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Research Article

Are beliefs believable? An investigation of novice mathematics teachers' beliefs and teaching practices

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ABSTRACT

Observing 17 teachers for a total of 116 hours, this paper examined whether a relationship between the beliefs of novice mathematics teachers and their classroom practices exists. A Spearman correlation analysis showed a modest relation between beliefs about the nature of mathematics and about learning mathematics, while the relationship between beliefs about mathematics achievement and these other two components was weak and statistically insignificant. Unexpectedly, the relationship between teachers' beliefs and their teaching practices has been found to be much different than anticipated, and in some cases, the relationships were even negative. The researchers conclude with a discussion of further implications.

Keywords: teaching practices, instructional activities, teacher beliefs, mathematics teaching, novice teachers

INTRODUCTION

For more than three decades, teacher beliefs have been an important topic of study in mathematics education (Ernest, 1989; Siswono et al., 2019). While this area of research has pitfalls to avoid and difficulties to overcome, it also has the potential to inform educational practices, and therefore, it is worth pursuing (Leatham, 2006; Lovin et al., 2012). One of the main reasons for continuing to investigate teachers' beliefs, according to Beswick (2006), is the view that what teachers believe has an impact on their teaching. In this regard, previous studies have identified complex relationships between teacher beliefs and how they relate to instructional practices (Purnomo, 2017; Raymond, 1997). While a substantial body of research suggests that the beliefs of mathematics teachers about teaching and learning affect their teaching practice (e.g., Fang, 1996), numerous other studies have reached conflicting results (Francis, 2015; Skott, 2001; Speer, 2005). For example, according to Richardson (1996), although beliefs are thought to drive actions, experiences and reflection on those actions may cause changes in and/or contributions to beliefs. In a similar sense, Ogan-Bekiroglu and Akkoc (2009) suggest that the early detection of the relationships and incompatibilities between beliefs and practices may play a critical role in interventions. With this in mind, this study was carried out to determine novice mathematics teachers' beliefs regarding nature of mathematics, mathematics learning, and mathematics achievement, and to examine the relationship between their beliefs and their teaching practices.

Beliefs and Teacher Beliefs

In the context of education, beliefs have been examined through the lenses of both cognition (Thompson, 1992) and affect (Furinghetti & Pehkonen, 2002). Those researchers who have emphasized the cognitive aspect define beliefs as "mental constructions of experience" (Sigel, 1985, p. 351) resulting from not only cognitive processes but also human experiences arising from social constructs. On the other hand, those who have emphasized the affective aspect define them as the understandings or propositions that an individual feels about the environment in which they exist (Richardson, 1996). Other researchers, however, have asserted that considering these two perspectives independently is problematic. For instance, Mcleod (1992) argued that beliefs are mostly cognitive by nature and are developed over a long time, but at the same time, they are emotional and may appear or disappear rapidly, as in the case of finding a solution to a difficult problem. In terms of mathematical beliefs, in particular, various sub-dimensions have been identified. McLeod (1992), for example, defines the following components from both cognitive and affective perspectives: beliefs about mathematics, self (e.g., self-efficacy, goal-orientation), the social context, and mathematics teaching. Uysal and Dede (2019) further clarify this issue, noting that beliefs about learning and teaching mathematics are shaped by the influence of cognitive processes, while individuals' beliefs about how they see themselves in mathematics and the social context of the society in which they live are shaped by affective processes.

In the current study, we focused on the cognitive aspect of beliefs and examined teachers' beliefs about the nature of mathematics, mathematics learning, and mathematics achievement. In this regard, beliefs about the nature of mathematics concern the ways that an individual describes mathematics. For instance, mathematics is defined in some cases as a dynamic and growing field of study according to Baki (2014), while others view it as a static set of concepts (Fisher, 1990). As Hersch (1986) points out:

> One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it … The issue, then, is not, What is the best way to teach? but, What is mathematics really about? (p. 13).

In parallel with the abovementioned statements, the classifications of beliefs about the nature of mathematics range from a static view that it is based on the execution of certain algorithms to a dynamic perspective based on pattern-seeking (Cooney, 2003; Ernest, 1989). For example, Ernest (1989) classified mathematics beliefs according to the instrumentalist view, the Platonist view, and the problem-solving view; while Dionne (1984) referred to them as traditional, formalist, and constructivist. While these categories are diversifiable, it should be noted that the classification is made with two extremes: namely, a traditionalist view on one side and a constructivist view on the other.

According to Ernest (1991), beliefs about the nature of mathematics also shape beliefs about how mathematics is most effectively learned. In his framework outlining the relationships between beliefs and their impact on practice, he indicated that from the Platonist view to problem-solving, the role of the teacher in teaching practices evolves from an explainer to a facilitator and the role of the learner evolves from the reception of knowledge to active construction of understanding. While beliefs in which learning is seen as the reception of knowledge are more aligned with the traditionalist approach, the idea that learning is the active construction of understanding shows similarities with the constructivist approach (Celik et al., 2018).

