly useful in problem-solving when a new problem is encountered.

The Role of Diagrams in Problem-Solving:

External representations, such as diagrams, can be a valuable strategy for solving word problems in mathematics. More importantly, diagrams allow students to think and solve problems in new ways. A diagram does more than visualize a problem; it is an essential tool for solving it. Creating diagrams to solve mathematical problems can help students in various ways (Stylianou, 2010). Early in the process, diagrams can be used to record information about the problem during the resolution process. Once the student begins conceptualizing the problem, diagrams can be a tool to explore alternative ways of understanding the problem. Even if a solution is found, diagrams can monitor and evaluate the solution. Traditionally, verbal analysis has been the primary method of solving mathematical problems. However, diagrams offer a new perspective that can lead to a deeper understanding and alternative problem-solving strategies.

Accurately and flexibly representing diagrams and other visual representations is critical to success in problem-solving, as pointed out by Lowrie (2020). Additionally, research has shown that adding diagrams related to the problem can improve learning and have cognitive benefits, as highlighted by studies by Mayer (1989, 2005). However, it is important to note that forming various diagrammatic reasoning theories and using diagrams depends on the problem at hand and the ability to solve it, according to Acevedo Nistal et al. (2009). Unsuitable representations presented to the student may not support the cognitive processing required to solve the problem, which is unlikely to help the student. Therefore, it is crucial to choose appropriate representations while solving problems to increase problem-solving success. Studies suggest that diagrams play a crucial role in problem-solving by enabling students to record information, explore alternative ways of understanding the problem, and evaluate their solutions, as demonstrated by Gültekin & Altun (2022) and Ertuna & Toluk-Uçar (2021). To summarize, diagrams play a key role in problem-solving, allowing students to visualize complex concepts and explore alternative solutions. Hence, educators should encourage using diagrams and other visual representations in problem-solving activities to promote deeper understanding and enhance students' cognitive abilities.

Diagrammatic Representations in Mathematics Teaching:

In mathematics teaching, diagrams are a frequently used tool in Singapore (Beckmann, 2004) and Japan (Murata, 2008), two countries known for their superior mathematics achievement compared to global standards (National Centre for Education Statistics, 2003; Lowrie, 2020). The purpose of diagrams is not to assist students in performing operations but to aid in selecting which operations to use and to comprehend why they are conceptually appropriate (Beckmann, 2004). Using diagrams at an early age helps students solve more complex problems later on and enhances their problem-solving abilities (Booth & Koedinger, 2012).

Teaching activities that incorporate diagrammatic representations in problems support young children who may struggle with representing problems accurately and independently or those who have low problem-solving skills (Booth & Koedinger, 2012). The use of diagrams plays a vital role in problem-solving activities, enabling students to visualize complex concepts, explore alternative solutions, and evaluate their solutions.

Using diagrams in mathematics teaching is an effective way to engage students in problem-solving activities and enhance their cognitive abilities. Moreover, research in cognitive science literature suggests that mental imagery and gestures also play a crucial role in problem-solving similar to diagrams (Tian et al., 2017). Incorporating diagrams in mathematics teaching can help students decide which operations to use, understand the conceptual basis of mathematical concepts, and solve more complex problems. Therefore, educators should encourage using diagrams and other visual representations in mathematics teaching to help students develop a deeper understanding of mathematical concepts and improve their problem-solving skills (Hatisaru, 2020).

The ability to produce accurate and effective diagrammatic representations is a significant skill for students, especially in the early stages of mathematical learning. Using concrete experiences and visual representations, primary school students can better understand abstract mathematical concepts. Research suggests that students who can produce correct visual-schematic representations are more likely to successfully solve verbal problems (Hegarty & Kozhevnikov, 1999; Lowrie & Kay, 2001). In contrast, students who use poorly constructed schematic or pictorial representations are more likely to produce incorrect solutions to verbal problems (Boonen et al., 2014). Therefore, it is crucial for students to be exposed to various diagrams and different forms of representation in their early years of education (van Garderen et al., 2013). It is recommended to focus on developing diagrammatic representation skills in primary school students to enhance their problem-solving abilities and deepen their understanding of mathematical concepts. This study is crucial because it highlights the role of diagrams and visual representations in helping students understand and solve mathematical problems. Studies have shown that the use of diagrams and other visual representations can enhance students' cognitive abilities and improve their problem-solving skills (Surya et al., 2013; Davenport et al., 2008). By encouraging students to use diagrams to represent mathematical problems, educators can help them develop a deeper understanding of mathematical concepts and approach problems in new ways. Moreover, studies suggest that students who can produce correct diagrammatic representations are more likely to solve verbal problems successfully, underscoring the importance of acquiring these skills at an early age (ex: van Garderen et al., 2013). Thus, this study emphasizes the significance of incorporating diagrams and visual representations into mathematics teaching and problem-solving activities. By doing so, educators can help students become more confident and successful problem solvers, which is critical for their academic and future professional success. It is important to conduct this study as it can provide valuable insights into the relationship between diagrammatic skills, problem-solving performance, and preferred representation types among early childhood education students. This information can be used by educators and policymakers to develop more effective teaching methods and learning materials that can enhance students' cognitive abilities and problem-solving skills. This study can contribute to the existing literature on the importance of diagrammatic representations in mathematics and problem-solving education.

