

High School Students' Beliefs about Mathematical Problem Solving: A Cluster Analysis

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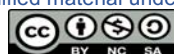
Abstract: This paper investigates profiles of high school students' mathematical problem-solving beliefs following the introduction of a new mathematics curriculum. In this quantitative study, 490 high school students responded to a mathematical problem-solving beliefs questionnaire to provide data about their beliefs related to mathematical problem solving. A cluster random sampling technique was used to select participants for the study. To accomplish the major goals of the study, a K-means clustering technique was conducted to analyze patterns that were discernible from their beliefs. In addition, a one-way ANOVA conducted to examine mean differences of their beliefs between clusters. Results revealed that, in general, students strongly believe that conceptual understanding is important in mathematics. In one of the clusters students hold strong beliefs about the usefulness of mathematics in their da-to-day lives while in another cluster it was strongly believed that effort was key for one to increase their mathematical ability. Results are important for students' confidence to solve mathematical problems and for implementation of a problem-solving approach in the new mathematics curriculum.

Keywords: Cluster analysis; High school; Mathematical problem-solving; Students' beliefs

Introduction

Problem solving is essential for mathematics conceptual development at all levels of education. It is the major reason for learning mathematics in all classrooms (Purnomo et al., 2022). Problem solving supports development students' critical thinking due to high order mathematical problems they encounter and enables them to think systematically and mathematically during the process of problematizing (Goutlet-Lyle et al., 2020; Rott et al., 2021). This view motivated curriculum reform in Zambia.

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Zambia embarked on reforming the mathematics curriculum whose implementation was launched by the Curriculum Development Centre (CDC) in 2019, and in which problem solving was adopted as one of the key teaching approaches (CDC, 2019). The reformed curriculum was named Science, Technology, Engineering, Mathematics (STEM) syllabus because it placed science, technology, engineering, and mathematics at the center of all classroom activities. Teaching and learning approaches were to integrate science, technology, and mathematics. CDC guided that problem-solving was important for fostering critical analysis of mathematical problems for learners and collaboration among teachers through sharing ideas about improvement of their classroom practice. CDC further guided that the new curriculum would provoke creativity, innovativeness, and critical thinking among students, and schools would train students to be problem solvers who would effectively apply mathematics to solving problems facing their society.

This view is anchored on the premise that mathematical problem solving remains the “*most important skill*” that students should acquire from school (Erlina & Purnomo, 2020) and must essentially be a core aspect of the mathematics curriculum in Zambia and beyond (Purnomo et al., 2022). However, Zambian high school students continue to exhibit lack of mathematical skills required for solving high order problems and they demonstrate low problem-solving abilities (Karimah et al., 2018; Piñeiro et al., 2021; Yeni et al., 2020). This has been attributed to the lack of mathematical creativity, poor choice of strategies, and inability to fully understand mathematical tasks especially word problems (Carotenuto et al., 2021; Payadnya et al., 2021; Purnomo et al., 2021; Supandi et al., 2021). Resolving these issues requires the intervention of teachers. Teachers are expected to intervene through the curriculum because curriculum implementation is a sole responsibility of teachers (Rafiepour & Farsani, 2021; Wang et al., 2022) and accounts for their key performance indicators in Zambia.

Teachers have faced challenges implementing the new STEM curriculum because it was not published together with revised or new mathematics textbooks that teachers would use to teach, and learners to learn. This is despite evidence that students and teachers heavily depend on materials like textbooks and worksheets for the learning and teaching process, and for development of students’ problem-solving skills (Jäder et al., 2020; Ulandari et al., 2019). Textbooks serve as the major reference materials for teachers’ lesson planning activities and for students exercises and notes. Sahendra et al. (2018) emphasized that textbooks, for example, can improve students’ mathematical word problem-solving abilities. This is supported by Yuanita et al. (2018) who posit that mathematical representations can enhance problem solving skills by acting as mediators between students’ beliefs and mathematical problem-solving.

As indicated earlier, teachers play a central role in students mathematics development. Several studies have reported about mathematics teachers’ beliefs in mathematical problem solving and the role teacher beliefs play in fostering students’ problem-solving abilities (Fatmanissa & Qomaria, 2021; Muhtarom, 2019; Siswono et al., 2019; Van Dooren et al., 2019). It has been argued that teachers should always consider and include students’ context and incorporate real life situations into mathematical problems they prepare as tasks for students (Fatmanissa & Qomaria,

2021). Such problems should be of high quality and demanding of deep intuition from students (Agustina et al., 2021). They may be time-consuming to solve but teachers should not confuse them with long procedural problems that may also take long to be solved. The quality of a mathematical problem should be emphasized when selecting tasks for students.

