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Teaching Math Online: Evaluating Access and Rigor in an Asynchronous, Online Algebra 1 Course

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Algebra 1 is a gatekeeper course with profound implications for students' academic and professional success. This study examines the implications of teaching Algebra 1 through a standardized, fully online, asynchronous course developed by one of the largest online course vendors in the United States in alignment with Common Core State Standards for Mathematics (CCSSM). Using an explanatory sequential mixed methods design, we evaluated the strengths, limitations, and leverage points for realizing the instructional rigor set forth in the CCSSM. Data were collected using a CCSSM-aligned observation rubric that required observers to rate each of the 34 online Algebra 1 lessons on Likert-type scale questions as well as provide a narrative description of the curricular content, instructional tasks, and assessment activities. The online course provided opportunities to demonstrate understanding and real-world application in a manner that demonstrated high surface-level fidelity to CCSSM but was limited by infrequent process reflective and communication-based tasks. We discuss implications and propose instructional strategies that could be implemented to enhance current limitations to the asynchronous, online setting studied.

Keywords: online learning, secondary math education, instructional strategies, Common Core State Standards

Algebra 1 is a gatekeeper to advanced mathematics course taking and entry to various higher-paying careers (Stein et al., 2011; Loveless, 2013). Differences in accessibility and quality have been shown to impact the trajectory of students' academic and professional careers (Heppen et al., 2017), establishing student access to algebra as an important equity issue (Allensworth et al., 2009). Further, as of 2015, approximately 850 thousand students enrolled in a fully online mathematics course with rapid growth experienced since that time and continued expansion of online enrollment in core subject areas projected (Gemin et al., 2015). Despite the well documented importance of Algebra 1 enrollment and success for future life success, less attention has been paid thus far to understanding what and how students are taught, particularly when it comes to increasingly prevalent online Algebra 1 courses.

Recognizing this, the adoption of Common Core State Standards for Mathematics (CCSSM) by 41 states starting in 2009 facilitated increased standardization of rigorous mathematics course content across the United States, focused on understanding, problem solving, and procedural fluency (CCSSI, 2020; McDonnell, 2014; NCTM, 2014). Potential benefits of CCSSM-aligned curricula include improved student engagement and performance due to the focus, rigor, and coherence of standards that resemble those of the highest achieving nations (Schmidt & Houang, 2012; Ross et al., 2015). However, few studies have explored what the CCSSM look like in practice. Further, these studies have been primarily small-scale and focused exclusively on face-to-face classrooms with traditional classroom-based instruction (Burch, 2018; McDuffie et al., 2017, 2018; Porter et al., 2015).

At the same time, several million public school students enrolled in online courses each year prior to the COVID-19 era, with around three-fourths of all online coursework completed in core content areas such as mathematics during that time (Gemin, Pape, Vashaw, & Watson, 2015). This topic is of even greater relevance during the COVID-19 era, as an increasing proportion of students receive at least a portion of their education virtually. Despite challenges associated with implementing virtual learning opportunities, research conducted during the COVID-19 era has demonstrated that some parents, teachers, administrators, and students perceive online learning to be a feasible tool in traditional K-12 learning environments (Simmons, 2022). Further, the extended restructuring of traditional schooling is likely to have some long-lasting effects on the K-12 education system, particularly for districts struggling to fully staff face-to-face classrooms and/or looking to capitalize on the cost-savings afforded by outsourcing teaching and learning processes (Heinrich et al., 2019; Heinrich & Darling-Aduana, 2021). As the reliance on online courses continue to grow, it is critical that we examine online course content and tasks to evaluate the viability of this instructional medium.

Building upon prior research, we examine the enacted CCSSM-aligned curriculum observed in an asynchronous Algebra 1 course developed by a vendor used in over 16,000 schools nationwide. Through an examination of detailed archival course data, we address the following research questions. (1) To what extent did one widely used online Algebra 1 course align with the CCSSM? (2) In what ways did the same online courses fail to fully deliver on the intentions of the CCSSM? Through the examination of these research questions, we document the successful practices and limitations of CCSSM enactment within a format of asynchronous online course-taking that is becoming increasingly widespread as a primary means of math instruction. More broadly, the documentation of one widespread enactment of CCSSM provides educators and policymakers valuable information on existing practices and challenges to ensuring access, rigor, and dependability when teaching math online.

ENACTING THE CCSSM ONLINE

The online Algebra 1 course examined was designed to align with the same CCSSM as the face-to-face Algebra 1 classes provided by the school district we studied. The eight mathematical practices (MPs) each CCSSM-aligned mathematics course should integrate include: make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning (CCSSI, 2020). For a complete list of actions that students should perform during class to demonstrate mastery in each MP, refer to Appendix A (CCSSI, 2020).

Although the goals and rationale of the intended CCSSM-aligned curriculum have been extensively analysed and discussed (e.g., Schmidt & Houang, 2012; Ross et al., 2015), research examining the extent to which these goals are delivered is limited. This is important to understand, as intended curricula (or standards) do not always translate into aligned practices and high-quality instruction (Hiebert, et al., 2005; McDuffie et al., 2017; Porter et al., 2015; Stein et al., 1996). Further, enacted curriculum has been historically challenging to measure, often requiring detailed observation of individual classrooms (Remillard, 2005; Remillard & Heck, 2014). This study expands upon the scope of artifacts reviewed at a larger scale than available in prior research. We accomplish this by evaluating all curricular content, instructional tasks, and assessment activities associated with an online Algebra 1 course implemented not just across multiple classrooms and schools but also across many school districts nationwide.