This study aims to examine the diagrammatic representation skills and problem-solving performances of primary school second and third-grade students according to their problem-solving representations. This study seeks to answer the following research questions:

- ✓ What types of representations and diagrammatic representation skills are preferred by second and third-grade students?
- ✓ Do the types of representations students prefer differ according to their grade levels?
- ✓ Do students' problem-solving performances and diagrammatic skills predict the types of representation students prefer to solve verbal problems?
- ✓ Do students' preferred representation types, problem-solving performances, and diagrammatic test scores predict their grade level?

METHOD

This study aimed to investigate the impact of diagrammatic skills and problem-solving performance on preferred representation types and grade levels among second and third-grade students in early childhood education at the start of primary school. For this purpose, a cross-sectional survey model, which is a quantitative method, was utilized. In such models, the data collection process is carried out once, and data collection is similar to taking pictures of the participants at any given moment (Metin, 2014). In this regard, this current study is limited with data quantitative in nature and conducted as a quantitative cross-sectional survey study (Fraenkel & Wallen, 2003).

Participants

The research data were collected in Turkey in the autumn term of the 2021-2022 academic year when face-to-face education resumed after the distance education period. This was a crucial period for students to adjust to being back in school, particularly as many parents were hesitant to send their children to school due to the pandemic. Consequently, obtaining consent from teachers, parents, and principals proved challenging, resulting in a limited sample size. To ensure accessibility and ease of participation, a convenience sampling method was used, meaning that participants were chosen based on availability and proximity to the researchers. No other criteria were considered, except for the school's willingness to participate in this study. Therefore, the socio-economic background of the school may not necessarily align with the researchers' preferences, although it is significant to note that the participating school had a middle socio-economic status. It is also worth noting that students with different mother tongues were in separate classes and did not participate in this study, thereby ensuring that students' reading-writing levels and reading comprehension development were age-appropriate. The study included 31 second-grade students (17 girls and 14 boys) and 41 third-grade students (25 girls and 16 boys) from two randomly selected branches in each grade. The students' ages ranged between seven and eight years, with an average age of seven years and seven months.

The decision to conduct this study with 2nd and 3rd-grade students has been based on several factors. Firstly, at this age, children are expected to have developed basic diagrammatic representation skills and problem-solving abilities, which can be further developed with appropriate instruction and practice. Secondly, younger children may be more receptive to learning new skills and strategies, making it easier for them to learn and apply new diagrammatic representations to solve problems. Finally, by focusing on two grades, the researchers may be to more closely examine the developmental progression of diagrammatic representation skills and problem-solving abilities over time. Research has shown that early mathematical skills are strong predictors of later academic success, and these skills are developed during the early elementary school years. Second and third-grade students are at an age where they are developing their diagrammatic representation skills and are becoming more aware of their preferred problem-solving representations (Newcombe & Frick, 2015). Therefore, studying these students can provide insight into how they use these representations and how their problem-solving performance is related to their diagrammatic representation skills. The 4th grade was not included because the researchers wanted to focus specifically on the early stages of diagrammatic representation and problem-solving development, and the 4th grade was considered beyond that scope.

Data Collection Tools

The diagrammatic representation test used in this study consisted of 24 questions and was originally developed by Frick and Newcombe (2015) for the 4-8 age group. The test assesses students' ability to manipulate and understand visual-spatial information and includes geometric objects, geometric objects with straight black lines drawn on their edges, and color photographs of the drawings created by following the borders. The test is designed as horizontally aligned, four-choice cards centered at the top of the letter-size sheet (four lines of 21.6 cm) presented in the horizontal orientation. In this study, the test was administered to the students in its original form (Figure 1).

Figure 1. Example of diagrammatic representation test questions.

The Mathematical Operations Test (MIT), developed by Suwarsono (1982), consists of 10-word problems selected according to students' levels. The test has been used in many studies to evaluate the effectiveness of external representations with different age groups, including studies by Lowrie & Clements (2001), Lowrie & Kay (2001), Boonen et al. (2013), and Boonen et al. (2014). The solution choices in the test were classified into two categories: diagrammatic (pictorial, schematic) and analytical. Two coders were hired to classify the solutions, and the inter-rater reliability coefficient was calculated as .90 for all solution options throughout the test. This method of coding student solutions has been used in the research literature for over 30 years. Examples of questions in MIT are given below.

