Journal on Mathematics Education
Volume 14, No. 3, 2023, pp. 395-414

The role performed by the teacher's question in the learning of quadratic function in an exploratory mathematics class

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Received: 22 March 2023 | Revised: 13 April 2023 | Accepted: 18 April 2023 | Published Online: 21 April 2023

Abstract

Communication plays a key role in teaching and learning processes. Questions are a communicational act greatly used by teachers to structure their discourse, establish dynamics, and foster interaction between the different participants in the classroom. In view of these potentialities of questions in the classroom context, we have developed a teaching experiment with the aim of understand the role of the teacher's question in the learning of topics on functions. Considering the nature of this aim, a methodology of qualitative and interpretative nature was used. The data collection was based on the students' written productions and on the audio and video recordings of a mathematics class of a Grade 10 educational group (in northern Portugal). Data analysis is based on content analysis techniques, crossing collected data and categories emerging from the literature. The study revealed that the teacher's questions alternated between confirmation, focalization, and inquiry, with inquiry prevailing. Questions aimed at testing the student's knowledge gave both the teacher and actual student important information. Questions that focused the student's attention on a particular detail enabled the students to organize their reasoning and structure their answer. Questions that required the students to explain or justify their thoughts were those that proved to most contribute to the development of the student's reasoning process.

Keywords: Communication, Functions, Learning, Tasks, Teacher's Questions

How to Cite: Silva, A. V., Viseu, F., & Menezes, L. (2023). The role performed by the teacher's question in the learning of quadratic function in an exploratory mathematics class. *Journal on Mathematics Education, 14*(3), 395-414. http://doi.org/10.22342/jme.v14i3.pp395-414

Communication plays a preponderant role in mathematics classes and, in particular, in the development of the students' reasoning (Baran & Kabael, 2021; Guerreiro et al., 2015; Lomibao et al., 2016; NCTM, 2007; Rodrigues et al., 2020). This entails the learning of a skill that helps students to explicitly detail their thought processes and clarify their ideas (Lomibao et al., 2016; Menezes et al., 2014). The development of this skill is highly influenced by the teacher's action in stimulating classroom activities (Chapman, 2004; Guerreiro et al., 2015). For this reason, the teacher performs a key role in the establishment of communication in the classroom and, especially, in the structuring of the discourse that can be generated and the learning opportunities that are created (Chapman, 2004). Among the discursive strategies available to the teacher, questioning stands out particularly (Guerreiro et al., 2015; Menezes et al., 2013; NCTM, 2007; Wachira et al., 2013). The type of questions that the teacher asks the students, the way that the students are heard and managed, and the way they are asked to explain and justify their ideas are decisive to the quality of the learning of mathematics. Considering that questioning has a major presence in the teacher's discourse (Menezes et al., 2013), that the questions can boost the students' engagement with mathematics, encouraging communication and interaction, this study examines the mathematics teacher's questions, seeking to understand the role of questions in fostering the learning of topics on functions by Portuguese students entering secondary education (10th grade). A teaching experiment based on an exploratory approach to the teaching of mathematics was developed as the context of this study. The classes that were taught involved challenging mathematics tasks, following this sequence: (i) Introduction of the task; (ii) Exploration of the task; (iii) Collective discussion (Stein et al., 2008).

Communication is a crucial part of mathematics, contributing to the sharing of ideas and to the negotiation and construction of mathematical meanings (Lomibao et al., 2016; Menezes et al., 2013; NCTM, 2007). Communication can be understood as a contextualized social process involving different players who use languages and representations to share their ideas (Guerreiro et al., 2015; Lomibao et al., 2016; Menezes et al., 2013).

Research studies have pointed to various benefits of promoting communication in the mathematics class, such as the development of mathematical knowledge and problem-solving skills, reasoning, and communication itself (Lomibao et al., 2016; Menezes et al., 2013; Rodrigues et al., 2020). In addition to this, the students' oral and written communication enables the teacher to assess the students' learning, their doubts and difficulties concerning mathematics, helping the teacher to construct strategies to overcome those difficulties. Communication plays a preponderant role in all teaching activity (Menezes et al., 2013; Wachira et al., 2013). The way that teachers facilitate communication and manage and structure their discourse in the classroom has a major impact on teaching and learning processes (Chapman, 2004; Wachira et al., 2013).

The discourse of the mathematics class is strongly influenced by the teacher's decisions, whether verbal or non-verbal. Questions stand out among the verbal (Chapman, 2004; Menezes et al., 2013). Questioning is one of the actions that are most associated with the teacher, embodying a fundamental aspect of communication in the mathematics classroom, as it corresponds to an invitation addressed to the other to enter the class discourse (Guerreiro et al., 2015; Menezes, 1995; Menezes et al., 2013; Wachira et al., 2013). Articulated with the tasks proposed by the teacher, questions challenge the students to engage in the activities, to conceptually design and discuss their strategies and the outcomes obtained (Menezes et al., 2020; NCTM, 2007). Aimed at promoting this discussion, the teacher is tasked with imbuing the classroom with a culture that cherishes and fosters questioning (Guerreiro et al., 2015; Moyer & Milewicz, 2002; Wachira et al., 2013), both on the part of the teacher and students. For Moyer and Milewicz (2002), a good question represents the difference between coercing the student's thinking as opposed to encouraging new ideas. Accordingly, the teacher's role consists of creating a 'discursive community', more specifically: (i) posing questions that challenge each student's thinking; (ii) listening carefully to the students' ideas and asking them to clarify and justify them; (iii) encouraging students to listen, answer and question the teacher and their classmates (Guerreiro et al., 2015; Menezes et al., 2013; NCTM, 2007).

