Mathematics Student-Teachers' Views of Teaching Practice in Classroom Context

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Abstract: This study aimed to analyze views of mathematics student-teachers on teaching practice in classroom contexts by subsuming following the new didactic triangle, which was used as a conceptual framework: 1) Teaching Process, 2) Learning Process, and 3) Thinking Process. A participative research design was employed for the research methodology. The participants included seven fourth-year student-teachers, a purposive group of a case study that had been teaching practice in schools of the mathematics teacher education program for two weeks. They voluntarily participated in schools using Lesson Study and Open Approach and collaboratively designed lesson plans coached by researchers. After two-week teaching practices in school contexts, they were asked to reflect on their teaching practice covering 1) whether their teaching practices are accomplished, show some evidence of students' ideas, 2) identify problems found in their teaching practice, and 3) identify improving aspects of teaching practice for subsequent lessons' improvement. Results of the study showed that the mathematics student-teachers' views are reflected on their teaching practice in the actual classroom context following the aspects of the new didactic triangle and correlate with teaching mathematics through problem-solving, which is a mainstream and widespread pedagogical approach.

INTRODUCTION

Teacher preparation is one of the crucial aspects of the educational reform movement in many countries. In Thailand, especially in the mathematics teacher education program, most teachers' colleges are attempting to improve the teaching practice of student-teachers by improving courses and supporting teaching practice (Ball & Cohen, 1999; Inprasitha, 2015; Hancherngchai, Inprasitha, & Thinwiangthong, 2017).

In the case of the mathematics education program of Lampang Rajabhat University, Thailand, there are some endeavors to improve its course and support its teaching practice. In addition, in the case of encouraging the teaching practice for its student-teachers, the Lesson Study and Open Approach are involved in this developmental process.

In this research context, the teaching practice was driven by Lesson Study and Open Approach, which aimed to improve teacher education programs and professional development in Thailand since 2002 (Inprasitha, 2011; 2015; 2022). Two innovations, Lesson Study and Open Approach, have been proposed to change the paradigm of teaching practices from a product-oriented approach to a product-process-oriented approach, and improve the teaching practices consecutively (Inprasitha, 2004).

This paper focused on the views of the student-teachers after their teaching practice in a school context of Lesson Study and Open Approach in relation to components of a didactic triangle in mathematics to demonstrate how the program encouraged the student-teacher's teaching practice. This process of research will be brought to improve a forthcoming (mathematics) teacher education program.

RESEARCH OBJECTIVE

This study aimed to subsume the reflections of the student-teachers relating their views of teaching practice derived from the new perspectives of a didactic triangle; the teaching processes, learning processes, and thinking processes. This teaching practice was in the classroom using an Open Approach as a teaching approach and Lesson Study as a way to improve the teaching approach of the student-teachers of the mathematics teacher education program, Faculty of Science, Lampang Rajabhat University, Thailand. For achieving this research objective, the after-teaching-practice reflections of the student-teachers and the new didactic triangle (Inprasitha, 2014) had been adapted to consider views of their teaching practice.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

A literature review was accomplished concerning the teaching practice, which is the foundation for the teacher education program (Jarrah, 2020). For this reason, the teaching practice could be reflected in the views of student-teachers, which are related to the didactics or teaching in their classrooms.

Teacher Education Programs and Teaching Practice

Teaching practice is a fundamental element of teacher education programs. It is not only because it provides student-teachers with experience in a real classroom context, but it is also an opportunity for student-teachers to implement pieces of stuff they have learned about their subject matters (Jarrah, 2020). Therefore, the teaching practice is also crucial for professional and personal development as the growth of student-teachers (Azhar & Kayani, 2017). In addition, effective teachers are expected to possess a richness of content knowledge, suitable theoretical foundation, and competence in pedagogical and instructional strategies (Cochran-Smith & Lytle, 1999).

Across the twentieth century, there is consistently a requirement for teacher education which has been severed by a persistent split between subject matter and pedagogy knowledge. In other words, fragmenting teaching is a gap in teacher education (Ball & Bass, 2000).

Consequently, to do the teaching practice successfully, teacher education programs need to provide knowledgeable teachers who are willing to learn about their students as learners. Moreover, the student-teachers must acquire the skills, disposition, and instructional strategies required to teach such subject matters. In addition, teacher education program needs to provide opportunities for student-teachers to connect different sorts of knowledge (Kilpatrick, Swafford, & Findell, 2001).