Mathematical Operations Test (MIT)

- Ahmet is taller than Meryem and shorter than Jale. Who is the tallest?
- The flying balloon first rose 20 meters from the ground. Then it moved 10 meters to the right, then fell 10 meters down. It then advanced 5 meters to the right and finally fell flat to the ground. How far is the balloon from the starting point?
- In an athletics competition, Ahmet is four steps ahead of Ayşe, and Ali is three steps behind Ahmet. How far ahead of Ayşe is Ali?

Data Collection

Before administering the tests to the students, preliminary preparations were made. For instance, a sample problem similar to those in the Mathematical Operations Test was presented to the students, and they were instructed to read the problem statement carefully. They were then asked to complete a table with the problem's purpose, estimation of the result, components, and given and requested information and provide solutions in their own words.

During the actual test, students were not given any instructions about whether or not they should use external representations. This allowed researchers to observe students' comprehension, reading and writing skills and problem-solving abilities.

Similarly, before administering the diagrammatic representation test, sample questions were studied to ensure that all students understood the task. A picture of a house and a child drawing the same house on paper was shown to the students. They were told that the child's name was Ali and that he enjoyed drawing. Next, the students were shown four different drawings of a ball and asked to choose which one they thought was drawn by Ali.

All of these activities helped ensure that the students understood the questions and were able to perform their best. This study, including the tests, preliminary information, and exercises, took two weeks to complete.

Data Analysis

When the data collected were transferred to Excel, the examination process began by coding the data obtained from the diagrammatic test as true or false and entering the scores obtained from the test. Then, the data obtained from the Mathematical Operations Test were coded as pictorially correct, pictorially incorrect, schematically correct, schematically incorrect, symbolically correct, and symbolically incorrect. These data were analysed using the Crosstab function in the SPSS 22 program. The students' correct answer rates in the diagrammatic test were analysed according to their grade level and the representation types they preferred. The students' preferred representation types were reduced to three groups (pictorial, schematic, and symbolic) using the dispersion interval method of Sevimli (2013), Taşova (2011), Krutetski (1976), and Galindo-Morales (1994). The distribution interval is the difference between the highest and lowest scores obtained from the test divided by the number of groups. Students' answers to the questions were coded as 0, 1, or 2 points, depending on the type of representation used (pictorial, schematic, or symbolic). The maximum score on the test was 20, and the minimum score was 0. According to the distribution range, those with 0-6 points were coded as using pictorial representation, 7-13 points were coded as using schematic representation, and 14-20 points were coded as using symbolic representation. The students' preferred types of representation were then examined according to their grade level using the chi-square test. The independent samples t-test was used to analyse whether the students' scores from the diagrammatic representation test and their problem-solving performance changed according to their grade level. Discriminant analysis was conducted to determine how well students' problem-solving performances and diagrammatic skills predicted the types of representations they preferred to solve verbal problems. Logistic regression analysis was conducted to determine how much students' preferred representation types and their problem-solving performances and diagrammatic test scores predicted their grade levels (being a member of a second or third-grade cluster).

RESULTS

What types of representations and diagrammatic representation skills are preferred by second and third-grade students? To answer this question, descriptive statistics were calculated and the Crosstabs data results were examined. Table 1 shows the correct answer rates for the diagrammatic test according to students' grade levels and preferred representation types.

Upon examination of Table 1 according to the problems in the Mathematical Operation Test, the frequency distributions of the number of correct responses and the preferred representation types in the diagrammatic representation test of second and third-grade students are shown. In this regard, for the first problem, it was observed that out of a total of 43 students, 40 students used pictorial representation (with 40 correct and three incorrect answers), six students used schematic representation (with six correct answers), and 23 students used symbolic representation (with 17 correct and six incorrect answers).

In the pictorial representation category, for the 9-14 correct answer range, there was one student from the second grade, for the 15-19 correct answer range, there were eight students from the second grade and five students from the third grade. For the 20-24 correct answer range, there were seven students from the grade and 19 students from the third grade. In the pictorial representation category, for the 15-19 correct answer range, there were three students from the third grade who provided incorrect answers. In the schematic representation category, there were no students who provided correct answers in the 9-14 correct answer range, and for the 15-19 correct answer range, there were two students from both the second and third grades. For the 20-24 correct answer range, there was one student from both the second and third grades. There were no students who provided incorrect answers in the schematic representation category.

In the symbolic representation category, for the 9-14 correct answer range, there were four students from the second grade. For the 15-19 correct answer range, there were four students from the second grade and five students from the third grade. For the 20-24 correct answer range, there were four students from the third grade. In the symbolic representation category, for the 9-14 correct answer range, there were four students from the second grade who provided incorrect answers. For the 15-19 correct answer range, there was one student from the third grade. For the 20-24 correct answer range, there was one student from the third grade.

