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Developing The Mathematical Thinking Scale for Gifted Students

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Abstract

The aim of this research is to develop a reliable and valid scale to determine the mathematical thinking skills of gifted students. In addition, with the developed scale, thinking skills of gifted students was examined in terms of various variables. In this context, the research was carried out on two different study groups. The first stage of this research was the development of the scale, and the data obtained from 410 gifted students .The second stage of this research was the testing process of the developed scale, and it was carried out on the data obtained from 393 gifted students. While developing the scale, confirmatory factor analysis, exploratory factor analysis, Guttman Split-half values reliability, Cronbach Alpha internal consistency coefficient calculations were performed. As a result of analyses, the total variance percentage of the scale consisting of 16 items and four factors was obtained as 60.781%. It was concluded that the model obtained from confirmatory factor analysis applied was at an acceptable level. For the whole scale, Cronbach's alpha value of the scale was determined as .841. As a result of this study, a reliable and valid scale was developed to determine mathematical thinking skills of students. In addition, through the scale developed, it was concluded that thinking skills of the students showed differences according to gender, group, grade level and education status.

Keywords; developing a scale; gifted students; mathematical thinking skill.

Introduction

Thinking is one of the basic characteristics of man. The most important difference from other living things is man's ability to think. The words 'thought' and' thinking' are the most frequently used words. However, it is not so simple to define these two words which are often encountered in daily life. Defining these words can be helped by examining frequently used examples. Everything that comes to a person's mind, that passes through his head, is called a thought. The term thought is expressed in relation to what is directly presented by Dewey (1910, p.2).

Humans think about a wide variety of situations. They think of many things they have not seen, tasted or heard before. To think of a situation is to somehow notice that situation, to review the situations that may occur and to act accordingly (Dewey, 1910). A person takes many actions related to thinking throughout his life such as problem solving, evaluating, making predictions, making comments (Hughes & Lavery, 2015). In this context, students also engage in many thinking actions. The justifications they give to a term, sentence, or text, offer solutions to a problem, or use to verify or reject a claim form the implicit or explicit basis of their general theory. Such actions are ways of thinking (Harel & Sowder, 2005). It can be said that mathematics is one of the most effective and intensive areas of thinking.

Mathematical Thinking

Thinking is one of the most important elements that distinguish human beings from other living things. Mathematics is one of the areas where thinking is used most effectively. In mathematics education environments, this point is expected to be considered as the main objective. In this context, Ayllón, Isabel and Ballesta-Claver (2006) state that the main purpose of mathematics teaching is to improve thinking. Thinking is most obviously a problem or problem-solving activity. However, it is not a correct assumption to say that the products emerging as a result of every thinking are injured. The usefulness of thought can be measured by its use in meeting needs and being productive in solving problems. We can express the ways of thinking that have these qualities as mathematical thinking (Alkan & Güzel, 2005). According to Sevgen (2002), mathematical thinking has been seen as the ability of the individual to approach the events he encounters in daily life in a systematic, fast and accurate way. This thinking manifests itself as illuminating a problem that is often encountered in daily

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life or bringing different solutions or clarifying an observation that contradicts the expectations of any event.

In the studies on mathematics teaching, it is seen that the definition of mathematical thinking is based on two approaches. One of them is the mathematical process-oriented approach (Isoda & Katagiri, 2012; Liu & Niess, 2006; Mason, Burton & Stacey, 2010; Schoenfeld, 1992). The second approach is the conceptual development-oriented approach (Dreyfus, 1991; Fruendental, 1973; Tall, 1995).

The mathematical process-oriented approach is based on the fact that mathematical thinking is related to the process related to mathematics. Working on mathematical tasks, expressing the mathematical information obtained can improve mathematical thinking. Working on challenging and contradictory, surprising tasks stimulates mathematical thinking. In addition, reflective thinking studies also contribute to the development of mathematical thinking (Mason, Burton & Stacey, 2010). Abstraction, symbolic representation, and symbolic manipulation are tools of mathematics (Schoenfeld, 1992). On the other hand, Schoenfeld (1992) states that being trained in the use of these tools of mathematics means thinking mathematically. Burton (1984) refers to mathematical thinking as the use of one's own unique methods to control and understand the environment. Operations, processes, and dynamics associated with mathematical thinking make up these methods. Devlin (2012) draws attention to the difference between mathematical thinking and doing mathematics. Doing mathematics mostly involves performing some processing steps and making some difficult symbolic associations. However, mathematical thinking is a specific way of thinking for everything that exists. In other words, mathematical thinking does not have to be a way of thinking about mathematics alone. However, some parts of mathematics offer very suitable content for learning to think mathematically. Polya (1954) gave importance to the concept of reasoning in mathematical thinking. Poincare (1907) proceeded from the concept of mind possessed by mathematicians related to mathematical thinking.

The mathematical thinking approach focused on the conceptual development of mathematics is mostly based on how the person structures mathematical concepts in the mind. In this context, Freudenthal (1973) introduced the expression mathematization. Mathematization is used to understand the workings of the process of objectifying mathematical activities. According to Freudenthal, the process that begins with real-life experiences results in mathematics. Mathematization serves as a bridge. This bridging task provides the transition from general thinking to mathematical thinking. Mathematical thinking takes place through the relationship established between the concrete world, the symbolic world, and the formal world (Tall, 1995). This relationship is explained by Isoda and Katagiri (2012) as follows. The world of mathematics consists of three headings including conceptual arrangement, manipulation and symbolic calculation, and axiomatic formalism. The world of axiomatic formalism consists of mathematical fields such as theoretical set, axiomatic algebra, analysis and axiomatic geometry. Symbolic calculation and manipulation covers trigonometry, matrix algebra, graphs, limit, analysis, algebra, functions, arithmetic and number. Conceptual embodiment also consists of geometry, plane and space. These three worlds in the world of mathematics are stated as an explanation focused on the conceptual development of mathematical thinking.