The questions posed by the mathematics teacher can be categorized in different ways according to their objectives. Love and Mason (1995) classify questions into three categories: focalization questions; confirmation questions; and inquiry questions. Focalization questions aims to focus the student's attention on a specific aspect, and in this way, help the student to follow a particular path of reasoning and/or overcome a specific difficulty (Guereiro et al., 2015; Love & Mason, 1995). Confirmation questions seek to assess the student's understanding and knowledge of a given topic when this student has already been taught or is in the process of learning the topic. Inquiry questions are genuine queries aimed at obtaining information from the student, such as about their reasoning, problem-solving strategies or difficulties experienced. To this end, the teacher may question by asking the student to comment on a solving strategy or explain and justify a rationale.

Considering the diversity of questions that can be made by the teachers, it is essential that they squeeze the most out of the questions posed and the answers given by the students, making the questioning productive (Guerreiro et al., 2015; Menezes et al., 2013; Newton, 2017). With this in mind, Newton (2017) drew up a cycle to support the preparation of questions (see Figure 1).

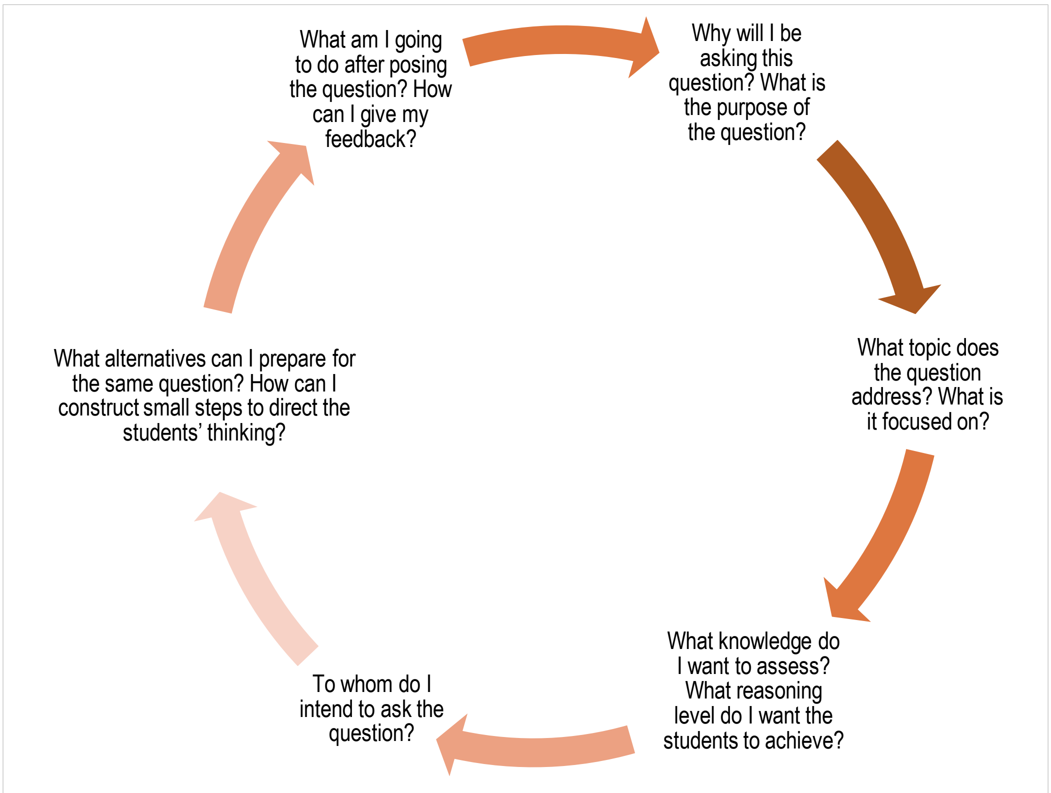


Figure 1. Sequence for preparation of questioning based on Newton (2017)

For the author, the questions should be elaborated bearing in mind their objective and the necessary mental processes for their answering. Thus, teachers should weigh up: (i) the reason underlying the question they intend to formulate and its purpose; (ii) which topic the question focuses on and how it is related to the study object or whether it focuses on some specific aspect; (iii) what type of reasoning is intended on the part of the student; (iv) who the question should be made to (group of students or a specific student) and what content is being worked on or in which context; (v) whether the question might need reformulating or require the preparation of some additional questions or strategies to enable guiding the student's thinking; (vi) what feedback will be given to the student.

Asking good questions is not a simple mission nor is the quantity of questions a criterion for the good use of questioning. The quality of the questions and their opportuneness are crucial for promoting communication interactions and for the productive learning of mathematics. Exploratory teaching is an educational format that boosts questions in the classroom. The exploratory teaching of mathematics (Guerreiro et al., 2015; Oliveira et al., 2013), inspired by inquiry-based learning (Artigue & Blomhøj, 2013;

Stein et al., 2008), is a powerful teaching approach for fostering the learning of mathematics, based on the solving of mathematical tasks, in three phases: (i) Introduction of the task; (ii) Exploration of the task; (iii) Collective discussion (Menezes et al., 2013). In the first phase, the teacher tends to ask confirmation questions to appraise the students' understanding of the task. In the second phase, the students work, normally in small groups, on solving the task, being monitored by the teacher through confirmation, focalization, and inquiry questions. During the collective discussion phase, the teacher's questions are primarily inquiry, designed to get the students to present and justify their ideas (Menezes et al., 2013). In this framework, in the next section, we recall the objectives of the study and the methods used.