In general, when examined, it was observed that students who scored high on the diagrammatic test (with a score range of 20-24) generally solved the problems using pictorial and schematic representations. In addition to the fact that the number of symbolic incorrect responses was higher than the incorrect responses in other representation types, these students were seen to be in a moderate score range in the diagrammatic test. It can be stated that, according to the grade level, pictorial and schematic representations were more frequently used, and these students were the ones who scored high on the diagrammatic test.

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Table 1. The rate of correct answers to the diagrammatic test according to students' grade levels and preferred representation types.

Problem Representation Types		Pictorially Correct			Schematically Correct			Symbolically Correct			Pictorially Incorrect			Schematically Incorrect			Symbolically Incorrect		
Diagrammatic test correct answer range		9-14 Correct	15-19 Correct	20-24 Correct	9-14 Correct	15-19 Correct	20-24 Correct	9-14 Correct	15-19 Correct	20-24 Correct	9-14 Correct	15-19 Correct	20-24 Correct	9-14 Correct	15-19 Correct	20-24 Correct	9-14 Correct	15-19 Correct	20-24 Correct
Problem 1	2nd Gr	1	8	7	0	2	1	4	4	0	0	0	0	0	0	0	4	0	0
	3rd Gr	0	5	19	0	2	1	0	5	4	0	3	0	0	0	0	0	1	1
	Total	1	13	26	0	4	2	4	9	4	0	3	0	0	0	0	4	1	1
Problem 2	2nd Gr	0	5	3	0	0	1	1	4	2	3	1	2	1	0	0	3	3	0
	3rd Gr	0	2	0	0	0	2	0	7	13	0	1	3	0	0	1	0	8	4
	Total	0	7	3	0	0	3	1	11	15	3	2	5	1	0	1	3	11	4
Problem 3	2nd Gr	2	8	4	0	0	1	1	1	3	4	5	1	0	0	0	2	0	0
	3rd Gr	0	5	14	0	0	1	0	2	1	0	7	6	0	0	0	0	2	3
	Total	2	13	18	0	0	2	1	3	4	4	12	7	0	0	0	2	2	3
Problem 4	2nd Gr	0	3	1	1	8	6	2	1	1	0	0	0	4	0	0	2	2	0
	3rd Gr	0	1	3	0	8	17	0	5	3	0	0	0	0	1	0	0	1	2
	Total	0	4	4	1	16	23	2	6	4	0	0	0	4	1	0	2	3	2
Problem 5	2nd Gr	0	2	3	0	0	0	0	4	4	0	4	4	2	2	0	4	3	0
	3rd Gr	0	1	7	0	1	2	0	3	7	0	2	3	0	0	2	0	9	4
	Total	0	3	10	0	1	2	0	7	11	0	6	7	2	2	2	4	12	4
Problem 6	2nd Gr	0	4	5	0	2	1	2	3	1	2	2	0	2	1	1	3	2	0
	3rd Gr	0	4	13	0	1	1	0	4	3	0	5	5	0	1	1	0	2	1
	Total	0	8	18	0	3	2	2	7	4	2	7	5	2	2	2	3	4	1
Problem 7	2nd Gr	0	1	0	0	3	7	0	0	0	1	5	0	2	4	1	4	1	0
	3rd Gr	0	0	1	0	3	8	0	0	0	0	3	3	0	5	9	0	5	4
	Total	0	1	1	0	6	15	0	0	0	1	8	3	2	9	10	4	6	4
Problem 8	2nd Gr	0	0	2	0	6	3	0	1	0	1	1	3	3	4	0	3	1	0
	3rd Gr	0	0	3	2	17	0	0	0	0	0	2	1	0	5	1	0	7	3

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	Total	0	0	5	2	23	3	0	1	0	1	3	4	3	9	1	3	8	3
Problem 9	2nd Gr	0	4	2	0	0	4	0	0	0	2	0	1	0	6	1	4	4	0
	3rd Gr	0	1	9	0	4	3	0	0	0	0	7	8	0	1	1	0	3	4
	Total	0	5	11	0	4	7	0	0	0	2	7	9	0	7	2	4	7	4
Problem 10	2nd Gr	0	1	1	0	2	5	0	1	1	0	0	0	2	6	1	2	4	0
	3rd Gr	0	0	3	0	3	8	0	3	2	0	3	3	0	4	4	0	3	5
	Total	0	1	4	0	5	13	0	4	3	0	3	3	2	10	5	2	7	5
General	2nd Gr	3	36	28	1	23	29	10	19	12	13	18	11	16	23	4	31	20	0
	3rd Gr	0	19	72	2	39	43	0	29	33	0	33	32	0	17	19	0	41	31
General Total		3	55	100	3	62	72	10	48	45	13	51	43	16	40	23	31	61	31

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In total, based on the answers provided by the students, 720 verbal problem representation data points were obtained. Figure 2 provides a general distribution of these representations by grade level.

