Primary School Students' Understanding of Four Operation Symbols (+, −, x, ÷, =) and Using Them in Arithmetic Operations and Word Problems

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Abstract: The aim of this study is to determine the primary school fourth-grade students' understanding of four operation symbols, using them in arithmetic operations and word problems. Phenomenology research design, one of the qualitative research methods, was used in the research. The participants of the research consist of 123 students, attending the fourth grade of primary school (9 years old). The participants of the study were determined by using convenience sampling, The "four operation symbol study form" developed by the researcher was used as a data collection tool. Content analysis method was used in the analysis of the data. According to the results of the research, it was determined that the students formed meanings in different categories for the meanings of symbols, they were more successful in writing arithmetic operations by using symbols than in solving arithmetic operations. It was concluded that the students showed lower success in writing and solving word problems by using symbols compared to writing and solving arithmetic operations. It was determined that the lowest success in the use of symbols in arithmetic operations and word problems was in the use of the "=" symbol.

INTRODUCTION

Mathematics can be considered as a way of thinking or a language, but the challenge for many people is what it usually looks like when it is written. As mathematics develops, communication tools become more concise through words, symbols and diagrams (Cooke, 2007). Making sense of mathematics is possible by structuring language symbols and concepts in students' minds (Yalvaç, 2019). The use and interpretation of mathematical symbols begin very early in school life, with arithmetic symbols forming the basis of most of the mathematics learned today (Angliheri, 2005). Mathematical signs and symbols have a decisive role in the coding, constructing and communicating of mathematical knowledge. Mathematical concepts are constructed as symbolic relational structures and encoded through symbols that can be logically combined in mathematical operations (Steinbring, 2006; Goldin, 2020). A symbol is needed to make it easier for students to recognize concepts and to express a concept.

Mathematical symbols (0, 1, 2, 3, +, –, <,>, %, etc.) are important tools used in the transmission of mathematical knowledge (Olkun & Toluk Uçar, 2018). It is impossible for us to manipulate concepts visually without symbols. The symbols of mathematics make it possible for us to discover and express relationships between various concepts. For example, when we write down the statement 4 +2 = 6 with symbols, we actually refer to the relationship between the concepts of four, two, six addition and equality (Haylock & Cockburn, 2014). Symbols form the basis of mathematical communication by conveying meanings and messages (Angliheri, 2005; Bardini & Pierce, 2015). Symbols also help to mediate thinking about mathematical concepts, enabling operations on concepts (Gray & Tall, 1994). According to Angliheri (2005), it is very important for children to be able to understand what the teacher says and how it relates to the symbols they use for calculations they see on a page, to understand the relationships that exist between numbers and the operations we use on numbers.

Since symbols may have different meanings depending on the mathematical context, the use and interpretation of symbols may not be easy for students. Students often have difficulties in attributing meaning to mathematical symbols (Powell, 2015; Powell & Driver, 2015). Although young children can identify and write symbols, this does not reflect an understanding of the mathematical meaning of symbols or their relationship to numbers (Ilany & Hassidov, 2018). In mathematics, there is common use of keywords denoting the four operations (addition, subtraction, multiplication, and division). Since mathematics includes symbols and signs, the inability to distinguish between them for visual reasons may hinder learning. For example; the symbols "<, >, +, –, x, ÷, =" may cause confusion between the numbers 2 and 4, 6 and 8 (Patkın, 2011). One of the main challenges at this stage of learning mathematics concerns children's misconception of the symbols used (+, −, x, ÷, =) as a result of their erroneous experiences (Baroody & Standifer, 1993). It is very important to teach the correct mathematical use of symbols from the pre-school period (Angliheri, 2005; Hassidov & Ilany, 2017).

As students progress from primary school to university in mathematics education, as a result of the discontinuity in the use of symbols, unlimited expansion, increased complexity in symbol load and not being familiar with the meanings of symbols, students lose their self-confidence in mathematics and may choose a way of study that minimizes their need for mathematics (Bardini & Pierce, 2015). Students must be able to decipher the meanings of symbols in order to effectively read mathematics (Adams, 2003). Students construct their algebraic understanding by associating them with their understanding in arithmetic (Akkan, Baki & Çakıroğlu, 2012; Yıldız & Atay, 2019). It is important to determine how the primary school senior students, who will encounter the field of learning algebra in the secondary school period and where arithmetic is mainly at the centre of mathematics teaching, make sense of the four operation symbols, their use in arithmetic operations and their use in word problems. It is important to determine the pre-knowledge of students regarding the meaning and use of the four operation symbols. For these reasons, the aim of this study is to determine the primary school fourth-grade students' understanding of four operation symbols, using them in arithmetic operations and word problems.

METHODS

The phenomenology design, one of the qualitative research methods, was used in this study, which aimed to get the thoughts of the fourth-grade students about their experiences, how the symbols of the four operations were interpreted by the students, how to write and solve arithmetic operations by using symbols and how to write and solve word problems by using symbols. The research was guided by the following questions.

Research questions

1. What are the opinions of the 4th-grade primary school students regarding the meanings of the four operation symbols (+, −, x, ÷, =)?
2. How are primary school 4th-grade students in writing and solving arithmetic operations by using the four operation symbols (+, −, x, ÷, =)?
3. How are primary school 4th-grade students in writing and solving word problems by using the four operation symbols (+, −, x, ÷, =)?

The participants of the research consist of 123 students (65 girls and 58 boys), attending the fourth grade (9 years old) in two different public primary schools in Ankara, Türkiye, in May and June of the 2021-2022 academic year. The schools of the participants are public schools located in the settlement where the families live at the middle socio-economic level. The participants of the study were determined by using convenience sampling, which is one of the purposive sampling methods.

In the research, the "four operation symbol study form" developed by the researcher was used as a data collection tool. The study form prepared to determine the level of understanding of the four operation symbols (+, −, x, ÷, =) of primary school fourth-grade students consists of three parts. In the first part of the data collection tool, there are questions to determine how the four operation symbols are interpreted by the students, in the second part, how to write and solve arithmetic operations by using four-operation symbols, and in the third part, there are questions to determine how to write and solve word problems by using four-operation symbols. The study form prepared by the researcher was sent to 1 mathematics education specialist and two classroom teachers, and expert opinion was applied to ensure the content validity. In line with the expert opinions received, the question sentences were rearranged by making arrangements regarding the suitability of the student level and supporting them with appropriate visuals. The reorganized study form was applied to 26 fourth-grade students studying in another primary school, which were not among the participants of the research, and a pre-application was made. As a result of the pre-application, it was determined that the questions were understood by the students, could be answered and were suitable for their level. The final form of the study form is given in Figure 1.

