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ANALYZING THE MEANINGS THAT FOURTH GRADE ELEMENTARY SCHOOL STUDENTS ASSIGN TO THE FOUR OPERATION SYMBOLS AND THEIR FORMS OF REPRESENTATION

Berat DEMİRTAŞ, Neşe IŞIK TERTEMİZ

Abstract: The aim of this study is to reveal the meanings that fourth grade students in elementary school attach to the four operation symbols (+, -, x, :) that are abstract, in different representations. The pattern of the study is a case study of qualitative research methods. In the study, the evaluation tool developed by the researchers was applied to elementary school fourth grade students. It was tried to present the situations of writing a mathematical sentence and expressing terms, formulating a problem, displaying the problem with a figure/diagram from students related to the symbols (+, -, x, :) for four processing skills. Based on the findings obtained in the study, it is seen that students attach concept and operation meanings to four operation symbols (+, -, x, :), create more routine word story problems related to symbols, and are more successful in writing a mathematical sentence appropriate for the problem they are writing than drawing a figure/diagram. In addition, it can be said that students made mistakes throughout the process from a linguistic point of view in terms of; using punctuations correctly, writing letters, numbers, units and expressions correctly, naming terms, and legible and understandable writing.

Key words: Multiple presentations, four operations, symbols, primary school mathematics courses

1. Introduction

We can look at the cognitive, affective, and motor skills that people are able to learn as learning products. Different from attitude, positive or negative experiences and motor skills; mental skills are skills that enable people to use symbols and communicate with their nature of forming an answer to the "...how?" question. We utilise mental conversions and calculations by using symbols to solve problems, and in this way, we indirectly interact with our environment (Busbridge & Özçelik, 1997). According to the Turkish Language Association (TLA) (2021), the word "symbol" was passed on to our language from the word (symbole), whivironment (ch has French roots, and it means "sign". Symbols constitute the basis of mathematical language. The meanings of mathematical symbols must be learned in order to understand and apply mathematics correctly.

1.1. Mathematical concepts and symbols

It is stated that one of the constant difficulties in learning mathematics is due to the consusion between mathematical concepts and symbols (Pimm, 2003). For this reason, the necessary importance and attention should be given to the teaching of concepts and symbols from the first years of mathematics teaching. Symbols can be used as a substitute for mathematical terms that can be expressed both as processes and as concepts. For example, the symbol for the addition operation, which we can express as an adding (action) or a sum (concept), is "+". when we see the "+" symbol, it is known to symbolize words like " addition, adding, etc.".

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According to Gray and Tall (1994), while stating that the symbols meet both process and concept expressions, they indicate that it can be called a “procept”. A single symbol represents many words at once, so the brain does not waste time reading and understanding words and concepts. This, in turn, empowers the individual substantially. While Haylock ve Cockburn (2014:13) answer to question "Are mathematical symbols just abbreviations?" as "In fact, they may be an abbreviation of mathematical ideas in a way”, they emphasize that this does not mean that a particular word or expression is just an abbreviation. When children begin to understand numbers and operations, considering their connections between concrete experiences, symbols, words and models/images, they start to understand that symbols are not just abbreviations of a word or a model/image. The children need to see this association. Considering elementary school children who are just beginning to learn mathematical terms, it is possible to say that symbols bring great benefits to these children.

1.2. The relationship between symbols, problems, models, and operations

When children engage in a mathematical activity, they utilize one or multiple components of the mathematical realm. Among these components, symbols and language have an important place. Mathematical symbols help us to decipher the relationships between various concepts. For example: when the 5+3=8 expression is written in symbols, it actually signifies the relationship between the concepts of five, three, eight, addition and equality. This situation can also create a complex network of connections with different images/models or concrete experiences (Haylock & Cockburn, 2014). In this case, mathematical operations are mathematical tools for solving problems in everyday life, which are the usage area for the mathematical operations. In the studies on the acquisition of concepts and symbols for operations, using models and problems together, can develop the skills of both the meanings of operations and their use in solving problems in everyday life (Baykul, 2005).

Through representations, the child also gets the opportunity to see the relationships between real life and mathematics. The only way for the child to answer the question "Where can I use what I learned in this class in real life?", is only possible trough seeing the connections between representations. In other words, the relationship related to an operation can be expressed with the help of a model, an oral or written expression of the problem, and symbols (Baykul, 2005). At the same time, strengthening the ability to switch between these elements, which make up different forms of representation of mathematical knowledge, will contribute to students' understanding of mathematical concepts and internalizing these understandings. The connections between the representation forms play an important role in learning mathematics by truly understanding it. The opportunity for students to discover mathematical concepts can be done by making conversions between representations (Sarı, 2020). When mathematical operations and symbols are associated with problem situations and models, they will be more meaningful for the child. The problem and the model expressions signify the relationship, and these two are sufficient to signify the relationship, while the symbols provide agreement in signifying concepts and relationships, in other words, universality. In order for the concepts related to symbols and operations to be truly acquired, students must navigate between models, expressions of problems and symbols often (Baykul, 2005). The model, problem, and symbol relationship does not work unidirectionally. There is a circular relationship between these concepts (See Figure. 1). When either one is given, you can convert to the other two.

