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RME-based local instructional theory for translation and reflection using of South Sumatra dance context

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Abstract

A learning trajectory as a sequence of mathematical activities that could facilitate the growth of students' understanding of learning goals. This research aimed to produce an instructional sequence to learn the concepts of translation and reflection and to investigate how students develop their understanding informally using a South Sumatra dance context. The research employed the design research method, which consisted of three stages: preliminary design, design experiment (pilot experiment and teaching experiment), and retrospective analysis. This research involved six students who have the high, medium, low capability at the first cycle and 32 students ninth graders at the second cycle at Junior High School 1 Palembang. Data from observations, tests, interviews, and documentation were analyzed descriptively. The tests used were aimed to determine the improvement made by the students; while the classroom observations, interviews, and documentations were used to develop the local instructional theory. This study produced a learning trajectory with three activities include: the students watched a dance video to describe the dancer's' positions in relation to floor tiles; the students were asked to determine the starting points of the dancers in Cartesian coordinates to learn concepts; and the students developed a formal concept based on their own knowledge.

Keywords: Design Research, Local Instructional Theory, Realistic Mathematics Education, South Sumatra Dance, Translation and Reflection

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One of the topics that has been push as an important part of geometry curricula is transformation geometry (Yanik & Flores, 2009). Learning about geometric transformations is important because it helps develop the geometric reasoning ability, spatial ability and mathematical proofs (de Villiers, 2017). Learning geometric transformation in mathematics is essential for three reasons: it gives students a chance to consider fundamental mathematical ideas (such as functions and symmetry), provides them with a context in which to see mathematics as a connected science field, and gives them a chance to engage in high level reasoning activities using a variety of representations (Trung et al., 2019). Geometric transformations have many roles in students' mathematical development (Fife et al., 2019)

However, several studies have shown that aspiring teachers may not always possess a comprehensive of the mathematics they are anticipated to teach (Yanik & Flores, 2009). The teachers and students lack the understanding of the concepts of reflection, rotation, and translation (Harper, 2003; Thaqi & Gimenez, 2012). To facilitate students in the concept of geometry transformation, a certain way is needed that is in accordance with student understanding, namely by doing activities realistically or related to daily life that can be used potentially for teaching and learning (Uygun, 2020). One of the supported learning activities to students' learning progress on transformation geometry based on Realistic Mathematics Education approach (Febrian & Astuti, 2018).

Realistic Mathematics Education (RME) as a learning theory or approaches that can help students since it links mathematics to all levels of complexity and a range of context that are applicable to student' daily lives (Armiati & Sari, 2022; Fauzan et al., 2020; Rawani et al., 2019). RME, is also used as a framework to build students' mental grasp from the concrete to the more abstract according to their degree of understanding. Learning geometry transformation requires mathematical visual thinking ability (Hermiati et al., 2021); reasoning ability (Febrian & Astuti, 2018) so that it can help students. Geometric transformation also is a beneficial to build spatial ability, geometric reasoning ability and strengthen mathematical proof (Sari & Putri, 2021). One of the three ways of thinking, visual thinking can be used as an alternative by students in solving mathematics especially those related to geometry transformation (Hermiati et al., 2021). Students are encouraged to share their ideas with others in order to contribute to the teaching and learning process (Sembiring et al., 2008). The concepts of transformation geometry are closely related to real world situations. This is consistent with the fact that mathematics is a discipline with a variety of practices, classifying it as a historically, culturally, socially, and politically human endeavor just like any others (Albanese & Perales, 2015; D'Ambrosio, 2016). Indonesia, rich and diverse in culture, has an opportunity to improve the mathematics education system in Indonesia through transformational efforts to bring mathematics closer to students' reality and culture (Prahmana & D'Ambrosio, 2020). The context in RME learning can be developed according to the local culture conditions in Indonesia (Sari & Putri, 2021)

The use of context is one of the characteristics in mathematics learning with RME (Rawani et al., 2019). Traditionally, mathematics as a school subject was positioned as both value and culture free (Lester, 2007) Beside that, get the impact of values in mathematics education on both cognitive and affective outcomes for students (Seah, 2018). Learning basic geometric shapes can be helped using the context of dance as an interesting and appropriate medium, to help students especially students who are visual and kinetics learner (Iyengar, 2015).Geometric shapes in space, patterns, symmetry and asymmetry, and phrase counting make math a wonderful partner for dance (Greer et al., 2009). Because dancing helps link more abstract ideas to real and basic movement principles, it is a particularly effective technique to enhance conceptual learning (Radiusman et al., 2021). Dance is a captivating medium that may help students picture problems while assisting visual and kinetic learning to understand fundamental geometric shapes; therefore we advocate using dancing to teach mathematics (Brenda Pugh, 2006). There are many different traditions and customs in Indonesia. Each province has its own unique culture. For example, the Kejei dance from Rejang Lebong district incorporates the concept of geometry in the form of kites (Ma'Rifah et al., 2019); the Saman dance, which opened the 2018 Asian Games, features mathematical concepts like 2 dimensional objects and sets (Maryati & Pratiwi, 2019); the Trenggal dance and the Madurese dance both use mathematical concepts to deepen students' understanding of number material (Ekowati et al., 2017); and the mathematical concepts taught in elementary schools can be found through the Gandrung Jejer Jaran Dawuk dance (Hariastuti et al., 2021). Culture is believed to be one of the aspect that can facilitate the growth of individual creativity to supported teachers can decide to do something special to achieve the best results (Rawani et al., 2023).