Further, prior literature on CCSS enactment has mostly ignored online learning. In the course studied, the online (vendor-hired) teachers were pro vided a pre-packaged (fixed) set of slides dictating curricular content, instructional tasks, and assessment activities approved by the vendor's curriculum development team, which the vendor-hired teachers delivered and facilitated asynchronously through pre-recorded videos. This type of curricula use prioritizes standardization and implementing with fidelity, increasing the extent to which the enacted curriculum mirrors the intended standards (Remillard, 2005). The high level of standardizations across contexts deepens the need for an examination of curriculum in this context due to the large number of students affected. For instance, around 350 students enrolled in the online Algebra 1 course each year in the district studied, with over 16,000 schools nationwide contracting with the same online course provider.

The Effectiveness and Limitations of Online Algebra 1 Course-taking

Additionally, although studies have begun to examine the effectiveness of online courses relative to face-to-face courses (i.e., Heinrich et al., 2019; Heppen et al., 2017; O'Dwyer et al., 2007; Pane et al., 2014), there are relatively few studies that explore the actual content and teaching practices of these online courses. Addressing questions related to effectiveness, Heppen et al (2017) randomly assigned 1,224 ninth graders who failed algebra to take either an online or face-to-face algebra credit recovery course and found that students were more successful in the face-to-face environment. Specifically, students in online credit recovery reported that the course was more difficult, were less likely to recover credit, and scored lower on an algebra post-test, which the authors linked to greater discretion available to teachers in the face-to-face environment. However, the study was not able to systematically examine curricular content and instructional strategies used in each setting to test this assertion. Additional research is needed to identify the instructional supports or strategies currently missing in the most commonly used online Algebra 1 courses, so the online courses themselves can be improved or supplemented through blended instruction.

Of the limited number of studies examining the content and teaching practices of online courses, Choppin and colleagues (2014) established the limited use of the multimedia, interactive, and social features in digital mathematics curricula and resources that could represent an enhancement of traditional, face-to-face instructional methods (see also An et al., 2022; O'Dwyer et al., 2007). For instance, research indicates that deeper learning is more effectively facilitated online via blended learning strategies through interactive tasks such as threaded discussions, partner-shared learning, and open-ended activities (An et al., 2022; Bergdahl & Hietajärvi, 2022; Harper, 2018; Jesson et al., 2018; Lai, 2017; Ukpokodu, 2008). Students also report higher satisfaction and learn better online with regular, timely, high-quality interactions with teachers (Bagdasarov et al., 2017; Hawkins et al., 2013; Jaggars & Xu, 2016; Shelton et al., 2017).

The prior literature that does exist has established that most vendor-developed online courses use the same type of asynchronous lecture, assignment, and assessment structure examined in this study that provide few opportunities for student-teacher or student-peer learning-related interactions (Choppin et al., 2014; Daniels et al., 2021). In general, instructional designers of vendor-developed online algebra courses report insufficient understanding, time, and resources to provide accessibility and personalization features (Rice, 2018). Instructional designers also reported employer pressure to design a single, standardized course that could be sold across the vendor market without modifications or the addition of more expensive (i.e., personalized) features (Darling-Aduana et al., 2022). Despite the resulting limitations to the subsequently developed online course system, in-person instructors most often conceptualize their role as providing administrative and technical (versus instructional) assistance, leaving students with only the online course system for content delivery with no (or minimal) access to supplemental content or instructional support (Darling-Aduana et al., 2019). The resulting predominance of didactic, teacher-centered online lessons in these settings fails to fully provide students with the learning environment, opportunities, and support required to be successful while also suggesting an underutilization of technology capabilities in online settings overall (An et al., 2022; Daniels et al., 2021; Darling-Aduana, 2021).

Gaps in the Literature

In summary, we reviewed prior literature on CCSSM enactment and online mathematics course-taking, including best practices as well as online course vendor norms and incentives. Despite an emerging research base on these topics, this study expands on current understanding by examining enacted practices versus only available features (Choppin et al., 2014) or stakeholders' perceptions (An et al., 2022). Further, while O'Dwyer et al. (2007) observed a handful of lessons as part of their research, our study provides a more in-depth look at the full curricular content, instructional strategies, and assessment tasks across the entire Algebra 1 course studied.

Given the current state of knowledge on these topics, we aimed through this study to extend documentation of the processes, strengths, and limitations of enacting the CCSSM to an asynchronous online learning environment. As such, this study contributes to understanding of instructional practices within mass-marketed online learning systems while also providing a nuanced examination of CCSSM enactment at greater level of detail than in previous studies. Specifically, based on our review of prior literature, we hypothesized that:

Hypothesis 1

The integration of technology-based tools and activities that facilitate interactivity, open-ended assignments, and/or leverage multimodal/multimedia features will support greater CCSSM-alignment (An et al., 2022; Harper, 2018; Jesson et al., 2018; Lai, 2017; Ukpokodu, 2008).

Hypothesis 2

Given the pressure to design one-size-fits-all, standardized online courses, we hypothesized that the online Algebra 1 course would provide limited opportunities for CCSSM elements most effectively facilitated through student-directed and/or multi-step, complex tasks (Choppin et al., 2014; Daniels et al., 2021; Darling-Aduana et al., 2022; Rice, 2018).