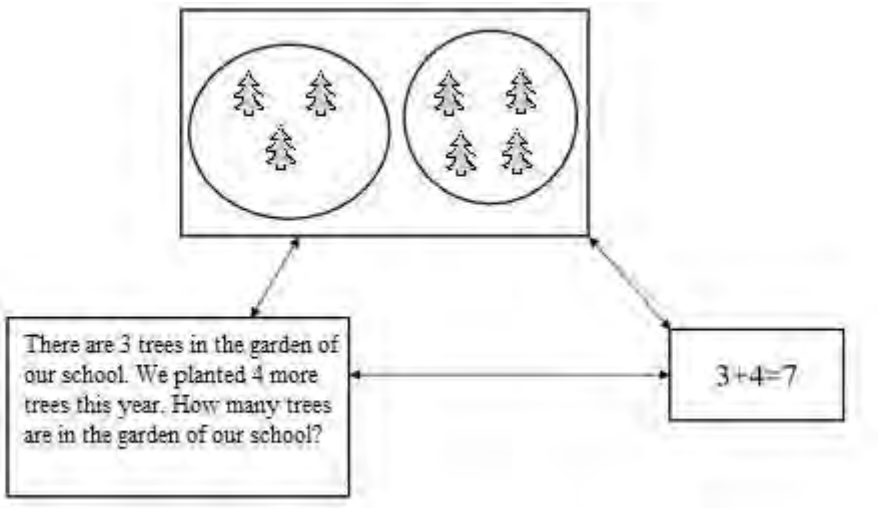


Figure 1. *Relationship between model, problem expression and symbol (Baykul, 2005, p. 248)*

When providing these transition studies in learning-teaching activities, children may be given a problem first and asked to draw the appropriate model and take action, or sometimes they may be given an action and asked to write an appropriate problem or create a model. The important thing is to help children see this relationship. Thus, the concept of operations and the symbols and terms in these operations will become more meaningful to children. In addition, the process will contribute to the development of children's problem-solving and thinking skills in the face of new situations and the ability to establish relationships (Baykul, 2005).

1.3. The Role of Language in the Learning of Mathematical Concepts and Symbols

Mathematics does not use the alphabet to communicate. Instead it uses symbols (such as 1,2,3, X, Y) for ideas, (such as =,<,>) for relationships, (such as+, -, x and :) for operations, (such as commas and parenthesis) for punctuation (Smith, 2016). Mathematics has its own unique sentences (for example: 3+2=5) and vocabulary, and it is also important for teachers use mathematical words correctly when teaching mathematics (Bali, 2002). Developing mathematical vocabulary is one of the first stages of teaching mathematics to children; this process continues throughout the children's mathematical education (Riccomini, Smith, Hughes, & Fries 2015). When students whose mathematical vocabulary is inadequate don't know how to express properly verbally what they understood, in mathematics in real life, their written works can be described as wrongly mistaken or confusing (Witzel, 2018). According to Van der Walt, Maree and Ellis (2008), the math learning speed of students whose language development is poor or incomplete is lower. This statement, which is especially true for teachers working in the primary school stage, can create great difficulties for students who are still in the period of concrete operations in a lesson that is already abstract. If children who have difficulty learning abstract concepts also get confused about the concepts, some problems may occur in their later educational life. According to Yeşildere (2007), one of the basic elements of acquiring concepts and knowledge related to mathematics and achieving mathematical thinking is the correct use of the language belonging to the field.

As a result of the literature review, various studies on four processing skills have been found, but these studies usually focus on studying the skills of four operations (Greer, 1992; LeFevre & Morris, 1997; Mulligan & Mitchelmore, 1997; Fosnot & Dolk, 2001; Toluk, 2002; Kula & Erdem, 2005; Sidekli et al., 2013; Önal, 2017; Yönal, 2018; Çıkılı & Gürbüz; 2019) or four operation problems (Kouba et. al., 1988; Albayrak et al., 2006; Jitendra, 2007; Ulu, 2008; Tertemiz et al., 2015; Tertemiz, 2017a; Ekici & Demir, 2018; Baber Elbistan, 2021; Yorulmaz et al., 2021). In this study, firstly, the meaning of the four operation symbols will be examined within the procept framework proposed by Gray and Tall (1994). After this stage, in the context of multiple representation, students will be analyzed this order, writing a math sentence, formulating a problem appropriate for the mathematical sentence, and showing the problem with a figure/diagram. At the final stage, the linguistic mistakes made by the students in the problems they have formulated will be determined. This study distinguishes itself from other studies by studying the multiple representations with a different order and the determination of the meanings assigned to the four operation symbols. In accordance with the findings obtained, it will be revealed what problems students have in converting between representations in classroom learning environments, and elementary school teachers will be provided with guiding information.

The aim of this study is to reveal how fourth grade students perceive the four operation symbols and their use of the representation forms of mathematics. For this purpose, the answers to the following questions were sought.

For fourth grade students in elementary school:

1) What meaning did they assign to the symbol of addition operation (+) and how are their skills of formulating a mathematical sentence, formulating a problem appropriate for the mathematical sentence, and drawing a figure/diagram appropriate for the problem?

2) What meaning did they assign to the symbol of subtraction operation (-) and how are their skills of formulating a mathematical sentence, formulating a problem appropriate for the mathematical sentence, and drawing a shape/diagram appropriate for the problem?

3) What meaning did they assign to the symbol of multiplying operation (x) and how are their skills of formulating a mathematical sentence, formulating a problem appropriate for the mathematical sentence, and drawing a figure/diagram appropriate for the problem?

4) What meaning did they assign to the symbol of division operation (:) and how are their skills of formulating a mathematical sentence, formulating a problem appropriate for the mathematical sentence, and drawing a figure/diagram appropriate for the problem?

5) What are the linguistic errors they make in the problems they formulate for the four operation symbols (+,-,x,:)?

2. Method

2. 1. Research pattern

In this study, the research pattern was determined as a case study. Qualitative research requires a multifaceted and long-term in-depth study of certain phenomena or events in their natural environment (Saban & Ersoy, 2017). According to Yıldırım (1999), qualitative research is a modeling study that describes a number of previously unknown results in relation to each other based on the collected information. A case study is a type of qualitative research that seeks answers to the researcher's “why” or “how” questions about a current situation in real life and provides the opportunity to collect in-depth data (Yin, 2003).

2.2. Study group

The study group consists of 30 students in the 4-B branch of a public school located at the middle class socio-economic level of the Southeastern Anatolia Region. These students were selected through convenience sampling. Convenience sampling is a method that speeds up the research. Because in this method, the researcher chooses a situation that is close and easy to access (Kılıç, 2013). One of the reasons for choosing this school in addition to the researcher working there, is to eliminate the schools that make up the extreme examples including very negative and positive factors (at least in terms of source factors in the same region) that can affect the practice, and to study and evaluate the data from the point of view of ordinary schools in the same region.