METHODS

The aim of this study is to understand the role of teachers' questions in the learning of topics on Functions. To achieve this objective, we used episodes of a class that addressed the topic of 'Problems involving the quadratic function' of a Portuguese secondary school class group (10^{th} grade). This class was part of a teaching experiment which took place during six classes, conducted by one of the authors of this article. This teaching experiment delineated strategies aimed at valorizing the student's activity through an exploratory approach (Stein et al., 2008). On the one hand, the choice of Functions was because it was the theme assigned by the supervisor of the school where the supervised teaching practice of one of the authors of this study took place. On the other hand, because Functions are a topic not well liked by students and because there are few studies that address the questions in the dynamization of teaching and learning processes in secondary education. By adopting in the supervisory process an exploratory teaching format, we seek to promote teaching practices that consider the students' activity (what they say and what they do), we consider it relevant to analyze the role of questions in each of the moments that integrate such method. The class group in which this experiment was held consisted of 27 students, 12 of whom were boys and 15 girls, aged between 15 and 17 years old. Among the students' preferred mathematics topics, Geometry ranked first (52.6%), while Functions was the topic least enjoyed in previous school years (61%).

In view of the nature of the delineated objective, we adopted a qualitative and interpretative approach with the objective of understanding the role of the teacher's questioning in the mathematical activity of the students in solving the proposed tasks (McMillan & Schumacher, 2014). To this end, the data were collected through written productions of the students made individually during the exploration of the 'André's temperature' task, and audio and video recordings of the class, to compile the discussions and dialogues between students (S) and teacher.

Data analysis is based on content analysis techniques, crossing collected data and categories emerging from the literature. For this, the collected information was transcribed and coded. The integration of the questions in the teaching strategies is evinced in the descriptions and interpretations of the information collected during the different moments of the class, having as reference the work developed by Stein et al. (2008) and Menezes et al. (2013): (i) introduction of the task; (ii) exploration of the task; and (iii) collective discussion. As the questions tend to have different objectives, we sought to characterize them to identify those prevailing at any given moment and their role performed in the class discourse and student learning. For each class phase, we identified the type of question, indicator, purpose, and frequency based on the analysis of the classroom, dialogues, and written productions.

RESULTS AND DISCUSSION

The topic of 'Problems involving the quadratic function' challenged the students to apply the knowledge acquired during their study of the quadratic function in, for example, the identification of the domain and counter-domain, the determination of the vertex of its graphic representation (parabola), in addition to requiring the development of mathematical reasoning and communication skills. In order to motivate the students to solve problems involving the quadratic function, showing them its applicability in daily life and fostering their taste for discovery, the teacher proposed a problem in a real-life context considered suitable to that year group's performance level:

> André woke up at 5 am shivering. He picked up a thermometer and found that he was feverish. Following his mother's advice, he recorded his temperature over the next 5 hours. As he was studying the quadratic function, he realised that his temperatures varied according to the function T, defined by $T(x) = -0.5x^2 + 2x + 38$, which started to drop 20 minutes after taking medication for fever. (T represents temperature, in degrees Celsius, observed x hours after the first temperature measurement).
> 1.1. What temperature was observed at 5 am? Justify your answer.
> 1.2. What was the maximum temperature reached in the observation period? Justify your answer.
> 1.3. At what time did André take the medication? Justify your answer.
> 1.4. Show that his temperature stayed above 39.5 ºC between 6 and 8 am.
> 1.5. Sónia, André's sister, also has a fever. Her temperature varied according to the function S, defined by $S(x) = T(x + 2)$. During which time interval was Sónia's temperature higher than 39.5 ºC? Justify your answer.

Through this problem, the teacher wanted the students to be confronted with different solving strategies by their classmates, and that several contents related to the quadratic function should be discussed, promoting different moments of discussion and, in particular, questioning.

Introduction of the Task

At a first stage, the task was given to the students, who read the text silently, followed by a reading out loud by one of the students and a class group interpretation of the information contained in the text. In the phase, the launching of the task, the teacher sought to clarify any doubts that had emerged during its reading.

Teacher: So, when reading this text, did any doubts arise? Let's see, what is x?
Students: x is the time of the first measurement.
Teacher: The time of the first temperature measurement. And at what time did André start to measure his temperature?
Students: At 5 am.
Teacher: Well, think about it, what does 5 am correspond to?
Students: It's $x = 0$.
Teacher: And what does $x = 1$correspond to?
S20: It's one hour later, 6 am.
Teacher: Do you all agree?
Students: Yes.
Teacher: Does anyone else have any doubts after having read the text?
[The students did not raise any doubts]
Teacher: Ok, so let's do it.

In the dialogue established in the class group context during this phase of the class, the prevailing questions were focalization and inquiry, with only one inquiry question having been made. Throughout the dialogue, the teacher essentially sought to focus the students' attention and direct their reasoning. After having understood that they were having difficulty in interpreting the meaning of the variable x, the teacher decided to formulate some questions that would enable them to 'progressively construct their thinking'. Table 1 presents the number of questions of each type posed during the different dialogues occurred in the task introduction phase.

Table 1. Frequency of the type of questions asked by the teacher in the Task Introduction phase

Type of question	Indicator	Purpose	Frequency
Confirmation	"What values can the variable x take?"	Test the students' knowledge	1
Focalization	"Do we not have any detail on the domain of T?"	Direct the students' reasoning	3
Inquiry	"Do you all agree?"	Understand the students' opinion and reasoning	3
Total			7

Following this stage of interpretation of the text and clearing up of doubts, the students were given a moment of autonomous exploration of the task.

Exploration of the Task

Despite having discussed various details of the work, in a class group context, during the task's introduction, the solutions of task item 1.1 reveal the students' difficulties in the interpretation of the text's information and in grasping that variable x corresponded to the time elapsed since the first temperature measurement. When moving around the classroom, the teacher caught onto this specific issue and, therefore, decided to ask the class some further questions, so as to help their comprehension of some of the text's details.

Teacher: What detail is implied in the text? Do we not have any detail on the domain of function T?

[Noise arises in the classroom, but no student explicitly answers the question]

Teacher: What values can the variable x take?

S6: It can take values between 0 and $+\infty$.