Symbol	Questions	Answers	Symbol	Questions	Answers	Symbol	Questions	Answers	Symbol	Questions	Answers	Symbol	Questions	Answers
+	1.What does this symbol mean?		−	1.What does this symbol mean?		X	1.What does this symbol mean?		÷	1.What does this symbol mean?		=	1.What does this symbol mean?	
	2.Write an arithmetic operation by using this symbol and find your result.			2.Write an arithmetic operation by using this symbol and find your result.			2.Write an arithmetic operation by using this symbol and find your result.			2.Write an arithmetic operation by using this symbol and find your result.			2.Write an arithmetic operation by using this symbol and find your result.	
	3.Write a word problem by using this symbol and find your result.			3.Write a word problem by using this symbol and find your result.			3.Write a word problem by using this symbol and find your result.			3.Write a word problem by using this symbol and find your result.			3.Write a word problem by using this symbol and find your result.	

Figure 1: Four operation symbol study form

The administrators and teachers in the schools where the application was carried out were informed by the researcher about the purpose of the study and necessary permissions were obtained. The necessary consent was obtained from the participants and their families to participate in the study. The data collection tool was applied to the fourth-grade primary school students during the mathematics lesson in their classrooms, and the students were informed about how to answer the questions in the data collection tool and what should be considered. Students were given an average of 30-35 minutes to answer the questions in the study form individually.

The data obtained through the four operation symbol study forms were analyzed with the content analysis method. Content analysis is an analysis method that focuses on how many times measurement units such as a particular speech pattern or phrase are used, summarizing some words of a text with smaller content categories with coding based on certain rules (Merriam & Tisdell, 2015).

The written answers given by students to the study form were arranged by labelling as S1, S2,…,S123 (S: students). The edited data (the answers) were coded by applying content analysis, then the similar codes were arranged in line with the emerging concepts, and the concepts were brought together under categories based on frequency and significance analysis. Using the four operation symbols of the students, arithmetic and word problems were digitized as the numbers of students who were successful, were not successful and were partially successful. Randomly selected samples from student worksheets were analyzed at different times and similar results were obtained to ensure reliability. In the study, the data were analyzed independently by the researcher and a mathematician and the researcher diversification was used, thus reliability was tried to be ensured. The list made by the researcher and the list prepared by the specialist were compared and the consistency of the results was calculated by applying the kappa statistics. Kappa statistic value (κ) was found to be 0.843. The fact that the Kappa statistic value (κ) is between 0,81 - 1,00 indicates that the level of agreement is very high (Landis & Koch, 1977 as cited in Bilgen & Doğan, 2017). In order to ensure the credibility of the analysis results of the data, the photographs from the raw data sources were included in the findings section.

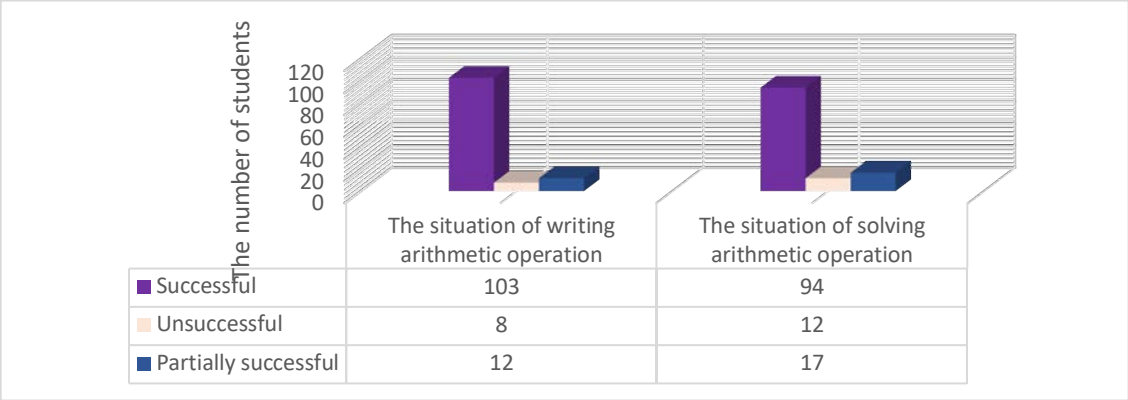
RESULTS

In this part of the study, the findings that emerged in line with the data obtained were presented in the form of tables, graphs and student response examples. The views of the fourth-grade primary school students on the meaning of the "+" symbol were given in Table 1.

Category	Code	f
Addition sign (43)	It's an addition sign	39
	It is the sign that we use to collect things	3
	It is the addition sign used when adding in problems	1
Addition (36)	It is used in addition	31
	It is the addition symbol used in the four operations	1
	It is used in mathematical operations	3
	To make an addition subject	1
To increase (27)	It is the symbol that makes the numbers increase	22
	It allows to increase the objects	5
Plus sign (21)	It's a plus sign	19
	Plus sign, that is (+)	2
To add (13)	It means to add	9
	Addition, that is, adding	3
	If a number is two, we add	1
To multiply (9)	It multiplies the numbers	7
	This sign serves to add, to multiply an object	2
To excess (6)	It means excessing	5
	It means an excess of something	1
To combine (4)	It is the symbol of combining	3
	This sign is a symbol for combining a number with a number	1
Correct sign (3)	It is the correct sign	2
	It is used if our answers to the problems are correct	1
Other (7)	It is similar to the multiplication sign	1
	Teachers want it, we do it	1
	It's the first operation we learn	1
	The most used symbol	1
	The symbol we learned in first grade	1
	It is forward rhythmic counting	1
	It is the sign of balancing	1

Table 1: The views of fourth-grade primary school students on the meaning of the “+” symbol.

When table 1 is examined, it is seen that primary school fourth-grade students stated the “addition sign” category most regarding the meaning of the “+” symbol, and the opinion of “it is an addition sign” was the most frequent expression within this category. The situations of writing and solving arithmetic operations by using the “+” symbol of primary school fourth-grade students were given in Graph 1.



Graph 1: The situations of students' writing and solving arithmetic operations by using the "+" symbol

When writing and solving arithmetic operations of the primary school fourth-grade students by using the "+" symbol were examined, it was determined that the majority of the students were able to write arithmetic operations and solve arithmetic operations. The example showing that the student with code S45 could write and solve an arithmetic operation using the "+" symbol is in Figure 2. The example, in which the student with code S68 could write an arithmetic operation but could not solve it correctly, is in Figure 3.