Students' mathematical beliefs are known to be the most central and powerful characteristics that enhance their learning and account for their performance in mathematics (Habók et al., 2020; Hidayatullah & Csíkos, 2022; Yin et al., 2020; Wang et al., 2019). When students exhibit positive beliefs in their mathematics problem solving abilities, they are motivated to work hard towards improving their mathematics competences. Evidence from the literature indicates that previous studies inquired on high school students' mathematical problem-solving beliefs focusing on different aspects of the topic, with some exploring gender differences (E.g., Awofala, 2017), while Rojo Robas et al. (2020) studied the role of beliefs on students' motivation to engage in problem solving. Others concluded that students' problem-solving abilities can be enhanced by their positive beliefs (NoprianiLubis et al., 2017; Özcan & Eren Gümüş, 2019; Prendergast et al., 2018; Surya et al., 2017; Zulkarnain et al., 2021).

In a study aimed at examining the effect of STEM project-based learning on secondary school students' mathematical problem solving beliefs, Kwon et al. (2020) found that mathematical problem solving beliefs were the best predictor of students' STEM career perception. The same study further found that mathematical problem solving beliefs and perceptions towards mathematics increased. In a related study, Hidayatullah & Csíkos (2022) found that students' beliefs in mathematics play a very important role in predicting their performance when solving word problems in mathematics.

We acknowledge that mathematical problem solving and student beliefs are among the widely researched areas of mathematics education in the world. However, little research related to these areas has been conducted in Zambia (E.g. Banda, & Mwansa, 2020; Chidyaka, & Nkhata, 2019). While these studies partly focused on problem solving, they did not address any aspect of mathematical problem-solving beliefs of learners. For example, Chidyaka, & Nkhata (2019) conducted a qualitative case study aimed at exploring metacognitive strategies employed by secondary school students in their mathematical problem solving. Results from their study showed that while students were involved in problem solving, they were not using metacognitive strategies because they were not aware of them.

In another study, Banda and Mwansa (2020) investigated factors that influenced female high school students' development of negative self-concept. They found that poor mathematics background and negative perception towards mathematics were the most prominent factors that contributed to girls' negative self-concept about mathematics. Thus, to contribute to reducing the gap in high studies in mathematics, the purpose of the current study was to explore profiles of high school students' mathematical problem-solving beliefs by clustering them into relatively

homogeneous groups based on their belief characteristics. In addition, the study aimed at highlighting implications of their beliefs for curriculum implementation. This was accomplished via answers to the following research questions:

1. What beliefs do high school students report about mathematical problem-solving?
2. What patterns can be discerned in high school students' beliefs about mathematical problem solving?
3. What differences exist between clusters of high school students' beliefs about mathematical problem solving?

Methodology

The current study follows a quantitative research design with a cross-sectional survey method. Precisely, a single, one-time-only, cross-sectional survey was administered to a sample of 490 high school students. The survey focused on high school students' beliefs about mathematical problem-solving.

Participants

Participants consisted of 490 Grade 12 students aged between 15 and 18 years who were selected from three public secondary schools in Chipata District located in the Eastern province of Zambia. Considering schools in the district are geographically spread, we used cluster random sampling, a probabilistic sampling method, to select the schools and participants. Within the established clusters, we requested all students to participate in responding to the survey upon explaining the purpose of the study. Only 490 students from the clusters volunteered to participate in the study. At the time of data collection, these students were following the newly introduced STEM Mathematics Curriculum and were expected to be examined based on the new curriculum. Convenience sampling, a non-probabilistic sampling technique, was used to select 490 students (288 boys and 202 girls) from three high schools. Grade 12 students were selected because they had followed the newly introduced mathematics Curriculum for more than one year. It was thus assumed that their mathematical problem-solving beliefs would be more representative of all senior secondary school students in the Eastern province.

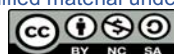
Data Collection Instrument

The data collection instrument used in this study was a survey adapted from Kloosterman and Stage (1992, p. 115) which was developed to measure students' mathematical problem-solving beliefs. The survey consisted of 36 items on a 5-point Likert scale (1 = *strongly disagree*, 2 = *disagree*, 3 = *not sure*, 4 = *agree*, and 5 = *strongly agree*). The items were evenly sub-divided into six beliefs. The beliefs and sample items are represented in Table 1.