2.3. Data collection tool

Firstly, the relevant literature was researched when preparing the data collection tool of the study. It was developed and adapted to this study by researchers using a study conducted by Cockburn and Nicholson (1976) (cited by Dickson et al., 1984), as a result of the literature research. The data were collected through the assesment tool given in Table 1 developed by the researchers. The researcher applied the assesment tool in their own class in the pre-trial. By taking note of situations that are not understood by the children, the necessary changes have been made and detailed explanations have been given in the actual application.

Table 1. *Data collection tool for the meanings assigned to the four operation symbols*

Symbols	What is the name of the given symbol/sign?	Write an appropriate operation for the symbol. Display the elements of the process by writing on the operation. (For example:3+2=5)	Create a problem that is suitable for the process you are writing (making one of them the required one).	Show the problem you have created by drawing a figure/diagram. (Draw the shape of the problem you wrote)
+				
-				
x				
÷				

2.4. Data collection process

The time at which the data will be collected has been determined by the elementary students' school teacher and it has been ensured that the it does not disrupt the students' lessons. The fact that the researcher was working as a teacher at the same school and that the children knew the researcher made it easy for the application to be performed. The assesment tool is given in two sheets of four pages with situations related to one symbol on each page. In order not to put stress on students regarding with time for completing the test, data were collected during two lesson hours, thus in one lesson the form for two symbols were completed. In addition, the teacher helped the children with any questions they had.

2.5. Data analysis

Descriptive analysis was used in this study. The data obtained in the descriptive analysis are summarized and interpreted according to previously determined themes (Yıldırım & Şimşek, 2018). In our study, the previously prepared questions were determined as the theme and the answers to these questions were examined with descriptive analysis.

In the study, first of all, it was ensured that the students responded to the form determined by the researchers. The titles of the names of the symbols appropriate for the symbols given to the students (+, -, x, ÷), writing the appropriate problem, displaying it with a figure/diagram, and writing a math sentence were determined, and the appropriate answers to these titles were analyzed in the determined themes.

The themes given in Table 2 were established by the researchers under the titles of meaning of the symbol, writing a problem appropriate for the symbol, creating a figure/diagram appropriate for the problem and expressing it in the form of a mathematical sentence:

Table 2. *Themes used in the data analysis*

The meaning assigned to the given symbol	Writing a mathematical sentence for a symbol and displaying its elements on the process	Formulating a problem appropriate for the mathematical sentence	Displaying the formulated problem by drawing a figure/diagram*	Linguistic Errors
a) Concept b) Action c) Mistake d) Blank	a) Formulating a mathematical sentence appropriate for	a) Writing a word story problem b) Writing a word practice problem	a) Drawing a figure/diagram appropriate for the meaning of the operation	a) Problems in which the writing is understandable and legible

	the symbol b) Writing the elements of a mathematical sentence correctly c) Incomplete writing of elements of a mathematical sentence d) Misspelling the elements of a mathematical sentence d) Blank	c) Writing an incomplete word story problem without a question d) Asking for only what is requested in the word story problem e) Writing a problem that requires different operations than the given symbol f) Invalid/unrelated statement	asked in the problem b) Expressing what is given in the problem with objects and the result with numbers c) Deciphering the problem by placing a symbol between the given and the required d) Drawing an incomplete figure/diagram e) Invalid/incorrect statement f) Blank	b) Problems written by an inverted sentence c) Problems in which punctuation marks are not used in appropriate places d) Problems in which the letters, numbers and units do not comply with spelling rules.

**In the theme of displaying the formulated problem by drawing a figure/diagram, the meanings of four operations are evaluated within the framework of addition (addition, part-whole), subtraction (separation, comparison), multiplication and division operations (peer groups, comparison) (Olkun and Toluk Uçar, 2018).*

There are several considerations that should be noted in the theme of creating a figure/diagram appropriate for the problem. When the generated codes were examined, it was determined by expert opinions that the code "expressing the data given in the problem with objects and the result with numbers" can only be accepted in the addition process. For this reason, the "correct" code was not used when making findings about the addition process, but the "correct" code was used in other operations. The code "drawing an incomplete figure/diagram" was used for the answers that the student started modeling but somehow did not complete, did not add the symbols or could not show the result.

The problems written by the students about each operation (problems related to addition, subtraction, multiplication, and division) were examined one by one and a total of 120 problems were analyzed. Based on the results, all of the students exhibited the same errors in 4 of the problems they wrote. For example, if a student has formulated a problem with an inverted sentence in case of the addition operation, he has also formulated a problem with an inverted sentence in problems related to other operations. For this reason, the analysis was made on the 30 problems given for the addition operation. In addition, the findings were supported by direct quotations from student responses. Codes (S1, S2, ... S30) were given instead of student names.

2.6. Validity and reliability

When determining the validity and reliability methods of the study, methods that are appropriate for the qualitative research were selected. From this point of view, the concepts of transferability, credibility, verifiability, and consistency were preferred as the validity and reliability methods of the study. In the study, it was aimed to increase the transferability of the research by choosing an easily accessible sampling method, which is one of the purposeful sampling methods for determining the study group. Student samples were included when transferring the findings obtained by the researchers. After determining the findings, expert opinions were taken and a reorganization was performed. As regarding consistency, researchers studied the codes together and tried to resolve the discrepancies. At the final stage, the codes obtained by the researchers are shown with a numerical frequency table.

3. Findings

The findings obtained as a result of the research were presented according to the addition (+), subtraction (-), multiplication (x) and division (÷) symbols related to the four operations in the light of the questions which answers were sought for in the study. The meaning of the symbol appropriate for each operation, writing a problem appropriate for the given symbol, creating a figure/diagram appropriate for the written problem, and writing a mathematical sentence appropriate for the problem and specifying its elements are discussed in a single table.