Effective learning must have a preparation or process design good learning (Lumpkin, 2020) and and exciting activity through the effort of teachers (Yuanita et al., 2018). Good learning design is a learning design that is tailored to the needs of participants educate for the achievement of learning objectives that have been set. The draft includes design of learning objective, learning strategies, teaching material used in learning, and assessment of learning (Isnawan & Wicaksono, 2018) Learning design is a plan related to student activities that are adjusted to the competencies possessed by students (Geitz & de Geus, 2019). Oriented learning approach in the students center, the learning design designed by teacher needs to pay attention to there is a student learning trajectories (Siemon, 2021).Learning trajectories (LT) has the potential to support a better understanding of student learning, effective teaching strategies, and guide better curriculum and standards design (Ellis et al., 2016). LT provides a plan or pattern which will be used as a reference to make learning plans in each process learning to be done. LT also are crucial resources for teachers to link domain specific knowledge about students' thinking with student centered (Wilson et al., 2015) LT is called Hypothetical Learning Trajectories (HLT) because its design is still in the form of a guess or hypothesis (Nuraida & Amam, 2019). A HLT is a tool for understanding how students think. It can be used to create assignments and activities that are based on assumptions about how students think. Provides conjectures of students' learning with a specific route when students learn a particular topic (de Beer et al., 2017). Learning trajectories are crucial resources for teachers to link domain specific knowledge about students' thinking with student centered education (Wilson et al., 2015). Learning objectives, learning activities, and a hypothesis about how students learn are the three key parts of a hypothetical learning trajectory (Simon & Tzur, 2004). Learners can emerge when an instruction is given in a designed environment from a sound theoretical point of view (Cobb et al., 2010). According to some research hypotheses, introducing students to the concept of vision lines through visual activities could aid in their understanding of transformation geometry. This study conjectured and expected that exploring the idea through visual field activities might result in meaningful insights for students in their understanding of the concepts of transformation geometry as well as their misconceptions relating to these concepts. This research aimed to produce an instructional sequence to learn the concepts of translation and reflection and to investigate how students develop their understanding informally using a South Sumatra dance context.

METHODS

This study employed the design research type validation study, which comprised three phases: (1) preliminary design, (2) design experiment (pilot experiment and teaching experiment), and (3) retrospective analysis (Akker & Gravemeijer, 2020; McKenney & Reeves, 2014; Putri & Zulkardi, 2017). This research involved six students who have the high, medium, low capability and had 3 male and 3 female students at the first cycle at Junior High School 1 Palembang, Indonesia. Six students participated in a pilot experiment aiming to produce an instructional sequence to learn the concepts of translation and reflection. 32 ninth graders and had 16 male and 16 female students at the second cycle at Junior High School 1 Palembang, Indonesia. 32 students participated in a teaching experiment aiming to investigate how students develop their understanding informally using a South Sumatra dance as a context and find formal forms of transformation geometry through learning with a designed instruction.

The group of six students and the group of 32 students were both from the same school, but from different classes. The implementation of the research was carried out in the even semester of the 2022/2023 academic year. The data from classroom observations, tests, interviews, and documentation were analyzed descriptively. The tests used were aimed to determine the improvement made by the students; while the classroom observations, interviews, and documentations were used to develop the local develop the local instructional theory. The production process of the instructional sequence to investigate students' understanding can be seen in Table 1

Table 1. The Method Process

Phase	Process	Objective	Instrument	Product Produced
Preliminary Design	An HLT (consisting of learning objectives; learning activities, and conjectures of students' learning process) was created and designed; a needs analysis of curriculum and a concept, a literature review, and interviews with teachers were conducted; and a worksheet, a lesson plan, a grid, a teacher instruction design, an observation sheet, and student answer predictions were created	To produce an instructional sequence to learn the concepts of translation and reflection.	Classroom observation, interviews, and documentation	Literature review; Design HLT; Get information by interview teacher to know students' ability; a worksheet, a lesson plan, a grid, a teacher instruction design, an observation sheet, and student answer predictions were created
Design Experiment	The HLT was implemented (i.e., pilot experiment and teaching experiment)			Result of discussions between teacher and researcher about upcoming activities and the end of each activity
1) Pilot Experiment	The researcher acted as a model teacher and the math teacher did as an observer. The subjects were six ninth graders of heterogeneous mathematics levels (high, middle, and low). The subjects were different from those involved in the teaching experiment.	To produce an instructional sequence to learn the concept of translation and reflection.	Classroom observation, interviews, and documentation	
	Thirty-two ninth graders of	To produce an instructional		