Table 1. Beliefs about mathematical problem solving with sample items (Kloosterman & Stage, 1992, p. 115)

Belief	Sample items
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Belief 1: I can solve time-consuming mathematics problems	Math problems that take a long time don't bother me I feel I can do math problems that take a long time to complete
Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures	There are word problems that just can't be solved by following a predetermined sequence of steps Memorizing steps is not that useful for learning to solve word problems
Belief 3: Understanding concepts is important in mathematics	A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem In addition to getting a right answer in mathematics, it is important to understand why the answer is correct
Belief 4: Word problems are important in mathematics	A person who can't solve word problems really can't do math Computational skills are of little value if you can't use them to solve word problems
Belief 5: Effort can increase mathematical ability	By trying hard, one can become smarter in math Working can improve one's ability in mathematics
Belief 6: Mathematics is useful in daily life	I study mathematics because I know how useful it is Mathematics is a worthwhile and necessary subject

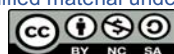
Data Analysis

Data were analyzed using a quantitative data analysis tool, Statistical Package for Social Sciences version 23 (SPSS 23). Considering that some items in the questionnaire were reverse coded (negatively worded), we started our analysis by reverse scoring all negatively worded items so that high values of the scale indicated responses of the same type. We then proceeded with the data analysis process by first calculating descriptive statistics (mean scores and standard deviations) to gain insight about participants' mathematical problem-solving beliefs. Descriptive statistics enabled us to adequately answer the first research question. Then, to analyze the existence of participants' mathematical problem-solving belief profiles, we performed a K-means cluster analysis procedure. The technique enabled us to generate homogeneous subgroups of participants based on the mean scores of their mathematical problem-solving beliefs. Finally, to examine differences in participants' mathematical problem-solving beliefs across the three clusters, a one-way analysis of variance (ANOVA) was conducted. In addition, a Bonferroni test for multiple comparison was performed for all significant results to determine the clusters that were significantly different.

Results

This section presents results following the order of the research questions. We begin by presenting results related to participants' mathematical problem-solving beliefs before delving into findings about the clustering of participants' mathematical problem-solving beliefs. We end the section by presenting results related to cluster differences of participants' mathematical problem-solving beliefs.

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High school students' beliefs about mathematical problem solving

To explore students' mathematical problem-solving beliefs, we based our analysis on mean scores and standard deviations of their beliefs (Table 2). Students were not sure about their beliefs related to conceptual understanding in mathematics is essential ($M = 3.79$, $SD = .58$) and that they can solve mathematics problems which are time-consuming ($M = 3.32$, $SD = .68$). However, they negatively believed that effort was key for the increase of one's mathematical ability ($M = 2.97$, $SD = .32$). Further, they were also uncertain about their beliefs concerned with the importance of word problems in mathematics ($M = 3.14$, $SD = .54$) although they exhibited negative beliefs about word problems in mathematics that can be solved using simple step-by-step procedures ($M = 2.55$, $SD = .54$). In addition, students exhibited negative beliefs about the usefulness of mathematics to their daily lives ($M = 2.97$, $SD = .46$) which potentially limits their application of mathematical concepts to solving problems in their immediate environment.

Table 2. Mean scores and standard deviations of students' mathematical problem-solving beliefs

Belief	N	M	SD
Belief 1: I can solve time-consuming mathematics problems	490	3.32	.68
Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures	490	2.55	.54
Belief 3: Understanding concepts is important in mathematics	490	3.79	.58
Belief 4: Word problems are important in mathematics	490	3.14	.54
Belief 5: Effort can increase mathematical ability	490	2.97	.32
Belief 6: Mathematics is useful in daily life	490	2.97	.46

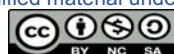
High school students' belief profiles about mathematical problem solving

To determine profiles of students' mathematical problem-solving beliefs we used a K-means clustering technique which revealed three subgroups of relatively homogeneous beliefs (Table 3). We then derived mean scores and standard deviations of students' beliefs from each cluster (see Table 3) for assessment. As we present results, we would like to clarify the contextual understanding on time-consuming mathematical problems referred to in beliefs 1. In the context of this study, time-consuming mathematical problems are high quality problems that require deep intuition from students. They are different from long procedural problems that may also be time-consuming. Sometimes such high-quality problems may require less time than these long procedural ones. Thus, in this study we emphasize the quality of a mathematical problem.