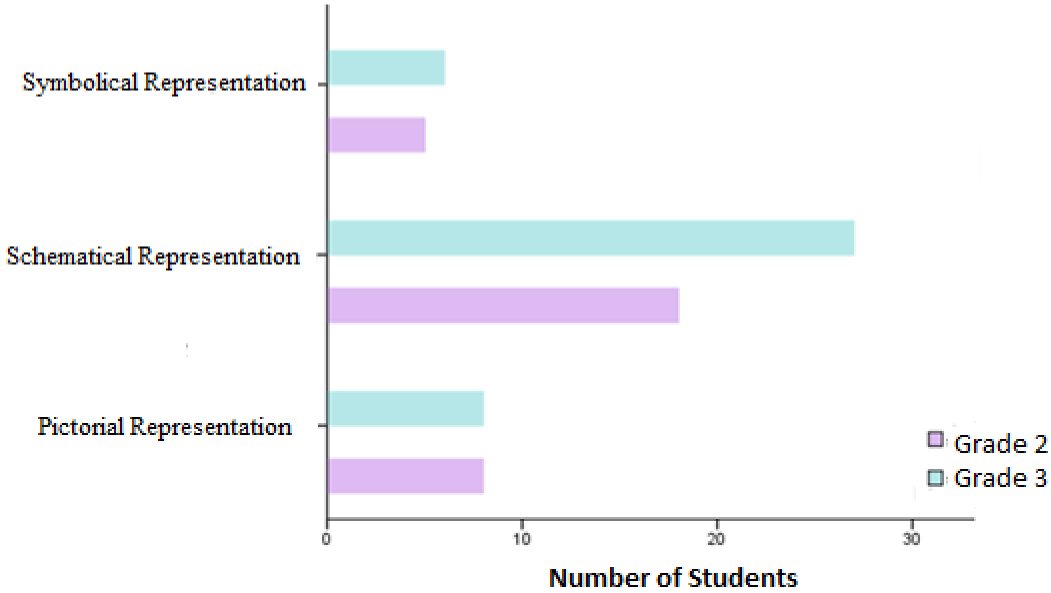

Figure 2. Types of representation preferred by students according to grade levels

Based on the data presented in Figure 2, it can be observed that second and third-grade students tended to use schematic representation more frequently than other types of representation. However, the frequency of using schematic representation by third-grade students was higher than that of second-grade students. Both grade levels used pictorial representation almost equally. It was seen that symbolic representation was less preferred than other types of representation. However, it can also be said that third-grade students used it more frequently than second-grade students. Table 2 presents the results of the chi-square test conducted to determine if there was a significant difference in the types of verbal problem representations preferred by second and third-grade students based on their grade levels.

Table 2. Chi-square test results regarding grade level and preferred representation types.

Variables	Pearson Chi-Square	df	p
Grade Level x Preferred Representation Type	.512	2	.774

As shown in Table 2, there was no significant difference in the types of representation preferred by students according to their grade level [χ^2 (2) = .512, p > .05]. Considering the number of students in this study and the distribution of their preferred types of representation, it can be concluded that the student's preference for a particular type of representation did not change based on their grade level. Table 3 shows the results of the independent sample t-test regarding whether the students' scores obtained from the diagrammatic test differed according to their grade levels.

Table 3. Students' diagrammatic test scores according to their grade levels

	Grade	N	Mean	Std.Dev.	t	df	F	p
Diagrammatic test	2	31	16.58	3.931	-4.367	70	3.610	.000*
	3	41	20.05	2.810				

p<.05*

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As shown in Table 3, there was a significant difference between the diagrammatic representation test scores of second and third-grade students, with third-grade students scoring higher on average ($Mean_{\text{second-grade}} = 16.58$; $Mean_{\text{third-grade}} = 20.05$; $t_{(70)} = -4.367$, $p<.001$). However, no significant difference was found between the mean scores of the two grades regarding problem-solving performance ($Mean_{\text{second-grade}} = 9.48$; $Mean_{\text{third-grade}} = 9.63$; $t_{(70)} = -163$, $p>.05$). Furthermore, a significant negative correlation was found between the students' problem-solving performances and the scores they received from the diagrammatic representation test, indicating that students who performed better in the diagrammatic test tended to have lower problem-solving performances (Correlation = -.464; $p<.001$).