2) Teaching Experiment	heterogeneous mathematics levels (high, middle, and low) were involved, with a cross adjustment of the seats of male and female students. The teaching experiment was conducted to see how the improved HLT affected students' understanding of translation and reflection. From their answer and discussions with students, the researchers observed students' thoughts. The purpose of the interviews was to examine the learning opportunities and learning obstacles.	sequence to learn the concepts of translation and reflection. To investigate how students developed their understanding informally using a South Sumatra dance as a context and find formal forms of transformation geometry through learning with the designed instruction	Classroom observation, interviews, documentation, and test	
Retrospective Analysis	The HLT was compared against the actual learning process of the students as the main focus. The researcher, model teacher and observers reviewed the lesson with the aim of knowing the strengths and weaknesses of the learning process. Their criticisms and suggestions were then used to improve the quality of learning.	To produce an instructional sequence to learn the concept of translation and reflection	Classroom observation, interviews, and documentation	Results of analyzed all the data and obtained the results of HLT evaluation. Then the HLT was developed and tested again in the next cycle. And the end of the second cycle, the researcher found the answer to the research question.

RESULTS AND DISCUSSION

This research aimed to produce an instructional sequence to learn the concepts of translation and reflection and to investigate how students developed their understanding informally using a South Sumatra dance as a context and then found formal forms of transformation geometry through learning with the designed instruction.

The activities were validated in a Focus Group Discussion (FGD) with lecturers, two PhD graduated, and teachers interested in RME and design research and then implemented as a preliminary design. The design instruments developed by the researcher distributed to be reviewed for several days. In a zoom meeting, the review's finding was spoken about and presented. Based on the advice and criticism of the experts and the students' comprehension of the issue, the prototype was deemed qualitatively valid (Zulkardi, 2002).

The initial ideas for a translation and reflection instructional activities were explored through a literature review and an interview with a ninth-grade teacher who as model teacher in this research. The implementation of the research was carried out in the even semester of the 2022/2023 academic year from the school where the test was done. The purpose of the interview was to determine the teacher's method of instruction, find out students' difficulties in learning translation and reflection, and examine student conjectures. The researcher designed a learning project using South Sumatra traditional dance as a context, including the development of a student worksheet, a lesson plan, a grid, a teacher instructional design, an observation sheet and student answer predictions. In line with McCuthchen stated that math is a good partner for dancer because of the geometric shape in space, patterns, symmetry, and counting of phrases (McCuthen, 2006). Contextual arises from student's immediate environment, especially a cultural and local wisdom content (Khusniati et al., 2017). One of principles reformulated RME as a learning approach is reality principle assuming that reality is not only discovered after studying mathematics in solving everyday problems, but also consider as a learning resource and context (Van den Heuvel-Panhuizen & Drijvers, 2014). A combination of theoretical and factual perspectives served as the basis for designing the hypothetical learning trajectory (HLT) (see in Figure 1).

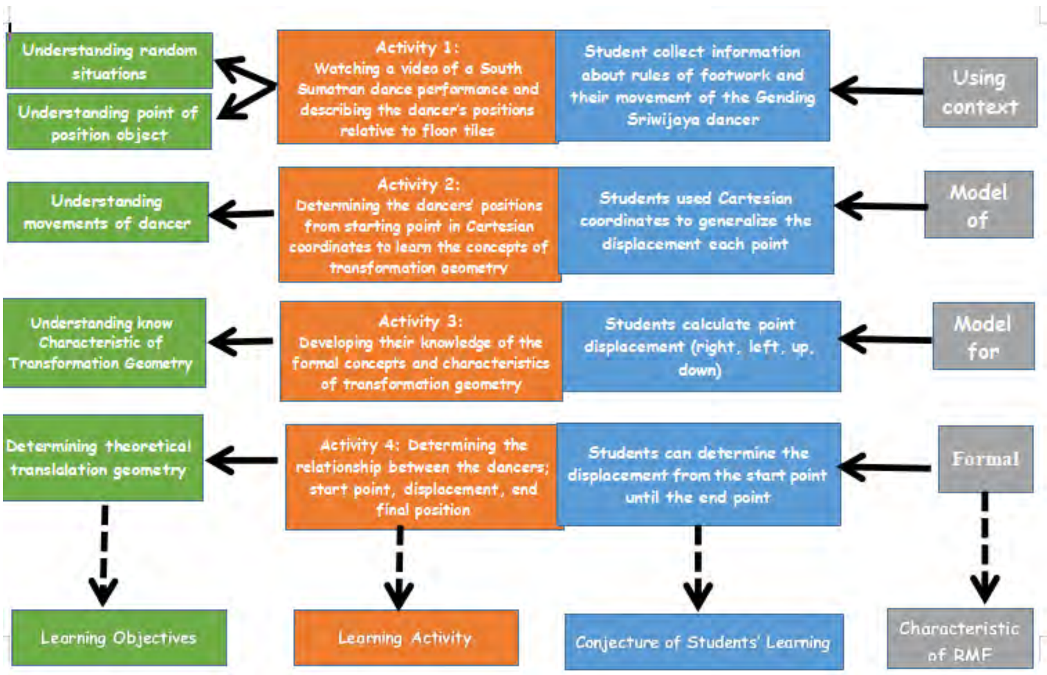
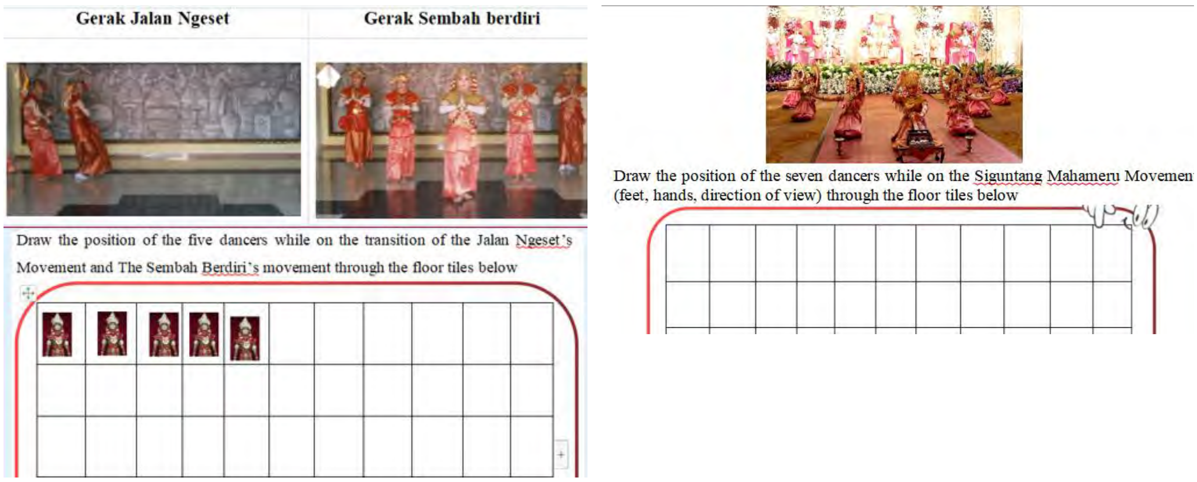