As with beliefs about the nature of mathematics, various classifications exist related to beliefs about mathematics learning and mathematics achievement. Beliefs about mathematics achievement, in particular, which is considered a factor in the current study, have been examined according to two common views: whether mathematics performance is a fixed ability or a learnable skill (Stipek et al., 2001). For those who see mathematics as a fixed ability, factors such as hereditary features are seen as impacting mathematics achievement; while for those who consider it as learnable, mathematics is a skill that can be improved (Tang & Hsieh, 2014). From this perspective, it can be asserted that beliefs about mathematics achievement fall within the framework of the nature of mathematics and beliefs about learning mathematics. In other words, it can be said that beliefs are shaped around constructivism on the one hand or by traditionalism or direct transmission on the other.

Teaching Practices

The nature of teaching practices and how they can be measured have been discussed by educational researchers according to various approaches and conceptual frameworks. Until recent years, the hypothesis

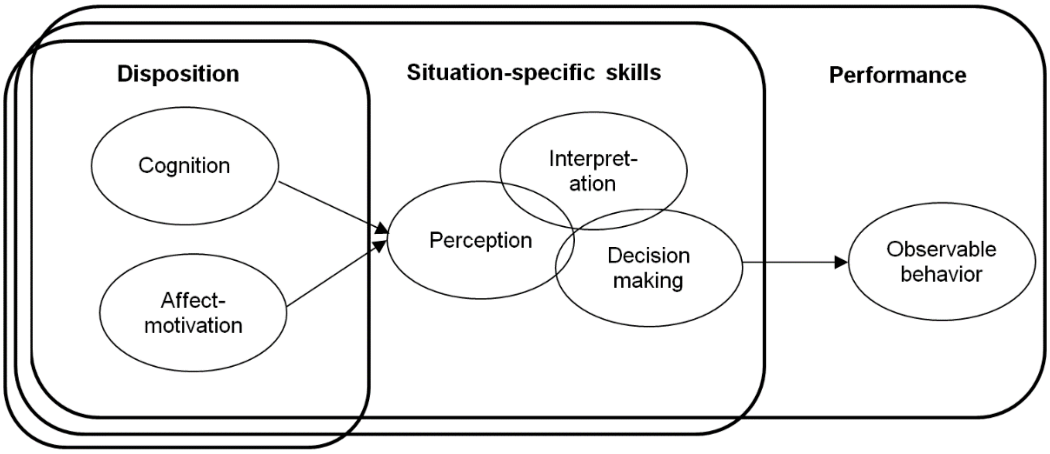

Figure 1. Modeling teacher competence as a continuum (Blömeke et al., 2015, p. 7)

that the knowledge of teachers has a major impact on their classroom practice has forced researchers to focus on the knowledge bases of teaching professionals (Krauss et al., 2018), such as pedagogical content knowledge (PCK) and mathematical content knowledge. Some researchers have found a close relationship between PCK and teaching practices (e.g., Baumert et al., 2010), yet others have noted the limitations of this standpoint and found no relation (Yang et al., 2020) between PCK and classroom practitioners' approaches to teaching. Given the reported limitations of measuring teacher competencies in this way, Blömeke et al. (2015) proposed a new model of teacher competence as a continuum (**Figure 1**). Within this framework, teacher competence is conceptualized based on observable behaviors in real classroom situations.

In the model created by Blömeke et al. (2015), teachers' dispositions are composed of both cognitive characteristics (e.g., PCK and MCK) and affect-motivational dimensions (e.g., teacher beliefs or attitudes), support the infrastructure of situation-specific skills. Blömeke et al. (2020) related these to the skills of noticing, which include the ability to "perceive and interpret what is going on in the classroom and then to make decisions" (p. 3). Drawing on these, as a means to assess teaching practices and evaluate the performance of a teacher, observable behaviors in terms of instructional processes are needed. Blömeke et al. (2020) discussed two facets of measurement, *generic* or *subject-specific,* for evaluating observable behaviors. While the generic aspect comprises the components of classroom management (e.g., using time effectively), student support (e.g., addressing student needs), and cognitive activation (e.g., presenting challenging content); subject-specific aspects comprise mathematics-related qualities such as the use of mathematics terminology and consideration of students' potential errors or misconceptions. Considering the upside of each approach, a combination of these two perspectives has come to the forefront in recent years (Lipowsky & Bleck, 2019, as cited in Blömeke et al., 2020) and has been adopted for this study. Accordingly, subject-specific and generic indicators were combined in a common framework. For example, in the context of effective use of time, the time given for students to think during a mathematical activity carried out in the classroom was considered.

Gaps in the Literature

Various studies in the literature have examined teachers' mathematical beliefs and instructional practices. There are two major concerns with these studies. First, in a significant number of these studies, teachers' instructional practices were measured through self-reporting. In other words, teaching practices were determined using their perceptions. For instance, a study conducted by Aljaberi and Gheith (2018) examined mathematics teachers' teaching practices through survey items in terms of their preferred approach to teaching mathematics in the classroom. They concluded that the perceptions of the teachers were mainly inclined toward the constructivist approach. Results from other studies about teaching practices are similarly based on teachers' self-reports (e.g., Polly et al., 2013; Yang et al., 2020). However, according to Guler (2019), when teachers are asked how they carry out classroom activities, they often describe ideal activities, rather than what they do. The basis for this may be that they are not aware of their own shortcomings, particularly in the case of novice teachers (Ngang & Chan, 2015), or that they report doing things that they want to do but do not do in reality (Guler, 2019). Grootenboer (2009) likewise points out that, although self-reporting may be an adequate way to determine teacher beliefs, it may not be effective for evidencing teachers' classroom practice. To fill this gap, this paper examines teaching practices of the teachers through in-class observations.

A second concern with the association between teachers' mathematical beliefs and teaching practices is inconsistent results (Vosniadou et al., 2020). Inconsistent findings suggest a need to conduct further research to understand this complicated relationship, as it may be affected by many factors, including years of experience (Isiksal-Bostan et al., 2015) and cultural norms (Xenofontos, 2014). As such, this study is important in that it focuses specifically on novice mathematics teachers, on one hand, and the other hand, on Turkish teachers, for whom the relationship between classroom practices and beliefs has not been well determined. This situation may be considered in the design of mentoring processes, with a particular focus on the relationships between teaching practices and beliefs in in-service courses during the induction process, as recommended by Ogan-Bekiroglu and Akkoc (2009). In conclusion, this study aimed to investigate novice mathematics teachers' beliefs related to the nature of mathematics, mathematics learning, and mathematics achievement, as well as to determine the relationship between their mathematical beliefs and their teaching practice. Therefore, the research question guiding this study was, "To what extent are teachers' mathematical beliefs related to their classroom practices?"

METHOD

As a part of a larger study focusing on the professional needs of novice mathematics teachers, this investigation was carried out in the form of a case study to determine the relationship between teacher beliefs and observed teaching practices. Since we focused on presenting an existing situation, a descriptive research design was adopted.

Participants

The participants were novice mathematics subject area teachers with one to five years of experience. In Turkey, subject area teachers only teach in their fields. Therefore, the participants taught mathematics only. In the process of selecting the participants, the equivalence of novice teaching in different theoretical frameworks was examined (Burden, 1979; Mok, 2005), and it was determined that teachers in the first five years of their careers were most appropriate for the study. The selection process was carried out in coordination with the Turkish Ministry of National Education (MoNE), which oversees all public schools in Turkey. A letter containing information about the study was sent to schools, and teachers who met the condition of years of experience were contacted and asked to participate voluntarily. Since the study was carried out with teachers from a region with a relatively lower population than other provinces, a total of 23 teachers were reached who fulfilled the aforementioned criteria. Of the 23 teachers who were contacted, 17 (aged 23 to 29) agreed to volunteer for the study. **Table 1** gives a brief description of the participants.

Table 1. Characteristics of participants

Participants	Educational degree	Years of experience	Place of duty	Taught classes
T1	Bachelor	3	Town	5th, 6th, 7th, & 8th
T2	Master in progress	4	Village	5th, 6th, 7th, & 8th
T3	Bachelor	2	Village	5th, 6th, 7th, & 8th
T4	Bachelor	1.5	Village	5th, 6th, 7th, & 8th
T5	Master in progress	4	Village	5th, 6th, 7th, & 8th
T6	Bachelor	1.5	Village	5th, 6th, 7th, & 8th
T7	Master in progress	4	Town	6th, 7th, & 8th
T8	Bachelor	1.5	Town	7th & 8th
T9	Master in progress (in a different field)	1.5	Town	5th, 6th, 7th, & 8th
T10	Bachelor	3	Village	7th & 8th
T11	Bachelor	3	Village	5th, 6th, 7th, & 8th
T12	Master in progress (in a different field)	1.5	Town	5th, 6th, 7th, & 8th
T13	Bachelor	3.5	City	7th & 8th
T14	Bachelor	3.5	City	5th, 6th, & 8th
T15	Bachelor	3.5	City	5th, 7th, & 8th
T16	Bachelor	4	City	8th
T17	Bachelor	3	City	6th

The characteristics of participants presented in **Table 1** indicate that most of the teachers work in villages. In addition, it is seen that almost all of the teachers work at different grade levels. The length of the observation period was determined in line with the suggestion of Wilkins (2008) that teachers should be observed for at least three hours to make inferences about their practice. However, to make an even more robust conclusion, the teachers, in this case were observed at least twice as long, from six to no more than eight hours; thus, a total of 116 hours of classroom observations were carried out. The observations detailed in the following sections were conducted by two research assistants who had received an MA in mathematics education, who were experienced in teaching and managing school practicum (internship), and who were working towards a doctoral degree in the same department.

Instruments

Belief scales

Before beginning the classroom observations, the participants were asked to complete a survey consisting of belief scales developed within the scope of the teacher education and development study in mathematics (TEDS-M) project (Tatto et al., 2012). The scales comprised teachers' beliefs, beliefs about the nature of mathematics, beliefs about learning mathematics, and beliefs about mathematics achievement and measured the degree of teacher agreement with 12, 14, and eight statements, respectively. Each item was rated on a 6-point Likert-type scale (1: strongly disagree to 6: strongly agree). **Table 2** shows sample items for each scale.