3.1. Findings on the addition operation symbol (+)

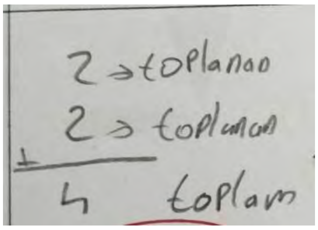
The findings regarding the meaning that the fourth grade students in elemantary schools assigned to the addition operation symbol (+), the problems they have formulated, the figures/diagrams they created appropriate for the problem and their mathematical expressions were given in Table. 3.

Table 3. *The meanings and forms of representation that students assigned to the addition (+) symbol*

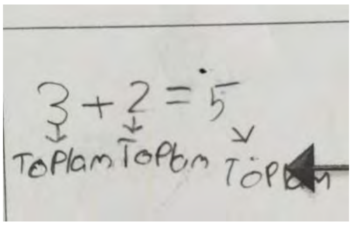
Themes	Codes	N
Meaning assignin to the “+” symbol	a) Concept	18
	b) Action	12
Writing a mathematical sentence for a symbol and displaying its elements on the process	a) Formulating a mathematical sentence appropriate for the symbol	30
	b) Writing the elements of a mathematical sentence correctly	25
	c) Incomplete writing of elements of a mathematical sentence	2
	d) Misspelling the elements of a mathematical sentence	2
	e) Blank	1
Formulating a problem appropriate for the mathematical sentence	a) Writing a word story problem	18
	c) Writing an incomplete word story problem without a question	10
	f) Invalid/unrelated statement	4
Displaying the formulated problem by drawing a figure/diagram	a) Drawing a figure/diagram appropriate for the meaning of the operation asked in the problem	14
	b) Expressing what is given in the problem with objects and the result with numbers	9
	d) Drawing an incomplete figure/diagram	1
	e) Invalid/unrelated statement	6

When the meanings the students assigned to the "+" sign which is the symbol for the addition operation was analyzed, in a group of 30 students, more than half of the students (N=18) expressed the symbol for the addition operation as a concept, and others (N=12) expressed this symbol as an action. All of the students who expressed the name of the symbol as a concept used the expression “plus”, while the students who expressed it as an action used the expressions “addition, addition operation and the addition symbol”.

In the mathematical sentences that students formulated for the addition (+) symbol and in the naming the elements of the addition operation (addend, addend and total); it was determined that all 30 students wrote appropriate math sentences and 25 students correctly named and 2 students incorrectly named the elements of the addition operation, 2 students named them incompletely, and 1 student left it blank (See Figure. 2).



Correct Example (S20)



Incorrect Example (S21)

Figure 2. *Student’s examples*

Analyzing the word problems that students have formulated appropriate for the given mathematical operation; it was determined that more than half of the students (N=18) created a word story problem, but 10 students created an incomplete word story problem that did not have a question, and 4 students used invalid/unrelated expressions.

Student's Examples

- Writing a word story problem:

"I had 7 liras that I had previously collected, my grandfather gave me another eight liras. How many liras will I have according to this?". (S17)

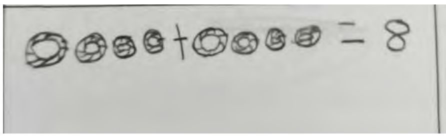
- Writing an incomplete word story problem without a question:

"I had 4 walnuts, I collected 4 more." (S24)

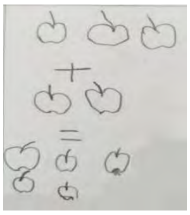
- Invalid/unrelated statement:

"I have eight pens, my mother gave me one more pen, I have a total of 18 pens." (S26)

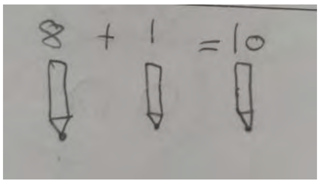
According to the analysis performed on the category of drawing a figure /diagram appropriate for the formulated problem, , it was determined that about half of the 30 (N=14) students drew a figure / diagram by placing symbols between the ones given in the problem and the ones required as the answer, rather than understanding what the addition operation carries (merging and fragment-whole), 9 students express what is given in the problem with objects and the result with numbers but it was thought that these situations could only be accepted for addition. In addition, it was determined that 1 student drew an incomplete figure/diagram, and 6 students drew an invalid/incorrect figure/diagram (See Figure. 3).



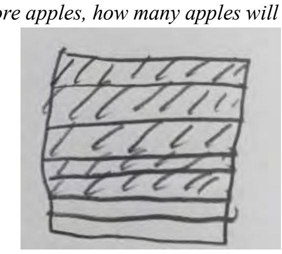
Expressing what is given in the problem with objects and the result with numbers (S14)
(Problem: *"I bought 4 bracelets, my mom gave me 4 more. How many bracelets do I have now?"*)



Displaying it by placing a symbol between the given and required ones in the problem (S18)
(Problem*: "My father gave me 3 apples. If my mom gives me 2 more apples, how many apples will I have ?"*)



Drawing an incomplete figure/diagram (S4)
(Problem: "*I have eight pens. My mom gave me another one. How many pens do I have now?"*)



Invalid/Incorrect (S20)
(Problem: *"I bought 6 apples and 3 oranges from the grocery store. How many fruits have I bought in total*)

Figure 3. *Student's examples*

3.2. Findings on the subtraction operation symbol (-)

The findings regarding the meaning that the fourth grade elementary school students assigned to the subtraction operation symbol (-), the problems they have formulated, the figures/diagrams they created appropriate for the problem and their mathematical expressions were given in Table 4.