Figure 1. A Dance based hypothetical learning trajectories for transformation geometry

Throughout the teaching experiment, this HLT was dynamically altered and adapted to students' actual learning processes. Learning objectives, learning activities, and conjectures of students' learning process were the three components of the HLT (see Figure 1). The learning activities consisted of three parts arranged from the simplest: deciding the footwork rule, identifying the movements of the Gending Sriwijaya dancers, and discussing the displacements made until the end point. This development process is described in future sub sections. This study produced a learning trajectory consisting of three activities.

Activity 1: The students watched a dance video and described the dancer's positions in relation to floor tiles.

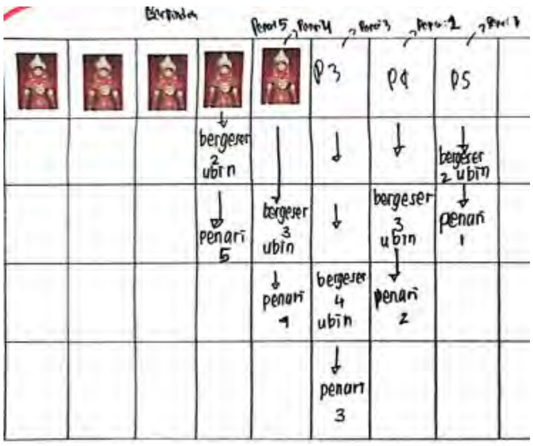
In the first activity, based on observation the students were presented with a video of Gending Sriwjaya dance through a projector. The students identified and collected information about the Gending Sriwijaya dance movements. Then, the students were given a worksheet instructing them to illustrate the positions of five dancers during the transition between the Ngeset movement and the Sembah Berdiri movement in respect to floor tiles that are depicted as unit boxes. The students calculated how many floors tiles the first dancer passed from the initial position, i.e., straight line floor pattern to the final position, i.e., V-shaped floor pattern as shown in Figure 2(a). Meanwhile, Figure 2(b) illustrates seven dancers' pose in the Siguntang Mahameru movement (i.e., the feet, hands, and the direction to face) in relation to the floor tiles on which to step. This can be seen in Figure 2.



(a) (b)

Figure 2. The Dancers' positions relative to the floor tiles

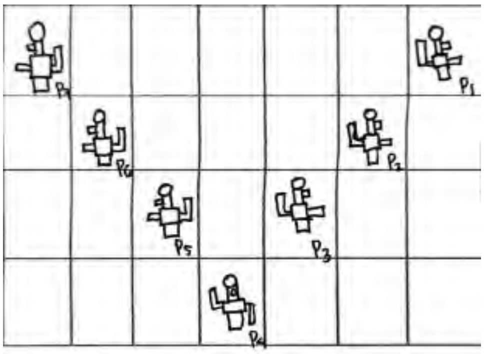
Figure 2 shows the students' understanding and experience of Gending Sriwijaya dance movements as a context as well as identification of the rules and characteristics of the dancer's' footwork. Figure 2(a) shows a student activity about translation. The transition from an initial position to another position indicates movement that the dancer. Meanwhile, Figure 2(b) shows a student activity related to reflection. In the Siguntang Mahameru movement, the dancers were in the same distance from a central axis. The video of the dance helped the students observe how dancers transitioned from one position to another. The students performed the activity in a variety of ways; some illustrated each dancer's step, and some did the final movement of the dancers only. This can be seen in Figure 3.