Table 2. Characteristics of belief scales (Tatto et al., 2008, p. 83)

Beliefs about	Items aims to determine	Sample Item
Nature of mathematics	Perception of mathematics as a domain	Mathematics is a collection of rules & procedures that prescribe how to solve a problem.
Learning mathematics	Appropriateness of particular instructional activities, questions about students' cognition processes, & questions about purposes of mathematics as a school subject.	The best way to do well in mathematics is to memorize all the formulas.
Mathematics achievement	Perception of teaching strategies used to facilitate learning of mathematics, as well as how mathematics learning may take place, & application of attribution theory to teaching & learning interactions	Since older pupils can reason abstractly, use of hands-on models & other visual aids becomes less necessary.

According to the theoretical framework on which the scales were based, beliefs on one hand reflect the traditional approach, and on the other hand, the constructivist approach. Detailed information about the construction of the scales, items, and validity and reliability studies is provided in Celik et al. (2018).

Observation protocol

Various instruments are available for measuring teachers' observable teaching practices. While most of the existing protocols have a *generic* structure that can be used for the observations of all teachers, regardless of their subject areas (MacIsaac & Falconer, 2002), some have been developed to evaluate teaching practices from specific aspects, such as interaction and engagement (Gleason et al., 2017). Furthermore, current approaches to assessing the professional competencies of mathematics teachers often use video clips as a tool, as they are useful for interpreting and making decisions about specific moments in the classroom (Guler et al., 2020; Guler & Celik, 2022). Considering the benefits and limitations of all three approaches, an observation protocol was developed by Guler (2019) to evaluate the teaching practices of the participants.

In developing the observation form, the PCK framework first introduced by Shulman (1987) was considered, and the components were adapted for mathematics education by the researchers (Ball et al., 2010). Accordingly, the teachers' classroom practices were assessed in terms of five facets: *student*, *content*, *strategies and techniques*, *measurement and assessment*, and *curriculum* (**Table 3**). The observation protocol consisted of a four-point rating scale where respondents rated the items, as follows: 1–the teacher did not display the expected competencies in the indicators, or what was displayed was completely wrong; 2–the teacher did not adequately demonstrate the expected competencies; 3–although the teacher largely met the

Table 3. Facets and sample items of the protocol

Facets	Sample item	Total items
Student	Considered preliminary information that student should have on subject. Considered possible misconceptions students may have.	6
Content	Gave students opportunity to present their ideas before making a statement about topic/concept. Followed a logical order in teaching of subject & examples given. Allowed sufficient time to answer questions used in classroom.	10
Strategies & techniques	Designed teaching environment to activate students effectively. Encouraged students to form different mathematical solutions.	6
Measurement & assessment	Asked questions that encouraged students to provide robust thinking. Gave feedback in accordance with student studies/responses.	4
Curriculum	Used examples that were within scope of objective to be taught. Considered previous achievements in selection of exercises & problems used in course or in presentation of different solutions.	5

behavior expressed in the indicator, this behavior could still be improved; 4–the teacher fully met the indicator described in the relevant item. For a detailed description of the protocol, as well as validity and reliability studies, see Guler (2019).

In the process of developing the observation protocol, which consisted of a total of 31 items, constructivist teaching inventories (e.g., Greer et al., 1999) were taken into consideration in addition to the components of PCK. Thus, decisions about the teaching practices of the participants were to be attributed to constructivist or traditionalist theory, while PCK components were to be considered in deciding what these teaching practices should be in terms of mathematics teaching.

Field notes

The classroom activities of the teachers within the scope of the study were primarily evaluated through the observation form. However, since these were limited to numerical and descriptive presentations of the observations, the findings from these were supported by field notes taken by the researchers. The field notes included anything from classroom discourse to teaching activities to dialogues. In addition, since the lessons were video recorded, the observer noted the important moments and then had the opportunity to review those moments in the analysis phase.

Data Analysis

Each teacher was observed for six to eight hours. For each objective the teacher covered, one observation form was completed. The main reason for this was that mathematics lessons are scheduled for a maximum of two hours a day per class, and the teachers distributed their teaching over the two lesson hours. In this regard, the pilot observations carried out before the main study revealed that the teachers generally focused on basic concepts in the first lesson hour, then included activities in the second lesson hour. As such, at least three observation forms were completed for each teacher. Given this, rather than scoring the teachers by observing a single objective, different objectives and sometimes different grades were considered. In assigning an overall score to each teacher in terms of the individual components of the observation form, their average scores were considered; and for the results from the entire form, all items in the observation form were likewise averaged.

In the analysis of both the observation forms and the belief scales, an equal interval classification approach was applied, as with Ogan-Bekiroglu and Akkoc (2009) (**Table 4**). For the scales, the mean values of the teachers' responses to the scalar items were categorized according to a 5-point Likert-type scale.

Table 4. Label ranges

Label	Range for belief scales	Range for observation protocol
Traditional	1.00-1.99	1.00-1.60
Close to traditional	2.00-2.99	1.61-2.20
Mixed constructivist and traditional	3.00-3.99	2.21-2.80
Close to constructivist	4.00-4.99	2.81-3.40
Constructivist	5.00-6.00	3.41-4.00

After determining the teachers' beliefs through the scales, as well as their teaching practices in terms of the components in the scales, a general picture was presented descriptively. Next, since the data were non-continuous, the Spearman correlations among these variables were reported. Finally, examples from the observations of the teacher's instructional practices were presented as raw data.

Reliability

Classroom observations of teachers have two major disadvantages in research. The first of these is the risk that the tension created in the teacher by being observed may affect his or her natural behavior; this risk is amplified if the number of observers is increased. On the other hand, observations carried out by only a single researcher may undermine the reliability when potential bias is considered (Brain, 2001). To mitigate this issue, the researchers decided to video record some of the lessons. This allowed a second coder to complete an observation form and the correlation value between the coders to be calculated. Before this undertaking, the two researchers involved in this study coded three lesson videos from a teacher who was not a participant in the current study and then reached a consensus by discussing the unanimous points on the scores on the scale. In addition, some research reports have noted that observing or recording teachers in the classroom may create tension and even make them feel nervous (Greenwalt, 2008; Musanti & Pence, 2010). To reduce these risks, the teachers were visited in their schools before the application, and preliminary observations and recordings were made that were not included in the study so that both teachers and students were familiar with the process.

Second, an additional study phase was conducted on the lesson videos belonging to teachers independent of the preset study group, and the correlation showed that there was an agreement between both coders. In the current study, twelve videos were coded by a second coder who was not an observer, and then a correlation was calculated. The results showed that the correlation value in each component of the observation form was larger than .60 and significant at the level of .05. When the social sciences literature that focuses on behavioral sciences is referenced, we can assert that this value is high since it is greater than .50 in consideration of Cohen's (1988) classification. With this in mind, it can be argued that because the two different coders obtained similar results, the study may be considered reliable.

RESULTS

Since the paper includes both quantitative and qualitative data, the descriptive results are presented first. After revealing general profile of the teachers and relation between teachers' beliefs and classroom practices, reflections are presented in the context of the classroom to illustrate typical situations that were observed.

Descriptive Results

The teachers' beliefs and teaching practices are visualized comprehensively in **Figure 2** in terms of each component. As can be seen in **Figure 2**, no teacher was directly involved in *traditional* or *constructivist* learning approaches, which are endpoints in the context of classroom teaching practices. However, their practices fell either in close range of one of these two approaches or in a *mixed* phase. Moreover, a similar distribution was revealed in terms of beliefs. In this respect, we found that concerning the beliefs about the nature of mathematics, two of the teachers displayed *constructivist* characteristics; on the other hand, in terms of mathematics achievement, two teachers displayed the *traditional* approach, while the others were either close to these two approaches or in the *mixed constructivist and traditional* phase.

As can be seen in **Figure 2**, in terms of beliefs about the nature of mathematics, none of the teachers fell in the close-to-traditional category, although some were in the mixed constructivist and traditional category. The teachers' beliefs about mathematics learning, on the other hand, were more evident; their beliefs fell in either the constructivist approach category or close to it. In this context, it can be said that these two beliefs have similar characteristics. However, when their beliefs about mathematics achievement were examined, it was noted that none of the teachers fell into the constructivist category. Two teachers were close to the constructivist category, while most of them were in the mixed constructivist and traditional phase, and a significant number fell either close to or within the traditional approach.

Figure 2. Distribution of beliefs and teaching practices (Source: Authors' own elaboration)

Considering the components of teaching practices, the most remarkable facet related to the curriculum; most of the teachers feel close to the traditional approach in this category. When the components aside from the curriculum were analyzed, no excessive differences were found in the categories. For example, a teacher who fell in the mixed constructivist and traditional phase in terms of one component of teaching practice either went up one level or down one level in the other components. In this respect, it can be said that more consistent results were obtained in terms of teaching practices.

A correlation analysis was conducted to examine the relationship between these results, which are presented in **Table 5**. Labels regarding the philosophy are displayed on the diagonal, whereas correlations can be found below that line.

Table 5. Labels and correlations for the facets and mathematical beliefs

	Nature of mathematics	Mathematics achievement	Mathematics learning	Student	Content	Strategy & technique	Assessment & evaluation	Curriculum
Nature of mathematics	Close to constructivist							
Mathematics achievement	0.21	Close to traditional						
Mathematics learning	0.49*	0.18	Close to constructivist					
Student	0.31	0.05	0.13	MCT				
Content	-0.10	0.10	0.24	0.65**	MCT			
Strategy & technique	-0.07	0.14	0.16	0.55*	0.87**	MCT		
Assessment & evaluation	0.28	0.34	0.19	0.61*	0.64**	0.58*	MCT	
Curriculum	0.26	0.35	-0.17	0.37	0.14	0.37	0.25	Close to constructivist

Note. *p<0.05; **p<0.01; & MCT: Mixed constructivist and traditional

The correlation analysis, which examined the relationship between teachers' beliefs and teaching practices in combination, provided interesting results. When examined along a general framework, teachers' beliefs about the nature of mathematics and mathematics learning felt close to the constructivist view, while their beliefs about mathematics achievement felt close to the traditional view. On one hand, the correlation values between these variables indicated a positive significant relationship between beliefs about the nature of mathematics and mathematics learning (r=0.49). On the other hand, a non-significant but positive relationship was found between teachers' beliefs about mathematics achievement and their beliefs about the nature of mathematics (r=0.21) and mathematics learning (r=0.18). In terms of the components of teaching practices, a consistent relationship between variables was found. Accordingly, the content was correlated highly with the variables of strategy and technique (r=0.87) and assessment and evaluation (r=0.64). Similarly, the student variable was found to be correlated with the variables of content (r=0.65), assessment and evaluation (r=0.61), and strategy and technique (r=0.55), respectively. The curriculum component, which seemed to be less associated with the other components, had no statistically significant relationship to any variable.

The analysis of the relationship between teachers' beliefs and their teaching practices revealed anomalous results. In this regard, consistent results were not found between the beliefs of teachers and their teaching practices. Moreover, when the relationship between beliefs about the nature of mathematics and teaching practices was analyzed, a weak negative relationship was found for two facets (content, and strategy and technique) and a positive correlation was found for the other facets. In terms of beliefs about mathematics achievement, all facets had a positive but non-significantly relationship. Finally, it was determined that the beliefs about mathematics learning had a weak and non-significant relationship with the facet of teaching practices and a negative relationship with the curriculum facet.

To summarize the quantitative data, the teachers' beliefs about mathematics were similar in terms of the nature of mathematics and mathematics learning and were categorized as close to traditional, while mathematics achievement differed from these as close to traditional. Considering the facets of teaching practices, a significant correlation was found between the dimensions, except student-curriculum and content-curriculum. While most of the teachers were coded in the mixed constructivist and traditional phase, it was observed that their practices approximated the constructivist approach for the curriculum facet.

Example Reflections from Observations

The descriptive analysis presented above revealed that teaching practices and beliefs were not related. In the following section, excerpts from the observations are presented as examples. For instance, the teacher referenced as T1, who characterized the general results, exhibited mixed beliefs about the nature of mathematics, feel close to constructivism in terms of beliefs about learning mathematics, and fell within the range of traditional beliefs about mathematics achievement. In other words, this teacher defined mathematics as neither static nor dynamic. However, she stated that the best way to teach mathematics is to involve students actively in mathematics activities, providing them with hands-on experience. In contrast, she believed that it is not possible to improve the mathematical achievement of every student, or that they can be improved only to a limited degree because only gifted students have access to higher-order thinking. In terms of teaching practices, she displayed mixed constructivist and traditional characteristics concerning curriculum facet, and close-to-traditional characteristics in the other facets, as illustrated from the following field notes from a lesson:

Grade: 5

Topic: Square and cube of integers.

> T1: (To herself: I wonder if there are any unit cubes in the school ...) She draws the shape of a 2×2 cube on the board and begins the following explanation: Now, someone asks me how many squares there are here (showing the front of the figure). What should I say? There are 2×2, 4 squares. Right?
>
> Students: Yes.
>
> T1: When we look now, did we find the number of cubes here (points to the front)? I have it on its back side as well; maybe that does not appear on the board, though? How many are there? Are not they in a queue? So, do not I multiply by 2?
>
> (Some students say yes, some students look confused).
>
> T1: Did you understand what I did here? So, I multiplied by 2 to express this cube; then I have another row in the back and multiply by 2 again. The product of multiplying by itself twice is called the cube.

> ...

Later in the lesson:

> A student: Teacher, I did not understand how we found that 8.
>
> T1: What we got when we multiplied 2 by 2 first? 4. Then multiply 4 by 2 again, and that is 8. We found it that way.

As seen in the example above, similar to other lessons noted by the observer, the teacher employs a mainly traditional way of teaching. In terms of belief, although T1 drew attention to the importance of structuring knowledge actively in mathematics learning, this is not reflected in her practice. Rather, she lectured directly and presented the lesson on the board instead of using instructional materials. Furthermore, the explanation given to address the student's question was a repetition of the original explanation and did little to provide conceptual understanding.

Another noteworthy conflict in the study was that most of the teachers' beliefs about the nature of mathematics felt close to either the constructivist approach or the mixed constructivist and traditional, but most of them fell in the mixed phase in terms of their teaching facets. In other words, instead of seeing mathematics as a structure consisting of a set of rules and a logical certainty, they defined it as explorable, open to new approaches, and testable through different solutions (or in part, for those who had mixed constructivist and traditional beliefs). However, a relationship between these beliefs and the actual teaching practices was not often observed. For example, an excerpt from the notes concerning participant T12, who displayed close-to-constructivist beliefs about mathematics learning, but mixed constructivist and traditional beliefs about the nature of mathematics and mathematics achievement, is provided here:

Grade: 6

Topic: Properties of adjacent, complementary, supplementary and opposite angles

> T12: (After the teacher reminded the students about the features of the adjacent, complementary, supplementary and opposite angles she had taught in the previous lesson, she wrote the following example): Two angles are complementary, and the larger angle is twice the smaller one. Find the measure of the smaller angle.
>
> Students: Shall we start?
>
> T12: Yes, please. I showed you on the board yesterday how to solve such questions. If you follow the same method, you'll do it right. If you do it differently, it will be wrong. Pay attention to that.
>
> Student: Is it necessary to do it the same way?
>
> T12: The same way is the most straightforward. If you do it another way, the result will probably change. (The teacher then solved the example on the board by assigning a variable to the angles).

As seen in the example above, the teacher asked the students to follow her exact approach to finding a solution, rather than encouraging them to explore different ways of solving the problem. From this point of view, it can be said that the teacher employed a teaching strategy that did not overlap very much with her stated beliefs. On the other hand, some teachers' beliefs and teaching practices did not differ completely. For example, participant T2 demonstrated beliefs close to constructivism in terms of all components; and the facets of student, strategy and techniques, and curriculum were also coded in the same category in terms of practice. An excerpt from T2's lesson is presented below.

Grade: 8

Topic: Number sets/real numbers

T2: Let's remember what sets of numbers we have seen so far. Yeah? (He started getting answers from the students).

Students: We saw natural numbers, we saw whole numbers, we saw rational numbers ...

T2: Well, let's remember what natural numbers, integers and rational numbers (After taking several examples from students, she wrote these sets on the board herself).

T2: Is $\frac{0}{7}$ a rational number?

Students: No, teacher!

T2: Okay, let's see. What do I multiply by 7 make 0?

Students: Zero.

T2: So, what does $\frac{7}{0}$ equal? (After the students said 0, she asked whether $\frac{7}{0}$ is a rational number).

Students: No, it's not!

T2: Why?

Students: Because when we multiply by 0, there is no number to give the result of 7.

In T2's lessons, as in the example above, it was observed that the teacher checked students' prior knowledge and considered possible misconceptions. However, the techniques applied were limited to short question-and-answer activities. It was also noted by the observer that discussion environments that would activate students such as group work were limited.

As a final remark, in terms of teaching practices, it is noteworthy that all but one teacher felt close to the constructivist approach. During the observations, it was determined that all of the teachers used primarily textbooks in their lessons or brought printouts from the MoNE's educational informatics network or various websites. As a result, the examples or questions they used were included within the framework of the objectives specified in the curriculum, and relationships were drawn to previous objectives. Although the reason for this was to maintain the activities at the operational level, the fact that the participants chose methods and techniques that were teacher-centered affected the other components negatively. However, it was found that the curriculum facet was less affected than others.

DISCUSSION AND CONCLUSION

Although researchers have long highlighted teacher beliefs and claimed a linear effect on teaching practices, the research that has tested this hypothesis and investigated whether mathematics beliefs are related to teaching practices is limited. The hypothesis in this study, which aimed to address this gap in the literature, was tested from the perspective of less-experienced novice mathematics teachers. Their teaching practices were examined concerning students, content, strategies and techniques, measurement and assessment, and curriculum; while their beliefs were explored in terms of the facets of nature of mathematics, mathematics learning, and mathematics achievement.

When the participants' beliefs were analyzed amongst each other, a significant relationship was found between the nature of mathematics and mathematics learning, albeit at a moderate level, while beliefs about mathematics achievement had a low-level relationship with these other two beliefs. Furthermore, the relationship between beliefs about mathematics achievement and mathematics learning was found to be weak. The fact that teachers' beliefs in one domain were not fully consistent in another domain reflects results reached by other researchers. For example, Siswono et al. (2019) examined the mathematics beliefs and teaching knowledge of teachers in the context of problem-solving over a two-year project and found that teachers' beliefs about the nature of mathematics did not support their views on mathematics teaching or learning. To exemplify, although teachers' beliefs about mathematics teaching and learning fell within the category of problem-solving, the indicators of the nature of mathematics, such as dealing with a mathematical formula or calculator, were Platonist or instrumentalist. In addition, although the teachers in their case emphasized the importance of understanding a mathematical problem, they stated that being "fast and instant" in problem-solving is especially important (Siswono et al., 2019, p. 503).

On the other hand, Uysal and Dede's (2019) work is contrary to our results. In their study with Turkish teachers with a year more of experience than those in the present study, it was found that that teachers had more static beliefs about the nature of mathematics, but more constructivist beliefs about learning mathematics. On the other hand, Tarasenkova and Akulenko (2013), in examining the beliefs of Ukrainian teacher candidates with the same data collection tool, obtained similar results about mathematics achievement while detecting more traditional beliefs about the nature of mathematics, likewise in contrast to our own findings.

In another study conducted by Celik et al. (2018) with the participation of 1,418 prospective mathematics teachers, the participants revealed inconsistencies in their beliefs about the nature of mathematics, mathematics learning, and mathematics achievement. As such, there are some conclusions to be drawn. The first is that teachers who have the same cultural background but differ in their years of experience may vary in their beliefs about mathematics. While this claim is supported by some researchers (Corkin et al., 2015), others have reached contradictory conclusions (Nisbet & Warren, 2000). The second is that different cultural structures may impact teachers' mathematical beliefs. As Wong et al. (2011) claimed, contextual variables such as the structure of the educational system or policies may be factors influencing beliefs about mathematics.

In terms of the relationship between teachers' mathematical beliefs and their teaching practices, unexpected results were revealed in this case. For instance, inconsistencies were identified in the current study, as with the inconsistencies of previous studies examining the relationship between teachers' beliefs and teaching practices. Many early researchers concluded that mathematical beliefs are one of the main factors affecting teaching practices (Fennema et al., 1989; Kaplan, 1991), and these results have created a new research area in mathematics teacher education (Di Martino & Sabena, 2010). Over time, new studies have emerged confirming the relationship between teachers' beliefs and teaching practices; however, other researchers have not supported this situation (Wilkins, 2008). Similarly, the results obtained from the current study reveal that teaching practices and beliefs are mainly unrelated and in some cases even opposite. For instance, it was observed that teachers who generally expressed close-to-constructivist beliefs about the nature of mathematics fell in the close-to-traditional category, outnumbering the mixed constructivist and traditional phase on average in terms of teaching practices. The correlation analysis we carried out revealed that none of the 15 correlation values between the five components of teaching practices and three beliefs were significant, most of them were weak, and some were negative.

Based on these results, it is difficult to argue a linear relationship between beliefs and teaching practices. Our findings support the claims made by numerous other researchers (e.g., Raymond, 1997; Toluk-Ucar & Demirsoy, 2010). For example, a study conducted by Toluk-Ucar and Demirsoy (2010) found that although teachers declare that they have non-traditional beliefs, they followed a traditional approach to teaching in their practice. Similarly, a study conducted by Raymond (1997) with a novice teacher found inconsistency between his beliefs and teaching practices. Contrary to the results of studies claiming that, although teaching practices do not overlap with beliefs, the teaching method preferred by a teacher is strongly grounded in beliefs (Wilkins, 2008), an inverse relationship was found between beliefs and the strategy and techniques used by the novice teachers in this case. In this regard, we contend that the strategies and techniques chosen in the classroom cannot be explained solely by beliefs.

According to Raymond (1997), the primary goal of teacher education programs should be to reveal the current beliefs of prospective teachers, to ensure that these beliefs are reviewed, and to develop beliefs that will be compatible with the new mathematics education program. The current study, conducted with novice mathematics teachers, reveals that although the development of beliefs has been partially achieved, in a system that has been centered on a constructivist educational philosophy, this has not been achieved in teaching practice, except beliefs about mathematics achievement. In this sense, many studies in the literature have shown that novice teachers experience problems relating to their teaching competence during the early years of their practice (Cankoy, 2010; Guler, 2019) and encounter issues arising from not being able to relate theory and practice (Solomon et al., 2017). This suggests that novice teachers may need professional development opportunities that can help them improve in terms of teaching practices.

As a final remark, the results obtained from the present study reveal that novice teachers' beliefs about how they perceive mathematics and their beliefs about teaching and learning mathematics do not serve as a reliable predictor of their teaching practices, as claimed by other researchers (e.g., Cross, 2009). This result shows that making inferences about teaching practices by considering the beliefs held will not always yield healthy results.

Limitations and Future Implications

There are several limitations of this research that should be considered when interpreting the results. The first and most important of these is the number of teachers who participated. Although seventeen teachers, who were each observed for at least six hours, are considered a sufficient number for descriptive research, there are limitations in terms of correlation calculations. Namely, although the correlation value does not change, the number of samples may affect the statistical significance (Aggarwal & Ranganathan, 2016). The second limitation of the study relates to measuring teaching practices through the PCK components. In this study, the researchers focused on the facets of student, content, strategies and techniques, measurement and assessment, and curriculum, and the relationship between beliefs and these components was examined. On the other hand, some researchers suggested that studies dealing with teacher beliefs should focus on building coherent models of teachers' belief systems (Leatham, 2006). In this regard, it can be said that models are needed to reveal the inconsistencies in the relationship between beliefs and practices and to uncover the missing factors in the equation. A final and important limitation of the study is the categories made to interpret the results gathered through instruments. Although there are similar studies in the literature for categorizing teaching practices, such approaches are faded the beliefs. Moreover, according to Tatto et al. (2012), although the belief scales used in this study might be seen as "loosely related to the calculational versus conceptual and the direct transmission versus cognitive- constructivist distinctions", they were defined not seen as equivalent to them (p.154). In this context, this limitation should be considered, and stronger methods should be adopted in categorizing beliefs.

Given the results, we may offer some suggestions for future research. The current study attempted to measure the observable behaviors of novice mathematics teachers as performance indicators of teaching practices. First of all, the relationship between beliefs and situation-specific skills such as perception, interpretation, or decision-making that feed these performance indicators, as suggested by Blömeke et al. (2015), may be examined in further studies. Research carried out in this way may offer significant results concerning the preparation of skill-oriented professional development models for novice mathematics teachers. Second, there is a need for studies on the source of inconsistencies between teachers' beliefs and practices. From this point of view, it is recommended to use data supported by interviews as a tool to reveal inconsistencies in greater detail. Finally, we made observations to evaluate the instructional practices of the teachers while determining their beliefs through self-reports. Given the difficulty of interpreting these two measurements in different situations together, it may be more effective to conduct video discussions (where teachers are shown their teaching video and discuss their beliefs and practices) as recommended by Francis et al. (2014) to investigate relationship beliefs and practices.

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