Teacher: Can it take values up to $+\infty$? Does it measure his temperature forever?

Students: No.

S6: André stopped measuring his temperature at 10 am.

Teacher: Do you all agree with student S6? Or does anyone not agree?

[The whole class nods their head, showing that they agree with their classmate]

Teacher: Ok, so André started measuring his temperature at 5 am and stopped at 10 am. What is the domain?

S1: The domain is from 5 to 10.

Teacher: From 5 to 10? What does the text say about variable x?

S26: What is x after the first temperature measurement?

[The classroom becomes noisy]

Students: So, the domain is from 0 to 5.

Teacher: Why?

S6: The first measurement is at 5 am which is when $x = 0$. And then it goes up to 5 because André measures his temperature up to 10 am, in other words, 5 hours later.

Teacher: Do you all agree? Do you understand what your classmate said?

[The students state having understood and the student who was called to the blackboard solves task item 1.1]

The transcribed dialogues enable understanding the difficulties shown by the students in identifying the domain of function T. In view of these difficulties, the teacher asks the class group a series of questions. Confirmation questions were those most evidenced, having been used to ensure that the class group understood the classmate's ideas and to test the students' knowledge. Focalization questions were essentially used to direct the students' thinking and make them focus on specific aspects of the text, such as the domain of the function. In turn, the inquiry question emerged due to the teacher's need to understand the rationale of the students who spoke.

Following this stage, various students probed the teacher, seeking indications about how to solve the task, if what they had done was appropriate and if it justified what was intended, as illustrated by the following dialogue:

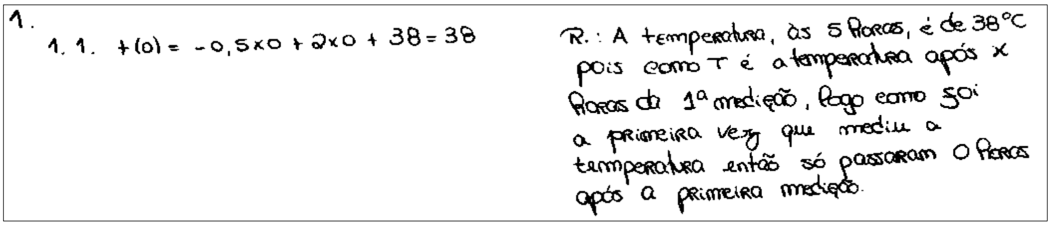
S26: I did it like this [points to her/his solution]. I calculated $T(0)$ and only put in that calculation. Is that right?

Teacher: I don't know. Can you justify why you put $T(0)$ and whether you think it makes sense?!

S26: Yes, I calculated $T(0)$ because at 5 am, as we have seen, $x = 0$. It's the first medication of André's temperature. That's it, right?

Teacher: We will soon see if that's the reasoning.

Through his answer and solution (Figure 2), the student revealed having understood the text and demonstrated the contribution of the previous questioning to his comprehension of the task text.


1.

1.1. $T(0) = -0,5 \times 0 + 2 \times 0 + 38 = 38$

R.: A temperatura, às 5 horas, é de 38ºC pois como T é a temperatura após x horas da 1ª medição, logo como foi a primeira vez que mediu a temperatura então só passaram 0 horas após a primeira medição.

Figure 2. Solution of item 1.1 by student S26 (The temperature at 5h is 38ºC because T is the temperature after x hours of the 1st measurement, so as it was the first time you measured the temperature then 0 hours passed after the first measurement).

Although the students who called out to the teacher had correctly answered this item of the task, it was found that some, despite harboring doubts, neither questioned nor expressed their doubts. For example, when the teacher approached the table of student S25, the teacher realized that the student had not solved item 1.1 and thus asked the following:

Teacher: Do you have any doubts?

S25: No.

Teacher: But you haven't yet done the first item. What is your reasoning?

S25: I think it's $T(5)$ so as to know the temperature at 5 am.

Teacher: We have already spoken about what x is. Go back and remember.

[S25 does not answer the teacher]

By questioning this student, the teacher asked two inquiry questions in order to understand whether the student was managing to organize his rational and how he was reasoning. Through these questions, the teacher understood that the student had not paid attention to the discussion about the item held at the beginning of the class or that he did not clarify his doubts at the time. In fact, the student then presents an incorrect solution of the item under consideration (Figure 3), revealing that the teacher might not have managed to capture that student's full attention during the different types of dialogues or that the type of questions made were not suited to the difficulties of this particular student.

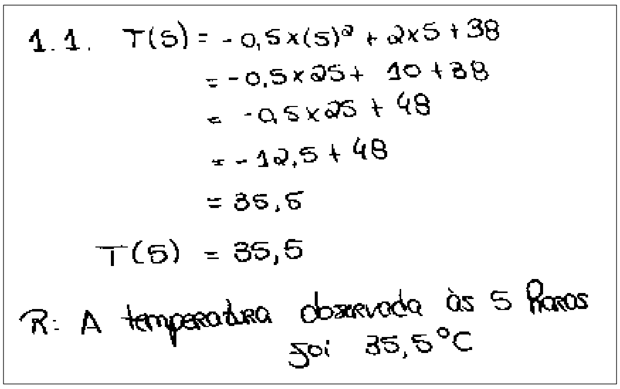


Figure 3. Solution of item 1.1 by student S25 (The temperature recorded at 5h was 35,5°C)

Another example of the contribution of questions in learning is the example of student S4 in task item 1.2, in which the students were asked to determine the maximum temperature reached by André during his temperature measurements. Although the topic had recently been studied in class, during the autonomous work, several students showed difficulty in expressing their doubts.

S4: Teacher, how do I do 1.2.? What was the maximum temperature reached?

Teacher: What is the maximum temperature?