Didactics of Mathematics

The ZDM-The International Journal on Mathematics Education in 2012 has a theme of "New Perspectives on the Didactic Triangle: Teacher-Student-Content" which called for various perspectives of mathematics education researchers about fundamental relationships within the didactic triangle or instructional triangle. This volume provides integration of technology roles in teaching mathematics, the researcher in mathematics teaching developmental research, and mediating complexes in the student-teacher-content vertices (Goodchild & Sriraman, 2012).

There are also researchers in the mathematics education community who have been interested in the research of the didactic triangle or didactic theory over decades ago and are still relevant today, for example, Brousseau (1997), Kilpatrick, Swafford, & Findell (2001), Cohen, Raudenbush, & Ball (2003), Straesser (2007), and Ruthven (2012). These studies are almost related to how teachers might be empowered to become aware of and work on relationships between themselves (the teacher), their students, and mathematics (Goodchild & Sriraman, 2012). Likewise, Brousseau (1997) raised a domain of the didactic theory regarding the theory of didactical situations in which the teacher creates a milieu in which students engage with mathematics. Regardless, there are some attempts to extend more vertex of the didactic triangle. For example, Ruthven (2012) and Rezat and Straßer (2012) stated that technology or artifacts were fundamental constituents of the didactic triangle, resulting in the didactic tetrahedron or socio-didactic tetrahedron.

Teaching through Problem-Solving

Problem-solving is central to mathematics knowledge construction (Pehkonen, 2019). In a learning and teaching atmosphere, in other words, mathematical problem-solving is significant and challenging as it is the heart of mathematics (Jarrett, 2000). Furthermore, teaching and learning mathematics through problem-solving support the development of learners with a deep understanding (Inoue et al., 2019).

In teaching through problem-solving, learning emerges during the problem-solving process. When the students solve the problems, they might use any procedures, recall any pieces of knowledge they have learned, and convincingly identify their ideas. In the learning atmosphere, the teacher should provide opportunities for the students to express various solutions to their class and learn

mathematics through meaning negotiation and grasping a shared understanding. These activities support the students in clarifying their ideas and acquiring different perspectives from their peers (Cai et al., 2003).

Moreover, Takahashi (2021) stated that teaching through problem-solving (TTP) is how Japanese teachers teach new mathematical notions by providing students with compelling mathematical challenges to solve on their own and discussing with the students to find shared conclusions, as in the TIMSS video. It was called Structured Problem-solving (Stigler & Hiebert, 1999). Additionally, teaching mathematics through problem-solving bestows educators an instrument for restructuring their lesson and curriculum design to create creative and adaptive problem-solving simultaneously (Takahashi, 2021).

Lesson Study and Open Approach

The traditional teaching approach in Thailand emphasizes on product-oriented approach as the teacher needs only an answer from the students. Consequently, there is an endeavor to shift a paradigm of teaching practices from a product-oriented approach to a product-process-oriented approach by introducing innovations since 2002; Lesson Study and Open Approach. In other words, the teacher needs to go beyond the answer. The process of solutions and the reason behind these solutions should also be emphasized. This endeavor has been consecutively improving teaching practices (Inprasitha, 2014).

In addition, the Open Approach is composed of four phases; 1) Posing Open-ended Problems, 2) Students' Self-learning, 3) Whole Class Discussion and Comparison, and 4) Summarizing through Connecting Students' Mathematical Ideas that emerged in Classroom. This teaching approach emphasizes individual differences, incredibly individual differences in students' thinking. Teachers try to collect their students' ideas to conduct a summarization in accordance with the student's ideas. The Lesson Study, moreover, focuses on improving the collaborative working of teachers for improving the teaching approach, composed of three steps; 1) Collaboratively Design Research Lesson (Co-Plan), 2) Collaboratively Observe Research Lesson (Co-Do), and 3) Collaboratively Reflect on Teaching Practice (Co-See), and done this collaborative work in a week or weekly cycle (Inprasitha, 2011; 2015; 2022). These two innovations are incorporated in the second step of the Lesson Study or Collaboratively Do, as in figure 1, called a Thailand Lesson Study incorporated Open Approach (TLSOA) (Inprasitha, 2022).