Table 4. *The meanings and forms of representation that students assigned to the subtraction (-) symbol*

Themes	Codes	N
Meaning assignin to the "-" symbol	a) Concept	15
	b) Action	15
Writing a mathematical	a) Formulating a mathematical sentence appropriate for the symbol	30

sentence for a symbol and displaying its elements on the process	b) Writing the elements of a mathematical sentence correctly	16
	c) Incomplete writing of elements of a mathematical sentence	9
	d) Misspelling the elements of a mathematical sentence	4
	e) Blank	1
Formulating a problem appropriate for the mathematical sentence	a) Writing a word story problem	14
	c) Writing an incomplete word story problem without a question	13
	d) Asking for only what is requested in the word story problem	1
	e) Writing a problem that requires different operations than the given symbol	3
	f) Invalid/unrelated statement	6
Displaying the formulated problem by drawing a figure/diagram	a) Drawing a figure/diagram appropriate for the meaning of the operation asked in the problem	1
	b) Expressing what is given in the problem with objects and the result with numbers	1
	c) Deciphering the problem by placing a symbol between the given and the required	9
	d) Drawing an incomplete figure/diagram	12
	e) Invalid/unrelated statement	7

When the meanings the students assigned to the "-" sign which is the symbol for the subtraction operation was analyzed, in a group of 30 students, more than half of the students (N=15) expressed the symbol for the addition operation as a concept, and others (N=15) expressed this symbol as an action. Students who express the name of the symbol as a concept used the “minus” expression; students who students who expressed it as an action have used the “subtraction” expression.

According to the analysis of whether the appropriate math sentence was written or not; it was determined that all of the students in a group of 30 formed an appropriate math sentence. It was determined that 9 students wrote the elements of the substration operation correctly, 16 students wrote them incorrectly, 4 students left it blank and 1 student named them incomplete (See Figure 4.).

Correct Example (S26) Incorrect Example (S17)

Figure 4. *Student’s examples*

According to the analysis related to the category of students formulating a problem, in a group of 30 students, it was determined that, 14 students formulated word story problems, 13 students formulated incomplete word story problems without questions, 6 students used invalid/irrelevant expressions, 3 students formulated a problem that required a different operation, and 1 student only asked what was required as an answer in the word story problem.

Student's Examples

- Word story problem:

“My uncle gave me 7 marbles. I gave 3 of the marbles to Ali. How many marbles do I have left? (S22)

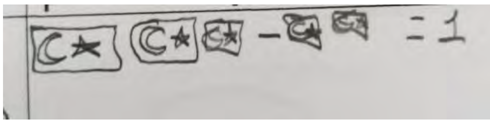
- Asking for only what is requested in the word story problem:

“My uncle gave me 2 Liras, my father gave me 1 lira What is the subtraction of these?” (S13)

- Writing a problem that requires different operations than the given symbol:

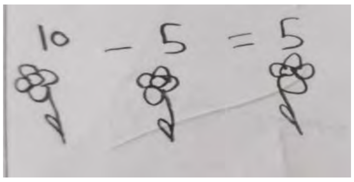
“My sister has 9 bracelets. And I have 6 bracelets. How many bracelets do me and my sister have?” (S19)

According to the analysis conducted on the category of drawing a figure appropriate for the formulated problem; it has been determined that in the displaying the operation related to the symbol by drawing it, amongst 30 students, 12 students have drawn incomplete figures/diagrams, 9 students drew a figure/ diagram by placing a symbol between the ones given in the problem and the ones required, 9 students drew an invalid / incorrect figure/ diagram, 1 student created a figure/ diagram appropriate for the problem, and 1 student expressed what is given in the problem with objects and the result with numbers (See Figure 5.)



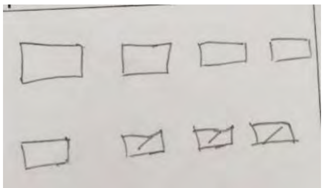
Expressing what is given in the problem with objects and the result with numbers (S14)

(Problem: *"I bought 3 flags, gave 2 of them to my friend. I had 1 left."*)



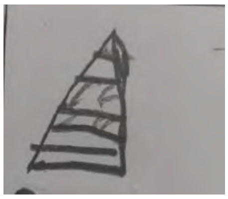
Drawing an incomplete figure/diagram (S24)

(Problem: *"I have ten flowers. I gave my mother 5 flowers. How many flowers do I have left?"*)



Drawing a figure/diagram suitable for the meaning of the operation asked in the problem (S25)

(Problem: *"I had 8 cubes. My brother took 3 of them. How many cubes do I have left?"*)



Invalid/Incorrect (S21)

(Problem: *"My mother gave me 2 openers. If I give 1 of them my father, how many openers would I have left."*)

Figure 5. *Student's examples*

3.3. Findings on the multiplication operation symbol (×)

The findings regarding the meaning that the fourth grade elementary school students assigned to the multiplication operation symbol (×), the problems they have formulated, the figures/diagrams they created appropriate for the problem and their mathematical expressions were given in Table 5.

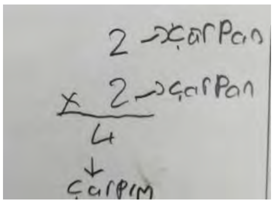
Table 5. *The meanings and forms of representation that students assigned to the multiplication (x) symbol*

Themes	Codes	N
Meaning assignin to the "x" symbol	a) Concept	19
	b) Action	10
	c) Mistake	1
Writing a mathematical sentence for a symbol and displaying its elements on the process	a) Formulating a mathematical sentence appropriate for the symbol	29
	b) Writing the elements of a mathematical sentence correctly	16
	c) Incomplete writing of elements of a mathematical sentence	7
	d) Misspelling the elements of a mathematical sentence	5
	e) Blank	2
Formulating a problem appropriate for the mathematical sentence	a) Writing a word story problem	7
	b) Writing a word practice problem	1
	c) Writing an incomplete word story problem without a question	12
	d) Asking for only what is requested in the word story problem	6
	e) Writing a problem that requires different operations than the given symbol	10
	f) Invalid/unrelated statement	6
Displaying the	a) Drawing a figure/diagram appropriate for the meaning of the operation	1

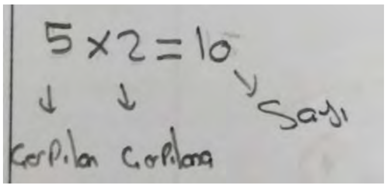
formulated problem by drawing a figure/diagram	asked in the problem	
	b) Expressing what is given in the problem with objects and the result with numbers	1
	c) Deciphering the problem by placing a symbol between the given and the required	12
	d) Drawing an incomplete figure/diagram	5
	e) Invalid/unrelated statement	11
	f) Blank	1

When analyzing the meanings that students assigned to the × sign, which is the multiplication operation’s symbol; it is seen that in a group of 30 students, 10 of the students expressed the multiplication operation symbol as a concept, 19 expressed this symbol as an action, and 1 student misrepresented the symbol's name. Students who express the name of the symbol as a concept used the “times” expression; students who students who expressed it as an action have used the "multplying" expression.