S4: It's the highest temperature that André recorded.

Teacher: And how do you determine that?

S4: It's in this expression. So, it's like this: André started at 5 am, plus the following 5 hours this gives 10 am. Should I do it like this or is it nothing like this?

Teacher: How are you going to determine the maximum temperature with that reasoning? Do you know at what time the maximum temperature was reached?

S4: No.

Teacher: The expression gives us a graph of what?

S4: Of a parabola.

Teacher: And the maximum of a quadratic function corresponds to what?

S4: To the vertex of the parabola.

Focalization questions were primarily made in the dialogue with student S4, but there were also confirmation and inquiry questions. The focalization questions were designed to direct the student's thinking, as the student proved unable to organize his reasoning and understand that the maximum of the function corresponds to the coordinates of the vertex of the associated parabola. The confirmation question was merely used to check that the student understood what was asked in the task item at issue, with the student having answered with a brief comment. In turn, the inquiry question was used to understand the student's rationale. However, it is important to note that the teacher did not pause between this question and the focalization question, thus not enabling the student to answer the teacher's question. Despite this detail, the student revealed having understood what was requested in the item and the path to be followed in order to answer it. Consequently, during the class group discussion phase, the student S4 was selected by the teacher to present his solution to the class (Figure 4).

temp. máxima $\rightarrow$ máx. absoluto $\rightarrow$ vértice(h, k)

$$h = \frac{-b}{2a} = \frac{-2}{2\times(-0{,}5)} = \frac{-2}{-1} = +2 \qquad a = -0{,}5 \quad b = 2$$

$$k = T(h) = T(2) = -0{,}5\times 2^2 + 2\times 2 + 38 = 40$$

Figure 4. Solution of item 1.2. by student S4

Item 1.3 requested the students to determine what time André took the medication. Knowing that André's temperature started dropping 20 minutes after having taken the medication, the students only had to use the previous item to calculate the time that André took the medication. Only one student called upon the teacher, seeking to validate his reasoning.

S18: We only need to know at what time his temperature dropped and subtract 20 minutes, right?

Teacher: Does the text state that his temperature dropped 20 minutes after André took the tablet?

S18: Yes.

Teacher: And how do you know that André's temperature started to drop?

S18: It's the vertex and I calculated it in the previous item.

Teacher: Ok. Try to apply your reasoning to see if we can manage to answer the question in that manner.

Student S18 wanted to validate his reasoning, probably due to thinking that his solution was too simple, which was why he used a question. In order to understand whether the student's reasoning was correct and not answer his question directly but seek to make the student validate his reasoning autonomously, the teacher decided to ask two confirmation questions. The student gave a brief answer to one of the questions and gave an explanation in response to the other question. The brief answer was already expected when the question was made, because it was a closed-ended question. In this dialogue, the student formulated a question, an explanation, and a brief answer.

During the autonomous work on item 1.4, the teacher realized that various students were making the same mistake: calculating the temperature at 6 am and at 8 am, observing that it was 39.5 ºC. Accordingly, the teacher decided to intervene to draw their attention to that detail.

Teacher: About item 1.4, how are you thinking of solving it?
S6: I am doing $T(1)$ and $T(3)$, is that right?
Teacher: I am seeing some incomplete solutions. By calculating $T(1)$ and $T(3)$, what are you determining?
S18: We are determining the temperature at 6 am and at 8 am.
Teacher: And those calculations must result in 39.5 ºC. But does that mean the temperature stayed above 39.5 ºC between 6 am and 8 am?
S1: Yes.
Teacher: Why?
S18: No, we only know that at 6 and 8 am the temperature was 39.5 ºC.
Teacher: Ok, but we want to show that the temperature stayed above 39.5 ºC. So, think about it.

As illustrated by this dialogue, the teacher sought to highlight the fact that it was not sufficient to calculate the temperature at 6 am and at 8 am in order to correctly answer item 1.4. To this end, the teacher used focalization and inquiry questions. The focalization questions enabled focusing the students' attention on what is obtained by calculating $T(1)$ and $T(3)$ so that the teacher could then pinpoint their attention to the fact that it was necessary to demonstrate that the temperature remained higher between that period of time. In turn, one of the inquiry questions enabled drawing the students' attention to the discussion that was sought to be promoted and grasping how they were solving that particular item, while the other inquiry question enabled getting student S1 to think about how he was going to justify his answer.

During the exploratory phase of the class, the teacher tried to support the autonomous work of the students, but did not answer their questions directly, so as not to reduce the task's cognitive requirements. For such, the teacher formulated questions to enable checking the students' level of comprehension, directing their reasoning or understanding their thought processes. During the task exploration phase, the teacher sought to stimulate the students' reasoning. The frequencies of each type of question posed in the dialogues are not very distinct, as presented in Table 2.

Table 2: Frequency of the type of questions asked by the teacher in the Task Exploration phase

Type of question	Indicator	Purpose	Frequency
Confirmation	"State that his temperature lowered 20 minutes after?"	Test the students' knowledge	9
Focalization	"And the maximum of a quadratic function corresponds to what?"	Direct the students' reasoning	6
Inquiry	"Why?"	Understand the students' reasoning	6
Total			21

By observing Table 2, it is found that, at this phase of the class, the different categories of questions are rather similar in frequency, with a rather higher number of confirmation questions.

Collective Discussion

During the collective discussion, one of the main goals of the teacher's intervention was to ensure that the students who were wrong in each item should understand their mistakes and clarify their doubts, and that different forms of solving the item should be presented so that the students would have the opportunity to develop their reasoning.

Although the majority of the students solved item 1.1 correctly and discussed important details in the task introduction phase, some students proved not to have paid attention to the discussion or expressed their doubts. For this reason, after choosing a student to go to the blackboard to solve the first item of the task, the teacher confronted the students about some of the solutions observed in their notebooks.