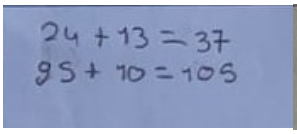


Figure 2: S45, an example of response

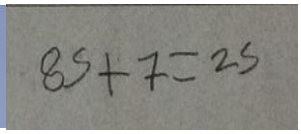
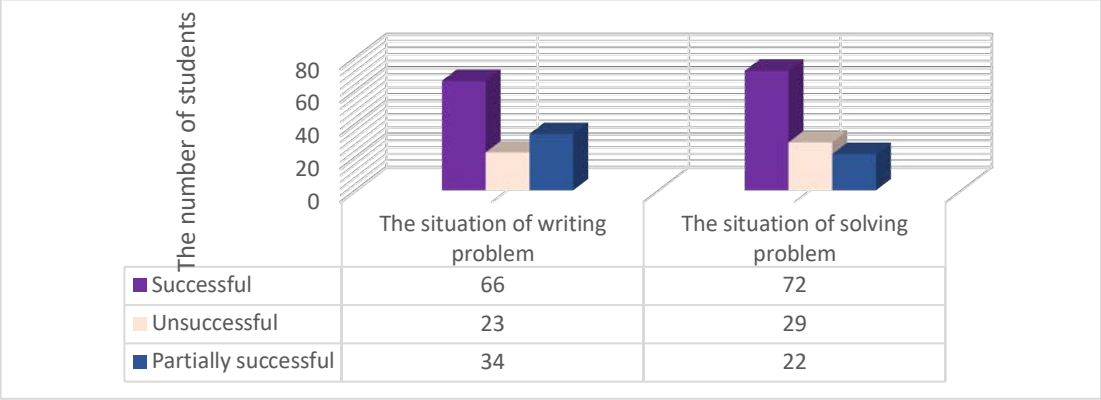


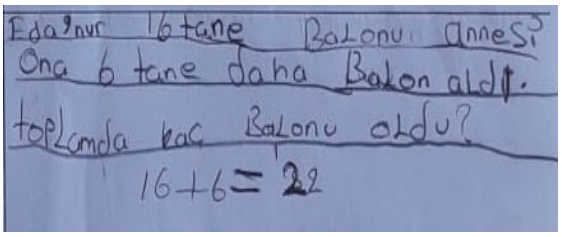
Figure 3: S68, an example of response

The findings on the writing and solving of word problems of primary school fourth-grade students using the "+" symbol were given in Graph 2.



Graph 2: The situations of students" writing and solving word problems by using the "+" symbol

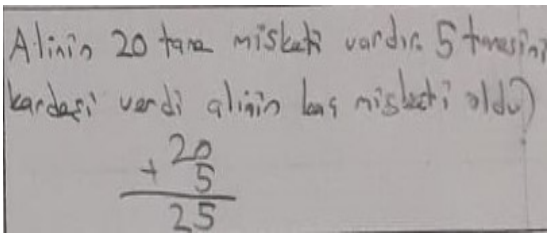
When the word problem writing and solving of primary school fourth-grade students by using the "+" symbol were examined, it was revealed that students are more successful in solving problems by using the "+" symbol than in writing. The example of the correct usage of the "+" symbol by the student coded S34 in problem writing and solving was included in Figure 4. The example of the incorrect usage of the symbol "+" in writing of the word problem and the correct usage in solving it by the student coded S96 was included in Figure 5.



Edanur had 16 balloons. His mother bought him 6 more balloons. How many balloons did he have in total?

16 + 6 = 22

Figure 4: S34, an example of response



Ali had 20 marbles. He gave 5 of them to his brother. How many marbles did Ali have?

20
+5
25

Figure 5: S96, an example of response

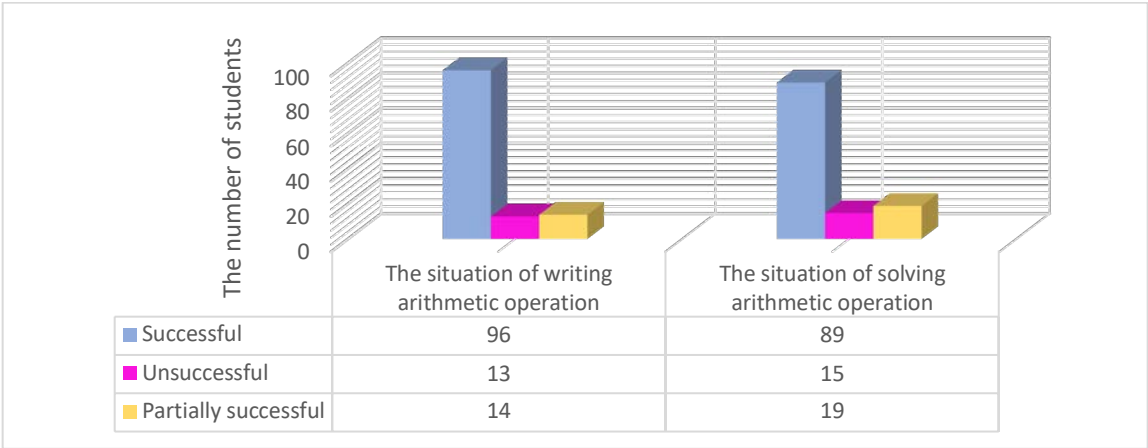
The views of the fourth-grade primary students on the meaning of the "–" symbol were given in Table 2.

Category	Code	f
Subtraction sign (41)	It's a subtraction sign.	36
	It is the subtraction sign that indicates the decrease.	2
	It's a subtraction sign in math.	3
Subtraction (34)	It is used in subtraction.	26
	It subtracts a number from a number.	4
	It means to subtract a number.	3
	Having a subject of subtraction	1
To reduce (26)	It helps to reduce the numbers.	18
	It reduces the objects.	4
	It means to reduce.	4
To decrease (23)	It means to decrease.	16
	It is used to decrease something.	4
	It's useful to decrease.	3
Minus sign (14)	It means minus sign.	13
	Minus sign, so small	1
Wrong sign (5)	It's the wrong sign.	4

	Wrong refers to controlling.	1
To separate (3)	It is the separation of one number and another number.	2
	Writing separately.	1
Other (6)	It means negative.	1
	It is used after addition.	1
	The sign next to words that have no endings.	1
	It means subtrahend.	1
	It means to delete.	1
	Getting a minus lowers points.	1

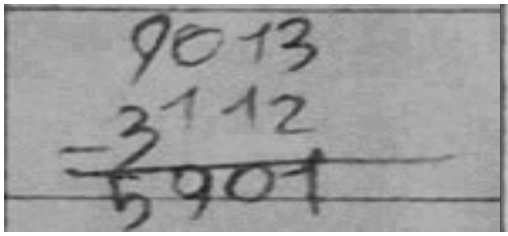
Table 2: The views of fourth-grade primary school students on the meaning of the "–" symbol

When table 2 is examined, it is seen that primary school fourth grade students stated the "subtraction sign" category most regarding the meaning of the "–" symbol, and the opinion of "it's a subtraction sign" was the most frequent expression within this category. The situations of writing and solving arithmetic operations by using the "–" symbol of primary school fourth grade students were given in Graph 3.