As a result of the analysis; in a group of 30 students it was determined that 29 of the students created the appropriate math sentence and 1 student created a wrong math sentence. It was determined that 16 students wrote the elements of the operation correctly, 5 students wrote them incorrectly, 2 students left it blank and 7 students have made them incomplete (See Figure. 6).



Correct Example (S21)



Incorrect Example (S25)

Figure 6. *Student’s examples*

According to the analysis related to the category of students formulating a problem, in a group of 30 students, it was determined that 12 students formulated incomplete word story problems without questions,10 students formulated a problem that required a different operation, 7 students formulated word story problems, 6 students used invalid/irrelevant expressions, 4 students only ask ed what was required as an answer in the word story problem and 1 student wrote a word practice problem.

Student's Examples

- Writing a word story problem:

“My mother gave me a 3-meter rope. My brother gave me twice the rope that my mother gave me. How many meters of rope did my brother give me?” (S18)

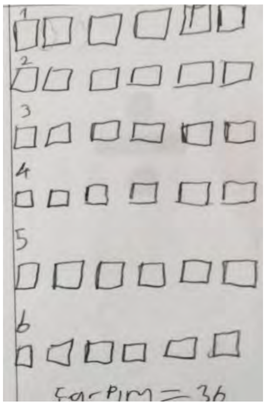
- Writing an incomplete word story problem without a question:

“There are nine liters of water in a bottle.”(S26)

- Asking for only what is requested in the word story problem:

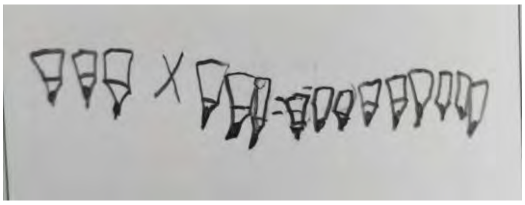
"When Erdem was at school, his teacher asked him what the multiplication of 1x1 was.” (S13)

Analyzing the drawings appropriate for the formulated problem; it has been determined that, amongst 30 students, 12 students drew a figure/ diagram by placing a symbol between the ones given in the problem and the ones required, 10 students drew an invalid / incorrect figure/ diagram, 5 students made incomplete modelling, 2 students left the relevant area blank and 1 student created a figure/ diagram appropriate for the problem (See Figure 7).



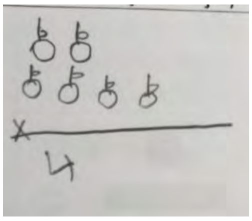
Drawing a figure/diagram appropriate for the meaning of the operation asked in the problem (S24)

(Problem: *"I read 6 books in 1 day. how many books can I read in 6 days?"*)



Displaying it by placing a symbol between the given and required ones in the problem (S11)

(Problem: *"I have 3 pens. My teacher gave me 3 times as many of my pens. How many pens did my teacher give me?"*)



Drawing invalid/incorrect figures/diagrams (S29)

(Problem: *"there are 2 oranges in 2 crates. How many oranges are in the crate?"*)

Drawing an incomplete figure/diagram (S25)

(Problem: *"My friend had 5 hair ties. My teacher gifted her 2 times as many of her hair ties. How many hair ties did my teacher give?"*)

Figure 7. *Student's examples*

3.4. Findings on the division operation symbol (÷)

The findings regarding the meaning that the fourth-grade elementary school students assigned to the division operation symbol (÷), the problems they have formulated, the figures/diagrams they created appropriate for the problem and their mathematical expressions were given in Table 6.

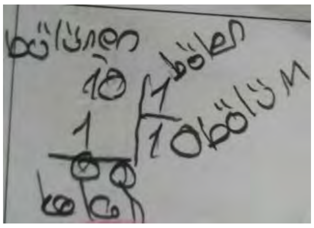
Table 6. *The meanings and forms of representation that students assigned to the division (÷) symbol*

Themes	Codes	N
Meaning assignin to the "÷"symbol	a) Concept	28
	b) Action	1
	d) Blank	1
Writing a mathematical sentence for a symbol and displaying its elements on the process	a) Formulating a mathematical sentence appropriate for the symbol	25
	b) Writing the elements of a mathematical sentence correctly	15
	c) Incomplete writing of elements of a mathematical sentence	9
	d) Misspelling the elements of a mathematical sentence	6
Formulating a problem appropriate for the mathematical sentence	a) Writing a word story problem	8
	b) Writing a word practice problem	1
	c) Writing an incomplete word story problem without a question	12
	d) Asking for only what is requested in the word story problem	4
	e) Writing a problem that requires different operations than the given symbol	6
	f) Invalid/unrelated statement	5
Displaying the	a) Drawing a figure/diagram appropriate for the meaning of the operation	1

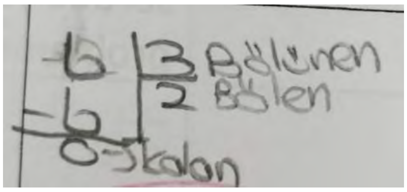
formulated problem by drawing a figure/diagram	asked in the problem	
	c) Deciphering the problem by placing a symbol between the given and the required	13
	d) Drawing an incomplete figure/diagram	4
	e) Invalid/unrelated statement	1
	f) Blank	11

When analyzing the meanings that students assigned to the (÷) sign, which is the division operation symbol; it is seen that in a group of 30 students, 1 of the students expressed the multiplication operation symbol as a concept, 28 expressed this symbol as an action, and 1 student misrepresented the symbol's name. Students who express the name of the symbol as a concept used the “divided by” expression; students who students who expressed it as an action have used the "division, division operation" expressions.