As the incorrect solutions all involved the same mistake – determining $T(5)$ –, the teacher considered that it was important to confront the class group. Accordingly, a discussion was started about this item of task, emphasizing the fact that not all the students had presented identical solutions.

Teacher: Student S26 did $T(0)$, but I saw many students doing $T(5)$. Do you think it makes sense to calculate $T(5)$?
Students: No.
S25: Yes.
Teacher: Why? Can anyone manage to explain to student S25 why it's not $T(5)$ or why it's $T(5)$?
S4: Because $T(5)$ corresponds to the last measurement, that's when 5 hours have elapsed since the first measurement.
S25: Ok. I get it.
Teacher: And what do we want?
Students: The first measurement.
Teacher: Exactly. And that is obtained by calculating what?
Students: $T(0)$.
Teacher: Can anyone solve this is another way?
[The students shake their head sideways, meaning no]

The main difficulty consisted of grasping that the variable x represents the hours elapsed after the first temperature measurement and not the numeric representation of the time at a which André measured his temperature. During the class group discussion more inquiry questions were used to understand the students' opinion and the manner in which they were reasoning. For this purpose, three inquiry questions and two focalization questions were posed. The focalization questions were used to direct the students' reasoning and focus their attention on specific aspects, making them re-read the task text, associating its information and the meaning of the variable x.

At this stage of the class, in order to clear up the doubts of the students who answered item 1.2 incorrectly or in a partially correct manner, and in order to understand whether all of the students had grasped the reasoning of the procedures, the teacher sought to confront them with some questions. Furthermore, the teacher asked student S4 to sketch the graph of the parabola, as he had not done so in his solution.

Teacher: The temperature is given by what?
S4: By the expression.
Teacher: By the expression of what kind of function?
Students: Quadratic.

Teacher: Student A4 is drawing a diagram of a parabola...

S1: With downward concavity.

Teacher: If we want to know when the maximum temperature was reached, what does that correspond to on the diagram?

Students: To the vertex.

Teacher: And would it always correspond to the vertex, even if the concavity were not downward?

Students: No, if the concavity were upward, it would not correspond to the vertex.

Teacher: So?

S26: Depends on the domain, but it would have to be the highest y value.

Teacher: Do you all agree?

[S4 sketched the following graph (see Figure 5)]

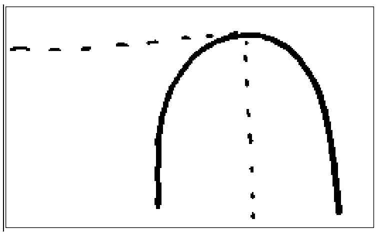


Figure 5. Representation of the graph of function T by student S4

After having observed the sketch drawn by student S4 on the blackboard, the teacher noticed that the representation of the graph was not mathematically accurate. Although this had not been requested in item 1.2 and was not necessary for solving it, as several students decided to sketch a graph, the teacher considered that it was pertinent to address this topic so as to overcome the difficulties experienced by the students. For the class group to help the student in question to improve his graphic representation and, at the same time, clarify his own doubts, several questions were made aimed at drawing their attention to its mathematical accuracy.

Teacher: Do you think that the representation of student S4 is correct?

Students: No, you must add the axes of the benchmark axes.

S4: I don't know, I didn't do that part of the representation.

S1: The graph doesn't begin like that, it begins at $x = 0$.

S1: You should erase the left part of the graph.

Teacher: Student S4, do you think the vertex could be there? Calculate the vertex first and then we'll see that representation.

[The student insists on continuing the representation of the graph]

Teacher: Please note that I saw lots of mistakes in the representation of the graph. Could you simply use the calculator and enter the expression?

Students: No.

Teacher: What do you need to take account of?

Students: The domain and the counter-domain.

Teacher: Ok. Do negative body temperatures exist? And what about the numeric representation of time?

Students: No.

In order for the students to understand the aspects that must be taken into account in the representation of the graph, the teacher decided to begin to draw a representation on the blackboard with some mistakes, namely in the domain (see Figure 6).

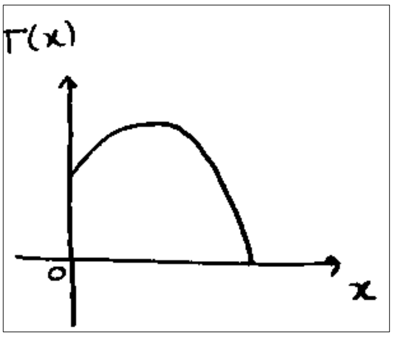


Figure 6. Graphic representation of function T drawn by the teacher

Teacher: Could the representation be like this?
[The students have different opinions]
S1: Could you not move the $y = 0$?
Teacher: I don't know, you tell me. What is the domain?
Students: From 0 to 5.
Teacher: Ok. Imagine that 0 is here [indicates a point on the graph] and 5 is here [indicates another point on the graph]. So, can we continue to draw the graph?
Students: No.
S4: No, we need to erase here [points to a part of the graph].
Teacher: Do you agree with student S4?
Students: Yes.
Teacher: Why?
Students: Because it's not included in the domain of the quadratic function.

This dialogue gave rise to the following graphic representation (Figure 7).