When these explanations made with mathematical thinking are taken into consideration, it is seen that there are various definitions related to mathematical thinking and components in this context. Liu (2003) evaluated the explanations and definitions that have been made about mathematical thinking. Accordingly, Liu expressed mathematical thinking as a combination set of complex processes such as prediction, generalization, deduction, induction, description, sampling, informal and formal reasoning, analogy, verification, and so on. These explanations suggest the question of how the mathematical projection levels of students with various characteristics will be. One of these characteristics is the students called gifted students. In this study, the mathematical thinking levels of gifted students were curious.

Special Talent

Special talent is defined as the ability to perform at a high level in abstract thinking and reasoning skills and to have an age of intelligence above normal (Gagne, 2004). Renzulli (1978) stated that the senses of duty and creativity skills of individuals with special abilities are at a high level and they have above-average academic ability. Sternberg (2002) expressed special talent as individuals with analytical creativity, intelligence and those practically talented. In Turkey, according to the Science and Art Centres (SAC) Directive, which was established for the education of individuals with special abilities, special talent is defined as "An individual who is faster learner than his peers, who have creativity, art, leadership capacity in the forefront, who has special academic ability, who likes to act independently in areas of interest, and who performs at a high level" (Ministry of National Education [MEB] Journal of Communiqués, 2016).

There are different practices in the world for the education of individuals with special abilities. It can be said that these applications increased with the sputnik event. The acceleration of the developments in the field of science and technology has led to an increase in the applications of gifted individuals, which have important effects on the progress of societies (Sak, 2020, p.217). Many countries (Canada, South Korea, Some European Countries), especially the USA, implement differentiated education programs for gifted individuals.

In Turkey, the diagnosis of special talent is made through two processes as screening and examination. Identification of students with special abilities is carried out according to the results of the exam in which the students participating in the group-screening exam are evaluated individually based on the observation forms filled out by the 1st, 2nd, 3rd and 4th grade classroom teachers. Individuals diagnosed with special abilities are educated in science and art centres. In SACs, interdisciplinary, project-based, differentiated education programs are applied according to the abilities of the students. These programs are carried out within the scope of adaptation, support, recognition of individual talents (RIT), special talent development (STD) and project production programs respectively. Students first participate in the integration program and continue until the last year of high school with the project production program.

Importance of the Research

The future may not be predicted with certainty, but it is a logical conclusion to predict that mathematical thinking will be valuable and continue to be important in the 21st century and beyond (Cimbricz, Stoll & Wilkens, 2015). Stacey (2006) explains the importance of mathematical thinking as follows. The fact that mathematical thinking has been determined as a goal to be achieved in schooling reveals its importance. In addition, Stacey (2006) emphasized the importance of mathematical thinking as a way to learn mathematics and its importance in terms of teaching mathematics.

In recent years, the mathematical thinking dimension of mathematics has started to be taken to the fore more in educational processes. Ayllón, Isabel and Ballesta-Claver (2006) have stated that the main purpose of mathematics teaching is to improve thinking. In this context, it can be said that the importance given to basic skills related to mathematical thinking has increased within the scope of the mathematics curriculum in force in Turkey (MEB, 2018). In this context, it is thought that there is a need for studies related to mathematical thinking. The importance of how the mathematical thinking levels of the students are and the measurement tools to be used in determining these levels are also revealed. Examining the relevant literature, it is understood that there are some studies aimed at determining the level of mathematical thinking (Artut & Bal, 2020; Ersoy & Başer, 2013; Harel & Sowder, 2005; Lincoln, 2008; Liu & Niess, 2006; Umay, 1992). In these studies, it has been suggested that more comprehensive studies should be carried out in which different variables are handled with different sample groups for mathematical thinking. On the other hand, in these studies, it was determined that there was no measurement tool whose validity and reliability were studied for students with special abilities. In this context, it was thought that there was a need to develop a mathematical thinking scale for gifted students. It is expected that the scale developed will contribute to the literature. In line with the explanations made, there was a need to develop a Mathematical Thinking Scale (MTS) for gifted students.In this context, the major purpose of this study is to develop MTS for gifted students. In addition, in this study, it was aimed to compare the mathematical thinking skills of the students according to some variables. In line with these purposes, answers were sought to the following questions.

1. Can the items that make up the MTS represent mathematical thinking according to the opinions of subject field experts?
2. Is the structure of the MTS simple and stable?
3. Within the scope of reliability;
   3.a What are the Cronbach Alpha and Guttman Split Half values of the MTS ?
   3.b What is the item-total score correlation for each item on the MTS?
4. Is the MTS able to distinguish between subverted groups of individuals?
5. Do the scores of gifted students on the MTS differ significantly in terms of gender, group, grade level, and educational status?