Table 3. Cluster profiles of students' beliefs about mathematical problem solving

Belief	Cluster 1 (N = 203)	Cluster 2 (N = 137)	Cluster 3 (N = 150)
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	M	SD	M	SD	M	SD
Belief 1: I can solve time-consuming mathematics problems	3.89	.43	3.24	.41	2.64	.46
Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures	2.43	.53	2.55	.55	2.71	.51
Belief 3: Understanding concepts is important in mathematics	4.12	.42	3.14	.34	3.92	.45
Belief 4: Word problems are important in mathematics	3.29	.49	3.08	.54	2.99	.54
Belief 5: Effort can increase mathematical ability	3.00	.29	2.87	.35	3.02	.32
Belief 6: Mathematics is useful in daily life	2.90	.39	3.01	.52	2.99	.48

Analyzing Table 3, students from Cluster 1 hold positive beliefs about the importance of conceptual understanding in mathematics ($M = 4.12$, $Sd = .42$), while those in cluster 2 ($M = 3.14$, $SD = .34$) and Cluster 3 ($M = 3.92$, $Sd = .45$) were not sure about their beliefs. Results also show that students in Cluster 2 ($M = 3.01$, $SD = .52$) were not sure about their beliefs related to the usefulness of mathematics in their daily lives as opposed to their counterparts in Cluster 1 ($M = 2.90$, $SD = .39$) and those in Cluster 3 ($M = 2.99$, $SD = .48$) who exhibit negative beliefs. Compared to those in Cluster 2 ($M = 2.87$, $SD = .35$), Cluster 1 ($M = 3.00$, $SD = .29$) and Cluster 3 ($M = 3.02$, $SD = .32$) were dominated by students who were not sure about their beliefs that one can increase his or her mathematical ability by applying effort to studying the subject. Students in clusters 1 and 2 were uncertain about their beliefs that word problems are important in mathematics while exhibiting negative beliefs that some mathematical word problems can be solved by simple, step-by-step procedures. It can also be deduced from Table 3 that students in clusters 1 and 2 were not sure about their belief that they can solve mathematics problems that are time consuming while those in cluster 3 clearly report negative beliefs. We further generated a bar chart to show a visual representation of the final cluster centers (Figure 1) of the clustering procedure.

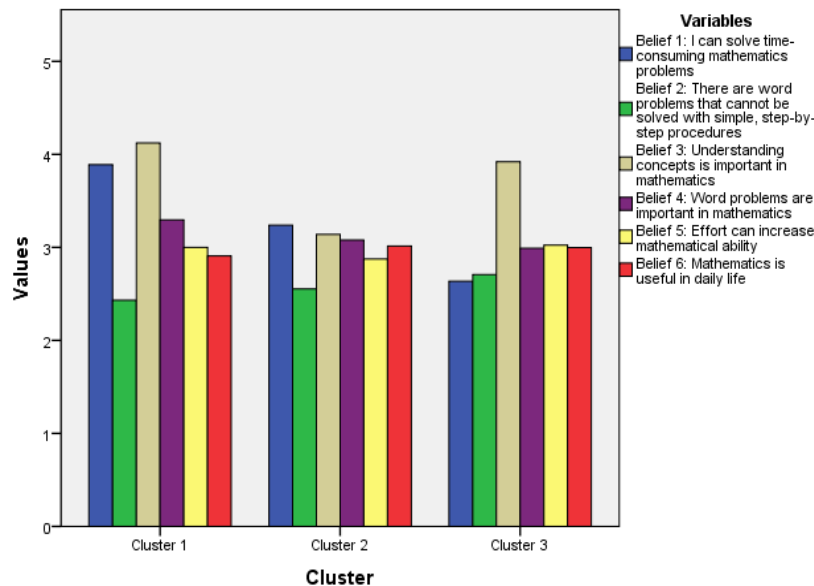


Figure 1. Visual representation of final cluster centers of students' mathematical problem-solving beliefs

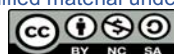
Cluster differences of high school students' belief profiles about mathematical problem solving

We also sought to examine differences in students' mathematical problem-solving beliefs across the three clusters. Except for the belief about the usefulness of mathematics in one's day-to-day life which was not statistically significant across clusters [$F(2,76)$, $p = .065$], all other beliefs held by students statistically differed across clusters (see Table 4). For the beliefs that were statistically different across clusters we went on to determine where the actual differences were using the Bonferroni post hoc test for multiple comparison (table 5).

Table 4. ANOVA for differences in students' beliefs about mathematical problem-solving beliefs across clusters

Belief		SS	df	MS	F	Sig.	$\eta^2_{Partial}$
Belief 1: I can solve time-consuming mathematics problems	Between Groups	136.52	2	68.26	363.23	.000	.599
	Within Groups	91.52	487	.19			
Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures	Between Groups	6.61	2	3.30	11.85	.000	.046
	Within Groups	135.75	487	.28			
Belief 3: Understanding concepts is important in mathematics	Between Groups	83.06	2	41.53	246.30	.000	.503
	Within Groups	82.12	487	.17			

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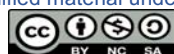
Belief 4: Word problems are important in mathematics	Between Groups	8.69	2	4.35	15.83	.000	.061
	Within Groups	133.72	487	.28			
Belief 5: Effort can increase mathematical ability	Between Groups	1.85	2	.93	9.06	.000	.036
	Within Groups	49.77	487	.10			
Belief 6: Mathematics is useful in daily life	Between Groups	1.16	2	.58	2.76	.065	
	Within Groups	102.19	487	.21			

In addition to the post hoc test on all significantly different results, we calculated the effect size of these differences to determine the practical significance of the ANOVA test results. Effect size adds value to statistical results by providing insight about the practical value of the said result for real world use. To accomplish this task, we used partial Eta squared value which reduces the bias of the Eta squared. Table 4 shows that the effect size ranges from $\eta^2_{\text{partial}} = .036$ to $\eta^2_{\text{partial}} = .599$. Students' belief that they can solve mathematics problems that are time consuming accounted for the highest effect size ($\eta^2_{\text{partial}} = .599$) followed by their belief that conceptual understanding is important in mathematics ($\eta^2_{\text{partial}} = .503$). this implies that 59.9% of the differences across clusters can be explained by students' belief that they can solve time consuming mathematics problems and their belief about the importance of mathematics accounts for 50.3 of the variability. The rest of the beliefs accounted for less than 10% of the variability in the differences across clusters. Thus, these beliefs had very low practical significance for the differences across factors.

Table 5. Bonferroni Post Hoc test for multiple comparison of significant cluster differences

Dependent Variable	Cluster (I)	Cluster (J)	Mean	Std. Error	Sig.
			Difference (I-J)		
Belief 1: I can solve time-consuming mathematics problems	1	2	.65*	.048	.000
		3	1.25*	.05	.000
	2	1	-.65*	.05	.000
		3	.60*	.05	.000
	3	1	-1.25*	.05	.000
		2	-.60*	.05	.000
Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures	1	2	-.12	.06	.109
		3	-.28*	.06	.000
	2	1	.12	.06	.109
		3	-.15*	.06	.041
	3	1	.28*	.06	.000
		2	.15*	.06	.041

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Belief 3: Understanding concepts is important in mathematics	1	2	.98*	.05	.000
		3	.20*	.04	.000
	2	1	-.98*	.05	.000
		3	-.78*	.05	.000
	3	1	-.20*	.04	.000
		2	.78*	.05	.000
Belief 4: Word problems are important in mathematics	1	2	.22*	.06	.001
		3	.30*	.06	.000
	2	1	-.22*	.06	.001
		3	.09	.06	.468
	3	1	-.30*	.06	.000
		2	-.09	.06	.468
Belief 5: Effort can increase mathematical ability	1	2	.13*	.04	.001
		3	-.02	.03	1.000
	2	1	-.13*	.04	.001
		3	-.15*	.04	.000
	3	1	.02	.03	1.000
		2	.15*	.04	.000

Examining Table 5, while the ANOVA test revealed statistically significant differences across all clusters about students' belief that there are word problems that cannot be solved with simple, step-by-step procedures, the post hoc test shows that clusters 1 and 2 did not significantly differ. Similarly, there was no statistically significant difference between students in clusters 2 and 3 about their beliefs related to the importance of word problems in mathematics. Table 5 further shows that students' beliefs about the need for effort to increase one's mathematical ability did not statistically differ between students in clusters 1 and 3.

Discussion and conclusion

With regards to students' mathematical problem-solving beliefs' profiles, we identified three distinct profiles, each of which exhibited characteristics that were relatively different from the other. Cluster 1, the largest of the three with 41.4% membership, is comprised of students that exhibit overall high motivation to engage in mathematical problem-solving than students in the other two clusters. Cluster 1 is characterized by students who demonstrated high positive beliefs that conceptual understanding in mathematics is important in mathematics than their colleagues in the other two clusters. This means that these students are motivated to engage with high quality mathematics problems and would persevere to understand the concept behind every mathematical problem they encounter. The result is consistent with findings of Mason (2003) and Surya et al. (2017) who also found that understanding mathematical concepts was important for students' development of mathematical ability.

This result is particularly crucial for the problem-solving approach to teaching and learning on which the new mathematics curriculum is anchored. This is because the aim of mathematical

problem solving is to enhance understanding of mathematical concepts as opposed to memorizing facts and procedures for solving problems. This is coupled with their strong belief that they can engage with mathematical problems that demand a lot of time to solve. The belief characteristics exhibited by these students are important as they imply that students in cluster 1 would embrace problem solving in their classroom activities that their teachers would plan for them. The other positive thing about students in cluster 1 is that they recognize the importance of word problems in mathematics and believe that simple steps would not be sufficient for solving word problems. This agrees with the findings of Mason (2003). The only worrying thing about cluster 1 students is that they do not strongly believe that mathematics is useful in their daily lives. This implies that they would not see the value of applying mathematical concepts in solving real life problems. It is expected that students who value conceptual understanding and believe that they can solve time-consuming problems. This result is in tandem with the findings of Ozturk & Guven (2016) who reported that students who did not find mathematics useful also found the subject to be difficult.

Pertaining to beliefs profiles of students in cluster 2, unlike their colleagues in clusters 1 and 2, they strongly believe that mathematics is useful in the daily life of individuals. This is especially important when it comes to application of mathematics concepts to solving real life problems. Thus, by recognizing the usefulness of mathematics in their daily lives, this group of students show potential to find solutions to problems facing their society using mathematics. This would also be supported by their strong beliefs about the importance of conceptual understanding in mathematics and that they can solve time-consuming mathematical problems. However, students in this cluster do not strongly believe that effort is important for enhancement of one's mathematical ability. This seems to contradict their belief that they can solve time-consuming problems because such problems would demand considerable amount of effort to be put in. In fact, these students would require effort as they face the process of mathematical problem solving because tasks that are likely to be given won't be easy and straightforward. They would demand a lot of effort from students.

Cluster 3 was dominated by students with weak characteristics about majority beliefs. Generally, they hold strong beliefs that conceptual understanding is important in mathematics, like their counterparts in the other two clusters, and that effort is a fundamental requirement for one to improve their mathematical abilities. These two beliefs are important for successful mathematical problem solving and this would potentially help them to engage with problem solving tasks that they would face (Simamora & Saragih, 2019; Son et al., 2020; Surya et al., 2017). However, like their colleagues in cluster 1 but unlike those in cluster 2, they do not strongly believe that mathematics is useful to one's daily life. This poses a challenge on their ability to apply mathematics to solving real life problems. In addition, they do not strongly believe that they can solve time-consuming mathematical problems.

When cluster differences were analyzed, we found that there were significant differences between clusters for five beliefs. However, further analysis revealed small effect size for majority of the

beliefs. We concluded that sample size may have caused significant some of these differences between clusters. Large sample sizes tend to make small differences present themselves as significant when their practical differences (effect size) may not be large enough. Thus, we observed from the effect sizes that apart from students' beliefs about solving time-consuming mathematical problems and that conceptual understanding being important in mathematics, the differences were practically insignificant (small). In conclusion we argue that it is important to consider psychological factors like students' beliefs, attitudes and perception about mathematics and mathematical problem solving and to prepare teachers to consider these factors when planning work for their students. The results of this study are important to Zambia's Ministry of Education as feedback on the newly introduced curriculum. The findings are also a contribution to mathematics classroom practice in general and contribute empirical evidence to the literature in mathematics education about high school students' beliefs about mathematical problem solving.

Limitations and future research

The limitation we note is about the data collection tool that was used to gather information upon which the analysis and inferences were based. The instrument was limited to collecting only self-reported beliefs which could not be probed by the researcher. This also limited the extent of the analysis to only statistical testing. We recommend the use of multiple instruments for future research which would enable the collection of data like observations and interviews that would allow for both qualitative and quantitative analysis. This would ensure rich and depth of findings. In future we also hope to conduct a study that includes participants from Grade 10 to 12 to assess student beliefs based on grade level and to examine whether their beliefs change as they advance in grade level. Finally, we recommend similar studies that would investigate teachers' beliefs about problem solving approach to teaching as this also has a bearing on how well they prepare tasks for their students.

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