Discriminant analysis was conducted to determine the extent to which students' problem-solving performance and diagrammatic skills predicted the types of verbal problem representations they preferred. This study met the necessary assumptions for conducting the analysis, including having at least 30 samples for the three independent variables. Seventy-two students participated in this study, and the independent variables showed a multivariate normal distribution (Kolmogorov-Smirnov $p>.01$). Homogeneity of variance-covariance matrices is required for conducting discriminant analysis. The homogeneity of matrices in this study was tested with the Box M test ($F_{(6\text{-}5.390)} = .841$, $p>.05$), which determined that they were homogeneously distributed. Extreme values were not observed, and there was no multicollinearity problem in this study. The eigenvalue of this study was 2.93, which is considered a good value (Kalaycı, 2005) with a score higher than .40. The canonical correlation between the groups formed by the dependent variable and the discriminant function was examined, and it was.863. The higher the value of the canonical correlation is, the stronger the relationship between the groups and the discriminant functions is. The model explains 75% of the change in the dependent variable, which is the square of the canonical correlation.

Discriminant analysis was conducted to determine the extent to which second and third-grade students' preferred representation types, problem-solving performances, and diagrammatic skills scores could differentiate them. Specifically, the analysis aimed to predict the types of representations preferred by the students based on their grade level, problem-solving performances, and diagrammatic skills scores. This study included 72 participants, and the necessary assumptions for conducting discriminant analysis were met, such as a multivariate normal distribution for the independent variables and homogeneity of variance-covariance matrices, which were tested using the Box M test.

The results of Wilks' Lambda ($\Lambda = .251$, $\chi^2 = 94.787$, $p<.001$) indicated that the model, which included two predictive variables, significantly separated the three groups based on their preferred representation types (i.e., schematic, pictorial, and symbolic). The standardized function coefficients and the relationships between the discriminant and independent variables are presented in Table 4.

Table 4. Standardized function and structure matrix (correlation) coefficients of variables.

Independent (predictor) variables	Standardized function coefficients		Structure Matrix (correlation) coefficients	
	1	2	1	2
Diagrammatic Test	-.131	1.007	.992	.129
Problem Solving Performance	.969	.301	-.297	.955

Based on Table 4, the highest correlation with the discriminant function was observed for problem-solving performance in the first function. On the other hand, the second function was mostly related to diagrammatic skills. This suggests that problem-solving performance has a stronger impact on the first function, while diagrammatic test scores have a greater influence on the second function. Moreover, according to the structure matrix coefficients, students' scores on the diagrammatic test were utilized to name the first function, whereas their problem-solving performance was used to name the second function. Figure 3 provides a graphical representation of the functions' points in each group (i.e., pictorial, schematic, and symbolic) based on their numerical values.

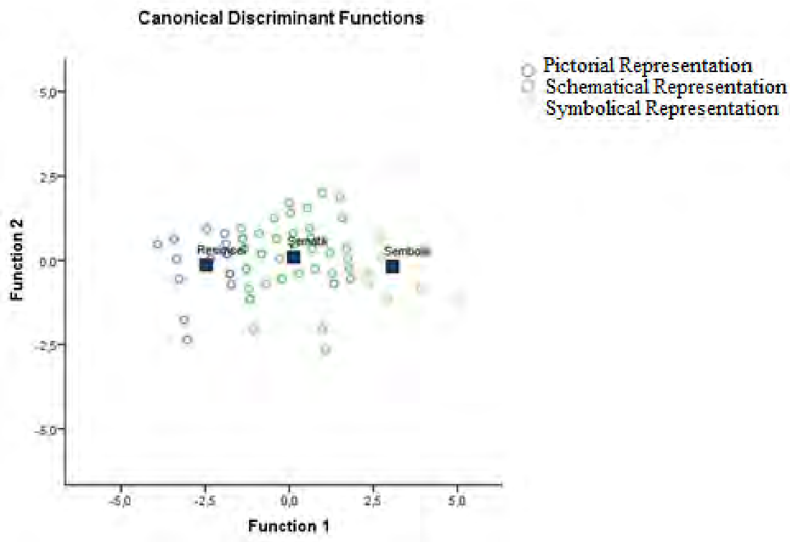

Figure 3. The place of each group in functions.

Based on Figure 3, both function 1 and function 2 effectively differentiate between the types of representations that students prefer. Table 5 provides the classification results, demonstrating the number of participants who were correctly assigned to their respective groups based on the functions.

Table 5. Discriminant analysis classification results.

Group	Pictorial		Schematic		Symbolic		Total	
	f	%	f	%	f	%	f	%
Pictorial	16	100	0	.0	0	0	16	100
Schematic	0	0	45	100	0	0.	45	100
Symbolic	0	.0	0	.0	11	100	11	100
Total Correct Classification Percentage: 100								

Table 5 shows the results of the classification, which indicates the number of students placed in each group. All 16 students (100%) who preferred pictorial representation were correctly classified, as were all 45 (100%) students who preferred schematic representation, and all 11 students who preferred symbolic representation (100%) were correctly classified. The overall correct classification rate for the discriminant function was 100%.