Methodology

This study is quantitative research conducted to develop scale to determine the mathematical thinking skills of gifted students and to examine the thinking skills of gifted students in terms of gender, group, grade level through the scale in this study. The study group consists of gifted students studying at the SAC in Adana in the Southern region of Turkey. The students constituting the study sample were determined according to typical situation sampling from the purposeful sampling methods. In this sampling method, the unusual typical situation is selected from the situation in the universe related to the research situation (Fraenkel, Wallen and Hyun, 2012).In this context, research was carried out with two different sample groups during the development process of the scale and in the process of testing the developed scale. Accordingly, the first sample consists of 423 gifted students studying at the Science and Art Center in the 1st semester of the 2021–2022 academic year. However, due to the lack of information in the process of filling out the first sample scale and the extraction of the extreme values, 13 scale data were excluded from the sample, and analyzes were evaluated over 410 data sets. Second sample constituted of 393 students attending the same school in the 2nd semester of 2021-2022 academic year. The personal information of the students in the first and second sample participating in the research is shown in Table 1.

According to Table 1, 44.9% of the students in the first sample group participating in the study are female, 55.1% of them are male. 28% of the students are primary school students, and 72% of them are secondary school students. In addition, 27.8% of the students are in support group; 50.4% of them are in RIT group, and 21.7% of them are in STD program. Among the student in the second sample group participated in the study, % 44.5 of them are female, and % 55.5 of them are male. 27.4% of the students are primary school students; 72.6% of them are secondary school students; 27.1% are in the support group; 50.8% of them are RIT group, and 22.1% of them are STD programs.

Data Collection Tools

In this section, first of all, the development process of the MTS was included, and then the changes of the scale developed according to the demographic characteristics (gender, group, grade level, education status) of the students in the second study group were examined.

Process of Preparing the MTS: The MTS, which constitutes the first part of the research, was prepared by the following steps (Devellis, 2016). In this context, the process steps consisting of the creation of the item pool, scope validity and reliability, construct validity, pilot application and the final version of the scale are summarized as in Figure 1.

Table 1: Distribution according to the Personal Information of Gifted Students

Variables N		*1st Group* %	N	*2nd Group* %	
Gender	Female	184	44,9	175	44,5
	Male	226	55,1	218	55,5
Group	Support	114	27,8	107	27,2
	RIT	207	50,4	199	50,6
	STD	89	21,7	87	22,1
Grade Level	3	45	11,0	45	11,5
	4	68	16,6	61	15,5
	5	146	35,6	140	35,6
	6	95	23,2	94	23,9
	7	39	9,5	37	9,4
	8	17	4,1	16	4,1
Education Status	Primary School	115	28,0	108	27,5
	Secondary School	295	72,0	285	72,5
	Total	410	100,0	393	100,0

Fig. 1: Development Process of MTS

Examining Figure 1, a pool of 58 items was created by scanning the field literature. Then, in the process of scope validity of the measurement tool, a pilot application was made by using the opinions of 5 experts. EFA and CFA were applied in the structure validity process in the third step. In the fourth stage Guttman Split-half analysis, Cronbach Alpha and upper lower group averages of 27% were compared in the reliability studies of the data collection tool. In the final stage, the scale consisting of 16 items and four factors was finalized.

Creating the Item Pool: First of all, the relevant field literature was scanned. In this context, the definition of mathematical thinking and the dimensions that make up mathematical thinking are examined. The studies carried out in this context were examined (Harel & Sowder, 2005; Lincoln, 2008; Lipman, 2003; Schoenfeld, 1992; Tall, 1995), and the first draft articles were prepared by considering both the process dimension and conceptual development dimensions of mathematical thinking. In line with the information obtained as a result of the literature search, an article pool of 58 items was prepared.

Content Validity

Submission of the Article Pool to Expert Opinion: The 58-item draft form prepared by applying to the expert opinion was examined in terms of its status of having the validity. The property to be measured by the content validity measurement tool should be measured without mixing it with other features (Balcı, 2001). In this context, firstly, the items in article pool were presented to the opinion of 3 experts in the field of mathematics education, 1 expert in the field of language education and 1 expert in the field of evaluation and assessment. Experts evaluated each item in the draft form as "appropriate", "corrected" and "inappropriate" according to the fact that the scale served the purpose. Opinions from experts were calculated using the formula prepared by Lawshe (1975). Each question prepared for content validity was evaluated individually. The calculation method for a single question is as follows. For the content validity rate, only the number of experts who marked the "appropriate" option for each question were collected, and then the number of experts whose measurement tool was sent was divided by two. The number of experts who say it is necessary and the number of experts who emerge as a result of the section are divided again and 1 is subtracted from the resulting number. As a formula $CVR = Na / (N/2) - 1$.

Pilot Application: The 58-item draft form was applied to 70 students. The draft form was also examined in terms of page layout for appearance validity, size of font sizes and usefulness. The 58-item essay scale form obtained as a result of the regulations was applied to 410 students studying at the science and art center in Adana province. The scale form consisting of 16 items obtained as a result of the analyzes was applied to the second sample consisting of 393 gifted students.