Translate in english

Dancer of 5	Dancer of 4	Dancer of 3	Dancer of 2	Dancer of 1
shifted 2 tiles	↓	↓	↓	shifted 2 tiles
Dancer of 5	shifted 3 tiles	↓	shifted 3 tiles	Dancer of 1
	Dancer of 4	shifted 4 tiles	Dancer of 2	
		Dancer of 3		

(a) MFA's strategies



(b) BAT' strategies

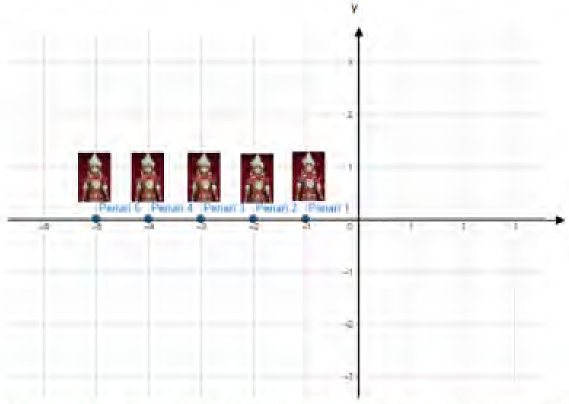
Figure 3. Student's strategies in describing the dancer's' position relative to the floor tiles

Figure 3 describes the student's' strategies to solve the problem in the first activity. Figure 3(a) shows a student's description of the dancer's stage entry. MFA's strategy was to use an arrow to illustrate the dancers' shift from a straight line forward with attention to their distances to each other to form a V-shaped pattern. MFA described that Dancer 1 shifted three tiles to the right and two tiles forward. Dancer 2 did three tiles to the right and three tile forward. Dancer 3 did three tiles to the right and four tiles forward. Dancer 4 did three tiles to the right and three tiles-forward. And dancer 5 did four files to the right and two tiles forward. Meanwhile, according to Figure 3(b), BAT solved the problem by illustrating the dancers with mathematical symbols. A circle denoted the dancer's head, a small square did the direction the dancer was facing, and a rectangle and an L shape did the hands.

Activity 2: The students were asked to determine the dancers' initial positions as starting points in Cartesian coordinates to learn the concept of transformation geometry.

In the second activity, the students illustrated the positions of five dancers in Cartesian coordinates. The steps of the dancers had been identified in the previous activity. The dancer's starting points were the points at which they were positioned before they made their first movement (seen in Figure 4).

The position of the five dancers before entering the stage is illustrated as shown below, which is located on the X axis with 1 dancer coordinated (-1,0) and each dancer has the same unit distance.



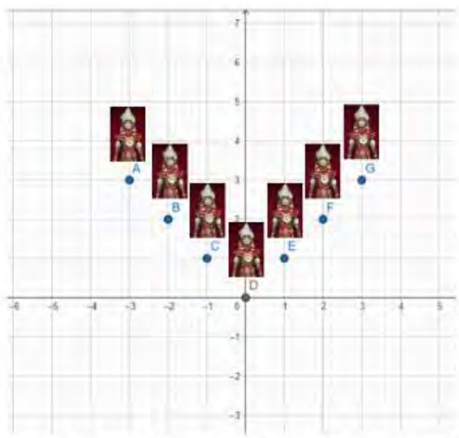
What are the coordinate points of dancer 1, dancer 2, dancer 3, dancer 4 and dancer 5?

If dancer 1 is shifted to the right (positive X- axis direction) by four units, where is the position of dancer 1 after being shifted? Is dancer 1 implicated with other dancers, if yes, where the dancer is positioned ?

If dancer 4 is shifted down (negative Y- axis direction) by one unit, where is the position of dancer 4 after being shifted? Is dancer 4 implicated with other dancers,, if yes, where the dancer is positioned ?

(a)

The position of the seven dancers in the Siguntang Mahameru movement is illustrated as shown in the picture below, which is located on the X axis with dancer A coordinated (-3, 3) and each dancer has the same unit distance.