Graph 3: The situations of students' writing and solving arithmetic operations by using the "–" symbol

When the primary school fourth grade students' writing and solving arithmetic operations by using the "–" symbol were examined, it was determined that the majority of the students were able to write arithmetic operations and solve arithmetic operations. The example showing that the student with the code S26 could write and solve arithmetic operation using the symbol "–" was included in Figure 6. The example of the student with the code S51, which shows that he could write arithmetic operation but could not solve the solution of the arithmetic operation correctly, was included in Figure 7.



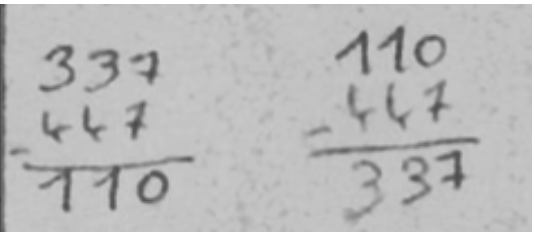
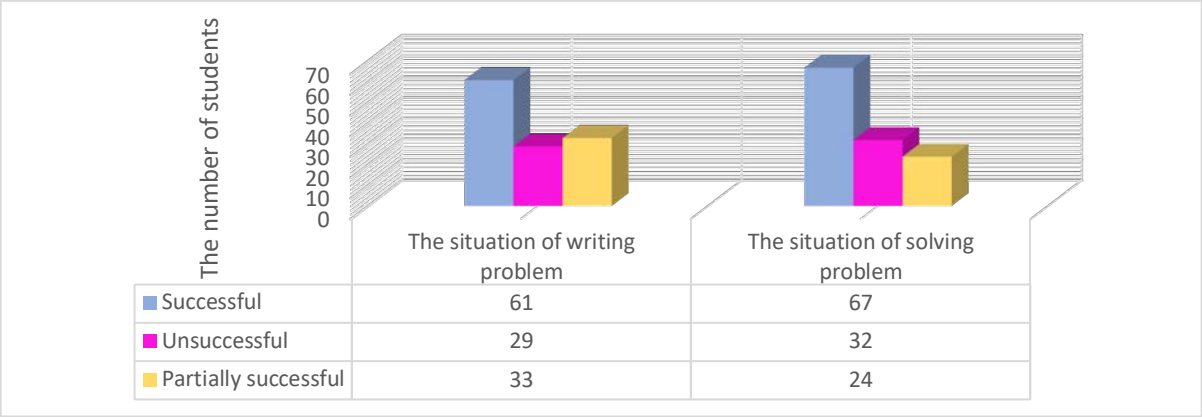


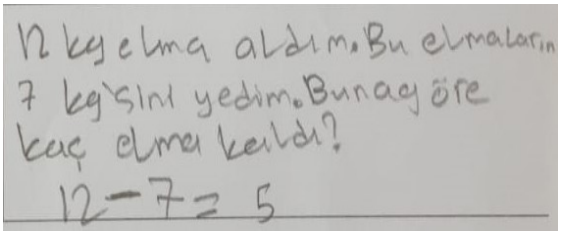
Figure 6: S26, an example of response Figure 7: S51, an example of response

The findings regarding the writing and solving situations of word problems of primary school fourth grade students by using the "–" symbol were given in Graph 4.



Graph 4: The situations of students' writing and solving word problems by using the "–" symbol

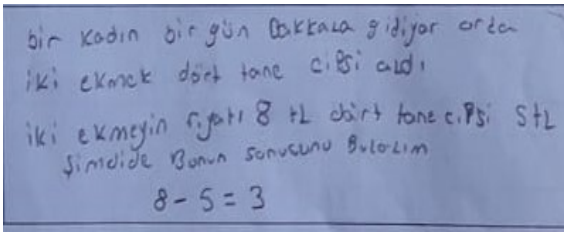
When the primary school fourth grade students' writing and solving word problems by using the "–" symbol were examined, it was determined that students are more successful in solving problems using the "–" symbol than in writing. The example of the correct usage of the symbol "–" of the student with the code S40 in writing and solving word problems was included in Figure 8, and the example of the incorrect usage of the symbol "–" in writing of the word problem but the correct usage in solving it by the student coded S03 was included in Figure 9.



I bought 12 kg of apples. I ate 7 kg of these apples. How many apples are left?

12 – 7 = 5

Figure 8. S40, an example of response



A woman is going to the grocery store one day. There he bought two breads and four chips. The price of two breads is 8 TL, the price of four chips is 5 TL. Now let's find the result of this.

$8 - 5 = 3$

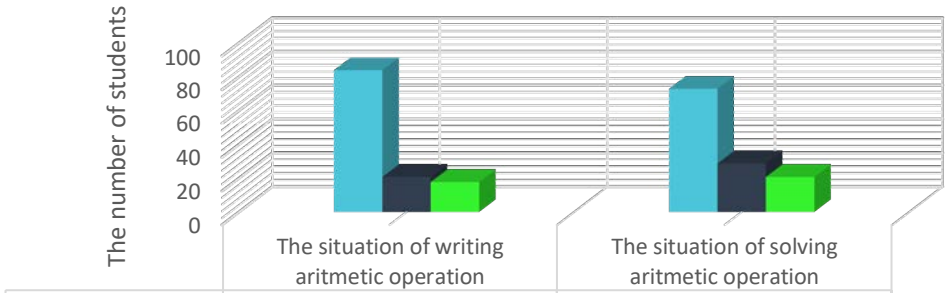
Figure 9. S03, an example of response

The views of the fourth-grade students on the meaning of the "x" symbol were given in Table 3.

Category	Code	f
Multiplication sign (32)	It's a multiplication sign.	27
	It is the multiplication sign that allows us to find the result in a short way instead of adding two numbers in a long way.	3
	It indicates the multiplication sign we use to multiply a number.	2
Multiplication (30)	It is used in multiplication.	20
	The short path for addition is multiplication.	8
	It only works for multiplication.	1
	Making a multiplication.	1
Cross (16)	It indicates a cross.	14
	It is the cross that increases the numbers.	2
Multiply (14)	It means multiply.	8
	This symbol means multiply two numbers.	6
Times (11)	It means times.	7
	It indicates operation of times.	4
Coefficient (8)	It tells how many times one number will be another number.	7
	I understand that it will be multiplied by the given coefficient number.	1
Wrong(4)	It means wrong.	3
	It is used if the problems we do are wrong.	1
Plus sign (3)	This sign means plus sign.	2
	It is the plus sign that increases the numbers.	1
Other (9)	We use it in important places.	1
	Obtaining different numbers by collision of numbers	1
	It means that something will multiply	1
	It's a no sign	1
	It means factor	1
	It does not change if the place of the numbers in the operation changes	1
	It teaches us multiplication	1
	It is forbidden to enter	1
	It is dangerous	1

Table 3: The views of fourth-grade primary school students on the meaning of the "x" symbol

When table 3 is examined, it is seen that primary school fourth grade students stated the "multiplication sign" category most regarding the meaning of the "x" symbol, and the opinion of "it is a multiplication sign" were the most frequent expressions in this category. The situations of writing and solving arithmetic operations by using the "x" symbol of primary school fourth grade students were given in Graph 5.