A logistic regression analysis was also conducted to determine the extent to which students' preferred representation types, problem-solving performances, and diagrammatic test scores predicted their grade levels (i.e., whether they were in the second or third grade). The results indicated that all the independent (predictive) variables significantly predicted the grade level ($\chi^2 = 25.932$, df = 3, N = 72, p<.001). Together, the predictive variables accounted for approximately 40% of the grade-level variance (Nagelkerke R2).

Table 6 presents the odds ratios [Exp (β)] of individual students, which show the degree to which their diagrammatic skills and problem-solving performances predict their grade level.

Table 6. Logistic regression rates for predicting grade level of students' diagrammatic skills and problem-solving performance.

Variable	B	Standard Error	Wald	df	P	Exp(β)
Diagrammatic Test	.522	.138	14.341	1	.000	1.685
Problem Solving Performance	.170	.162	1.109	1	.292	1.186
Representation Type	.868	.896	.939	1	.333	2.382
Constant	-11.732	3.386	12.006	1	.001	.000

As shown in the results in Table 6, if a student's diagrammatic test score is known, it is likely to predict that they belong to the third-grade cluster with a probability of 68.5% (I1- 1.186I x100). In other words, an increase of one unit in the diagrammatic test score leads to a 68.5% increase in the odds of being a member of the third-grade cluster.

DISCUSSION, CONCLUSION, and SUGGESTIONS

The development of student’s ability to solve verbal, mathematical problems is considered important in mathematics education (NCTM, 2000). Over the years, many learning approaches and strategies have been developed to improve this, including the use of diagrams. Studies have shown that using diagrams has many benefits, including helping students succeed in problem-solving (Cheng, 2004; Mayer & Massa, 2003; Stenning & Oberlander, 1995). This study aimed to determine the use of diagram representation, problem-solving performance, preferred representation types, and problem-solving representation types of students at different age levels based on their ability to use diagram representation and their problem-solving performance. While most studies focus on problem-solving strategies (Cooper et al., 2018), this study is unique in its approach to student development and its association of representation types preferred by children in early childhood in primary school with diagrammatic representation skills.

When primary school third-grade students used schematic and pictorial representations and solved the problem correctly, they were observed to be in the group with a high rate of correct answers on the diagrammatic test. Similarly, if they used schematic representation and solved the problem correctly in the second grade, they were also in the high correct answer rate group on the diagrammatic test. On the other hand, when they used pictorial representation and solved the problem correctly, the correct answer rate on the diagrammatic test was moderate, despite pictorial diagrams being known as ineffective diagrams in the literature. Pictorial diagrams are usually used to describe the visual appearance of the variables in verbal problems or to make drawings that do not help solve the problem. However, in this study, it was seen that students who drew pictures and answered the question correctly had higher scores in the diagrammatic representation test. This is because pictorial representations are considered a step toward transitioning to more advanced schematic representations, which are known to be effective.

Using pictorial representations, a person describes the visual appearance of the variables in verbal problems (van Garderen & Montague, 2003) (for example, a one-to-one drawing of a cat in a verbal problem) or makes drawings that will not help solve the problem (Presmeg, 2006). However, in this study, it was seen that the students who both drew pictures and answered the question correctly had higher scores in the diagrammatic representation test. This is because pictorial representations are thought to be a step toward transitioning to more advanced schematic representations. After all, schematic diagrams are known to be effective. With schematic representation, the student goes beyond visualizing the objects in the problem; they represent the content of the problem, use graphs, diagrams, and meaningful tables, and depict relational information by employing more metacognitive skills (van Garderen, 2007).

With schematic representation, students go beyond visualizing the objects in the problem, represent the content of the problem, use graphs, diagrams, and meaningful tables, and depict relational information by employing more metacognitive skills. According to Zahner and Corter (2010), schematic diagrams or schematic representations are extremely useful for verbal problems in mathematics, and this benefit transfers to other areas of mathematics learning, such as geometry and probability. Rellensman et al. (2016) also noted that both pictorial and schematic representations positively affect modeling performance and problem-solving performance, emphasizing that such representations are central to abstraction and generalization in mathematics.

Interestingly, when third-grade students solved the problem using symbolic representation and answered incorrectly, they scored low on the diagrammatic test, while second-grade students received a moderate score on the diagrammatic test. This finding contradicts Lowrie's (2020) study, which suggests that students may not need diagrammatic representation when problems are easy, as they can use computational and analytical methods to generate solutions. However, according to this study, if the problems in which the students used symbolic representations were easy, the students should not have answered these problems incorrectly. In other words, it was observed that students at both levels who incorrectly answered the problems they solved without using diagrams were not successful in the test of diagram representation skills.

In this study, it was found that there was no significant difference in the types of verbal problem representations preferred by third and second-grade students based on their grade levels. It was observed that both groups preferred schematic representation over other types of representation. Moreover, there was no significant difference in the problem-solving performance of students between the two grade levels, which contradicts the findings of Cooper et al. (2018). These researchers reported that students with higher mathematical skills are more likely to use visual representation to solve problems and that students' skills, interests, attitudes, and motivation toward mathematics also affect their use of diagrams.