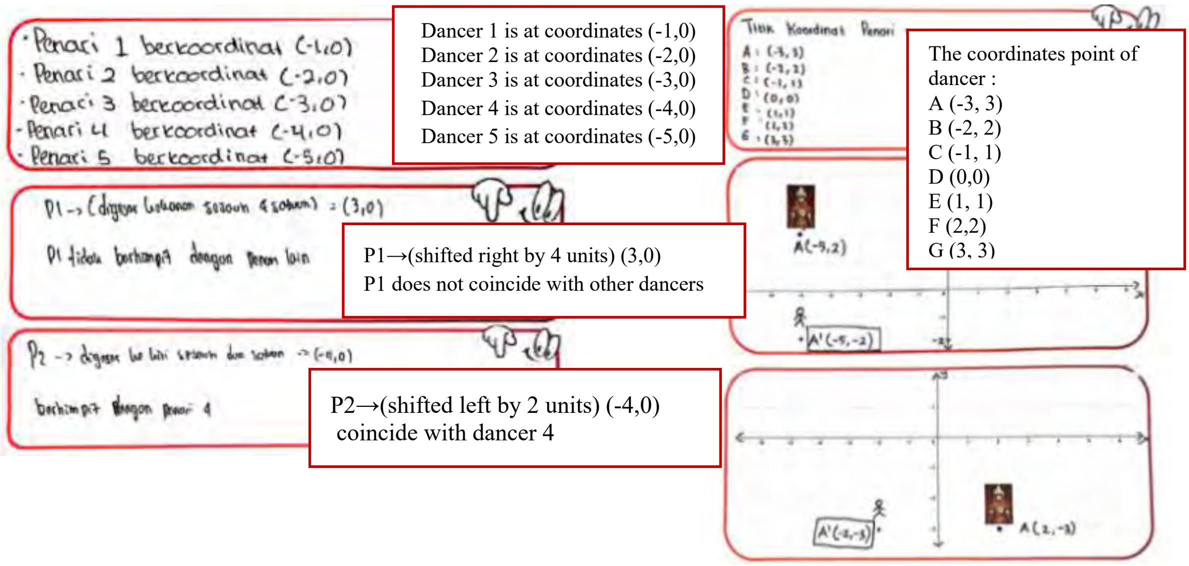
What are the coordinate points of dancer A, dancer B, dancer C, dancer D, dancer E, dancer F and dancer G ?

(b)

Figure 4. The dancers' s position in Cartesian coordinate

Figure 4 depicts the students understanding in illustrating the dancers' positions in Cartesian coordinates. Figure 4(a) shows a student activity related to translation. The students indicated the directions in which the dancers moved, i.e., to the right/left or up/down and displacement in Cartesian coordinates. The four translation processes of source allocation, initial coordination, targets construction, and equivalence determination were clearly represented in the graph (Bossé et al., 2014). Meanwhile,

Figure 4(b) shows a student activity related to reflection. The students were asked to determine a dancers' reflections with respect to the x-axis and the y-axis if she was at a certain point. In this case, the students could clearly classify the movement directions and alter the position according to their own knowledge. A number of student answers are provided in Figure 5.



(a) DDZ's strategies (b) NS's strategies

Figure 5. The student's' strategies in determining the dancers' positions in Cartesian coordinates

Figure 5 describes the student's' strategies in determining the dancers' positions in Cartesian coordinates. As shown in Figure 5(a), DDZ indicated the direction of each dancer using an arrow and wrote the dancer's Cartesian coordinate. The student wrote that dancer 1 (-1, 0), dancer 2 was at (-2, 0), dancer 3 was at (-3, 0), dancer 4 was (-4, 0), and dancer 5 was at (-5, 0). Then, dancer 1 shifted to the right by four units to (3, 0) without an overlap with other dancers, and dancer 2 shifted to the left by two units to (-4, 0) dancer 4. Meanwhile, Figure 5(b) show how NS solved the problem by an arrow to indicate the direction of each dancer and writing down the coordinate position of each dancer. The student wrote that dancer A was at (-3, 3), dancer B was at (-2, 2), dancer C was at (-1, 1), dancer D was at (0, 0), dancer E was at (1, 1), dancer F was at (2, 2), and dancer G was at (3, 3). The students were asked to draw a mirror image of an object in relation to the x-axis, which in this case was at the point (-5, -2). They were subsequently asked to draw a mirror image of the object in relation to the y-axis, which in this case was at the point (2, -3).

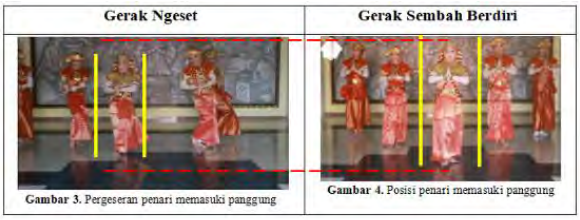
Activity 3: The students developed a formal concept based on their own knowledge and identified the characteristics of transformation geometry.

The third activity aimed to find out the students' understanding of the relationship between the dancer's starting position, displacement, and final position as well as the characteristics of the movements. The students wrote the translations in a formal form in general.

Soal Nomor	Posisi Awal penari	Pergeseran ke-				Posisi Akhir penari
		Atas	Bawah	Kanan	Kiri	
1.	(-1, 0) (-2, 0) (-3, 0) (-4, 0) (-5, 0)	0	0	0	0	(-1, 0) (-2, 0) (-3, 0) (-4, 0) (-5, 0)
2.	(-1, 0)	0	0	4	0	(3, 0)
3.	(-2, 0)	0	0	0	2	(-4, 0)
4.	(-3, 0)	2	0	0	0	(-3, 2)
5.	(-4, 0)	0	1	0	0	(-4, -1)
6.	(-1, 0) (0, 0)	0 1	0 0	-1 0	0 0	(0, 0) (0, 1)
7.	(-2, 0)	1	0	0	0	(-3, 0)