As a result of the analysis; in a group of 30 students it was determined that 25 of the students created the appropriate math sentence and 5 of the students created the wrong math sentence. It was determined that 15 students misspelled the elements of the operation, 9 students spelled them correctly, and 6 students left it blank (See Figure 8).



Correct Example (S13)



Incorrect Example (S9)

Figure 8. *Student’s examples*

According to the analysis related to the category of students formulating a problem, in a group of 30 students, 12 students formulated incomplete word story problems without questions, 8 students formulated word story problems, 6 students formulated a problem that required a different operation, 5 students used invalid/irrelevant expressions, 4 students only ask what was required as an answer in the word story problem and 1 student wrote a word practice problem.

Student's Examples

- Writing a word story problem:

“I have 8 marbles. I divided them between 2 people. How many marbles does each person get?” (S1)

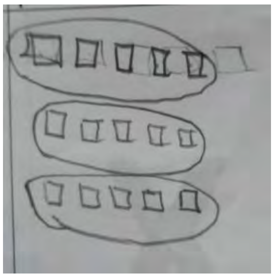
- Writing an incomplete word story problem without a question:

“I have a brother and 2 marbles. We divided it in half to share with my brother.” (S24)

- Invalid/unrelated statement:

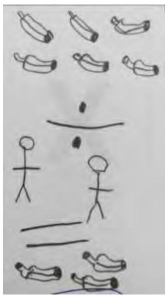
“I have 6 erasers. I lost three of them, I have three erasers left.” (S4)

Analyzing the drawing made for illustrating the formulated problem; it has been determined that, amongst 30 students, 13 students drew a figure/ diagram by placing a symbol between the ones given in the problem and the ones required, 11 students left the relevant area blank, 4 students drew an incomplete figure/ diagram, 1 student drew an invalid / incorrect figure/ diagram and 1 student created a figure/ diagram appropriate for the problem (See Figure 9).



Drawing a figure/diagram appropriate for the meaning of the operation asked in the problem (S7)

(Problem: *"I have 15 books. I want to distribute it equally between 3 people how many books would each of them get?"*)

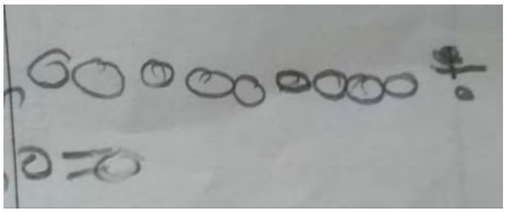


Displaying it by placing a symbol between the given and required ones in the problem (S18)

(Problem: *"I had 6 bananas. I want to share it with my brother. How many bananas should I give my brother?"*)

Incomplete modeling (S21)

(Problem: *"I had 32 apples. Dad divided my apples into 2, how many apples are there left?*)



Incorrect modeling (S6)

(Problem: *"I have 2 flags, he gave me 1 more."*)

Figure 9. *Student's examples*

3.5. What are the linguistic mistakes made by fourth grade elementary school students when writing word problems

Of the 30 problems that students wrote, only 5 of them had understandable, legible, and error-free writing, while the following general errors were diagnosed in other problems (See Table. 7).

Table. 7 *The table of linguistic mistakes made by fourth grade students in elementary schools when writing problems*

Linguistic Errors	N
a) Problems in which the writing is understandable and legible	5
b) Problems written by an inverted sentence	7
c) Problems in which punctuation marks are not used in appropriate places	23
d) Problems in which the letters, numbers and units do not comply with spelling rules	18

According to the results of the analysis, out of the 30 problems examined, it was determined that in 23 problems, punctuation marks were not used in the appropriate places; in 18 problems, letters, numbers and units were not in accordance with spelling rules, in 7 problems, inverted sentences were used, and in 5 problems, the writing was understandable and legible.

Student's Examples

- Problems in which punctuation marks are not used in appropriate places:

"I have eight pens my mother gave me one more pen I have a total of 18 pens" (S4)

- Problems in which the letters, numbers and units do not comply with spelling rules:

"I had 5 smiley faces. then my teacher gave me 5 more smiley faces. how many smiley faces do I have in total?" (S1)

- Problems written by an inverted sentence:

"5 pens I had. 2 more my sister gave me. Now I have 7 pens in total." (S27)

- Problems in which the writing is understandable and legible:

"I had 5 plums. I bought 7 more. How many plums do I have in total?" (S3)

4. Conclusion and Discussion

The aim of the study conducted was to examine the meanings and forms of representation that fourth grade elementary school students assign to the four elementary operation symbols:; how they name these operations and their elements, their mathematical expressions, the word problems they formulate for these operations, the figure/ diagram drawn according to the formulated problems, and the linguistic mistakes they make within this framework. The obtaind results are discussed as follows.

It seems that fourth-graders assign meaning as either a concept or an action when naming the four operation's symbols. Students called the operation more as a concept in addition and subtraction (plus, minus, cross, divide), while they called it an action in multiplication and division (addition, subtraction, multiplication, division). Mathematical symbols are important concepts for creating mathematical thinking (Yeşildere, 2007). The correct learning and use of symbols by students is one of the first stages of mathematics. Because a student who does not perceive all the mathematical meanings placed on a symbolic expression cannot fully realize the learning process (Aydın & Yeşilyurt, 2007). The adventure for this study begins precisely at this stage. In order to reveal how students perceive the four operation symbols, they were shown the symbols and asked to write the names of these symbols.

As a result, it was observed that almost all of the students wrote the names of all the symbols correctly, but they assigned different meanings in their namings. Some of the students perceive symbols as concepts, and some perceive them as actions. Considering the meanings of the words concept and action, the word concept according to TLA (2021) is defined as follows; "The abstract and general design of an object or thought in the mind...". Action on the other hand has been defined as "Acting, movement, action." (TLA, 2021). As can be seen, a concept indicates the name of the object, while an action speaks of a process. According to the definition of Gray and Tall (1994), symbols are expressions containing both a process and a concept. When it comes to writing the names of the elements of operations, it is noteworthy that students are more successful in addition and multiplication operations than subtraction and division operations. In addition and multiplication operations, the fact that there are 2 element names (addend and total; multiplier and multiplication) in other operations, the fact that there are more element names (output, decrease, difference; divisor, division, difference) may have created this difference. In his study, Kızıltoprak (2014) found that students cannot express the elements of operations (addend, total, output, minus, difference, divisor, division, multiplier, multiplication) formally, which supports the current study.