Findings can be used to clarify the instructional practices and additional supports students enrolled in Algebra 1 online require to succeed and identify any inherent strengths of the online environment or course-taking system that could be integrated in face-to-face instruction to enhance students' educational experience.

METHODS

We implemented an explanatory sequential mixed methods study whereby qualitative analysis was used to provide additional nuance to findings that emerged through the initial quantitative analysis (Creswell & Clark, 2018). Data were collected by observing online and face-to-face Algebra 1 lessons using a CCSSM-aligned rubric that captured both rating scale items and narrative comments. Initial descriptive analysis was supplemented by qualitative analysis using thematic codes. Additional information on the sample and setting, data collection, empirical strategy, and limitations follows.

Setting and Sample

We conducted observations of online course-taking in a large, urban district in the Midwest where approximately 20 percent of students enrolled in the online Algebra 1 course sometime over the course of their high school career. This translated into around 350 students enrolling in the online Algebra 1 course each year within the district studied. Within the district studied, most students enrolled online after failing the course in a face-to-face setting (i.e., for the purpose of credit-recovery). However, the course was designed as a standalone course that could be (and was occasionally in the district studied) used by students enrolling in Algebra 1 for the first time as a substitute for the face-to-face alternative. Importantly, in communications with the vendor, the communications team emphasized that their course should be integrated within a blended learning model. This stands in contrast to the message our district partner received from the vendor marketing and professional development staff and the observed reality of how classes were administered within the district (see Heinrich et al., 2019 for more information). Among students attending the district, over 80 percent qualified for free or reduced-priced lunch, approximately half of all students identified as Black or African American, and another one-fourth identified as Hispanic or Latino/a.

We focused our observations on the first semester of the online Algebra 1 course, since slightly more students enrolled in that semester. The district provided us access to the online portal where students accessed course content. When logging into the course, students saw a webpage with the course divided into sections, each containing three to four lessons. After observing the 34 lessons contained within the Algebra 1 course (and tracking the time spent on each activity), we note that each of the lessons within the course took 30 to 55 minutes to complete, with the modal lesson requiring a little over 40 minutes. All followed the same general format and contained several instructional tasks. Students watched an approximately 20-minute-long lecture. During these lecture videos, the remote, pre-recorded teacher appeared in a box in the top right-hand side of the screen, where they led the student through information-based slides and practice problems. Next, the course provided students more practice problems followed by a multiple-choice quiz on which students needed to earn a minimum grade to receive credit for completing the lesson; Students who failed the quiz could retake it until they attained the minimum passing grade. Students needed to achieve a passing grade on each individual lesson to receive credit for the course. All but (at most) one of the items on these summative assessments were closed response versus open ended.

Data Collection

Attempts to measure enacted curricula in published education literature included quantifying features such as demonstration, practice, and recall of concepts (Hiebert et al., 2005). Other research focused on analysing archival documents, such as classroom assignments (Joyce et al., 2018; Porter et al., 2015) or assessment tasks (Hunsader et al., 2014). We combined these approaches, reviewing archival documents in the form of assignments and assessments and observing classroom instruction by watching the same online video lectures as students enrolled in the course. All observations were conducted during the 2019-20 school year.

We used a rubric to indicate the approximate percentage of total lesson time (including lecture, practice problem, assessment, and activity time, rounded to the nearest ten) devoted to each of the actions identified by the CCSSM (see Appendix A) (CCSSI, 2020). We used language directly from the standards in our rubric before adding examples and definitions as needed to provide additional clarity on the authors' conceptualization of each component. In addition, we each provided narrative descriptions of the curricular content, instructional tasks, and assessment activities employed in each lesson.

The authors established interrater reliability at the beginning of and throughout the coding process, discussing and agreeing upon any discrepancies before proceeding. Both authors coded the first eight (23 percent of) lessons in a process designed to clarify definitions and establish reliability through iterative individual rating, discussion, and reconciliation. Of the remaining 26 lessons, each rater completed the rubric for nine lessons individually, while we used eight lessons throughout the course to confirm continued interrater reliability. We achieved an interrater reliability rate of 87 percent when we considered any response within one point on the ten-point scale to indicate consistency. When we considered any response within two points on the ten-point scale to indicate consistency, we achieved an interrater reliability rate of 96 percent.

As a means to consider alternative hypotheses and perspectives during the analytic process, we considered our primary data in conjunction with observational data collected in eight traditional, face-to-face Algebra 1 classrooms. These observations allow us to more accurately identify practices that are unique to the asynchronous online learning environment compared to those that are common across all instructional modes. Additionally, this comparison aided us in highlighting the potential of online instruction that may not be as feasible in a face-to-face setting. We used a purposive sampling method that oversampled classrooms in schools where a larger proportion of students enrolled in online courses to improve comparability across instructional settings. We also made sure to observe traditional, face-to-face Algebra 1 classrooms geared towards credit-recovery along with classrooms geared toward ninth grade students enrolled in Algebra 1 for the first time. We collected observation data guided by the same instrument used to evaluate online lessons. Class sizes of observed face-to-face classrooms ranged from approximately 15 to 35 students per class. All face-to-face classes were delivered within 90-minute blocks.

Empirical Strategy

Using ratings from the Likert-type scale rubric, we created descriptive figures that displayed the frequency at which each MP occurred by lesson for the online course. From these figures, we identified modal lessons as well as high and low rated outlier lessons. We used emergent, thematic qualitative coding to identify commonalities in the narrative vignettes within each group of lessons (modal, high outliers, low outliers). Sample codes included requiring student interaction, encouraging higher order thinking, and using technology. We compared and discussed themes within and across groups, reviewing narrative vignettes to better understand when and why certain tasks occurred and to highlight counterexamples.