Figure 6. The question and student answers on the dancers' displacement

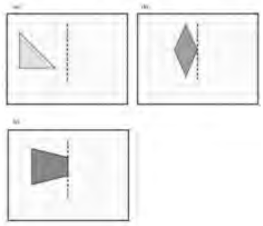
Figure 6 describes students' answers from the student's activity sheet 2. The students were required to calculate the magnitude of displacement of the dancers and determine the direction in which they moved. The use of the South Sumatra dance contexts and the floor tile Cartesian coordinate models helped the students understand the concept of transformation geometry. The conjecture in the last activity was the students determining the geometric translation of dancers with changes in shape, size, and position. The student gained information on the relationship between the dancer's starting position, displacement, and the final position. Based on the coordinates of the starting point and the magnitude of displacement the mirror position of the dancer was obtained. In addition, the direction of the dancer's displacement was also determined. FA' s answer, which was based on the information from the preceding activities. To the first question, FA that their displacement between the starting position and the final position. In other words, the starting position and the final position were the same. In answering the second question, FA identified a shift to the right by four units, from (-1, 0) to (3, 0). In answering the third question, FA identified a shift to the left by two units, from (-4, 0) to (-2, 0). FA answered the fourth question by writing there was an upwards shift by two units from (-3, 2) to (-3, 0). FA' s answer to the fifth question was that there was a shift downwards by one unit. In answering the sixth question, FA stated that there were two shifts, i.e., to the right by one unit and upwards by one unit. The shift was from the starting point (-1, 0) to the final point (0, 1). The answer to the last question was an upwards shift by one tile to the final point (-3, 0) from the starting point (-2, 0). In addition, following the worksheet, the students discovered the characteristics of translation and reflection (see Figure 7).



Characteristic	Yes or Not
The translational objects undergoes a change of shape	……………………
The translational objects undergoes a change of size.	……………………
The translational objects undergoes a change of position	……………………

(a)

Draw the mirroring of each flat wake below on the mirror line

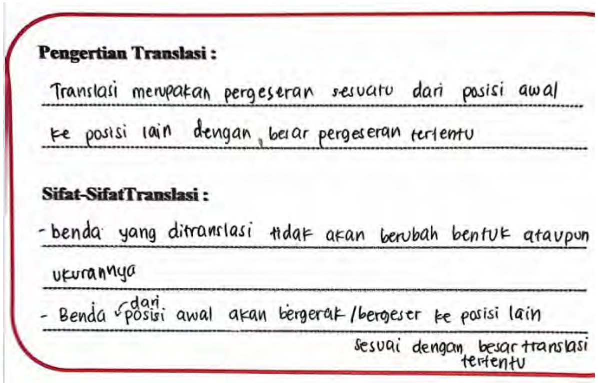


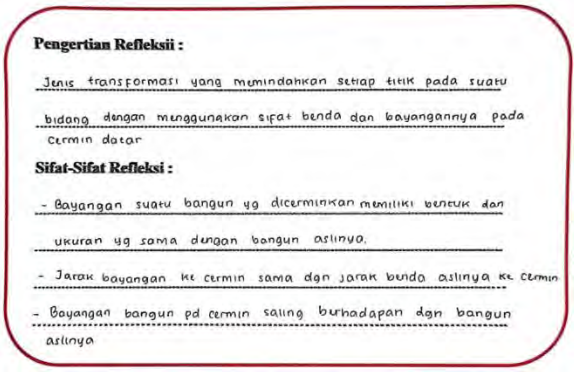
Characteristic	Yes or Not
The distance of the object to the mirror line and the distance of the image to the mirror undergo a change in position	……………………
The distance of the object to the mirror line and the distance of the image to the mirror undergo a change in size	……………………
The distance of the object to the mirror line and the distance of the image to the mirror undergo a change in shape	……………………
Thee shape and size of the object to the mirror line and the distance of the image to the mirror facing each other	……………………

(b)

Figure 7. Determining the characteristics of translation and reflection

The last activity was assisted by an illustration showing the movement of an object for understanding the concept of translation. It is as can be seen in Figure 7(a). As for the concept of reflection the students were asked to describe how a reflection occurs through the mirror line. On the right side is the mirror image of object on the left. The dotted line is called the line of reflection. It is also sometimes referred to as the axis of reflection or the mirror line. Notice that the object and its mirror image are at the same perpendicular distance from the mirror line. It is as can be seen in Figure 7(b). Student answers to the problem on the concepts of translation and reflection can be seen in Figure 8.





Translate in English

Definition of translation: the shift of something from the initial position to another position with a certain magnitud of shift.
Then the characteristic of translation:
-Translation objects will not change their shape or size and objects from the initial position will move/shift to another.
-Objects from the initial position will move / shift to another position according to a certain translational magnitude

(a)

Translate in English

Definition of Reflection: a type of transformation that moves every point on a plane by using the properties of objects and images on a flat mirror
Characteristic of Reflection:
-The shadow of a mirrored wake has the same shape and size as the original build.
-The distance of the image to the mirror is equal to the distance of the original object to the mirror.

(b)

Figure 8. Student answers on concepts and characteristics of translation and reflection

Figure 8 presents student answers related to the concepts and characteristics of translation and reflection. Figure 8(a) is the answer of QM regarding the definition and the characteristics of translation. QM defined that translation is the shift of an object from an initial position to another position by a certain magnitude of shift. The student then described the characteristics of translation as follows: an object will not change in shape and size in translation from an initial position to another and the will moves from the initial position to the final position by a certain translation magnitude. Figure 8(b) is the answer of RT. RT explained that reflection is a type of transformation that moves every point on a plane by using the properties of objects and images on a flat mirror. The student describes the characteristics of reflection as follow: the mirror image has the same shape and size as the original object; the distance of the mirror image to the mirror is equal to the distance of the original object to the mirror; and the mirror images and the original object face each other.

In the second stage, the hypothetical learning trajectory was implemented in two stages of experiment, i.e., pilot experiment and teaching experiment. The pilot experiment was conducted to assess the viability of the initial HLT design involving six ninth graders representative differing mathematical ability levels (i.e., high, middle, and low). During the pilot experiment, the researcher acted as a model teacher and the math teachers did an observer, they then switched their roles for the teaching experiment, for the purpose of assessing how the learning process was run. The concepts of translation and reflection were used in the teaching experiment. The teaching experiment involved 32 ninth graders of heterogeneous mathematical ability levels (i.e., high, middle, and low) seated in a cross-seating arrangement between male and female students.

In the final stage, all the data collected during the experiment stage were retrospectively analyzed with a focus on the comparison between the HLT and the actual learning process of the students. In order to identify the strengths and flaws of the learning process as they reflect the success of the teaching materials created and used, model teacher, the researchers and observers examine the lesson (Zulkardi & Putri, 2019). The learning process was made better by implementing their concerns and recommendations.

The second aim of this research was to investigate how students developed their understanding informally using a South Sumatra dance as a context and find formal forms of transformation geometry through learning with the designed instruction. The HLT that had been designed was tested to see the extent to which the conjecture and instruments that the researcher had designed were carried out. The teaching experiment was conducted to see how the improved HLT affected students' understanding of translation and reflection. From students' answers and interviews with them, the researcher observed the students' thoughts. The purpose of the interviews was to examine the learning opportunities and learning obstacles. Therefore, the researcher conducted interviews to confirm students' understanding, especially on translation and rotation and how to express notation of their formation. The following transcription describes this situation.

Researcher	: What do you think of the dancers' movements shown on the video?
Student	: The movement between its parts is clearly visible. Through hand, feet, eye view.
Researcher	: How do you describe this movement object in Coordinate Cartesians?
Student	: First identify the initial position of the dancer's object. Then observe how much movement the object through at once in its direction. After that, explained them in detail movement between translation or reflection.

This object Coordinate Cartesian represents that specifies each point uniquely by a pair of real number.

After the experiment stage, the researcher and model teacher reviewed the lesson. The observation and reflection phase aims to discover the strengths and weaknesses of the learning implementation were identified. The model teacher as a subject initiated the discussion by sharing their impressions, experiences, constraints, and opinions about the implementation of the learning. The activity starts with delivered the impression of knowledge, then the observer responds. Their suggestion geared to improve the quality of learning and offered wisely without degrading and hurting the teachers. It was concluded that students worked collaboratively within groups to solve problems, but several errors were found. Some students make mistakes when drawing object before translation. This is in line with Mulwa that one form of student difficulty is interpret mathematical symbols (Mulwa, 2015). Lower ability students received a guidance and collaborated with higher ability peers to solve problems (Putri & Zulkardi, 2018). Therefore, the duty of a teacher of mathematics as educator, to develop the ability to think from a low level at a high level (Susanti et al., 2019).

Classroom events were recorded, group works were video-taped and student materials were collected. The interaction and collaboration between a student and another student in every lesson enables the learning process to run well. Collaborative learning is strategy associated with RME theory because learning activities should engage an interactive principle. In this research, the use of a South Sumatra dance as a context and the floor tile Cartesian coordinate model helped students understand concept of transformation geometry. In general, the South Sumatra traditional dance as a context could inspire students to actively participate while learning and thinking mathematically. Using a context encourages students to think mathematically because their potential for mathematical thought processes arises from a given real situation (Kohar et al., 2019; Rawani et al., 2019; Zulkardi et al., 2020). Additionally, the use of a context encourages students to participate in collaborative learning (Putri & Zulkardi, 2020).

CONCLUSION

The learning trajectory can be used by teachers to create learning activities that assist the growth of their students' conceptual grasp of mathematical concepts. The use of a South Sumatra dance as a context to discover the formal forms of translation and reflection through learning with a designed instruction based on the RME approach helped students improve their' mathematical ability. The three-activity learning trajectory designed guided students to figure out the characteristics of translation and reflection. The three activities included the students watching a video of a South Sumatra dance performance and describing the dancer's positions relative to floor tiles; the students determining the dancers' positions from starting point in Cartesian coordinates to learn the concepts of transformation geometry and the students developing their knowledge of the formal concepts and characteristics of transformation geometry. It was found that the use of the South Sumatra dance context and floor tile Cartesian coordinate model helped the students understand the concepts of transformation geometry. Limitations of this research some students make mistakes when drawing object before translation and reflection. For future research to design the learning activities that using the worksheet to improve student ability of recognition the mathematical symbols.

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