When the mistakes made by the students in the problems they set up for the four operation symbols (+,-,× ,:) are examined ; it is seen that about half of the students wrote "incomplete word story problems without a question" in in case of the addition and subtraction operation, and even if its little bit "invalid / unrelated expression" "asking only what is required in the word story problem" for multiplication and division, more "problem that requires different processing than the given symbol" . Problem-building is more than being able to do mathematics (Pirie, 2002) and it is an activity that arouses interest in the student, makes him think, question, and interpret (Brown & Walter, 1993, Zhang & Zu, 2021). In their study, Bozkurt and Ergin (2018) examined the problem-building skills of fourth, fifth and sixth grade students and determined that more than half of the fourth grade students participating in the study had non-mathematical and irrelevant problems. This situation is in parallel with the work at hand.

When examining the figure/diagram illustrating the problems that fourth grade students have formulated for the four operation symbols (+,-, ×,:), it is seen that the students are quite unsuccessful in representing the problem with a figure/diagram. It is seen that only two students drew figures/ diagrams suitable for the meaning of the operations in case of the addition operation and only one student drew figures/ diagrams suitable for the meaning of the operation in case of the subtraction and division operations. Looking at the mistakes made by students when drawing a figure/ diagram, it seems that they represent it in the form of “placing a symbol between the ones given in the problem and the ones required”, rather than in the sense that the problem carries. In other words, it seems that children tend to draw only the numbers of objects they are given in the problem, and they cannot display the relationship between the objects. In addition, drawing the number of objects for an operation is an acceptable model in addition but it does not constitute a valid situation in subtraction, multiplication and division operations. Students should be able to use information in different formats and explore a different aspect of a mathematical concept with multiple representations in the problem solving process (Ipek & Okumuş, 2012; Abu Bakar et al., 2020). However, with the results of our study, we can say that the representative abilities of the participating students are very weak and they are trying to reason. One of the interesting representations in our study is the figure drawn by Student 18 regarding the division operation. In the image where he divided 6 bananas into 2 people, the student drew as many bananas as the number of bananas he had and then wrote the division symbol (÷) and drew 2 stickman on the back. Here it can be said that the student draws a banana/human image taking into account the numerical data in the operation rather than conceptual understanding. Van Garderen, Scheuermann and Poch (2014) stated that low-achieving students cannot create a suitable structure for representing problems. Failures in using visual representations also negatively affect problem solving performance (Dofour-Janvier et al., 1987; cited by Ergan, 2018; Sinha & Kapur, 2021).

Looking at the linguistic mistakes made by students in the problems they have formulated for the four operation symbols (+,-,×,:) in general, it is observed that it is mostly in the use of punctuation marks, followed by misspelling of letters, numbers and units, inadequate / broken expressions and incomprehensibility of the writing. Previous studies on the use of punctuation marks (Bağcı, 2011; Çetin, 2013; Hamzadayı & Çetinkaya, 2013) show parallels with the results of this study and show that students are inadequate in using punctuation marks. Problem-building activities not only develop linguistic ability, but also provide special ways to explain solutions (Tertemiz, 2017b). The fact that a problem can be solved does not always mean that the problem is fully understood. In order to understand the problem deeply, it is also important to establish problems that can be solved by the solution method of the problem (Korkmaz & Gür, 2006). Considering this sub-factor, which examines the linguistic errors in the problems written by the students, it is seen that a very small part of the students create understandable and legible problems. This result is in parallel with the studies of Işık et al.(2012) and Gökkurt et al. (2015).

In conclusion, it is seen that students assign concepts and process meanings into four operation symbols (+, -, ×, :), create more routine word story problems related to the symbols, and are more successful in writing an appropriate mathematical sentence suitable for the problem they are writing than drawing a figure/diagram. However, , it can be said that although children were successful in writing a math sentence, they had a decrease in their problem-building skills suitable for it, and they were unsuccessful in showing their problem with a figure/diagram. It can be said that children have difficulties in representing their thoughts verbally and with a figure/diagram in other representations. In addition, it can be said that students made mistakes in this whole process from a linguistic point of view; using punctuation correctly, writing letters, numbers, units and expressions correctly, naming terms, and making the article legible and understandable. Overall, van de Walleet al. (2014) has noted, in mathematics, expressions such as symbols that we don't encounter much in everyday life won't be used much, so it requires intermediaries such as manipulatives or more of a structure and help. For this, the lesson should begin with the disclosure of further, previously acquired experiences, and the necessary meanings and contexts for the lessons should be given. In addition, according to Piaget, if the symbols used in mathematics (+, -, x, : or =) are given to students without questioning, logic is not required and they are transferred as social information without any meaning. However, children interact with their surroundings and form abstract structures based on concrete experiences. Direct, firsthand experiences of children during the concrete processing period are of great importance in creating abstract concepts. Children seeing the relationship between concept and process in understanding concepts and operations in mathematics will enable them to think deeply (contemplation) in an abstract way. This will also create a jump in the child's transition from the real world to the world of mathematics. This is the only way for the child to progress in the world of mathematics. In addition, the most important conclusion from the study due to the linguistic errors made is that the errors made when converting the verbal representation form of mathematics to writing are quite a lot. It is considered important not to neglect linguistic issues in both verbal expression and written expression.

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Authors

Berat DEMİRTAŞ, Vice Principal, Mehmet Zeki Obdan İlkokulu, Republic of Turkey Ministry of National Education, İstanbul, Turkey; beratdemm@gmail.com. ORCID: 0000-0001-9875-3147

Neşe IŞIK TERTEMİZ, Prof. Dr. Elementary Education, Gazi University, Ankara, Turkey; netertemiz@gazi.edu.tr. ORCID: 0000-0001-7033-7192