Limitations

We chose to examine in-depth a single course developed by one of the largest vendors in the United States. While this strategy allowed for a nuanced evaluation of the enactment of one CCSSM-aligned online course to which thousands of students are exposed each year, this may limit generalizability to other learning contexts. Similarly, we chose to focus on the curricular content and instructional tasks to which students were exposed versus documenting how students interacted with that content. This means that we cannot claim that students either attempted or performed successfully the CCSSM-aligned actions. We pursued this analytic strategy because we were interested in evaluating the instructional strategies and tasks that could facilitate CCSSM-aligned actions in an asynchronous, online course environment. Prior research provides insights into the likelihood that students will fully engage with the tasks integrated in the online course and how to develop a classroom environment that will support student success in those activities (Heinrich et al., 2019; Wang et al., 2020; Zheng et al., 2020). Future research could examine student engagement and learning more directly in response to the different types of instructional tasks that facilitate CCSSM-aligned actions across settings.

FINDINGS

To examine the extent to which one widely used, online Algebra 1 curriculum enacted CCSSM-aligned MPs, we first ran descriptive statistics on each rating scale item across all observed online and face-to-face classrooms (see Table 1). We calculated mean differences and ran t-tests to determine whether there was a statistically significant difference in the proportion of time spent accomplishing each action between instructional modes. We used the resulting estimates to identify initial themes that guided subsequent qualitative analysis as well as to triangulate emergent findings.

Table 1
Proportion of Time Spent by Action between in Online and Face-to-Face Lessons

		Face-to-Face		Online		Mean Difference
MP	Action	Mean	Std. Error	Mean	Std. Error	
1	Explain meaning of the problem.	13	7	4	2	9*
	Analyse givens and constraints.	13	6	13	2	0
	Make conjectures.	5	5	2	4	3
	Consider analogous problems.	63	2	35	11	28*
	Monitor and evaluate progress.	14	4	0	0	14*
	Transform algebraic expressions.	36	10	22	9	14
	Explain correspondences.	14	11	6	4	8
	Use a different method.	5	5	2	3	3
	Ask, "Does this make sense?"	6	4	0	1	6*
	Understand other approaches.	5	5	5	4	0
2	Decontextualize.	16	12	22	1	-6
	Contextualize.	11	7	20	5	-9
	Attend to quantity meaning.	11	7	20	5	-9
	Vary properties of operation.	15	6	14	4	1
3	Use stated assumptions.	11	7	4	3	7*
	Make and test conjectures.	19	17	9	1	10*
	Break into cases.	2	3	9	2	-7*
	Justify conclusions.	19	5	6	2	13*
	Reason inductively about data.	1	3	10	4	-9*
	Compare arguments.	5	5	3	3	2
	Determine domains.	3	3	6	5	-3
	Improve arguments of others.	9	4	0	1	9*
	Use counterexamples.	4	6	2	3	2
4	Solve real-life problems.	22	25	49	5	-27*
	Map important quantities.	23	25	29	0	-6
	Interpret results in context.	14	5	3	3	11*
	Make assumptions.	8	8	4	5	4
5	Consider available tools.	4	3	6	5	-2
	Demonstrate tools familiarity.	20	24	4	5	16*
	Understand technology.	0	0	3	5	-3

Table 1, *Continued*

6	Communicate precisely.	13	12	7	2	6
	Use clear definitions.	9	7	1	1	8*
	State symbol meaning.	9	7	0	0	9*
	Specify units and label axes.	4	9	0	0	4*
	Calculate accurately/efficiently.	7	4	3	3	4
	Express answers with precision.	8	4	3	3	5*
7	Discern a pattern.	16	12	15	8	1
	Shift perspectives.	6	2	1	3	5*
	Break into pieces/steps.	23	11	15	1	8
8	Look for shortcuts.	9	6	4	7	5
	Evaluate reasonableness.	9	5	0	1	9*

* Significant at the 0.05 level

At a high level, we observed variability in the proportion of each lesson that provided opportunities for students to perform the actions contained within each MP, as shown in Figure 1. In general, students enrolled in the online course were provided the greatest opportunity to model with mathematics (MP4), to reason abstractly and quantitatively (MP2), and to make sense of problems (MP1). In contrast, the online course interface devoted consistently less time for students to look for and express regularity in repeated reasoning (MP8), use tools strategically (MP5), or attend to precision (MP6). When examining across instructional modes, face-to-face lessons devoted a larger proportion of time to demonstrating actions associated with making sense of problems (MP1) and attending to precision (MP6). In contrast, online lessons devoted a higher proportion of time to reasoning abstractly and in context (MP2) and modeling with mathematics (MP4).