	The situation of writing aritmetic operation	The situation of solving aritmetic operation
Successful	84	73
Unsuccessful	21	29
Partially successful	18	21

Graph 5: The situations of students' writing and solving arithmetic operations by using the "x" symbol

When the primary school fourth grade students' writing and solving arithmetic operations by using the "x" symbol were examined, it was determined that students are more successful in writing arithmetic operations using the "x" symbol than in solving them. The example showing that the student coded S103 can write and solve arithmetic operations using the "x" symbol is in Figure 10, and the example that the student with the code S23 can write arithmetic operation but cannot solve the arithmetic operation, is in Figure 11.

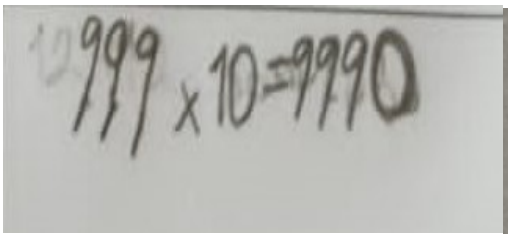


Figure 10: S23, an example of response

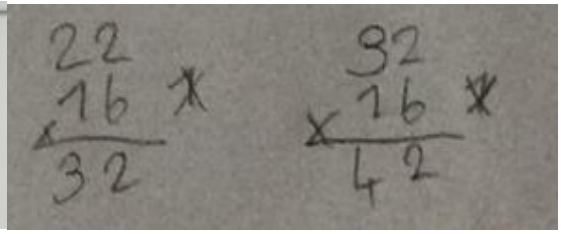
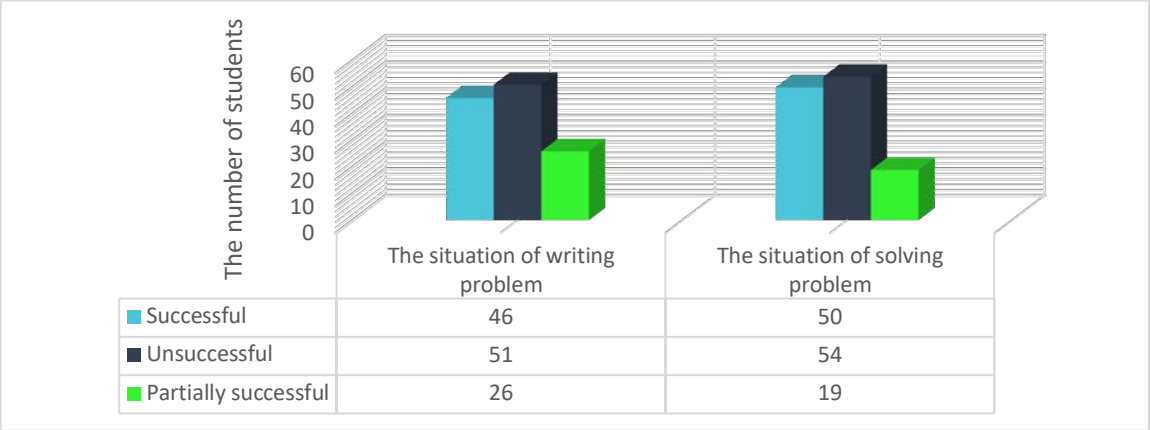


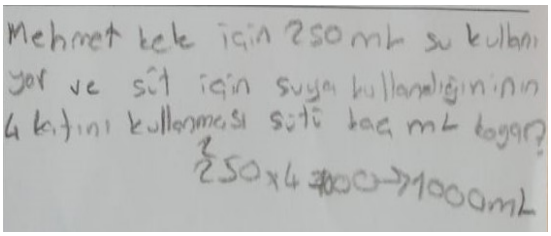
Figure 11: S103, an example of response

The findings regarding the writing and solving situations of word problems of primary school fourth grade students by using the "x" symbol were given in Graph 6.



Graph 6: The situations of students' writing and solving word problems by using the "x" symbol

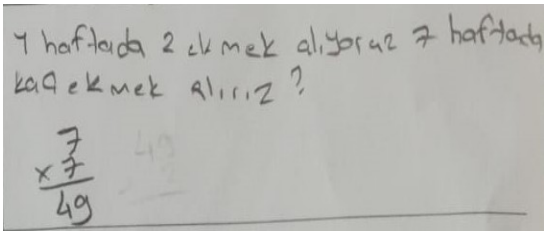
When the primary school fourth grade students' writing and solving word problems by using the "x" symbol were examined, it was determined that students are more successful in solving problems by using the "x" symbol than in writing. The example of the correct usage of the "x" symbol of the student with the code S71 in writing and solving word problem is in Figure 12, and the example of the correct usage of the symbol "x" in writing of the word problem but the incorrect usage in solving it by the student coded S42 is in Figure 13.



Mehmet uses 250 ml of water for cake and uses 4 times more water for milk than he uses water. How many ml does he put in the milk?

250 x 4 = 1000 ml

Figure 12: S71, an example of response



We buy 2 breads in 1 week. How many breads do we buy in 7 weeks?

$$\begin{array}{r} 7 \\ x7 \\ \hline 49 \end{array}$$

Figure 13: S42, an example of response

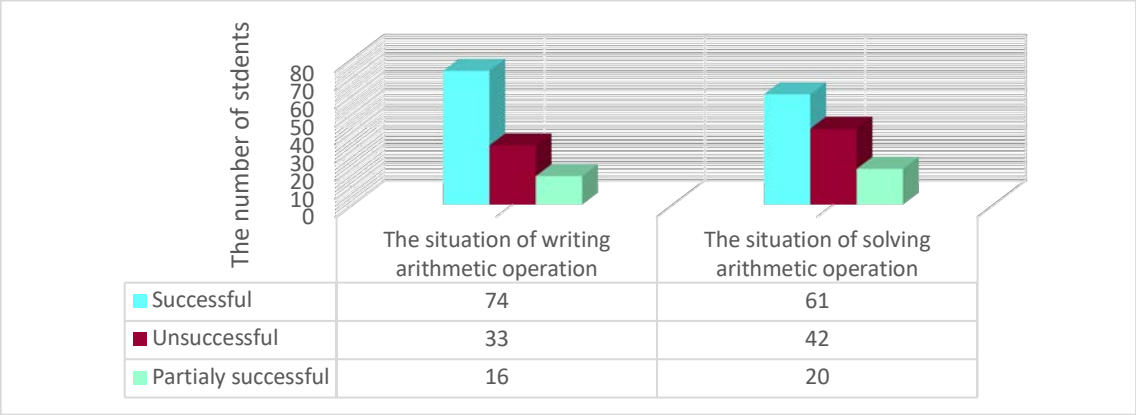
The views of the fourth grade students on the meaning of the symbol "÷" were given in Table 4.