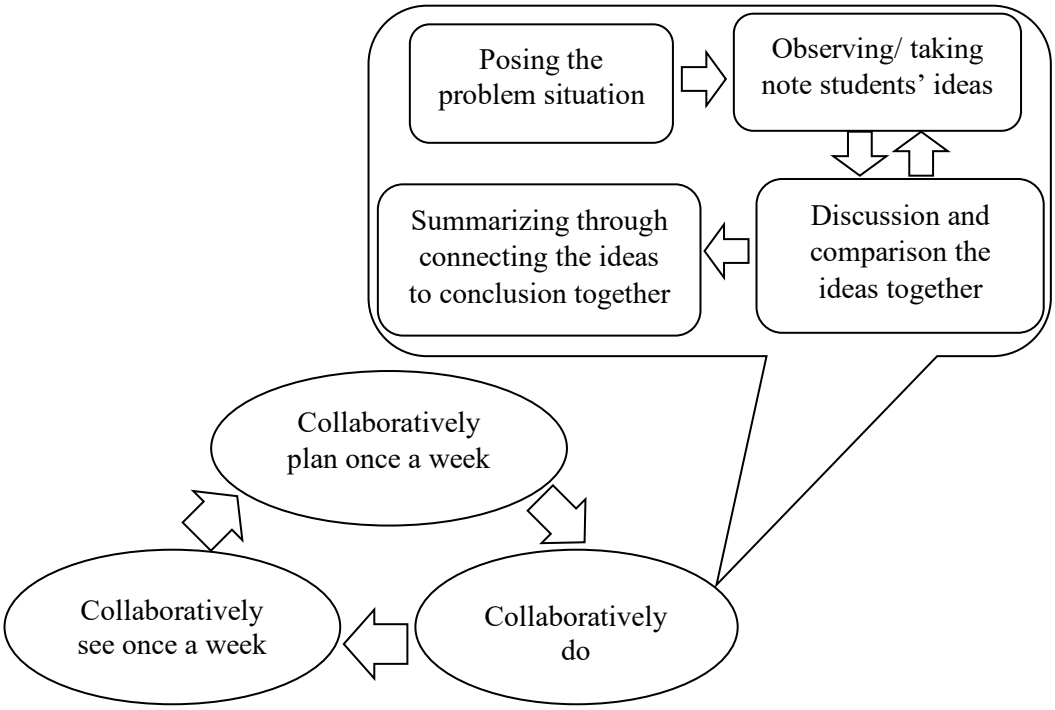

Figure 1: The cycle of Lesson Study and Open Approach
(Adapted from Inprasitha, 2011; 2015; 2022)

Additionally, the Lesson Study encourages the teachers to have a Community of Practice (CoP), described as the community as a way of talking about the social form in which members of the community, defined as participation in the community members, is recognized as competence (Wenger, 1998).

In the new context of teaching and learning mathematics, Inprasitha (2014) has proposed three classroom components as a new didactic triangle: the Teaching Process, Learning Process, and Thinking Process. These components are related to the student's ideas used for accessing the student's learning or thinking processes.

There is, consequently, an extended perspective on the three components of the didactic triangle: teacher, student, and content concerning the teaching approach that emphasizes teaching through problem-solving and utilizes the students' ideas in steps of reflecting, planning, and observing as the Lesson Study step: Teaching Process, Learning Process, and Thinking Process, as in figure 2, respectively. The new didactic triangle (Inprasitha, 2014) was employed as a theoretical framework to subsume views associated with teaching practice in the classroom of the student-teachers in the mathematics teacher education program. Details of three constituents in the new didactic triangle are as follows.

1) Teaching Process is related to the teacher's capabilities to engage the students to have their problems from problem situations, preferring to use semi-concrete aids to extend the students' ideas that occurred in the classroom and encouraging the students with questions.

2) Learning Process is related to how each student will master their experience with the condition and context of a problem situation. The students afterward could solve the problem situation in various ways. In addition, the students could share their ideas with their friends in the classroom.

3) Thinking Process is related to the student's engagement with the problem situation and encountering a problematic condition. The students will proceed with ways of solving the problem. The students then formed their ideas using 'how to' from previous lessons to think about and solve such problematic conditions to generate mathematical ideas.

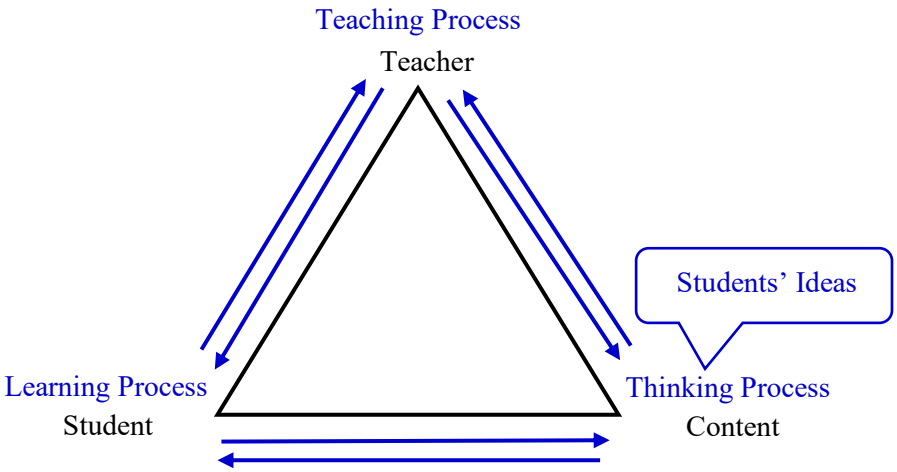

Figure 2: New didactic triangle (Adapted from Inprasitha, 2014)

RESEARCH METHODOLOGY

Research Design

This study was a case study of student-teachers in a teacher education program who had two-week teaching practice in a school context. The study employed a qualitative research methodology as a participative research design in which the researchers participated in a collaborative design (Collaboratively plan) and a collaborative reflection (Collaboratively see) of the Lesson Study processes with the mathematics student-teachers who served as a purposive group of this study.

Participants

In this study, the researchers surveyed all 52 student-teachers in the fourth year of the five-year teacher education program in the 2020 academic year. This study was done in the course of mathematics education called 'Teaching Experiment'. This course provided an experience for student-teachers in a school context as they will have their mathematics classroom at the primary or lower secondary school level for 8-12 hours a week (Mathematics Program, Faculty of Science, Lampang Rajabhat University, 2014). Seven fourth-year student-teachers voluntarily participated (McLain & Kim, 2018) in the schools that use the Lesson Study and Open Approach. They were a purposive group that had been doing the Lesson Study and Open Approach for two weeks in a school context. To protect the identities of the purposive group, the researchers abbreviated each participant by applying a code student-teacher added by a capital letter, i.e., student-teacher A-G.

In addition, the mathematics student-teachers of Lampang Rajabhat University learn about subject matters in courses of a five-year mathematics teacher education program. They had opportunities to attend extra-curricular activities during four years of the teacher education program. The extra-curricular activities, such as National Open Class, were part of school mathematics (Klein, 1982). These activities applied the Lesson Study and Open Approach processes; co-plan, co-see, and co-reflect, to cooperate with in-service and pre-service teachers of the mathematics teacher education program working in the school context.

Data Collection

These mathematics student-teachers used Japanese textbooks translated into Thai versions (Inprasitha & Isoda, 2019) to collaboratively design lesson plans coached by the researchers on their lesson study team at the university. They then went to the schools with the lesson plans and taught in the classroom context using the Open Approach as a teaching approach. Furthermore, they and in-service teachers used a weekly cycle of the Lesson Study to collaboratively reflect on getting aspects used to improve the students' ideas via collaboratively designing activities used in their lesson plans.

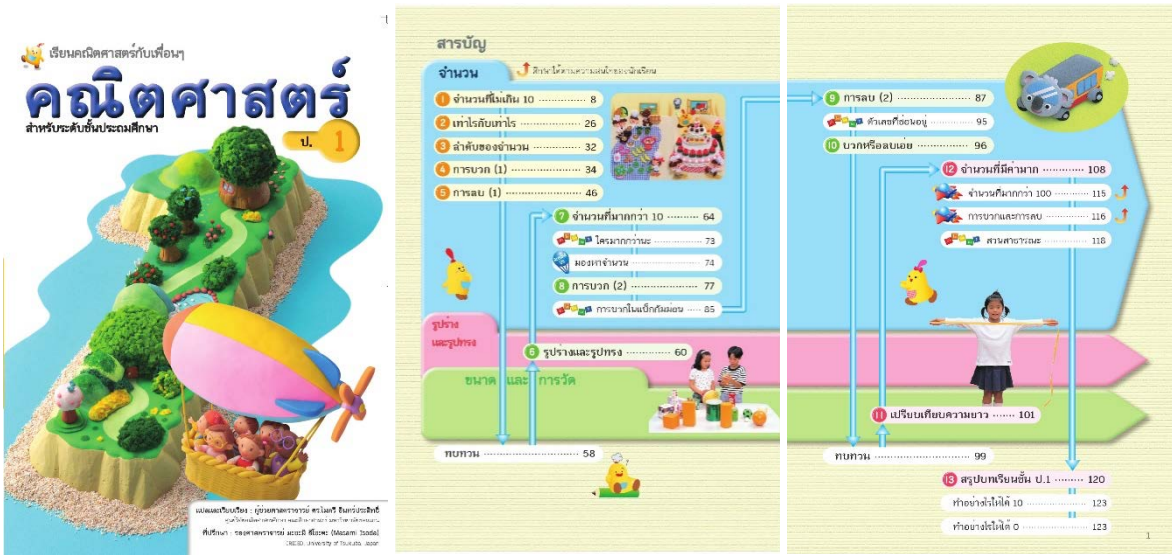

Figure 3: Japanese textbooks translated into Thai versions since 2010
(Inprasitha & Isoda, 2019)

After the two-week teaching practice in the classroom, the student-teachers of the mathematics teacher education program reflected on their teaching practice according to the following aspects.

1) whether their teaching practices are accomplished, show some evidence of students' ideas,

2) identify problems found in their teaching practice,

3) identify improving aspects of teaching practice for the following lesson's improvement,

with evidence such as two consecutive lesson plans, photos of students' ideas or worksheets, bansho (blackboard used in the classrooms), and students' solving problems.

Data Analysis

The data from reflections about teaching practices for two weeks in a school context by the student-teachers and their pieces of evidence accompanied the data from the collaborative planning phase. The research lessons of the student-teachers and the researchers, and the phase of collaborative teaching and observing the research lessons of each student-teachers with their partner in the school context were analyzed using content analysis (Bengtsson, 2016). Then, they were classified into the following components of the new didactic triangle (Inprasitha, 2014).

RESEARCH RESULTS

After the two-week teaching practice in the actual classroom setting of school contexts, the seven student-teachers were asked to reflect on the abovementioned aspects of their teaching practice in mathematical classrooms.

1) Reflections of student-teachers related to the teaching process

(1) engaging the students to have their problems from problem situations

"When posing an open-ended problem, students couldn't access a problem situation. This situation makes them confused with the teacher's direction, and students couldn't see what their problems are."

Student-Teacher A (January 13, 2021)

"To improve in teaching is how to transform problem situations into the students' problems because the students don't realize that is the problem to solve."

Student-Teacher C (January 13, 2021)

The student-teachers reflected on an awareness of the problematic conditions by identifying a state in each student that could not solve the problems or obstacles that the students confronted with the problem situation students had been solving. Moreover, the student-teachers realized how to improve their teaching practice, emphasizing teaching through problem-solving, conveying the problem situations as their students' problematic conditions or problematization.

(2) preferring to use semi-concrete aids to extend the students' ideas that occurred in the classroom and how to connect them

"The students couldn't move erasers in a worksheet, so they can't find the total number of those erasers. A teacher should have additional material (blocks) for helping students in counting the erasers."

Student-Teacher A (January 13, 2021)

"The students couldn't realize that 9 is missed 1 to become 10 because the teacher had only blocks or missed a ten-bar. So, the teacher should prepare the tens-bar for review this lesson before starting the next lesson."

Student-Teacher D (January 13, 2021)

The student-teacher reflected on supporting materials that the student-teachers need to use to broaden the students' ideas out to other students in the classroom to learn together, e.g., blocks, tens-bar, and figures that can be moved on the blackboard. Additionally, the student-teachers realized that they had prepared to get enough of these supporting materials to improve the students' ideas by ultimately connecting them to mathematical ideas.

(3) teachers' role in encouraging students with questions

"The teacher asked students the most straightforward way how to find the answer of 9+4. The students could realize that they should make 10 first and the answer a product of the addition of 10 and 3 to become 13."

Student-Teacher D (January 13, 2021)

The student-teacher reflected on how to grasp the lesson's purposes by asking critical questions. These questions should be prepared as a sequence of the lessons and considered as the specific students' ideas occurring in the third phase of discussion and comparison of the student's ideas. Finally, the student-teachers realized that these critical questions would support the students to think about "how to learn" on their own for the upcoming lessons.

2) Reflections of student-teachers related to the Learning Process

(1) Students can solve the problem situation in various ways

A grade-1 problem situation of how many stamps there are related to numbers more than 10 and less than 100.

"The students could be able to count by 1, 2, 3, 7, or 10."

Student-Teacher A (January 13, 2021)

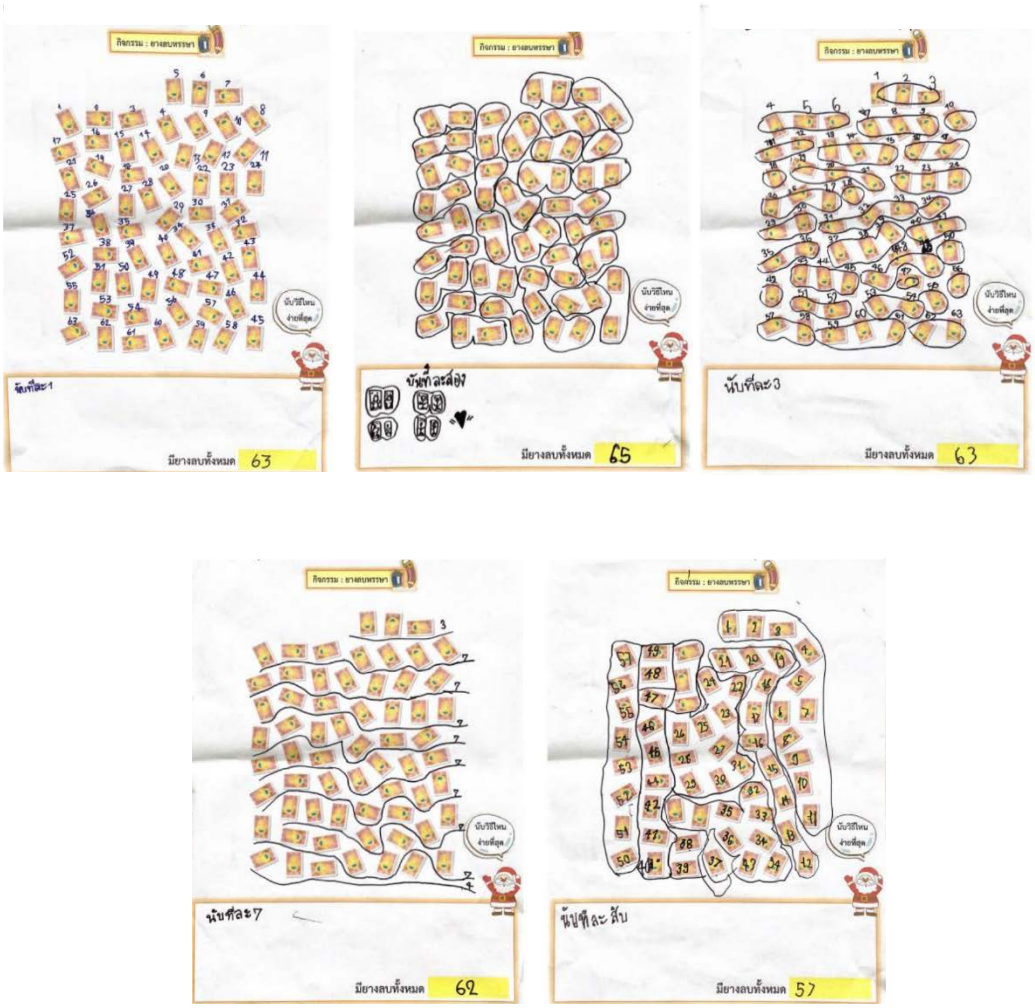

Figure 4: Students' worksheets representing ideas of numbers more than 10 and less than 100 (a) count by one (b) count by two (c) count by three (d) count by seven (e) count by ten

From a grade-2 problem situation of length, the students will learn how to compare the length of pieces of stuff. The students will be forced to use things around them as measuring tools.

> *"The students used a palm, a pencil, and an eraser to be measure tools. Furthermore, one of the students used his body as a measuring tool."*
>
> *Student-Teacher E (January 13, 2021)*

The student-teachers reflected on the students' various tools to solve the problematic conditions. Eventually, the student-teachers realized that the opportunity they provided for their students was in the phase of students' self-learning by solving the problem with themselves or

their peers. In addition, the student-teachers only observed or took note of the student's ideas, whether those ideas were as had been anticipated thoroughly.

(2) Students share their ideas with their friends

For a grade-4 problem situation of pouring water by estimating 1 liter into a kettle without any measuring devices, the student-teacher let their students solve the problem by themselves and share their ideas in groups. In the third classroom phase, student-teachers asked their students to share their ideas on representing the amount of water in front of the class.

Figure 4: Atmosphere of classroom
taught by Student-Teacher F (January 13, 2021)

The student-teachers reflected on the opportunities they provide for their students to share the ways used to solve the problem situations with all peers. Accordingly, the student-teachers could record the students' ideas on a blackboard, and the students could visualize back and forth along the lessons. The student-teachers realized that these opportunities would encourage the students to be conscious of their ideas and learn with peers' ideas to examine whether they are reasonable.

3) Reflections of student-teachers related to the Thinking Process

From a grade-4 problem situation of an unknown unit (deciliter), the students used their prior knowledge to access the meaning of deciliter.

> *"The students adapted 'how to learn' from the previous lesson (group of ten) to be a tool to access an unknown water level by separating the beaker into 10 parts equally, to deliver 10 dl as 1 L."*
>
> *Student-Teacher F (January 13, 2021)*

From a grade-1 problem situation of addition (2) in which two numbers are less than 10, and the addition result is more than ten, student-teacher G reflected on the necessity of reviewing

using a diagram of adding two numbers that are less than 10. Moreover, the addition result is also less than ten.

> *"The students' difficulty was incorrectly using diagrams such as composing and decomposing. The student-teacher realized that there should be reviewing of the diagram in the introduction part of the lesson."*
>
> *Student-Teacher G (January 13, 2021)*

The student-teachers reflected on the students' ways of solving the problem situation after the students engaged with the problematic conditions by preferring "how to learn" from previous lessons. Consequently, the student-teachers realized that the students would solve the problem properly when they are conscious of their own "how to learn" and eventually use them to solve the problem.

DISCUSSION AND CONCLUSION

Based on research results, a discussion could be divided into three parts as the research results followed the theoretical framework of a new didactic triangle: the view of student-teachers related to the teaching process, learning process, and thinking process.

1) View of student-teachers related to the Teaching Process: awareness of problematic conditions that the student-teachers should consider when they design problem situations. In the teaching phases, the student-teachers should support their students to express ideas and encourage them to deliberate their ideas for solving problems.

The student-teachers reflected on the teachers' roles in engaging the students to have their problems from problem situations or problematic mentioned by Isoda (2010) that there is a local theory of problem-solving approach, for instance, the difference between problem situation (task) and problematic (problem). Moreover, these corresponded to Isoda and Katagiri (2012) as "problematic" is an essential element of the problem-solving approach because it is necessary for children to learn by/for themselves, and it is also related to the objective of the lesson. Consequently, the teachers will collect answers only good answers if there are without the problematic condition. However, with various kinds of answers, children can discuss which answers are appropriate by themselves.

The connection between real-world experience and formal mathematics concepts or the mathematical world is enormous and complex (Bransford, Brown, & Cocking, 1999). Therefore, the student-teachers reflected on using semi-concrete aids to clarify and extend the students' ideas to consecutively establish mathematical ideas starting from the students' real-world experiences. They also used them to create the problem situations, using semi-concrete aids, and forming the students' mathematical ideas by comparing and discussing the students' ideas or "flow of lesson" (Inprasitha & Isoda, 2019; Intaros & Inprasitha, 2019).

The student-teachers reflected on questioning to encourage the students' thinking about the problem situations to recall how to solve problems by utilizing what they have learned. In collaboratively planning the lessons, the student-teachers anticipated the students' ideas or ways of solving problems and prepared questions that would be used when the student's responses followed the students' anticipated ideas. These student-teacher roles align with Ulep (2015) that the teachers accommodate students to consider how they knew if their answers were correct. They let students evaluate which correct solutions they preferred and share their reasons with other students using questions. In addition, these ideas are consistent with Takahashi (2021), who states that during the lesson study processes are ongoing, the teachers conduct an in-depth study of the mathematics and curricular material related to the lesson's objective. The teacher must, moreover, consider the student's prior knowledge to choose an appropriately challenging problem that will accomplish the lesson's objective and anticipate the students' possible response to the problem to plan how to engage those students' ideas. This teaching approach is teaching through problem-solving, and the role of teachers in guiding mathematical discourse is a remarkably complex activity (Cai et al., 2003). Furthermore, by devoting a suitable time to discuss ways of solving the problem, teachers should decide what aspects of a task would be highlighted and how to orchestrate the students' ideas. Teachers should also decide what questions are used to challenge those with different expertise of the students and how to support students without taking over the thinking process for them and therefore eliminating the challenge of the problem (National Council of Teachers of Mathematics, 2000).

2) View of student-teachers related to the Learning Process: awareness of providing the opportunity for their students to solve the problem in various ways and share these ideas with the other students to grasp the purpose of the lessons.

The student-teachers reflected on students' ways of solving problems in various ways by using the students' prior knowledge, experiences, and what they have got, such as a palm and a pencil, to solve the problems. These scenarios would happen when the teachers provide the opportunity for the students to solve by themselves and prepare problem situations related to the student's experience. This is consistent with Gueudet et al. (2017), which mentioned transitions in mathematics education, e.g., a transition between in- and out-of-school mathematics, which is connected by including cultural contexts or sociocultural perspectives of students' contexts in school mathematics.

The student-teachers who used the Open Approach as their teaching approach will provide opportunities for students to share ideas after the students solve the problems on their own or by themselves. This step of the Open Approach is similar to the Japanese teaching approach called 'Nariage' (Shimizu, 1999), which is dynamic and collaborative during the class discussion by looking back on students' ways of solving problems. This teacher's role is worth it as they help the students derive the essential facts, concepts, and procedures. Therefore, it differs from the

teachers' role in a traditional teaching approach in which the students do not explore any new mathematical concepts.

3) View of student-teachers related to the Thinking Process: awareness of 'how to learn' that the students have to develop through their ways of solving problems and become aware of these ideas used to solve the problems.

The student-teachers reflected on "how to" what is in accordance with Isoda and Katagiri (2012), who stated that teaching children how to develop mathematics by nurturing children who think and learn mathematics by and for themselves. Well-nurtured children are given problem situations to consider the next step by themselves and imagine the next step for themselves. Likewise, these are affected by the translated Japanese textbooks, which are based on a 'think about how to' characteristic of problem situations. When the students learn mathematics as a sequence of textbooks, it could be called 'today's learning is preparing for the next day's learning (Inprasitha & Isoda, 2019).

The study could be concluded that the views of student-teachers reflected from their teaching practice in the actual classroom context follow the aspects of the new didactic triangle, which is composed of the teaching process, learning process, and thinking process. The awareness of the student-teachers in each aspect of the didactic triangle is taken from the teaching practice using the Open Approach. These aspects of the new didactic triangle are correlated with teaching mathematics through problem-solving, which is a mainstream mathematics teaching approach and a widespread pedagogical approach (Takahashi, 2021).

IMPLICATION AND FURTHER RESEARCH

The results of this research shed some light on teaching mathematics through problem-solving as a new mathematics teaching and learning approach for school context and mathematics teacher education programs. Nevertheless, the research is a case study that has a limited number of participants because it focused on only seven student-teachers in the mathematics teacher education program. Further research should level the number of participants up to other mathematics teacher education programs' students who are encouraged to apply the Lesson Study and Open Approach as a way of teaching practices in school contexts.

The two-week teaching practice is the experience in an actual classroom context that the student-teachers could learn before being a pre-service teacher for a year in the next academic year. There should be more than two weeks, i.e., four weeks, for teaching practice in a school context. This will enhance the teaching practice of student-teachers by using the Lesson Study and Open Approach, which emphasizes students' ideas to collaboratively plan, teach and observe, and reflect to improve the problem-solving lessons. The at-least-four weekly cycle of the Lesson Study will be an effective flow for the student-teacher to perceive a suitable tendency to use students' ideas to improve their lesson plans and their teaching practice in the school context.

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