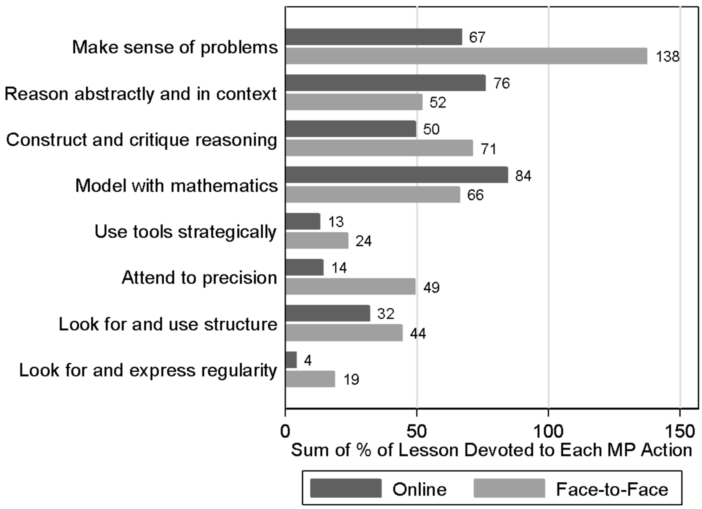

Figure 1. Summative Proportion of Lesson Devoted to Each MP Action.

Note that because a given instructional activity could accomplish more than one MP action, values above 100 percent are possible.

When interpreting the figure, it is important to note that each lesson was not expected to provide opportunities for students to demonstrate each action, and it was not expected that students would have opportunities to engage in each action throughout an entire lesson. For instance, where the action was more specific (i.e., "state the meaning of the symbols they chose"), less representation in the course was desirable. Instead, when evaluating the asynchronous, online course, we looked primarily at whether, on average, students were provided ample opportunity to engage in an action (after considering the relative importance of the action to CCSSM) when using the descriptive statistics to inform the subsequent qualitative analysis.

CCSSM-Alignment

When identifying trends and patterns across lessons, we identified two places of strong alignment with CCSSM. First, the general course structure encouraged students to practice newly learned skills often. Second, lessons often moved beyond symbolic or abstract mathematical examples to relate coursework to real-world scenarios. We discuss each of these strengths in greater detail below, documenting how each strength was achieved in the online instructional setting. We also highlight opportunities to further enhance these strengths by expanding the use of promising strategies.

Opportunities for independent practice

The online Algebra I course provided frequent opportunities for students to practice newly learned skills. Lessons devoted an average of 35 percent of instructional time to activities that allowed students to consider analogous problems independently (MP1.4). In a representative example from a lesson on dimensional analysis, the vendor-hired teacher walked the student through multiple examples related to determining how much paint is needed to paint a wall, providing the opportunity for students to practice a similar problem on their own before introducing a new concept. After the instruction section of each lesson, students were required to complete independent assignments and a quiz. The high proportion of instructional time dedicated to independent practice encouraged active learning in the online instructional setting.

Interestingly, teachers in the face-to-face Algebra 1 classrooms observed devoted an even higher proportion of time (over 60 percent of class time observed) to practice. Part of this gap can be explained by the fact that some of the face-to-face lessons contained no new material and instead acted as opportunities for students to complete worksheets and review problems. In contrast, the online course did not contain any modules that acted solely as review, although if the online course was implemented within a blended model (as recommended by the vendor) this would have likely increased opportunities for practice. The online lessons also likely devoted less time to practice, as the online course was designed to be completed in around 23 hours (if no blended component was added) compared to the semester long face-to-face course, which included approximately 60 hours of in-class, instructional time.

One of the most engaging, although less frequently observed, ways that students were asked to practice new skills within the online courses involved the use of interactive tools. The use of these tools demonstrated the ability of the course to provide students with opportunities to understand how technology could enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data (MP5.3). One lesson used an animation of a turtle walking across a ruler to graph distance and time. Another lesson asked students to fill in blanks to create intervals for a histogram and then drag boxes with values on them into the appropriate intervals. After completing these steps, the boxes connected and created a histogram. Another interactive tool allowed students to practice their analytical skills by adjusting the slope and y-intercept of lines and taking note of how these changes effected the graph. While not frequently integrated, these tools demonstrated possible ways an online interface could encourage exploration and interaction using technology to support student-generated knowledge and greater integration of CCSSM-aligned actions.

Real-world application

Instructors in the videos also often related examples, practice problems, and quiz questions to real-world scenarios. The average lesson focused on applying mathematics to solve real-life problems (either student or teacher-directed) for 49 percent of instructional time (MP4.1), and students were asked to spend 20 percent of instructional time, on average, contextualizing abstract concepts into a relevant context (MP2.2). Additionally, tasks that required students to map important quantities (MP4.2) occurred in 29 percent of lessons on average. Most often, these real-world applications were facilitated through word problems. Most word problems focused on daily life situations (e.g., calculating the cost of phone plans) or integrated content from other subject areas (e.g., calculating the exponential rate at which bacteria colonies multiply in a petri dish). These problems allowed students to see the direct value of the algebraic concepts introduced and practice them in meaningful ways. In contrast, while almost half of online lessons were devoted to using mathematics to solve real-life problems, this was only true for around 22 percent of face-to-face instructional time. For example, during one face-to-face lesson, students spent the entire class completing three order of operations worksheets where they utilized their calculators to work through multi-operational expressions. The sixty problems varied only slightly from each other and were presented in purely symbolic (versus applied) terms.

The final ten lessons on applied statistics did a particularly good job of using real-world scenarios. Sixty-nine percent of instructional time during these final ten lessons was rooted in real-world scenarios (MP4.1), requiring students to apply newly learned statistical concepts to varied topics such as sports, economics, and education. For instance, students were asked to explain the utility of a statistical model or evaluate the representativeness of model outputs through essay questions. Assessment questions also asked students to select which conclusions were supported by the data provided or explain what information could be obtained by analyzing a distribution curve. By shifting the focus towards interpretation, the final ten lessons of the course integrated instructional activities that encouraged deeper understanding by prompting higher-order thinking through student-directed tasks.