Category	Code	f
To share (43)	It means to share	30
	It ensures that something is shared equally.	9
	This sign is the sign of the sharing of two numbers.	3

	It's to share.	1
Division sign (24)	It is a division sign	21
	We make a division with division sign	2
	The sign we use when we solve a division problem	1
Division (22)	It is used in division	12
	It indicates by how many times we can divide a number	6
	Division is a shortcut for subtraction	3
	Making a division	1
To group (17)	It means to group.	12
	It indicates how many groups a number will have.	3
	It means to divide into groups.	2
Slash sign (11)	It is a slash sign.	8
	It is a sign that means to divide one number by another number.	3
To divide (7)	It means to divide.	5
	It divides two things between each other.	2
Minus sign (4)	It means minus sign.	3
	Abbreviation for subtraction	1
To decrease (3)	It means a decrease.	2
	This symbol means decrease.	1
Other (6)	We use it in important places.	1
	We use it in numbers	1
	It cannot be found in the quotient of any number.	1
	It helps to distribute the numbers equally.	1
	It is the most difficult operation.	1
	It means to separate.	1

Table 4: The views of fourth-grade primary school students on the meaning of the symbol "÷"

When table 4 is examined, it is seen that primary school fourth grade students stated the category of "to share" the most regarding the meaning of the "÷" symbol, and the most frequent expressions were the opinion of "it means to share" in this category. The situations of writing and solving arithmetic operations by using the symbol "÷" of primary school fourth grade students were given in Graph 7.



Graph 7: The situations of students' writing and solving arithmetic operations by using the "÷" symbol

When the writing and solving arithmetic operations of the primary school fourth grade students by using the "÷" symbol was examined, it was determined that students are more successful in writing arithmetic operations using the symbol "÷" than in solving. The example showing the student coded S98 can write and solve arithmetic operations using the symbol "÷" is in Figure 14, and the example that the student coded S39 can write arithmetic operations but cannot solve the arithmetic operation correctly, is in Figure 15.

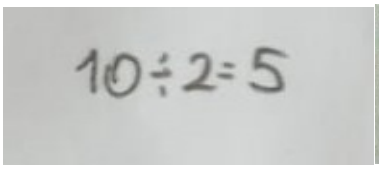


Figure 14: S98 an example of response

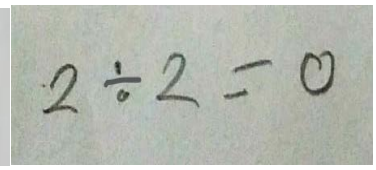
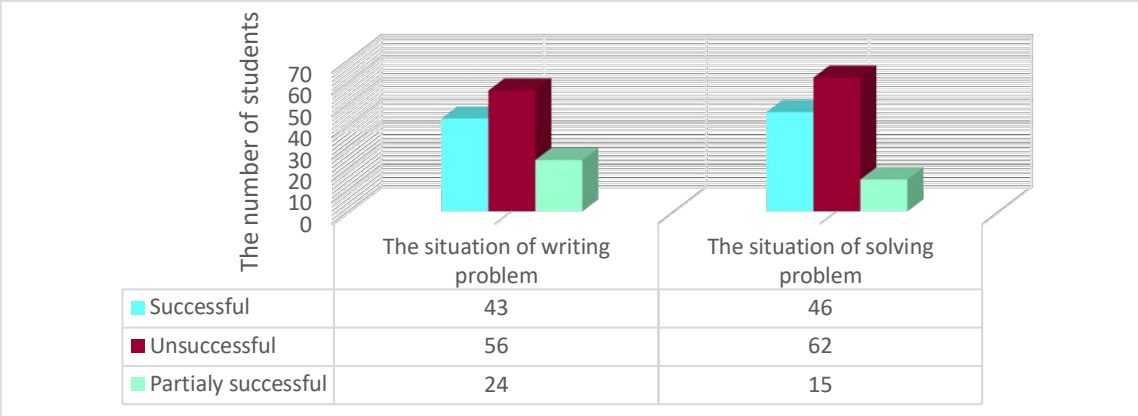


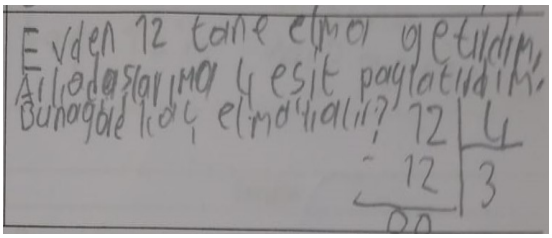
Figure 15: S39 an example of response

The findings regarding the writing and solving situations of word problems of primary school fourth grade students by using the symbol "÷" were given in Graph 8.



Graph 8: The situations of students' writing and solving word problems by using the symbol "÷"

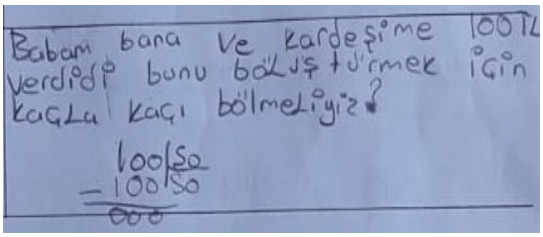
When the writing and solving word problems of the primary school fourth grade students by using the "÷" symbol was examined, it was determined students are more successful in solving problems by using the symbol "÷" than in writing. The example of the partially correct usage of the "÷" symbol of the student with the code S05 in writing and the correct usage in solving word problem is in Figure 16, and the example of the correct usage of the symbol "÷" in writing of the word problem but the incorrect usage in solving it by the student coded S102 is in Figure 17.



I brought 12 apples from home. I shared 4 equal parts with my friends. How many apples are left?

12 | 4
-12 | 3
00

Figure 16: S05 an example of response



My father gave me and my brother 100 TL. How many should we divide by how many?

100 | 50
-100 | 50
000

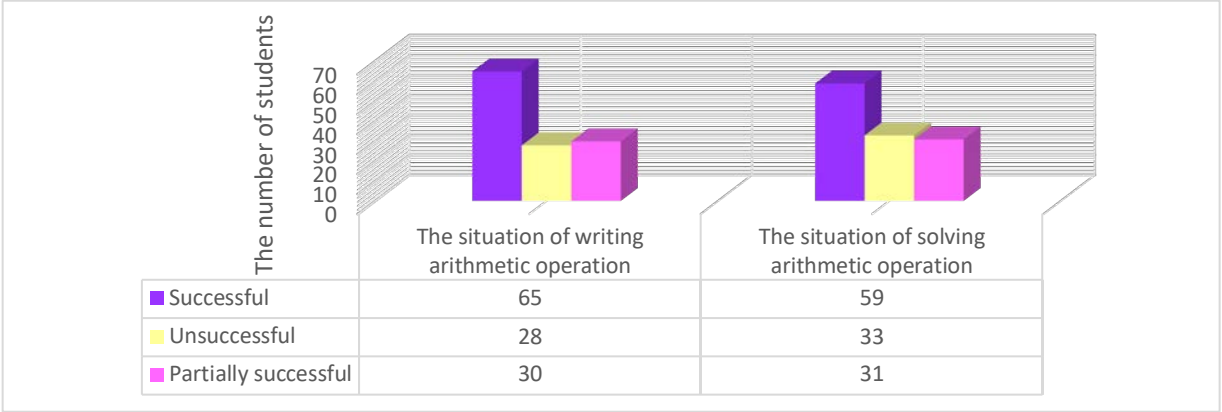
Figure 17: S102 an example of response

Category	Code	f
Result (37)	It is used to find the result.	21
	It shows the result of each problem.	10
	It is equal to the result.	6
	When we add something with something, we write it to the result.	1
Equal sign (23)	It's an equal sign.	19
	It is the equal sign found in all operations.	3
	It is an equal sign that indicates the equality.	1
Answer (22)	It means to find the answer	14
	It shows the answer in addition, subtraction, multiplication, division.	8
Same (19)	It means the same.	13
	It is said that if all numbers are the same, they are equal.	4
	It indicates that two numbers are the same.	2
	It is put between the same numbers.	2
Equivalent (13)	It means equivalence.	8
	If two things are equal, It says they are equivalent.	4
	It means equivalent to something.	1
Across (10)	The sign written across the question.	6

	The across side of the solution.	4
The end (8)	We use it at the end of the numbers	5
	It is used at the end of the operation	2
	It is put at the end of +, -, x, and ÷	1
Next to (7)	It is written next to the operations.	4
	It is placed next to the numbers.	2
	The sign that precedes the result of adjacent operations	1
To start (4)	It is the signal to start the operation.	3
	It means to start.	1
Other (6)	It is found everywhere.	1
	It is used in tests and books.	1
	It evens everything out.	1
	It means common.	1
	I think of this symbol as the sign between "< and >".	1
	It provides the crosscheck of an operation.	1

Table 5: The views of fourth grade primary school students on the meaning of the "=" symbol

When table 5 is examined, it is seen that primary school fourth grade students stated the category of "result" the most regarding the meaning of the "=" symbol, and the most frequent expressions were "it is used to find the result" in this category. The situations of writing and solving arithmetic operations by using the symbol "=" of primary school fourth grade students were given in Graph 9.



Graph 9: The situations of students' writing and solving arithmetic operations by using the "=" symbol

When the writing and solving arithmetic operations of the primary school fourth-grade students by using the "=" symbol was examined, it was determined that students are more successful in writing arithmetic operations using the "=" symbol than in solving. The example showing the student coded S63 can write and solve arithmetic operations using the symbol "=" is in Figure 18, and the example that the student coded S84 tried to write an equation using the symbol "=" but made an incorrect solution is in Figure 19.

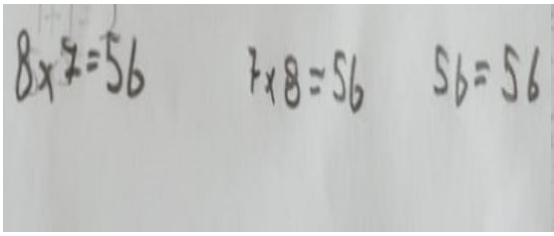


Figure 18: S63 an example of response

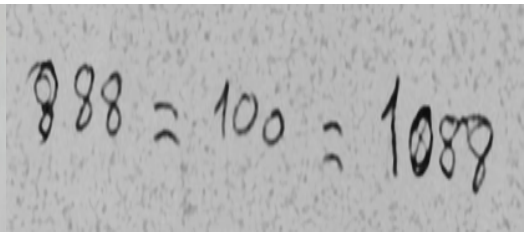
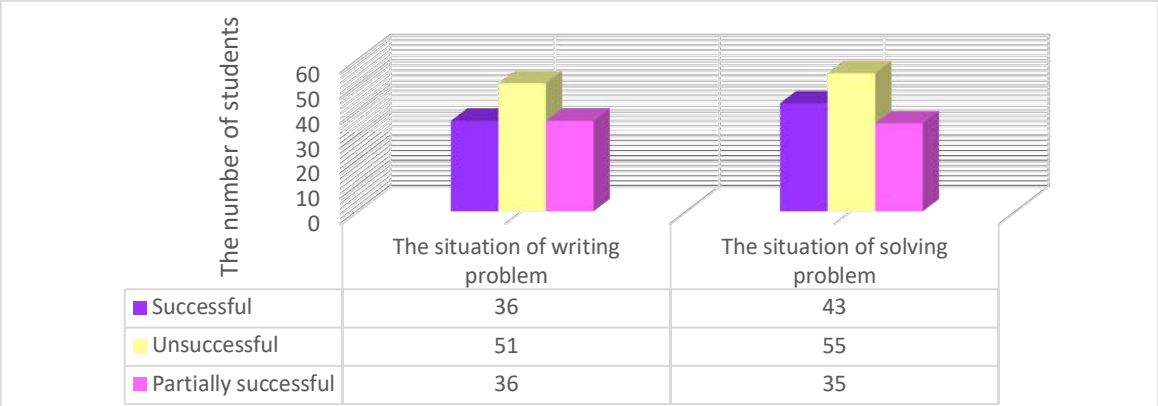


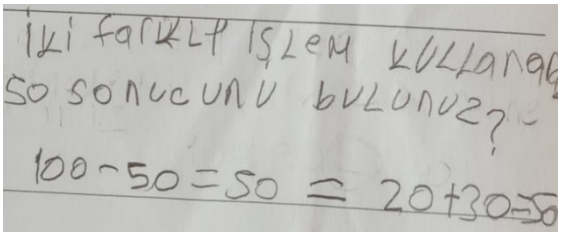
Figure 19: S84 an example of response

The findings regarding the writing and solving situations of word problems of primary school fourth grade students by using the "=" symbol were given in Graph 10.



Graph 10: The situations of students' writing and solving word problems by using the "=" symbol

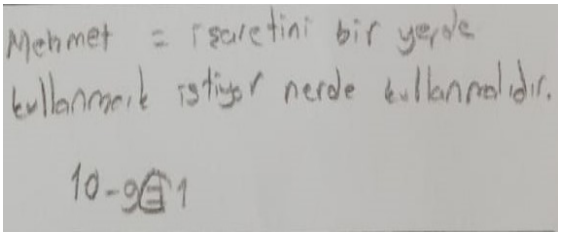
When the writing and solving word problems of the primary school fourth-grade students by using the "=" symbol was examined, it was determined students are more successful in solving problems by using the "=" symbol than in writing. The example of the correct usage of the "=" symbol of the student with the code S79 in writing and solving a word problem is in Figure 20, and the example of the correct usage of the symbol "=" in writing the word problem but the incorrect usage in solving it by the student coded S58 is in Figure 21.



Find the result 50 using two different operations?

100 – 50 = 50 = 20 + 30 = 50

Figure 20: S79 an example of response



Mehmet wants to use the = sign somewhere. Where should use.

10 – 9 = 1

Figure 21: S58 an example of response

CONCLUSIONS AND DISCUSSIONS

In this study, which was conducted to determine how the four operation symbols are interpreted by the students, how to write and solve arithmetic operations using symbols, and how to write and solve word problems using symbols, it was concluded that the fourth graders of the primary school made sense of the "+" symbol in connection with the categories of "addition sign, addition, to increase, plus sign, to add, to multiply, to combine and correct sign". According to Anghileri (2005), children should learn that the "+" symbol can be read as "and", "add" or "plus" and that they need to put meaning into the symbol to solve written sums. According to Altun (2018), in order to gain the knowledge of the addition operation at the primary school level, the expressions such as "and, more, total, plus, added, multiplied" should be included in the problem sentences given and the students should be provided with meaning to the addition. As another result of the research, it was determined that the fourth-grade students of primary school put forward the categories of "subtraction sign, subtraction, to reduce, to decrease, minus sign, wrong sign and to separate" regarding the meaning of the "–" symbol.

It was concluded that primary school fourth-grade students put forward the categories of "multiplication sign, cross, multiply, times, coefficient, wrong and plus sign" regarding the meaning of the "x" symbol. According to Pesen (2020), it should be emphasized that 4 x 2 =? can be expressed as "4 times 2 equals 8, 4 by 2 is 8", the words "by", "times" are related to multiplication, the symbol of multiplication is "x" and it is read as "cross", and it should be explained to students that this expression is written as 4 x 2 = 8. The fact that multiplication also means repeated addition explains the coefficient meaning of multiplication. It was determined that the fourth-grade students of primary school put forward the categories of "to share, division sign, division, to group, slash sign, to divide, minus sign and to decrease" regarding the meaning of the symbol "÷". Angliheri (2005) states that the "÷" symbol used in the expression "12 ÷ 3" can be read in many ways. The word "sharing" is often associated with division. Although the word "divide" may be less familiar to most young children, there is an increasing acceptance and use of this interpretation as children grow up. It was determined that the fourth-grade students of primary school put forward the categories of "result, equal sign, answer, same, equivalent, across, the end, next to and to start" regarding the meaning of the "=" symbol. Under these categories, it was found

that the symbol "=" was used the most in the "result" category in the sense of "it is used to find the result". Students often interpret the equal sign as an operational symbol. They do not see the equal sign as a relational symbol that requires the use of numerical relations between the two sides of the equation (Carpenter & Levi, 2000; Knuth, Alibali, McNeil, Weinberg & Stephens, 2005; Mirin, 2020; Powell, 2015; Yıldız & Atay, 2019).

When the primary school fourth-grade students' writing and solving arithmetic operations using four operation symbols were examined, it was concluded that students were more successful in writing arithmetic operations using symbols than in solving arithmetic operations. In addition, it was determined that students were more successful in writing and solving arithmetic operations using "+" and "–" symbols than in writing and solving arithmetic operations using "x", "÷" and "=" symbols. Önal (2017), in his research concluded that misinterpretation of symbols and texts is one of the important factors that cause students to make mistakes in the four operations. According to the research findings, it was concluded that the students had the most difficulty in using the "=" symbol in writing and solving arithmetic operations. It may not seem important for teachers to focus on the relational meaning of the equal sign. However, when students move to secondary school and high school and begin to solve mathematical (or., $9 = 81 \div x$) and algebraic equations (or., $3a = b + 12$), the sign of equality needs to be understood relationally (Powell, 2015). Misconceptions developed in primary school, especially for concepts that form the basis of algebraic thinking such as equality prevent students from understanding algebra issues correctly (Byrd, McNeil, Chesney, & Matthews, 2015; Knuth et al., 2005; Mirin, 2020).

When the writing and solving word problems of primary school fourth-grade students using four-operation symbols was examined, it was concluded that students were less successful in writing and solving arithmetic operations using symbols, and they were more successful in problem-solving than in problem-writing. It can be said that students are not able to fully convey their thoughts into words as a reason why they are more successful at problem solving than writing problems. Angliheri (2006) states that the use of the subtraction symbol "–" to represent the words "take away" in some problems and "difference between" in others causes difficulties for children. Where a child knows subtraction only as 'take away', confusion may arise in representing the "difference between" using exactly the same symbol. Children should learn that there are many meanings associated with arithmetic symbols, the certain interpretations will better match the different contexts and the different solution procedures that are appropriate.

RECOMMENDATIONS

Students do not always attribute the correct meaning or any meaning to their mathematical symbols. Teachers need to provide opportunities for their students to learn the correct definitions of symbols and to apply symbol understanding in a variety of contexts (Powell, 2015). In the study of Kabael and Ata Baran (2016), the participants suggested that a section called "the corner of symbols and their meanings" should be created on the blackboard or on the school math boards during the course process in order to gain the meanings of symbols.

Misconceptions that may occur in the four operation symbols can reveal conceptual problems in mathematical studies. In line with the findings of the study, different examples and exercises can be included in mathematics lessons, textbooks and workbooks regarding the meanings of the four operation symbols (+, −, x, ÷, =), their use in arithmetic operations, and their use in word problems. In the teaching of symbols, speeches and concrete objects containing daily life situations can be used. Research can be conducted on the meanings attributed to other symbols (<, ≤, >, ≥, %, etc.) used in primary school mathematics lessons, the determination of their use in arithmetic operations and word problems, and the difficulties experienced in the use of symbols.

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