Data Analysis

Firstly, the data were numbered, transferred to the computer environment. Then, a total of 13 forms were excluded from the evaluation due to missing information in the data collection tool, and endpoint values were determined by Mahalanobis Distance. In the first stage, 410 data obtained from the first working group were evaluated during the development of the scale. In this context, the construct validity of the data by applying EFA in IBM SPSS 26.0 program of the obtained data set and the reliability of the data were examined by Guttman Split-half, Cronbach Alpha analysis. In addition, arithmetic mean values and standard deviation values and item total score correlations of the expressions on the scale were examined, and the item discrimination powers were calculated by independent samples t-test analysis. In the second stage, the scale consisting of 16 items was reapplied to 393 students, and CFA was performed with Lisrel software. Since the validity and reliability studies provide normality of the scores obtained from the completed mathematical thinking scale, in case of comparison of two groups such as gender and education status of the students, ANOVA and independent groups t-test were performed in situations where more than three groups faced such as group, grade level. Before the analyzes, the homogeneity of the variances was checked, and it was seen that the homogeneity prerequisite of the variances was met. To check whether the normality condition is met, Kurtosis and Skewness values were examined. It was observed that Skewness values were -.204 and -.276 for the gender variable, while Kurtosis values were -.149 and -.568, respectively. In terms of education status variable, it was observed that Skewness values were .012 and -.340, while Kurtosis values were -.633 and -.312, respectively. It was observed that Skewness values ranged from 014 to -.348 for the group variable and Kurtosis values ranged from -.655 to -.158; for the class variable, Skewness values ranged from 176 to -1.278, and Kurtosis values ranged from -1.001 to 2.202. Since the number of samples was greater than 300, the Skewness and Kurtosis values obtained were sufficient for normal distribution (Kim, 2013). In the process of comparing the groups, Scheffe test was performed. In this process, p=.01 was accepted for the significance value in the analysis of all statistical analyses.

Findings

In this section, the findings are given in two parts. In the first section, the findings regarding validity and reliability of "Mathematical Thinking Scale" applied in the development process were included. In the second section, the findings of the scale developed according to gender, group, grade level and education status of the students were examined.

Findings on Construct Validity of MTS

Factor analysis was made to build the construct validity of MTS. EFA was made at first and some information was obtained about the number of the factors at first. Then, the appropriateness of the construct was tested through CFA.

Findings on EFA of the MTS

For EFA applied to the MTS, the suitability of the obtained data to both factor analysis and the adequacy of the number of samples were made with the KMO test and the Bartlett Sphericity test (Büyüköztürk, 2011). The results of the analysis applied are shown in Table 2.

As can be seen in Table 2, KMO value is determined as .851. In line with this result, it was concluded that the sample fitness was "very good" for EFA (Sharma, 1996). In addition, Bartlett sphericity test result was calculated as x^2=2238.178, and it was seen that it was significant at .01 level. These results show fitness for factor analysis.

While performing EFA on MTS, promax rotate operation was implemented. According to this analysis, four factors, eigen value of which was above 1, were obtained for 16 items. The construct of the factors are considered as stable when their eigen values were 1 or above. Scree plot graph, which is one of most frequently used criterion while determining the number of factors, can be seen in Figure 2.

Table 2: Barlett Sphericity Test, KMO Values

KMO	*,851*	
Barlett Sphericity Test	*X2*	2238,178
	P	,000

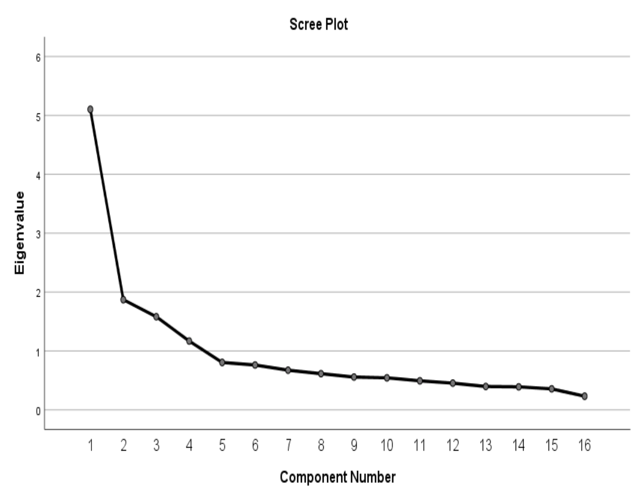

Fig. 2: Scree and Plot Graph

Table 3: Items and Factor Loads of the MTS

Items	*F1*	*F2*	*F3*	*F4*
I57: I reach the general opinion based on the examples.	,800			
I55: I try to reach general information by examining the examples.	,796			
I42: I try to see the possible outcomes.	,702			
I45: When I encounter a difficulty, I first try to understand it.	,664			
I58: First I sense the rule and then I check its accuracy.	,591			
I38: I try to get to the truth with evidence.	,574			
I46: Understanding a problem situation is important to me.	,573			
I11: Mathematical skills make my daily life easier.		,921		
I12: Mathematical skills allow me to be more productive in my daily life.		,834		
I10: I use mathematical knowledge in daily life.		,778		
I29: I organize my daily work in order of priority.			,884	
I30: I can easily implement the plans I make in my daily life.			,819	
I27: I like to plan my work in advance.			,731	
I48: Problem solving is not an important part of mathematics.				,840
I54: Investigating why the solution to the problem is the right one is a waste of time.				,765
I44: I avoid dealing with difficult problems.				,737
Eigen value	5,103	1,872	1,582	1,168
Percentage of Variance Explained	31,892	11,698	9,888	7,303
Number of items	7	3	3	3

As seen in Figure 2, scree plot takes a horizontal shape after four factors. Therefore, it is observed that the scale gathers under four factors. A four-factor construct, which is reached in four iterations, was obtained as a result of the analysis. Item factor loads, eigenvalues of each factor are seen in Table 3.

When Table 3 is examined, as a result of the applied analysis, the total variance percentage of the scale consisting of 16 items and four factors is 60.781%. Of this variance, 31.892% is explained by the sub-factor of inductive and deductive thinking; 11.698% is explained by utilitarian thinking sub-factor; 9.888% is explained by sub-factor of planned thinking; and 7.303% is explained by sub-factor of thinking based on problem-solving. Factor loads of the scale vary between .57 and .92. In determining the items measuring the same sub-factor, it was noted that the substance factor loads were included in a single value and had a high load value. In addition, it was taken into account that the substance factor loads should be at least .30 (Seçer, 2013). The correlation values of the MTS are shown in Table 4.

Table 4. Correlation Coefficients Values of Total Score and Sub-Factors of the MTS

Item sub-factors	*F1*	*F2*	*F3*	*F4*
IDT	1			
UT	,424**	1		
PT	,445**	,346**	1	
TBPS	,321**	,296**	,054	1
Total	,844**	,702**	,645**	,583**

**p<.01

Examining Table 4, it was concluded that there were statistically significant correlations between the total score of the MTS and the sub-factor scores (**p<.01). The f1 sub-factor showed a significant positive relationship with other sub-factors in a value of .424, .445, .321 respectively, and in a value of .844 with the total score; the f2 sub-factor showed a significant positive relationship with other sub-factors in a value of .346, .296 respectively, and in a value of .702 with the total score; the f3 sub-factor showed a significant positive relationship with other sub-factors in a value of .054, and a value of .645 with the total score; the f4 sub-factor also showed a significant positive relationship in a value of .583 with the total score. Büyüköztürk (2011) stated that if the correlation value is between .30 and .70, there is a moderate relationship, and if it is less than 30, there is a small relationship. Accordingly, while the sub-factors of the scale are statistically significant among themselves at positive, low and medium levels, it can be said that there is a positive, moderate and statistically significant relationship between the sub-factors and the total score.

In the process of naming the sub-factors of the MTS, both item contents and the relevant literature were taken into consideration. Accordingly, the names of the factors were determined as "Inductive and Deductive Thinking (IDT)", "Utilitarian Thinking (UT)", "Planned Thinking (PT)", "Thinking based on problem-solving (TBPS)".

Findings on CFA of MTS

Confirmatory factor analyses were obtained from the second sample group data. Compliance index values and limit values obtained during the analysis process are shown in figure 3.

Goodness of Fit indices	Calculated Value	Acceptable Threshold Values	References
$\chi 2/df$	227/98=2,31	≤3= perfect fit	Hooper, Coughlan and Mullen(2008), Kline (2005)
RMSEA	0,058	≤.08= good fit	Hooper et al.(2008)
SRMR	0,050	≤.050= perfect fit	Kline (2005)
NNFI	0,96	≥.95= perfect fit	Kline(2005), Hu and Bentler (1999), Tabachnick and Fidell (2001).
GFI	0,93	<.90= good fit	Cole (1987)
CFI	0,97	≥.95= perfect fit	Tabachnick and Fidell (2001)
IFI	0,97	≥.95= perfect fit	Hu and Bentler (1999)

Fig. 3: Fit Index Values

Table 5: T Values from CFA for the MTS

Item No	*T*	*Item No*	*T*
I57	10,92*	I12	18,21*
I55	10,70*	I10	14,77*
I42	9,84*	I29	14,91*
I45	14,68*	I30	13,56*
I58	12,30*	I27	13,13*
I38	10,34*	I48	10,59*
I46	14,30*	I54	13,07*
I11	20,42*	I44	12,70*

*p<0.01

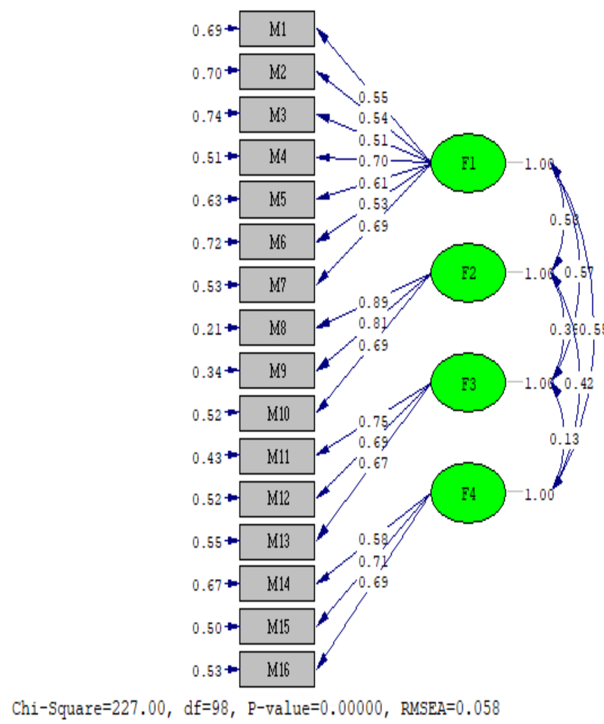

Fig. 4: The standardized values of the proposed MTS

Examining figure 3, fit indices were determined as χ2/sd=2.28 RMSEA=.058; SRMR=.050; NNFI=0.96; GFI=0.93; IFI= 0.97;CFI=0.97. RMSEA and SRMR range from 0 to 1 value. It is desirable to give values close to "0" (minimum error between observed and produced matrices).A value equal to or less than 0.05 indicates a perfect fit, and a value up to 0.08 indicates acceptable good fit. According to these results, it can be said that SRMR and RMSEA values are well and have perfect fitness. GFI ranges from 0 to 1 value. 0.90 and above is considered a good fit, while values above 0.85 are also considered acceptable values. It is influenced by the sample size. It returns smaller values in large samples. Accordingly, it can be said that obtained GFI values are good fitness. CFI above 0.90 indicates sufficient fitness, and a 0.95 above, it is a sign of perfect fitness. Accordingly, it can be said that obtained CFI values are excellent fitness. According to the results obtained, it is observed that the values are located between good fitness and perfect fitness. In other words, it confirms the factor structure of MTS, which consists of four factors. As a result of CFA, the t-values for four-factor model are shown in Table 5. Items are listed by factors in Table 5.

According to Table 5, the t-test values of the MTS are between 9.84 and 20.42. Accordingly, when the t values for all substances are examined as a result of the analysis, it is seen that the level of significance is .01. These findings confirm the factor structure of the MTS. The standardized values of the proposed MTS are shown in Figure 4.

When Figure 4 is examined, the factor loads for the proposed model are between .51 and .89. When error variances of the observed variables are examined, it is seen that the error variance of the scale items is not high.

Findings on Reliability of the Scale

To determine the reliability of the MTS, the Cronbach Alpha value and the Guttman Split-half test were calculated. The analyses related to the reliability of the scale were obtained from the first sample group of data. Analysis results are presented in Table 6.

When Table 6 is examined, these values were calculated as .818 in the sub-factor of factor 1, as .842 in the sub-factor of factor 2, as .756 in the sub-factor of factor 3, and as .695 in the sub-factor of factor 4. Cronbach Alpha value for the entire scale is .841. In addition, the GuttmanSplit-half test was calculated. Accordingly, it was calculated as .794 for the sub-factor of factor 1;as .675 for the sub-factor of factor 2; as .647 for the sub-factor of factor 3; .630 for the sub-factor of factor 4, and as .733 for the entire scale. Accordingly, the fact that the obtained values are greater than .70 indicates that the scale is reliable (Tavşancıl, 2010). In addition, as a result of the analysis conducted to determine the reliability of the mathematical thinking scale, the corrected item total correlation values and the Cronbach Alpha values when the item is discarded are included in Table 7.

As can be seen in Table 7, according to the results of the applied item analysis, the item total correlation coefficient values vary between .243 and .605. Büyüköztürk (2002) stated that these values should be at least .30 and above and should not be negative. As a result, it was seen that the scale met these criteria. In addition, when the substances are discarded, the Cronbach Alpha value obtained is between .825 and .846.

Characteristics of the Items of the MTS

The developed measurement tool is expected to distinguish between the lower and upper group (Can, 2013). The total scores of the data obtained from 410 students, that is, the first sample group, were listed and the upper and lower 27% groups were determined. Since the scores of these groups were normally distributed, independent groups were t-tested. Analysis results are shown in Table 8.

According to Table 8, it was concluded that there was a statistically significant difference between the average scores received by the upper group and the mean scores received by the lower group ($p<.01$). Accordingly, on the mathematical thinking scale, it can be said that the average scores of the students in the lower group and upper group from the items are distinctive (Büyüköztürk, 2002).

Answering and Scoring the MTS

As a result of this research, a reliable and valid measurement tool has been developed that aims to measure the mathematical thinking level of students with special abilities. The scale, which consists of four factors, has a total of 16 items. Scoring in the responses to the negative items in the SPSS program after the compilation of data entry was converted in the form of "1-5; 2-4; 3-3; 4-2; 5-1". The points that can be obtained from the five-item Likert-type scale vary between 16 and 80 points (16 * 5 = 80). To facilitate the interpretation of the MTS, the scores from sub-factors or total score can be divided by the total number of items. The high scores obtained from this scale mean that the students' level towards the relevant dimension is high. The scale can be applied to students with special abilities from different grade levels studying in primary and secondary school. The results of the analysis related to the answering and scoring of the scale were obtained from the data in the second sample group.

Table 6. Analysis Results to Determine the Reliability of the Scale

Sub-Factors	*Cronbach Alpha*	*Guttman Split-half*
F1	,818	,794
F2	,842	,675
F3	,756	,647
F4	,695	,630
Total score	,841	,733

Table 7. Corrected Item Total Correlations of the MTS and Cronbach's Alpha Value when Item is Discarded

Items	*Corrected Item Total Correlations*	*Cronbach's Alpha Value when Item is Discarded*
I1	,487	,831
I2	,447	,833
I3	,449	,833
I4	,597	,826
I5	,528	,829
I6	,482	,832
I7	,605	,826
I8	,525	,829
I9	,574	,825
I10	,499	,830
I11	,428	,834
I12	,418	,835
I13	,437	,834
I14	,243	,846
I15	,417	,835
I16	,362	,839

Table 8.MTST-Test Results for Upper and Lower Group Scores

		Lower Group		Upper Group		
Item No	*N*	$\bar{x}$	*Sd*	$\bar{x}$	*Sd*	*T*
I1	111	3,2883	1,03033	4,4595	,71075	,000**
I2	111	3,3243	1,01051	4,4865	,84062	,000**
I3	111	3,6577	1,02243	4,6216	,58859	,000**
I4	111	3,3784	1,05360	4,8468	,36177	,000**
I5	111	3,1441	1,15874	4,5045	,64489	,000**
I6	111	3,7477	,99516	4,8559	,37772	,000**
I7	111	3,3604	1,02507	4,7928	,42889	,000**
I8	111	3,2973	1,20299	4,7658	,55522	,000**
I9	111	3,0631	1,30926	4,7387	,56748	,000**
I10	111	3,3333	1,20856	4,7477	,62477	,000**
I11	111	3,0090	1,34499	4,4955	,84071	,000**
I12	111	2,8378	1,28304	4,1622	,93944	,000**
I13	111	2,8919	1,26743	4,4775	,76094	,000**
I14	111	3,2793	1,27330	4,5495	,98847	,000**
I15	111	2,9730	1,28247	4,6757	,78812	,000**
I16	111	2,7568	1,27362	4,5766	,79257	,000**

**p<0.01

Table 9: T Test Results on the Total Score of MTS of Students according to Gender

Gender	*N*	$\bar{x}$	*SD*	*d*	*T*	*P*
Female	75	63,50	8,67	391	,255	,799
Male	218	63,27	9,29			
Total score	393	63,37	9,01			

Table 10.T Test Results on the Total Score of MTS of Students according to Education Status

	Education Status	*N*	$\bar{x}$	*SD*	*d*	*T*	*P*
Total score	Primarly	108	63,20	8,80	391	,237	,813
	Secondary	285	63,44	9,10			

Table 11: One-way Variance Values on the Total Score of MTSof Students according to Grade Level

	Grades	*N*	$\bar{x}$	*SD*	*d*	*F*	*P*	*Meaningful difference*
Total sore	3	45	62,91	9,15	5	3,388	,005	7<5
	4	61	63,49	8,73				
	5	140	65,52	8,27				
	6	94	61,89	10,46				
	7	37	59,89	7,16				
	8	16	62,25	6,83				

Findings on Students' Levels of Mathematical Thinking According to Gender: The results of the independent groups t-test analysis conducted to determine whether the students participating in the study pointed to a significant difference according to gender in terms of the total scores of the MTS are shown in Table 9.

Examining Table 9, it is seen that there is no significant difference between the total score according to gender [t(391)=.799 p<0.01]. In terms of arithmetic averages, it can be said that female students (63.50) have higher mathematical thinking levels than male students (63.27). In addition, it can be said that all students have a good level of mathematical thinking (63.37/16=3.96).

Findings on Students' Levels of Mathematical Thinking According to Education Status: The results of the independent groups t-test analysis conducted to determine whether the students participating in the study pointed to a significant difference according to education status in terms of the total scores of the MTS are shown in Table 10.

Examining Table 10, it is seen that there is no significant difference between the total score according to education status [t(391)=.813 p<0.01]. In terms of arithmetic averages,

Table 12: One-way Variance Values on the Total Score of MTS of Students according to Groups

	Groups	*N*	$\bar{x}$	*Sd*	*d*	*F*	*P*	*Meaningful difference*
	Support	107	63,19	8,84				
Total score	RIT	199	65,40	8,24	2	16,644	,000	STD< Support, RIT
	STD	87	58,97	9,38				

it can be said that secondary school students (63.44) have higher mathematical thinking levels than primary school students (63.20).

Findings on Students' Levels of Mathematical Thinking According to Grade Level: Whether there is a differentiation between the mathematical thinking levels of the students according to the grade level was tested by one-way analysis of variance. These results are presented in Table 11.

Examining Table 11, it is seen that there are differences in the mathematical thinking levels of the students according to their grade levels (F[5–3.388], p<.01). Scheffe test was used from multiple comparison tests and according to the results of the analysis; the mathematical thinking level of 5th grade students (65.52) differs significantly from the mathematical thinking level of 7th grade students (62.25).

Findings on Students' Levels of Mathematical Thinking According to Groups: Whether there is a differentiation between the mathematical thinking levels of the students according to the groups was tested by one-way analysis of variance. These results are presented in Table 12.

Examining Table 12, it is seen that there are differences in the mathematical thinking levels of the students according to their groups (F[2–16.644], p<.01). Scheffe test was used from multiple comparison tests and according to the results of the analysis; the mathematical thinking levels of students from support (63.19) and RIT (65.52) groups differs significantly from the mathematical thinking level of students from STD group (58.97).

Discussion, Conclusion and Recommendations

This study is quantitative research conducted to develop a valid and reliable scale to determine the mathematical thinking skills of gifted students and to examine the mathematical thinking levels of gifted students in terms of various variables through the scale developed in this study. In the first stage, in the process of developing the scale, a pool of 58 items was created by scanning the field literature, and the scope validity was ensured by presenting it to expert opinions. As a result of the applied EFA, the scale explained 60.781% of the total variance. As a result of the analyzes made for the reliability of the mathematical thinking scale, it was seen that the scale was quite reliable. In addition, item discrimination power was calculated within the scope of reliability. There was a significant difference (p<0.01) between the total scores of the upper group (27%) and the lower group (27%).In line with these results, it can be said that the scale is a reliable and valid data collection tool that can measure the mathematical thinking levels of primary and secondary school students who are diagnosed as gifted students.

To test fitness of the structure of the mathematical thinking scale, data was obtained from the second study group, and when the fit indices were examined as a result of the DFA, it was concluded that the structure obtained was between good and perfect fit values. In the second part of the research, the functioning of the scale after development was tested in the context of gender, group, grade level and education status variables of the students.

As a result of the research, it was found that the students' mathematical thinking levels were high in terms of the total scores of the mathematical thinking scale. Sak (2011), in a study, concluded that the academic success of the student can increase and develop creative and questioning thinking skills with an effective program. In the field literature, Renzulli (2014) defines gifted individuals as having high-level thinking skills, bringing creative solutions to the problems they encounter by considering from different angles, willing to take responsibility and being able to fulfill their responsibilities with high motivation. Such a finding may have emerged as a result of the fact that the majority of students in science and art centres were subjected to intelligence tests and included in education. There are scales with high reliability, which were developed to help identify individuals with special abilities in mathematics (it is not appropriate to use likert type scales alone) supported by leading researchers of the literature. These scales are the Purdue academic rating scale and the scales for evaluating the behavioral characteristics of gifted students. In the expressions in the mathematics sub-scale items of these scales, it was stated that they could generalize mathematical expressions for gifted individuals and develop creative solutions while solving mathematical problems (Tan, 2021, p.23-44). In this study, it is thought that the high scores and total scores obtained from the items included in the deductive, inductive thinking, and problem-based thinking factors in the scale, which developed to determine the mathematical thinking skills of the students diagnosed as gifted, are due to the fact that the students are individuals diagnosed with giftedness. In addition, this finding may have emerged, which is found as a result of the education and training received by the students within the scope of the article "In education and training activities shall be included in the Science and Art Centers Directive published in 2016, applications that will enable students to gain high-level thinking skills are included".

In this study, in terms of the total scores of the MTS of the gifted students, it was analyzed according to gender, grade level, and education status and group variables. In this study, there was no significant difference according to the gender variable in terms of the total scores of the MTS of the gifted students, but it was seen that the arithmetic average of the scores of the female students on the scale was slightly higher than the arithmetic average of the male students. Tüzün and Cihangir (2019) found that there was a significant difference between gender and mathematical thinking stages and that there was an intermediate correlation among them. Mubark (2005) concluded that female students scored significantly higher than boys in three of the six dimensions of mathematical thinking and in total test scores. Duran (2005) concluded that the mathematical thinking skills of male students were better than female students in his study with 15-year-old students. Karakoca (2011) stated that in her study with 6th graders, there was no change in mathematical thinking situations according to gender in problem-solving. In the literature, it was seen that there were differences in the studies where gender and mathematical thinking levels were discussed. From here, it can be said that this finding of the research is in part similar to the field literature.

In this study, no significant differences were found in terms of the total scores of the MTS of the gifted students according to grade level, group and education status and group variables. In addition, since these three variables were related to the age and development levels of the students, the findings showed similarity. As students' grade level increased, their levels of mathematical thinking also increased. However, it was observed that there was a decrease after the 5th grade up to the 7th grade level, and an increase again at the 8th grade level. Besides, it was seen that there was a significant difference in favor of 7th grade students in terms of mathematical thinking levels at the grade level at the 5th and 7th grade level. Yılmaz (2019) found significant difference in favor of the 5th grade statistically between 5th and 9th grade in terms of grade level variable on students' mathematical thinking. In Kaya (2009)'s research on the analysis of the thinking styles and mathematics academic achievements of primary school 6th-7-8th grade students according to school type, gender and grade level, it was found that there was a significant difference between the 7th and 8th graders in favor of the 7th graders between students' scores in the law-making thinking style according to grade level. In executive thinking style, the researcher found a significant difference in favor of 6th graders between 6th and 8th graders, and in favor of 7th graders between 7th and 8th graders. Finally, the researcher found that there was a significant difference in judgmental thinking style in favor of 7th graders between 7th and 8th graders. In the field literature, it was seen that there were differences in the studies where the relationship between developmental features of students and their mathematical thinking levels were examined. Similarly, Demirtaş (2018) investigated whether the mathematical thinking levels of classroom teachers differ significantly according to the grade level variable in which they work. As a result of the analysis, there was a significant difference in favor of the teachers teaching in the 4th grade. The researcher explained that this situation was due to individual differences and the fact that the teacher gave education to the same grade level every year. The decrease in mathematical thinking with the increase in the grade level of the students may be due to the fact that the curriculum gives less space to activities aimed at increasing the power of mathematical thinking according to the grade levels. For this, comprehensive results can be obtained by examining the content of the curriculum at different grade levels in terms of developing mathematical thinking.

Consequently, as a result of this study, a reliable and valid scale was developed to determine the mathematical thinking level of the students. Through the scale developed, it was concluded that the mathematical thinking levels of the students differed significantly according to the grade level and the group. This scale was conducted only to determine the mathematical thinking levels of primary and secondary school students diagnosed with special abilities. However, it may be recommended to carry out validity and reliability studies by considering different sample groups (primary school, teacher candidates) of the prepared scale. In addition, within the scope of the study, the differentiation of students' mathematical thinking levels was examined in terms of gender, group, education status, grade level variables. In future studies, the differentiation of students' mathematical thinking levels according to academic success and type of school can also be examined. In this study, a scale was developed to measure the mathematical thinking level of individuals diagnosed with giftedness. In addition, it can be suggested that detailed studies should be carried out by comparing the mathematical thinking levels of students with normal development and those diagnosed with special talent. Thus, the reasons for the mathematical thinking level of individuals can be revealed.

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