An additional note on CCSSM alignment

Comparing the online versus face-to-face Algebra 1 lessons we observed brought to light several structure limitations to CCSSM-alignment within face-to-face classrooms in the district that merit mention. Obviously, because of the asynchronous nature of the online course, as well as the fact that each individual lesson followed the same format, each student enrolled in the online course was exposed to the same exact lesson as every other student who took the course. Whether that is a strength or not is contingent on the quality of instruction received online, but it also depends on the quality of instruction received in face-to-face classrooms. For instance, two of the eight observed face-to-face classes were run by substitute teachers, which reflected the high rate of substitute teachers observed across the district. In classrooms taught by substitute teachers, students were expected to work on practice problems during the entirety of the class.

We also observed frequent distractions during face-to-face classes that limited learning. In one observation, a student left the classroom without permission. The teacher paused the lesson to resolve the issue and needed to backtrack afterwards to remind students what they were discussing prior to the break. Some students regained their focus after the incident, whereas others appeared off-task for the remainder of the class. Although distractions also occurred in the classrooms in which students took the online course, the asynchronous, re-playable nature of the course allowed students to access the same activities and information as each other whenever required. From a curriculum alignment standpoint, this meant that the online format ensured more consistent delivery of materials and opportunities for students to engage with the materials in a uniform manner.

Areas that Lacked CCSM-Alignment

Limitations to the course design also became apparent through the analytic process. Specifically, process reflective tasks occurred infrequently. Additionally, there were limited opportunities for students to communicate with and critique the reasoning of others. Beyond discussing these limitations, we highlighted counterexamples to demonstrate the types of tasks that could be integrated to support greater CCSSSM-alignment and to distinguish between those actions which could be (but were not often) integrated in an online, asynchronous learning environment versus those actions which could not be integrated in such an environment.

Limited process reflective tasks

Several CCSSM-aligned actions required students to reflect during the problem-solving process. However, opportunities for students to perform these tasks occurred infrequently. Students were asked to monitor and evaluate progress (MP1.5), consider whether the problem makes sense (MP1.9), step back for an overview and shift perspective (MP7.2), or evaluate the reasonableness of intermediate results (MP8.2) in two percent or less of instructional time. The lack of process reflective tasks meant that while students needed to know how to solve problems, they were not often required to demonstrate that they understood why.

The practice problems that came closest to being process reflective required students to submit intermediate answers or identify steps required to complete a problem. For instance, one problem asked students to identify, "What mistake did Anya make when finding the slope of a line?" The inclusion of this type of question represented one strategy to encourage students to demonstrate a deeper understanding of the concepts in the asynchronous, online instructional setting observed.

In comparison, face-to-face classes were substantially more likely to facilitate process reflective tasks. In one face-to-face class, opportunities for process reflective actions were facilitated through an independent assignment that required students to check in with the teacher at certain points before they were able to progress to the next part of the assignment. During the individual meetings, the teacher gauged student understanding by checking the students' work and asking them to explain the rationale behind the steps they took to reach their intermediate results. In another classroom, a teacher in a face-to-face classroom paused frequently during guided practice to ask students questions such as "does her answer make sense… what is the problem asking us… did we find what we expected?" Due to the format of the asynchronous course studied, there was limited opportunity for those types of responsive, follow-up questions, and when reflection questions were posed within the online environment, there was limited (or no) social or academic pressure to respond thoughtfully. Although the limited open-ended questions included on the summative assessment were graded by the school-based lab monitor, these professionals viewed their role as predominately administrative versus instructional, checking more often for correctness versus depth (Heinrich et al., 2019).

Limited communication-focused actions

Opportunities for students to communicate mathematical concepts within the course were also rare. Several CCSSM practices require students to communicate a deep understanding of newly learned concepts, such as by explaining the meaning of the problem (MP1.1), justifying conclusions (MP3.4) and improving the arguments of others (MP3.8). Each were observed in six percent or less of instructional time. Communication-focused activities were limited due at least in part to the particular structure of the asynchronous online course studied, which did not provide any mechanisms for students to interact with each other or the teacher. Instead, teachers sometimes asked in the pre-recorded video lectures rhetorical questions that were meant to encourage students to think more deeply about a concept. For example, one teacher asked in the pre-recorded video, "Can you think of how you might use slope to solve problems in your own life?" While these lecture-based questions might encourage students to consider how they might apply the newly learned concepts to real-world scenarios, students lacked the means to practice communicating their ideas to others verbally.

Instead, we only identified written opportunities to practice communicating mathematical concepts. Essay prompts were integrated in 17 out of 34 lessons and usually required students to provide some explanation as to why they answered the question the way that they did. One example included a prompt from the analyzing graphs lesson: "Must a function that is decreasing over a given interval always be negative over that same interval? Explain." Unfortunately, lessons contained at most two short answer problems. The rarity and the lack of variety of communication-focused, as well as process reflective tasks, in the Algebra I course illustrated several key limitations to the asynchronous, online course system studied.

