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Nana Boahen Mensah
Montana State University, United States

Emmanuel Barton Odro
University of Nebraska Lincoln, United States

Derek A. Williams
Montana State University, United States

To cite this article:

Mensah, N., Barton Odro, E., & Williams, D.A. (2023). Examination of 9th graders' levels of geometric thinking. *International Journal of Research in Education and Science (IJRES), 9*(3), 688-703. https://doi.org/10.46328/ijres.3184

International Journal of Research in Education and Science
2023, Vol. 9, No. 3, 688-703 https://doi.org/10.46328/ijres.3184

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Article Info

Article History

Received:
11 March 2023
Accepted:
12 June 2023

Keywords

van Hiele Theory
Geometric thinking
Junior high school

Abstract

Robust understanding of geometry is important for students' future studies in mathematical sciences, many careers, and for understanding the world around them. Therefore, the aim of this study was to understand the geometric thinking levels of 9th graders in Ghana before they enter the Senior High School. This study used the van Hiele theory (Van Hiele-Geldof, 1957) to understand geometric thinking levels of 400 9th graders in western Ghana. These students were given the van Hiele Geometry Test (VHGT) developed by Usiskin (1982). The results showed that 56.2% of 9th graders demonstrated thinking at the pre-visualization level, 30.75%of the graders attained level 1(visualization), 12.5% reached level 2 and only 0.5% demonstrated level 3 thinking. These findings indicate that students are struggling to meet the Ministry of Education's curriculum standards. Additional analysis points to reasoning about properties of geometric figures using definitions and informal deductive reasoning, instead of relying on visual characteristics of diagrams as a specific obstacle for students. Recommendations are made for improving the teaching and learning geometry based on our findings.

Introduction

In everyday life, people regularly encounter geometric figures and routinely reason about or model the world using geometric thinking. Further, many careers involve professionals to have robust understanding of geometry concepts. In fact, the word "geometry" means "Earth measure." To understand the wonder of the world, we need to understand and have knowledge of geometry. By studying geometry, students can apply it to their real life. When students learn geometry, it enhances their logical reasoning and critical thinking capabilities (see also, Bayaga et al., 2019). Developing logical reasoning and deductive thinking is paramount for further study in mathematical sciences (Barkl et al., 2012), and school geometry offers students foundational exposure to these important skills (Sinclair et al., 2012).

Geometry is a natural place to develop students' reasoning and justification skills (National Council of Teachers of Mathematics [NCTM], 2000). In addition to being used daily by present-day architects, engineers, and physicists, geometry has had great historical importance in people's lives, originating with human beings' need to specify quantities; measure figures, land, and earth; and make maps (Sunzuma et al., 2013). Due to its importance in our lives, geometry has been an integral component in school mathematics curricula globally. Research on the teaching and learning of geometry has been prominent for decades (Amidu & Nyarko, 2019; Armah & Kissi, 2019; Armah et al., 2018; Asemani et al., 2017; Baffoe & Mereku, 2010; Clement & Battistts, 1992; Crowel, 1987; Gutiérrez et al., 1991; Lappen et al., 1996; Mason, 1998). More recently, research has shown that students' ability to succeed in mathematics has been closely linked to their geometric understanding (e.g., Möhring et al., 2021).

Research on teaching and learning geometry in African countries has focused on high school students' thinking and reasoning (e.g., Bayaga et al., 2019; Mthethwa et al., 2020). However, Ghana's junior high school mathematics curriculum emphasizes geometric thinking as one of its objectives (Ministry of Education of Ghana, 2007). More precisely, the Ghanaian geometry curriculum is based on the van Hiele theory of geometric thinking (van Hiele-Geldof, 1957), where students are expected to routinely demonstrate thinking at van Hiele level 3 by the end of junior high school (9^{th} grade, ages 14-15) (Ministry of Education of Ghana, 2007). Similarly, by the end of senior high school (12^{th} grade, ages 17-18), students are expected to regularly demonstrate thinking at level 4 of van Hiele's theory (Ministry of Education of Ghana, 2007). This study focused on Ghanaian 9^{th} graders, so attention is paid to the first three levels of van Hiele theory (Van Hiele-Geldof, 1957).

The van Hiele theory of geometric thinking is the most established and widely used model by researchers in the teaching and learning of geometry (Amidu & Nyarko, 2019; Armah & Kissi, 2019; Bulut & Bulut, 2012; Fuys et al., 1988; Hershkowitz & Vinner, 1984; Knight, 2006; Mayberry, 1983; Shaughnessy & Burger, 1985). Currently, Ghanaian mathematics education researchers and teachers are concerned about the performance of geometric thinking students demonstrate on national assessments (Armah & Kissi, 2019; Asemani et al., 2017; Baffoe & Mereku, 2010). Additionally, studies analyzing Ghanaian students' thinking through van Hiele levels offer inconsistent results, especially for senior high school students. Some researchers report the most demonstrated thinking at level 1 (recognition of shapes) (Amidu & Nyarko, 2019; Baffoe & Mereku, 2010), while others indicatethat students are only routinely showing thinking at level 0 (pre- recognition of shapes) (Asemani et al., 2017; Yalley et al., 2021). In either case, these studies justify the concerns over teaching and learning of geometry in Ghanaian schools, as the desired thinking at level 4 (proof of theorems) by graduation is currently not being met.

In Ghana, aside the research of Amidu and Nyarko (2019), research on teaching and learning geometry has focused senior high school students or pre-service teachers (Asemani et al., 2017; Armah & Kissi, 2019). However, discovering the implications of the van Hiele theory, particularly for junior high school students is also very crucial. Currently, research assessing students' geometric thinking in Ghana is scarce (see Asemani et al., 2017). As a result, there is a need for more comprehensive descriptions of Ghanaian students' geometric thinking to inform teacher preparation and pedagogy. Limited attention to junior high school students' learning of geometry has rendered previous interventions ineffective. Furthermore, since mathematics at the senior high school level builds on the knowledge and competencies developed in junior high school, it is essential to understand the geometric thinking of junior high school students. For these reasons, this study sought to determine 9^{th} graders' levels of geometric thinking for students in the western part of Ghana.

The objective of this study was to understand the geometric thinking level of 9^{th} graders in Ghana using the Van Hiele Theory. In pursuance of this objective, the following question was formulated to guide the study: At what level of the Van Hiele Geometric thinking do Ghanaian 9th graders reach before entering the Senior High School?

Theoretical Framework

The van Hiele model of geometric thought emerged from the doctoral works of Dina van Hiele-Geldof and Pierre van Hiele (1957), completed simultaneously at the University of Utrecht. Dina died shortly after finishing her dissertation, so Pierre clarified, amended, and advanced the theory. Even though the Soviet Union revised its curriculum in the 1960s to conform to the van Hiele model, the theory was slow to gain international attention until the 1970s and it was particularly enhanced by the 1984 translations into English (Crowley, 1987).

The model consists of five hierarchical levels (see Figure 1), including behavioral characteristics of each level and closely associated with phases of learning (Atebe & Schafer, 2010; Clement, 2004; Crowley, 1987; Hoffer, 1983; Lumbre et al., 2023; Usiskin, 1982; van de Walle, 2004).

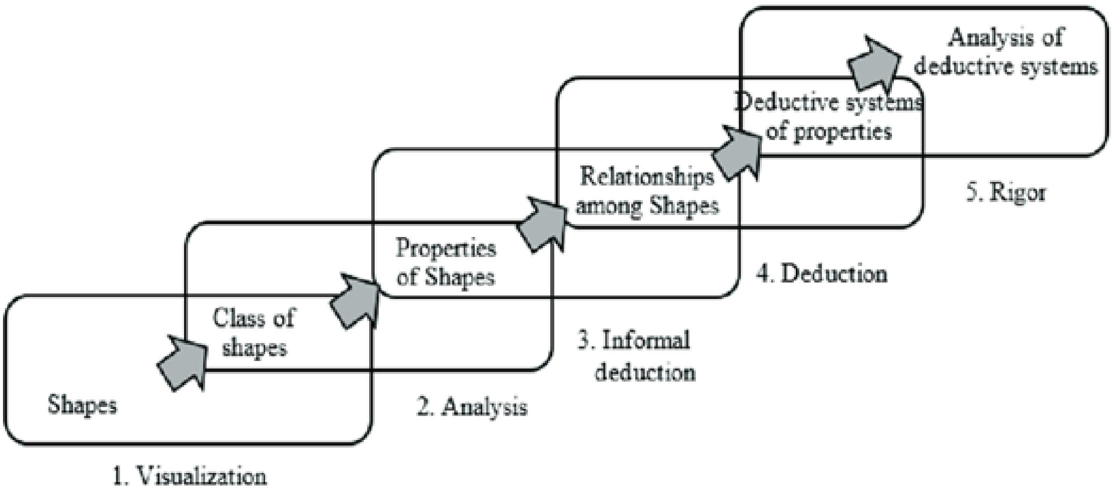


Figure 1. The van Hiele Theory of Geometric thought (van de Walle, 2004, p. 347)

Researchers have labeled these five levels in two ways: Level 1 through Level 5 or Level 0 through Level 4. Researchers have yet to conclude which one to use; however, there is empirical evidence to show that there is a pre-recognition level (Clements & Battista, 1992). This study has adopted the labeling system of Levels 1 – 5 to be consistent with the Ghanaian Education Ministry's use. To be consistent with the development of the theory, we describe Level 0 (pre-recognition); however, this study focuses on Levels 1 – 3 because of the curricular goals in Ghanaian junior high schools. Drawing from existing literature (Clements & Battista, 1992; Crowley, 1987; Hoffer, 1983; Karakus & Parker, 2015; Mayberry 1983; Shaughnessy & Burger, 1986) these descriptor levels are described as follows:

Level 0 (Pre-recognition/Pre-visualization): At this level, learners may only recognize some visual characteristics of shapes, such as distinguishing between triangles and quadrilaterals, but are not yet attending to characteristics to distinguish between classifications like rectangles and general parallelograms within quadrilaterals, for example.

Level 1 (Recognition/Visualization): At this level, learners recognize or identify figures by their visual appearance. Learners recognize figures by their shape as "a whole" and compare figures with prototypes or everyday things ("it looks like window or door"). Learners may use basic geometric vocabulary. Learners use visual considerations without explicit regard for geometric properties. Learners can reproduce (draw) a given geometric figure shown to them.

Level 2 (Analysis): Learners can analyze the properties of figures and establish necessary and sufficient properties. At this level, learners can begin generalizing about shapes and their properties. For example, learners can say that all the edges of a square are congruent and perpendicular to each other with opposite edges parallel. They can classify shapes according to properties. Learners may not yet be able to analyze properties across figures. For example, they may not yet conclude a square to be a rectangle based on properties.

Level 3 (Informal Deduction): At this level, learners can establish interrelationships of properties both within figures (e.g., in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and among figures (e.g., all squares are rectangles because squares possess all the properties of a rectangle); class inclusion is understood. Definitions are meaningful, and learners can use definitions as justification in informal arguments. Learners are not yet constructing proofs.

The van Hiele model is adopted for this study because of its emphasis on developing successively more sophisticated thinking and the close alignment with learning, which appears to signal direction and potential for improving teaching (Fuys et al., 1988). Also, the model has been used by numerous researchers to analyze students' reasoning on geometry tasks (e.g., Amidu & Nyarko, 2019; Armah & Kissi, 2019; Fuys et al., 1988; Mayberry, 1983; Senk, 1985; Shaughnessy & Burger, 1985). According to Pegg and Davey (1991), the van Hiele model is helpful in that it may assist us in determining just how much learners understand geometry and inform us on how to teach more effectively. It is important to note that tasks play an important role in students' ways of reasoning (Shaughnessy & Burger, 1985) and that the van Hiele levels of geometric thought describe thinking, not students.

Method

Setting and Participants

For this study, descriptive research was used since the study sought to describe the geometric thinking level of 9th graders. The population of interest for the study was 7,500 9th grade students enrolled in the 196 junior high schools in the Sekondi-Takoradi Metropolis in the Western Region of Ghana. These students were about to complete junior high school in the 2019-2020 academic year. Schools in the Sekondi-Takoradi Metropolis were used for the study because schools in this metropolis enroll students from diverse backgrounds and have qualified teachers. The metropolis also offers a good representation of junior high schools in Ghana.

The data for this study was collected from 400 9th graders in 4 schools in the Sekondi-Takoradi Metropolis—two public and two private schools. These 4 schools were selected using the convenience sampling technique and all

9th grade students present on the days of data collection participated. This method was used to select the four schools to be representative of junior high schools in the region because, the COVID restrictions in place at the time inhibited our ability to collect data more broadly or through use of randomized selection processes.

At the end of the data collection there were 213 students from the public schools and 187 from the private schools, yielding a total of 400 students. We selected 9th graders for the study because they are believed to have completed the geometry curriculum for junior high school.

Data Collection

The van Hiele Geometry Test (VHGT) developed by Usiskin (1982) was adopted and administered to students to measure their geometric thinking. Students had 25 minutes to complete the assessment. The VHGT is organized into five subgroups of five multiple choice items, where each subgroup is based on the levels of geometric thought associated with the theory. Thus, the VHGT has five items describing each level of the van Hiele geometric thinking. The assessment begins with items to assess thinking at level 1 and progresses to level 5. Ghanaian 9th graders are expected to regularly demonstrate thinking at level 3; hence the test administered to students for this study consisted of 15 items associated with these first three levels. The VHGT has been used by many researchers (e.g., Abdullah & Zakaria, 2013; Amidu & Nyarko, 2019; Armah & Kissi, 2019; Anas, 2018; Asemani, 2017; Fuys et al., 1988; Halat, 2008; Hofferr, 1983; Mayberry, 1983; Usiskin, 1982).

Five mathematics teachers compared the items of the VHGT to the content standards indicated by the junior high school geometry curriculum and cleared the assessment as appropriate. All five teachers had at least six years of teaching experience in junior high schools. Four of these teachers worked at the site schools, and the fifth worked at a school where pilot testing took place. Pilot testing of the VHGT instrument with twenty 9th graders, from a different school than where data collection took place, was conducted. The pilot instrument also used 15 multiple-choice items associated with the first three van Hiele levels. This pilot yielded a Cronbach alpha reliability coefficient of 0.62, which means the subsections of the VHGT are reliable at assessing thinking associated with the different levels. Other researchers have reported reliability estimates of the VHGT ranging from 0.69 to 0.78 and have determined the instrument to be reliable and valid (e.g., Anas, 2018; Armah & Kissi, 2019; Atebe & Schafer, 2008; Baffoe & Mereku, 2010; Usiskin, 1982). Moreover, existing literature has determined that the items of the VHGT accurately reflect the hierarchical nature of the theory. That is, from a theoretical standpoint, students should only be able to demonstrate thinking at level N once they routinely demonstrate thinking at all previous levels. Finally, that the Cronbach alpha coefficient with the pilot students indicated the assessment is reliable also suggests that implementing the assessment with 9th graders in the Sekondi-Takoradi Metropolis is appropriate.

The VHGT was administered at each of the four junior high schools on days when a member of the research team could be present. Mathematics teachers at each of the schools supported the administration of VHGT to the 400 participants in this study.

Data Analysis

The data collected were coded and processed using SPSS to build a database of responses that was used to determine the van Hiele level of geometric thinking demonstrated by each student as indicated by the VHGT. Specifically, percentages and frequencies of correct responses were used to analysis the level of geometric thinking of 9th graders in the region, as a way of evaluating the effectiveness of teaching and learning of geometry through junior high school.

Two methods of scoring responses were used: an overall score, and a method to assess the van Hiele level of thinking demonstrated by each student. For producing an overall score, each correct response to the 15-item multiple choice test was assigned 1 point. Hence, scores could range from 0 to 15. This scoring method allowed us to quickly ascertain students' performance on individual items. The second scoring method was used to determine the van Hiele level of thinking demonstrated.

Usiskin (1982) established "success criteria" for a student to demonstrate thinking at each of the van Hiele levels if they correctly respond to at least 3 of the 5 items associated with each level. By this criterion, any student who correctly answers at least 3 out of the 5 items in any of the 3 subtests within the VHGT was deemed to have achieved that level of geometric thinking. The second method for scoring VHGTs involved scoring 1 point for assessments that met the success criteria for Level 1 (recognition/visualization), and then adding 2 points for assessments that met the criteria for Level 2 (analysis), and finally adding 4 points for assessments that met the criteria for Level 3 (informal deduction). The sum of these values would then uniquely indicate the level of geometric thinking demonstrated by students' responses because of the binary nature of point accrual. For example, a score of 3 = 1 + 2 + 0, would indicate that the success criteria for Levels 1 and 2 had been met, but not for Level 3. Similarly, a score of 7 = 1 + 2 + 4 would indicate that the success criteria for all three levels assessed had been met, and a 7 is the maximum score attainable for this study. The second scoring method was used to determine each 9th graders' level of geometric thinking based on their responses. This approach is consistent with processes of other researchers (e.g., Anas et al, 2018; Asemani et al, 2017; Atebe, 2008; Baffoe & Mereku, 2010; Mayberry, 1983; Usiskin, 1982).

According to van Hiele theory (1986), the levels of geometric thinking are intended to be hierarchical, which is also reflected in the VHGT instrument. One empirical consequence of the hierarchical nature of the levels is that scores indicating students' thinking to any level should also indicate thinking at all previous levels, so a score of 2 = 0 + 2— indicating success criteria have been met for Level 2 but not Level 1— does not align with theoretical underpinnings of the instrument and such scores should not be considered in analysis (Mayberry, 1983). Our data did not encounter this situation.

Results

This research study aimed to determine the level of geometric thinking demonstrated by 9th graders in western Ghana using a VHGT instrument (Usiskin, 1982). Figure 2 presents results from the first scoring method to share the spread and frequencies of number of correct responses (out of 15) produced by the 400 student participants. After discussing the overall raw scores presented in Figure 2, we discuss the results of the second scoring method.

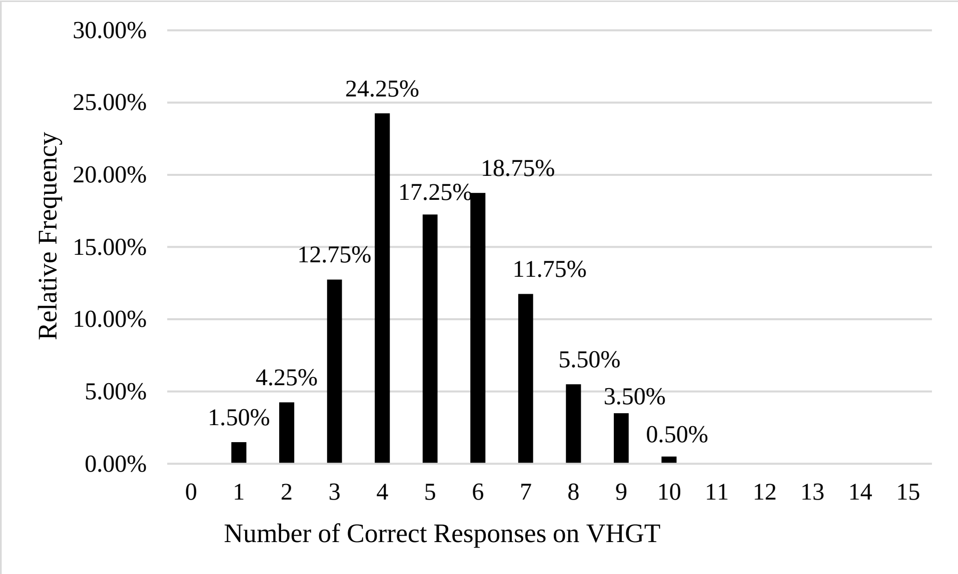


Figure 2. Relative Frequencies of Number of Correct Responses on the VHGT

Figure 2 shows that 362 out of 400, representing 90.5% of the 9th graders who took the VHGT, correctly responded to 7 or fewer items, meaning just 38 (9.5%) of the students correctly responded to 8, 9 or 10 items. Figure 2 also shows that every student correctly responded to at least 1 item; however, no student correctly responded to more than 10. Figure 3 presents the number of students whose responses indicated geometric thinking at the levels assessed.

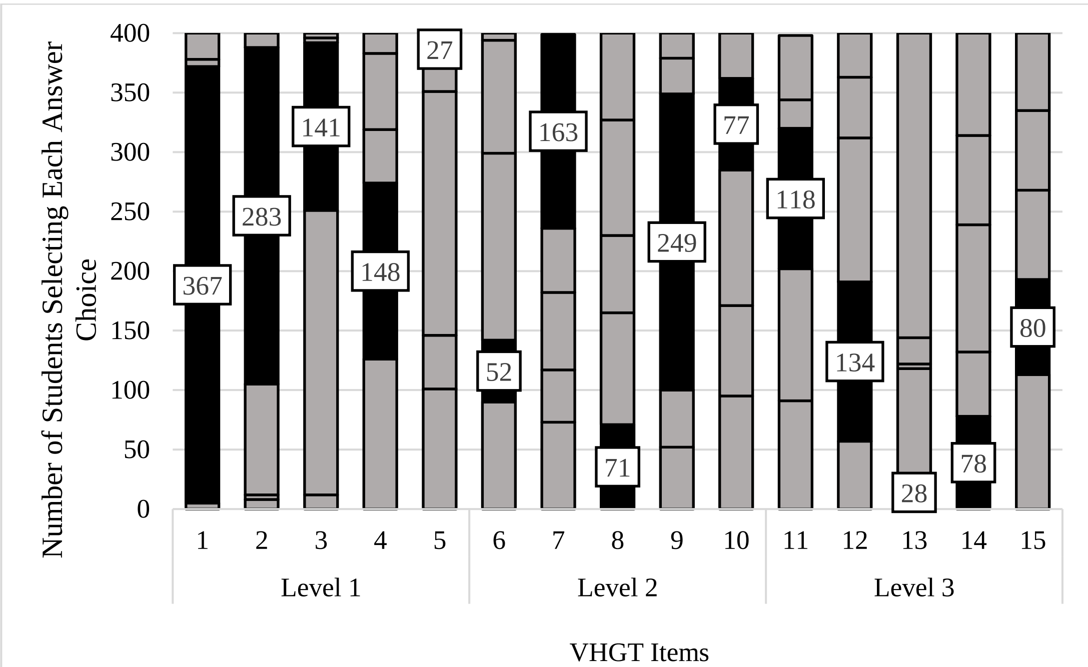


Figure 3. Number of Students Selecting Choice (darker bar indicates the correct choice, number of students responding correctly displayed in labels). Separated by van Hiele Level

Recall each level had five multiple-choice items. Figure 3 shows the proportion of students who selected each answer option for each item and is sub-divided according to the associated van Hiele levels. The boldfaced information represents the correct answer choice. Overall, just 2 (0.5%) students demonstrated thinking at Level 3 based on their responses to the VHGT, 50 (12.5%) students demonstrated thinking at Level 2, 123 (30.75%) demonstrated thinking at Level 1, and the remaining 225 (56.25%) students' responses did not meet the success criteria for any level assessed. The following paragraphs discuss the participants' overall performance on the items for each level in more detail and describe implications from the assessment.

Further analysis of students' responses to Level 1 items revealed an interesting phenomenon regarding their recognition of figures. For instance, it was observed that students could visually identify quadrilaterals; however, once these quadrilaterals were re-oriented visually or tilted, students could no longer recognize these shapes as a quadrilateral. For example, almost all students (367) correctly responded to item 1, where the quadrilaterals are oriented horizontally or vertically (see Figure 4).

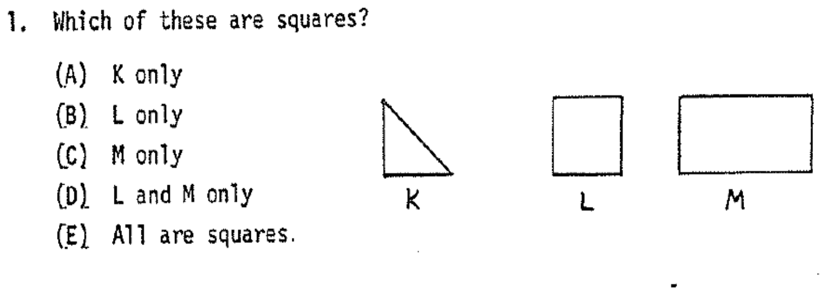


Figure 4. Item 1 of the VHGT

However, overall, the group of 9th graders were less successful at visually identifying quadrilaterals that were no longer oriented as in item 1. For example, 148 students correctly responded to item 4 (see Figure 5).

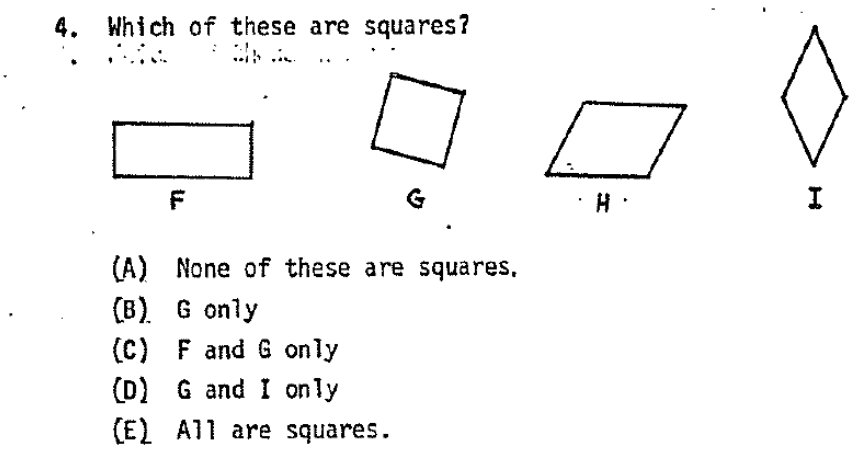


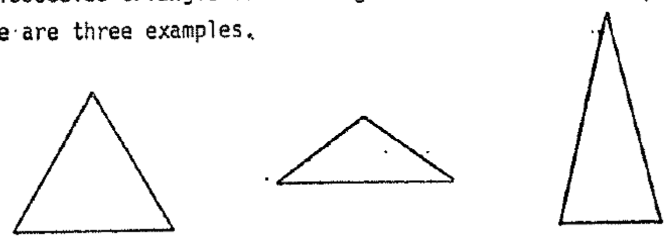
Figure 5. Item 4 of the VHGT

This further confirms the finding that tilting quadrilaterals potentially influenced students' ability to visually recognize shapes. In fact, roughly two-thirds of students (252) did not visually identify any square in item 4.

Hence, regarding students' ability to visually recognize shapes, students could easily identify quadrilaterals such as squares based on visual characteristics, provided they were oriented vertically or horizontally.

Further analysis of students' responses to items on Level 2 indicated that students were reasonably successful at thinking about properties of triangles and could use more instruction associated with properties of quadrilaterals. Figure 3 shows that the students performed reasonably well on item 9 (see Figure 6), where 248 of students were correctly able to analyze properties of triangles.

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.

(B) One side must have twice the length of another side.

(C) There must be at least two angles with the same measure.

(D) The three angles must have the same measure.

(E) None of (A)-(D) is true in every isosceles triangle.

Figure 6. Item 9 from the VHGT

However, the same cannot be said about how students responded to item 6 (see Figure 7), which asked them to analyze properties of quadrilaterals.

6. PQRS is a square.

Which relationship is true in all squares?

(A) $\overline{PR}$ and $\overline{RS}$ have the same length.

(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.

(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.

(D) $\overline{PS}$ and $\overline{QS}$ have the same length.

(E) Angle Q is larger than angle R.

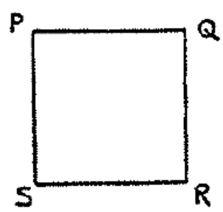


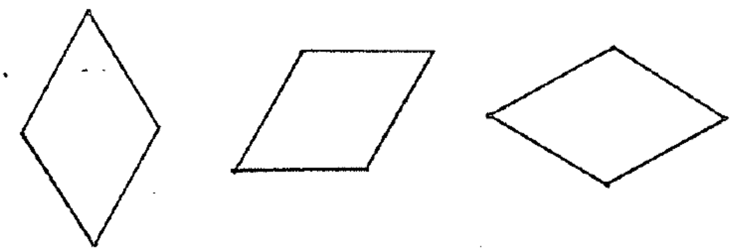
Figure 7. Item 6 from the VHGT

Most students did not know the properties of the square since only 52 answered item 6 correctly. Some students need help understanding the concept of perpendicular lines because 157 students chose C, which states that PS and QR are perpendicular. To further illustrate students' difficulty in analyzing properties of quadrilaterals, we discuss item 8 (see Figure 8).

Responses to item 8 suggest students need more instructional support for analyzing properties of quadrilaterals beyond squares and rectangles. Overall, 329 students did not answer item 8 correctly. Altogether, it is evident that students demonstrated a better understanding of analyzing properties of triangles than quadrilaterals.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.



Which of (A)-(D) is not true in every rhombus?

(A) The two diagonals have the same length.

(B) Each diagonal bisects two angles of the rhombus.

(C) The two diagonals are perpendicular.

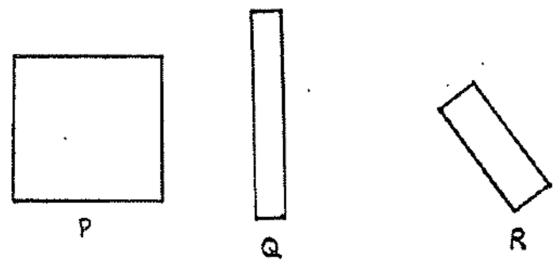
(D) The opposite angles have the same measure.

(E) All of (A)-(D) are true in every rhombus.

Figure 8.Item 8 from the VHGT

Finally, students' performance on Level 3 items further indicates that 9th grade students in western Ghana would benefit from instruction on analyzing the properties of quadrilaterals. For example, 28 students correctly responded to item 13 (see Figure 9). This situation means that 372 students did not know that squares have common properties with rectangles. This suggests that students have difficulties in understanding class inclusion and attending to properties of geometric figures instead of just to visual characteristics.

13. Which of these can be called rectangles?



(A) All can.

(B) Q only

(C) R only

(D) P and Q only

(E) Q and R only

Figure 9. Item 13 from the VHGT

In conclusion, 2 of the 400 9th graders demonstrated thinking at van Hiele Level 3, which is the curricular goal for all junior high school students by the time they complete 9th grade. A more detailed analysis of the individual items reveals some plausible targeted starting points for instruction to support students' geometric thinking in the region.

Discussion

This study investigated the van Hiele levels of geometric thinking of 400 9th graders in Sekondi-Takoradi Metropolis in western Ghana. The results of the VHGT reveal that 225(56.25%) students demonstrated thinking consistent with the pre-visualization level, 123 students demonstrated Level 1 (visualization/recognition), 50 students demonstrated Level 2 (analysis), and 2 students demonstrated Level 3 (informal deduction). The results show that students are not yet meeting the level specified in the curriculum. These findings are in line with previous studies at the high school level and for K-9 pre-service teachers (e.g., Amidu & Nyarko, 2019; Armah et al., 2017; Asemani et al., 2017; Atebe & Schäfer, 2008; Baffoe & Mereku, 2010; Fitriyani, Widodo & Hendroanto, 2018; Hassan, 2015; Usiskin, 1982).

If van Hiele Level 3 is the pre-requisite for students entering senior high school and the vast majority of 9th grade students are below this level, as revealed in this study and others (e.g., Amidu & Nyarko, 2019; Usiskin, 1982), then there is the need for an intervention about teaching geometry in K-9 schools. Because learners who start high school geometry at the van Hiele level 3 have a greater chance of succeeding (Senk, 1989). For example, the studies by (Atebe & Schäfer 2008 and Baffoe & Mereku, 2010) revealed that students in the higher grades did not have the prerequisite skill to follow the content of higher-grade geometry curriculum affirming the assertion by Senk (1989).

Altogether, the results from this study and what has been presented in the research literature suggest that teaching and learning of geometric concepts in K-9 should emphasize supporting students' abilities to create meaningful definitions and give informal arguments to justify their reasoning. It is worth mentioning that studies by Robichaux-Davis and Guarino, (2016); Armah et al. (2017) and Anas (2018) observed that van Hiele Level 3 is the most difficult for K-9 pre-service teachers because class inclusion was not understood by many teachers. Understanding class inclusion relies on reasoning about properties of objects instead of visual characteristics and is at the forefront of deductive reasoning (Usiskin, 1982; van Hiele, 1986). Also, few pre-service teachers demonstrated geometric thinking at Level 4 (Armah et al. (2017)). K-9 teachers need to be at level 3 and above if they should function at the Junior High School level (Crowley, 1987; Usiskin, 1982; Armah et al. 2017).

Given that geometry is challenging for learners to learn and teachers to teach (e.g., Sinclair et al., 2012), and that research suggests geometry is an obstacle for students, pre-service teachers, and in-service teachers, then additional attention from researchers, mathematics teacher educators, and curriculum developers is warranted. Much work needs to be carried out to help K-9 teachers teach the geometry curriculum. Thus, to improve students' understanding of geometry, there is a need to address both in-service and pre-service teachers' understanding as well.

Finally, these findings have implications for mathematics teacher educators (MTEs) to understand the mathematical knowledge for teaching geometry. MTEs can support pre-service teachers in developing their own content knowledge of geometry as well as their pedagogical content knowledge for teaching geometry. MTEs can also support pre-service teachers in designing lessons that intentionally capitalize on students' prior understanding through student-oriented approaches. For continuing education, other stakeholders such as the Ghana Ministry of Education and Heads of Department in schools can organize professional development for in-service teachers in western Ghana to address the need described by this study. Below, we provide some details on recommendations based on the findings of this study.

Conclusions and Recommendations

Based on the findings, most 9th grade students in Western Ghana are operating at the pre-visualization and visualization levels by the time they enter senior high school; however, as previously explained, the expectation based on the Ghanaian curriculum for these students goes beyond pre-visualization and visualization levels. This means that the current teaching and learning experiences for K-9 students are not sufficiently preparing students to meet the curriculum standards in Ghana. Since 225 of the 400 9th grade students demonstrated thinking at the pre-visualization level (Level 0), it is recommended that the Five Phases of Learning Process—Information or Inquiry; Guided Orientation, Explanation, Free Orientation, and Integration suggested by van Hiele (1957) should be adopted into the teaching and learning of geometry. This will help promote students' reasoning about properties of geometric figures, and to de-emphasize visual characteristics. As Robichaux-Davis and Guarino (2016) recommended, mathematics teacher educators must identify the van Hiele level of thinking attained by each pre-service teacher and facilitate progress throughout his\her teacher preparation program through meaningful experiences until van Hiele level 3 is demonstrated and then continue to support this level of thinking through focused professional development during the early years of teaching after the pre-service teachers have graduated.

We suggest that there should be teacher professional development programs that will train teachers using the van Hiele Phases of learning because it is evident that geometry is an area where both teachers and students are struggling to succeed. Furthermore, to analyze properties of geometric shapes and interrelationship between different types of shapes, hands-on activities are a prerequisite (Abu & Abidin, 2013; Kutluca, 2013). Therefore, there is a need to use manipulatives while teaching concepts regarding geometry. As observed from this study, students had difficulty visually identifying quadrilaterals depending on the orientation on the page. We posit that perhaps if students were exposed to manipulatives of shapes such as squares and rectangles and, they may have been able to visually recognize quadrilaterals regardless of the orientation and could more fluently transition into reasoning about properties.

We also suggest that further studies are needed to better understand teacher preparation for pre-service teachers' content knowledge and pedagogical content knowledge (Ball et al., 2008). As studies over the years have shown that both pre- service and in-service mathematics teachers at the elementary, middle and high schools do not have sufficient geometry knowledge to teach at these levels (Gutierrez, Jaime,& Fortuny,1991; Chappell,2003; Knight,2006; Robichaux-Davis & Guarino, 2016; Armah et al. 2017 and Anas;2018).Three years ago, Armah and

Kissi (2019) found that much of the geometry teaching and learning strategies of the mathematics tutors were not structured in a way that support the development of geometric thinking as described in van Hiele Levels 3 and 4. Could teachers' lack of understanding in geometry potentially be affecting students' ability to attain the level specified in the curriculum?

Furthermore, we suggest that additional studies investigate how instructors could use technology to support teaching and learning of geometry concepts. Lastly, the Ghana Ministry of Education recently instituted teacher licensure exams for K-12 teachers in Ghana. Results from these exams hold significance to inform MTE's and teacher licensure decision-makers of the weaknesses and challenges associated with the teaching and learning of geometry for K-12 education. While we continue to strive to develop students' geometric thinking in school, we also need to provide students with the appropriate learning opportunities.

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Author Information

Nana Boahen Mensah
https://orcid.org/0000-0001-7238-4494
Montana State University
United States
Contact e-mail: *nanamensah@montana.edu*

Emmanuel Barton Odro
https://orcid.org/0000-0002-5881-9663
University of Nebraska Lincoln
United States

Derek A. Williams
https://orcid.org/0000-0002-4553-4132
Montana State University
United States