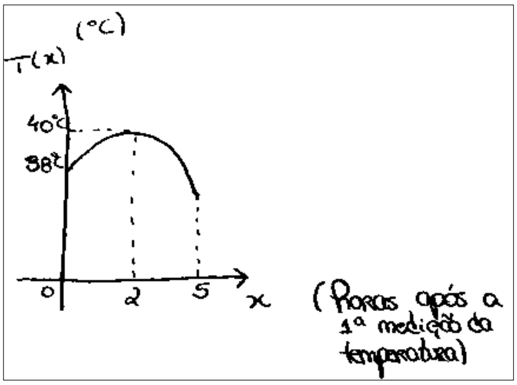


Figure 7. Graphic representation of function T on the blackboard with the students' assistance (hours after first temperature measurement)

The class group had difficulty in solving this item, showing some lack of mathematical accuracy, and struggling with the graphic representation of the quadratic function in question. Therefore, the questions were primarily asked to help overcome these difficulties. To this end, the teacher formulated confirmation questions, aimed at testing the students' knowledge, enabling them to assess their own level of knowledge, and so that the students with greater difficulties could, both through their own answers and those of their classmates to the questions, clear up any doubts and surmount those difficulties. The inquiry questions were essentially used to understand the students' reasoning and check that their answers had been underpinned by a correct rationale. With these two categories of questions, the teacher wanted the students to manage to organize their reasoning and better understand some concepts that had not yet been fully taken on board, such as, for example, domain and counter-domain, which seems to have been achieved by the majority of the class group.

Although the questioning proved to be important in all the items, whether in the identification of difficulties, in the clarification of doubts or in the development of reasoning, it was in item 1.4 that the questions demonstrated being particularly crucial. This item required the students to show that the temperature stayed above 39.5 ºC between 6 and 8 am. Although different problem-solving strategies could have been presented, the students had to show that the temperature remained above that figure and could not be limited to calculating the temperature at 6 am and 8 am. Here, the teacher intervened and sought to provide a moment of discussion during the autonomous work. While this strategy proved to be effective in overcoming the difficulties of various students, others revealed that they had not understood the difference between the various scenarios addressed by the teacher. Hence, and in order to promote a class group discussion, the teacher decided to request one student with a high level of performance and ability to express himself to solve this item on the blackboard. Student S24 presented his solution on the blackboard, explaining his reasoning (see Figure 8).

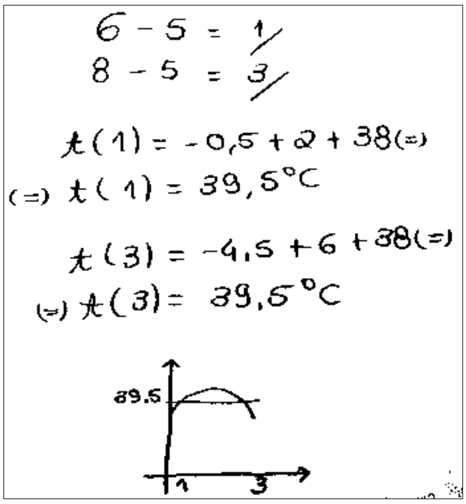


Figure 8. Solution of item 1.4 by student S24

Teacher: Did everyone understand the solution presented by student S24? Does everyone agree or did anyone solve it in a different way?

[Student S22 calls the teacher and say he does not agree with his classmate's solution]

Teacher: Student S24, your classmate has an issue. I don't think he agrees with you.

S24: Go on, say it.

S22: In the question, they give us the data that between 6 am and 8 am, the temperature is above 39.5 ºC. And you are basing yourself on what they give you in the question, knowing that it's correct, to check. Shouldn't you discover the answer based on the text and not on the question?

[Student S24 shows that he did not understand his classmate's question and the classroom becomes noisy]

S22: If you didn't have the question, you wouldn't know that it was between 6 am and 8 am that the temperature was above 39.5 ºC.

Students: Can't it be done like that? It's also in the text of the question.

Teacher: What your classmate is saying is that, in his opinion, what student S24 did is not a demonstration, a "shows that", but rather a sort of checking. Is that what you are saying, student S22?

S22: Yes, precisely. You are checking that $T(1)$ gives 39.5ºC and $T(5)$ does too.

S24: No, I demonstrated it with the sketch of the graph because the graph has downward concavity.

[Student S24 solves the item in a different way, using second degree inequalities (see Figure 9)]

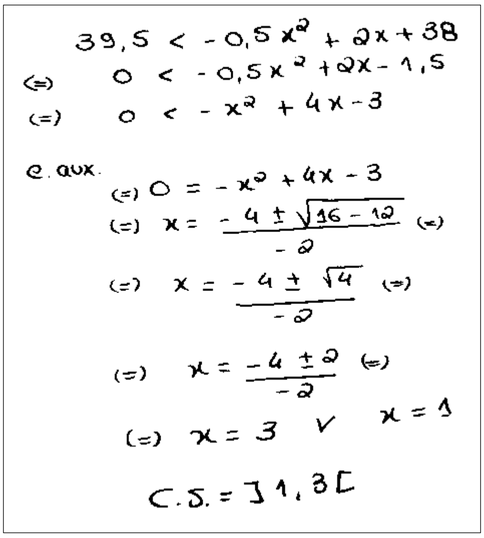


Figure 9. Solution of item 1.4 by student S24 using a different method

Teacher: What is missing here, something that you learnt when solving second degree inequalities? [pointing to the solution on the blackboard]

Teacher: Calculate the zeros of this expression, making them equal to zero, but how do you know when the function is positive? Do you do it mentally?

Students: The sketch of the graph.

Teacher: To identify just what in this sketch?

Students: The sign of the function.

[The teacher completes the solution on the blackboard, sketching the graph with the help of the class group]

Teacher: From $-\infty$ to 1, is the function positive or negative?

Students: The function is negative.

In this task item, the discussion centered on two students who had solved it using different strategies and who did not agree about the validity of the strategy adopted by one of them. As the discussion was progressively and spontaneously further enriched by the students, the teacher did not ask many questions, merely intervening to kick off the initial discussion and manage the students' interventions. The majority of the questions were made later, after the student who was at the blackboard presented a second solution, using his classmate's problem-solving strategy, with some lacunas in that solution such as, for example, the lack of a sketch of the graph. During that part of the class, the teacher formulated focalization questions that helped to focus the student's attention on the graph required for the study of the monotonicity of a function, one inquiry question – which enabled understanding the students' opinion –, and one confirmation question. The confirmation question was used for the purpose of testing the students' knowledge about the monotonicity of functions, although it could have been enriched if the teacher had merely asked: "What is the monotonicity of the function from $-\infty$ to 1?". Thus, the teacher could have required the student to apply all of his knowledge on the concept of monotonicity.

Table 3 presents the frequencies of each type of question made by the teacher during this phase of the class.

Table 3. Frequency of the type of questions asked by the teacher in the Task Exploration phase

Type of question	Indicator	Purpose	Frequency
Confirmation	"from $-\infty$ to 1 is the function positive or negative?"	Test the students' knowledge	7
Focalization	"To identify just what in this sketch?"	Focus the students' attention on particular details	10
Inquiry	"why?"	Understand the students' reasoning	12
Total			29

Table 3 illustrates that the inquiry and focalization questions were the most numerous, showing somewhat analogous frequencies. The confirmation questions were those less used by the teacher at that phase of the class.

In general, it was found that, during the three phases of the class, the types of question show a rather similar frequency, albeit more particularly the inquiry and confirmation questions. The confirmation questions were essentially aimed at testing the students' knowledge and, simultaneously, with their answers, clearing up the students' own doubts or those of their classmates. The focalization questions were used to direct the students' reasoning, focusing their attention on particular aspects of the task text, for example. In turn, the inquiry questions served to understand the students' reasoning, motivating them to justify their strategies, and understand whether the students agreed with their classmates, had decided to follow different strategies, or were struggling with doubts or difficulties. However, it was also found that the questions were not always suited to the intended aim or did not manage to fully capture the students' attention. Despite this fact, over the different stages of the class, the potentialities of questions became evident and likewise, their contribution to effective learning, when appropriate to the moment, to the student and to the intended aim.

CONCLUSION

Questions formulated during classes perform a preponderant role in teaching and learning processes and are a means to maintain the pace of the class and engage the students in the class activity. The teacher's formulation of a high number of questions reveals the importance given by that teacher in fostering interactions between and with the students, as indicated by Newton (2017). The questions posed by the teacher of this study aimed to detect difficulties among the students and help them to overcome those difficulties, clear up doubts, promote the sharing of ideas and encourage participation and discussion (Menezes et al., 2013; Rodrigues et al., 2020; Wachira et al., 2013).

While the teacher's discourse was characterized by questions, they had different purposes, according to the class phase and the way that they were formulated. In the task introduction phase, the teacher wanted the students to interpret the task text and talk about it. For this reason, the key purpose of the questions was to check that the students understood the task text and what was required of them. In this phase, checking questions were evident, primarily aimed at appraising whether the students were correctly interpreting the task text and whether they agreed with their classmates' ideas. As argued by several authors (Oliveira et al., 2014; Stein et al., 2008), this phase is enormously influential in the success of solving the task.

In the following task exploration phase, the students expressed their doubts and sought to construct distinct problem-solving strategies. As the majority of the students' interventions aimed to check whether a particular solution was correct or clarify doubts, the teacher formulated questions that enabled her to support the students without that assistance reducing the level of the cognitive challenge of the proposal (Chin, 2006; Moyer & Milewicz, 2002¸ Stein et al., 2008). Although the questions fluctuated between the three types, confirmation questions stand out particularly.

During the class group collective discussion phase, the students showed that they were motivated to grasp the strategies used in the solutions by answering the questions that were made and assessing the answers given by their classmates. The most striking feature of this phase, highlighted by Ponte (2005) and Canavarro et al. (2014), is the dynamics of interaction between the different players who expressed ideas, answered, and addressed questions to one another. In this phase, the questions were mainly intended to fire up the discussion, encouraging the students to participate in the class discourse, sharing and justifying their ideas (Lomibao et al., 2016; Stein et al., 2008). To this end, the teacher formulated inquiry questions, above all, also with the support of some confirmation and focalization questions.

In the overall analysis of the different phases of the class, it was found that the inquiry questions were predominant, followed by focalization questions and confirmation questions, with very similar frequencies. For this type of class, this reveals the need to combine these different types of questions with varying frequencies.

The teacher showed a tendency to pose a high number of questions and, in this way, boost the discussion. Although the teacher was concerned with asking questions and getting the students to participate, there were moments when the sheer quantity of questions and the time between them did not allow the student to think and have time to respond.

In a nutshell, the study revealed that the questions formulated in the classroom by the teacher are a powerful instrument to engage the students in the class discussion and, thus, contribute to the learning of mathematics. Nevertheless, it is not easy to ask the right questions, at the right times and to the right students, meaning that it is imperative that the teacher's discursive skills must be more thoroughly worked on during teacher training, both the initial and continuous.

Acknowledgements

We would like to thank the Centre for Research in Education (CIEd) of the University of Minho, the school that allowed the pedagogical practice, the Centre for Studies in Education and Innovation (CI&DEI) and the Polytechnic of Viseu for their support.

Declarations

Author Contribution	:	AVS: Conceptualization, Writing - Original Draft, Editing and Visualization. FV: Writing - Review & Editing, Formal analysis, and Methodology. LM: Validation and Supervision.
Funding Statement	:	This work is funded by CIEd – Research Centre on Education, Institute of Education, University of Minho, projects UIDB/01661/2020 and UIDP/01661/2020, through national funds of FCT/MCTES-PT. This work is also funded by National Funds through the FCT – Foundation for Science and Technology, I.P., within the scope of the project Ref. UIDB/05507/2020. Furthermore, we would like to thank the Centre for Studies in Education and Innovation (CI&DEI) and the Polytechnic of Viseu for their support.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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