In contrast, it was common practice in the face-to-face classrooms for students to explain nearly every answer during a class period. Many of the opportunities for students to perform communication-focused actions in the face-to-face classrooms came during the guided practice portion of the lessons. Where these opportunities were present, teachers in face-to-face classrooms collaboratively dissected word-problems with students rather than asking students to simply solve a problem. This often resulted in short classroom discussions about the problem and underlying concepts. Probes such as, "You're shaking your head; what would you have done differently?" were used effectively to hear from dissenting students who did not immediately volunteer to speak. This line of questioning, as well as teachers in the face-to-face classrooms regularly clarifying and re-asking questions, allowed students to practice communicating their understanding of newly learned concepts to others in a manner that could not be facilitated within the existing online course structure.

DISCUSSION

The adoption of CCSSM by a majority of state legislatures nationwide encouraged increased rigor and a focus on conceptual understanding, problem-solving, and procedural fluency in mathematics instruction (McDonnell, 2014; NCTM, 2014; CCSSI, 2020). Yet, few prior studies have examined the implications and feasibility of these MPs in an asynchronous, online setting despite millions of students completing courses in this manner (Gemin et al., 2015). This study leveraged artifacts from an asynchronous, online Algebra 1 course developed by one of the largest online course providers in the United States, resulting in a more detailed and larger-scale analysis than present in prior literature.

When examining our first research question – to what extent did one widely used online Algebra 1 course align with the CCSSM? – we hypothesized based on prior literature that the integration of technology-based tools and activities that facilitate interactivity, open-ended assignments, and/or leverage multimodal/multimedia features will support the greatest CCSSM-alignment (An et al., 2022; Harper, 2018; Jesson et al., 2018; Lai, 2017; Ukpokodu, 2008). Consistent with our hypothesis, when implemented, these elements showed high alignment with CCSSM. Specifically, the final ten lessons in the courses focused on statistical analysis integrated student-directed tasks that prompted higher-order thinking (see also Jesson et al., 2018). Many lessons accomplished this by using interactive tools (i.e., graphing software, visualization interfaces, and animations), which have been shown to enhance students' learning experiences in mathematics (Choppin et al., 2014; Dinov et al., 2008; Harper, 2018; O'Dwyer et al., 2007; Ukpokodu, 2008). Greater use of these types of technology-based tools represents one of the potential strengths of digital learning in mathematics, as their integration has fewer hurdles in an online learning environment (Choppin et al., 2014).

In contrast to our hypothesis, the most **consistent** strengths of the online course structure and content included regular opportunities for students to practice newly learned skills and apply mathematical concepts to real-world scenarios (O'Dwyer et al., 2007). Less variability in instructional quality, the repeatability of online lectures, and anytime, anywhere access may also be particularly important strengths in districts, such as the one studied, with high rates of substitute teachers and frequent classroom disruptions (Darling-Aduana et al., 2019). Consistent access to the instructional environment does not in and of itself ensure CCSSM-alignment, but it is a prerequisite to learning that was not guaranteed within the face-to-face classrooms observed. Additional research is needed to determine the extent to which (and in which contexts) this strength of online learning systems outweighs any potential, associated cost (in more passive, lower cognitive demand instructional activities) when it comes to improving student learning.

When examining our second research question – in what ways did the same online courses fail to fully deliver on the intentions of the CCSSM? – we hypothesized that given the pressure to design one-size-fits-all, standardized online courses, the online Algebra 1 course would provide limited opportunities for CCSSM elements most effectively facilitated through student-directed and/or multi-step, complex tasks (Choppin et al., 2014; Daniels et al., 2021; Darling-Aduana et al., 2022; Rice, 2018). Consistent with our hypothesis, activities that required students to communicate mathematical concepts and perform process reflective tasks were infrequently facilitated within the online learning environment. This is consistent with prior research showing that the use of CCSSM-aligned curriculum designed for instructional delivery like that integrated into the online learning platform studied more often results in direct instruction, despite the intended goals of CCSSM to facilitate dialogic instruction (Daniels et al., 2021; McDuffie et al., 2018). In particular, the primary limitation to demonstrating higher rates of conceptual understanding and procedural fluency online appeared to be the absence of a mechanism to facilitate collaboration and discussion among students and the pre-recorded, vendor-hired teacher (An et al., 2022).

However, despite being observed infrequently, we did observe some techniques that were used to provide opportunity engage in some process reflective tasks online. For instance, some practice problems required students to identify the steps required to solve a mathematical problem or evaluate the work of a fictional individual. Considering that students completing Algebra 1 online did not have opportunities to interact with each other or the pre-recorded teacher, questions that required students to produce a written explanation became the only opportunities to express thoughts beyond just answering a question with a number or mathematical equation. This is consistent with prior research indicating that technology-based instruction may be more effective in facilitating writing-based versus oral communication (Bagdasarov et al., 2017). By integrating more prompts to think deeply (along with sufficient time for student reflection) in the pre-recorded videos and problems that require students to produce an explanation, the course could provide more opportunities for students to engage in sense-making and communicate mathematical concepts (Jesson et al., 2018). However, for the full enactment of CCSSM-aligned process reflective and communication-based tasks, supplemental instruction outside the type of mass-marketed, standardized online course studied will likely be required (Harper, 2018; Hawkins et al., 2013; Ukpokodu, 2008).

CONCLUSION

The findings from this study contribute to conversations on CCSSM enactment in the classroom through the analysis of instructional artifacts at a level of detail not previously available. This study also represents one of the first examinations of the curriculum (and subsequent opportunities for student learning) in a widely used asynchronous, online Algebra 1 course. Specific, high-impact instructional practices – such as the use of data visualization tools to support student investigation – are reproduceable across a wide range of settings, including through the integration of digital tools within traditional, face-to-face classroom settings. At the same time, inherent limitations of the asynchronous structure (i.e., around process reflective or communication-based tasks) can be minimized by expanding the use of the promising strategies and supplemental resources identified. Findings have relevance for educators and online course developers looking for ways to enhance the rigor and depth of mathematical practices facilitated by asynchronous, online learning systems.