This study also revealed that there was a significant difference in diagrammatic representation test scores between students of different grade levels. It was observed that third-grade students were more successful in the diagrammatic test, suggesting that diagrammatic representation skills develop as age progresses. This finding is consistent with the work of Frick and Newcombe (2015), who reported that children's diagrammatic representation skills improve until the age of eight and that they can achieve near-perfect performance only above the age of 8.

It has been found that students' problem-solving performances and diagrammatic representation skills can predict the types of representations that students prefer to solve verbal problems. According to Johnson-Laird's (1983) mental models theory, people construct representations or models in their minds while solving problems, and the underlying cognitive processes can influence these representations. Therefore, it has been emphasized that the decision to use pictures or diagrams in problem-solving representations may be related to the person's underlying cognitive processes. Hence, this theory supports the results of this study.

The study findings suggest that successful problem-solving and the level of diagrammatic representation skills can affect the types of representations students use. Murayama (2003) also revealed that the use of representations in problem-solving can be influenced by cognitive processes. In Rellensmann et al.'s (2016) study with high school students, the types of representations used by students were found to affect their problem-solving performance and mathematical modeling skills, and the use of schematic representation, a mathematical drawing, was associated with successful problem-solving. In Van Garderen et al.'s (2012) study with different age groups, the findings showed that students who used schematic diagrams were more likely to solve the problem correctly than those who used picture diagrams.

Uesaka et al. (2010) suggested that the use of diagrams in problem-solving does not come naturally and requires exposure and experience. Therefore, it can be inferred that students who have more opportunities to use diagrams in their learning, through either teacher instruction or real-life experience, are more likely to have stronger diagrammatic representation skills. This study also found that students' diagrammatic test scores were a significant predictor of their grade level (second or third grade), while their problem-solving performance and preferred types of representation were not. Booth and Koedinger's (2012) study found no significant differences in verbal problem-solving performance among sixth, seventh, and eighth-grade students but noted significant developmental differences when these problems were supported with diagrams. They also suggested that diagrams can help transition to more abstract, symbolic representations depending on age and mathematics skills. As children get older, they may be better at making connections between different elements of a problem, but it is essential to relate these connections to other areas of mathematics and problem-solving. Student guidance may be necessary to facilitate these connections and increase awareness.

The study's findings highlight the importance of diagrammatic skills and problem-solving performance in predicting students' grade levels. Specifically, the results indicate that students who perform better on diagrammatic tests are more likely to be in the third-grade cluster. This study shows that preferred representation types, problem-solving performances, and diagrammatic skills are important factors in predicting students' grade levels.

The study's focus on early childhood and primary school students' problem-solving and representation skills is particularly relevant for mathematics education. By identifying the factors that contribute to students' problem-solving abilities and representation preferences, educators can develop more effective strategies for teaching mathematics. Moreover, the study's findings suggest that diagrammatic representations can be a powerful tool for helping students improve their problem-solving skills. As such, educators can use diagrams as a way to support students' learning and development in mathematics. Overall, it is important to prioritize the development of diagrammatic representation skills in students, as it has been shown to significantly impact their problem-solving performance. Classroom practices should be designed to help students improve their skills, and teachers should be knowledgeable in this area to provide guidance and support to students. Additionally, students should be taught how to construct and use different types of representations effectively. By doing so, they will be better equipped to generate appropriate representations and use them to solve problems accurately. Teachers should also be mindful that pictorial representations should serve as a stepping stone toward understanding abstract mathematical problem structures. Encouraging the use of schematic representations as students' diagrammatic and mathematical skills develop can be a helpful strategy in this regard.

Limitations of the Research

This study aims to examine the diagrammatic representation skills and problem-solving performances of primary school second and third-grade students according to their problem-solving representations. The data obtained from the students were analyzed using descriptive statistics without manipulating any variables that could influence their mental processes, such as prior knowledge, guidance, or hints. The purpose was to determine the current situation of the students by observing their problem-solving strategies. However, this study has an important limitation, as it lacks experimental and causal interpretations. While trying to determine the representations preferred by the students in problem-solving, the researchers coded the students' representations based on their initial choices. However, it is possible that students may mentally create schematic representations before choosing a pictorial representation or solve the problem symbolically in their minds. This is a critical limitation that researchers cannot control.

Ethics and Conflict of Interest

This study was designed in accordance with ethical rules, and the necessary approvals were obtained, including parent consent forms, school board information forms, and university ethics board report [45346595 – Date: 05/07/2021 No: 131383]. Students were informed that their responses would be kept confidential and used for research purposes only, and they were told that they could withdraw from this study at any time. The authors declare that they have no competing interests.

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