DECLARATIONS

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APPENDIX A: ALGEBRA 1 STANDARDS FOR MATHEMATICAL PRACTICE OBSERVATION PROTOCOL

Observer Name: ______________________________
Lesson Name: _______________________________

Directions: Rate each of the following actions on an 11-point (0 to 100 percent rounded to the nearest 10) scale to indicate the approximate proportion of each class devoted to each action.

MP1: Make sense of problems and persevere in solving them.

Action	Rating
Explain meaning of the problem (i.e., report back pertinent information and explain how they are supposed to use it to solve a problem).	
Analyze givens, constraints, relationships, and goals (Must be process reflective. Only solving the problem does not qualify).	
Make conjectures about the form and meaning of the solution.	
Consider analogous problems, and try special cases and simpler forms of the original problem (Completing a practice problem after guided practice qualifies).	
Monitor and evaluate progress. Change course if needed.	
Transform algebraic expressions to get necessary information (i.e., simplifying, isolating x).	
Change the viewing window on graphing calculator to get necessary information.	
Explain correspondences between equations, verbal descriptions, tables, and graphs; or draw diagrams of important features and relationships, graph data, and search for regularity or trends (solving isn't sufficient. Need to PRODUCE an explanation).	
Check answers to problems using a different method.	
Continually ask, "Does this make sense?"	
Understand other approaches to solving complex problems (i.e., more than one method). Identify correspondences between different approaches (Different data visualizations, i.e., tables, graphs, equations do NOT count).	

Additional MP1 relevant information, thoughts, or analytic notes:

MP2: Reason abstractly and quantitatively.

Action	Rating
Make sense of quantities and their relationships in problems: Decontextualize (student only, not teacher talking about) – to abstract a given situation and represent it symbolically and manipulate the representing symbols without necessarily attending to their referents.	
Make sense of quantities and their relationships in problems: Contextualize – to pause as needed during the manipulation process in order to probe into the referents for the symbols involved (there needs to be a transfer from abstract to contextual).	
IF YOU DON'T RATE THE SAME AS ABOVE, MAKE NOTE. Attend to the meaning of quantities.	
Know and flexibly use different properties of operations/objects (select from array of tools, demonstrating ability to use each tool; might be accomplished across different problems).	

Additional MP2 relevant information, thoughts, or analytic notes:

MP3: Construct viable arguments and critique the reasoning of others.

Action	Rating
Understand and use stated assumptions, definitions, and previously established results in constructing arguments (i.e., must CONSTRUCT an argument. Using the solution from a word problem to make an argument would qualify).	
Make conjectures and build a logical progression of statements to explore the truth of their conjectures (student would have to demonstrate breaking problem into steps).	
Analyze situations by breaking them into cases.	
Justify and communicate conclusions and/or respond to the arguments of others.	
Reason inductively about data, making plausible arguments that take into account the CONTEXT from which the data arose.	
Compare the effectiveness of two plausible arguments. Identify flaws in reasoning.	
Determine domains to which an argument applies.	
Ask questions to clarify or improve arguments of others.	
Recognize and use counterexamples.	

Additional MP3 relevant information, thoughts, or analytic notes:

MP4: Model with mathematics.

Action	Rating
Apply mathematics to solve real-life problems (includes teacher discussed real-life problems).	
Identify and map important quantities in a practical situation, map their relationships, and analyze the relationships to draw conclusions (i.e., in most cases, solving a word problem would qualify.	
Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense (requires explaining/reflecting on answer).	
Demonstrate comfort making assumptions and approximations to simplify a complicated situation and/or revise assumptions as needed.	

Additional MP4 relevant information, thoughts, or analytic notes:

MP5: Use appropriate tools strategically.

Action	Rating
Consider the available tools when solving a mathematical problem (deciding among various mathematical techniques to solve, has to be multiple options discussed).	
Demonstrate familiarity with tools appropriate for grade level (i.e., statistical calculator).	
Understand how technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Identify and use relevant external mathematical resources to pose or solve problems.	

Additional MP5 relevant information, thoughts, or analytic notes:

MP6: Attend to precision.

Action	Rating
Communicate precisely to others (MC and T/F would not qualify for any of these actions).	
Use clear definitions.	
State the meaning of the symbols they chose.	
Use care in specifying units and in labeling axes.	
Calculate accurately and efficiently.	
Express numerical answers with an appropriate degree of precision.	

Additional MP6 relevant information, thoughts, or analytic notes:

MP7: Look for and make use of structure.

Action	Rating
Look closely to discern a pattern or structure.	
Step back for an overview and shift perspective.	
Demonstrate ability to see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects (have to talk through or show intermediate steps, such as breaking down and analyzing another student's work OR labeling all parts of a graph).	

Additional MP7 relevant information, thoughts, or analytic notes:

MP8: Look for and express regularity in repeated reasoning.

Action	Rating
Notice if calculations are repeated, and look both for general methods and for shortcuts.	
Maintain oversight of the process, while attending to the details. Continually evaluate the reasonableness of intermediate results.	

Additional MP8 relevant information, thoughts, or analytic notes: