

Philosophical inquiry for critical mathematics education

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Abstract: *This paper argues that critical mathematics education requires reflective knowledge, which lies outside of mathematical and technological knowledge, and which can be generated through philosophical inquiry in the classroom. Philosophical inquiry can provide “thinking tools” for questioning, challenging and critiquing implicit assumptions and misconceptions and for the reconstruction of concepts. As such, it can offer a space for critical reflection on mathematics, for the development of an epistemological approach that encourages an enriched, overarching view of mathematics and its connections to the other school disciplines, society and self. It also offers space for the deconstruction and reconstruction of beliefs about mathematics as a form of knowledge, about the social value of mathematical practice, and beliefs about oneself as a mathematics learner/thinker.*

INTRODUCTION

Critical mathematics education researchers (e.g., D’Ambrosio, 1999; Skovsmose, 1994) understand the development of “mathemacy” as an essential mathematics competence, and one of the crucial dimensions of authentic democracy, which goes beyond mere competencies associated with computational and mathematical problem-solving skills, the application of mathematics in complex contexts, and the interpretation of outcomes. It extends to “reflective knowledge,” which involves the making of critical judgments about their social consequences of using mathematical tools (Skovsmose, 1994). As such, critical mathematics education advocates for an understanding of mathematics that encompasses its relationship with everyday life, its uses and applications in the world, and its role as a productive element of human culture. However, Skovsmose (1994) points out that reflective knowing--which he argues is an important aspect of “mathemacy”--is not an ingredient of mathematical or technological knowledge, as its focus lies outside them, on the understanding of the role of mathematics in society, and the implications of using mathematical and technological knowledge in addressing social problems. Thus, we need to look “beyond” mathematics for discursive environments that cultivate reflective knowing, and for instruments that can be helpful in understanding the role of mathematics in reading and writing the world.

What I am suggesting is the opening of an additional space for philosophical dialogue, in order to address on a deep-structural level questions related to social issues that emerge when we read the world through mathematics, and to examine the relationships between those social issues and

students' own experiences (Kennedy, 2018). In this paper I explore this interface between using mathematics for understanding social issues, social critique, and philosophical dialogue in a group setting. I offer a brief description of what philosophical inquiry is, the context in which it is conducted, and outline the potential role of philosophical dialogue as a vehicle and space for critical reflection in the classroom on contestable questions related to social issues, particularly as related to mathematics.

INQUIRY DIALOGUE WITH BIG QUESTIONS IN A COMMUNITY OF INQUIRY

Reflective knowing is “the competence needed to be able to take a justified stand” in a discussion, and an awareness of the implications or consequences of applications of mathematics and technology for society (Skovsmose, 1994, p. 101). Reflective knowing is akin to Dewey’s “reflective thinking”—a specific kind of ratiocination that is aware of its means and consequences. Apart from being a major instrument in mathematical inquiry per se, it can serve as a vehicle for an inquiry focused on complex questions that are not discipline-specific, but act to question the outcomes of the uses of mathematics in society. It includes such questions as “What part does mathematics play in the way we organize our lives?” or questions related to social justice, such as “What is fair wealth distribution? How should wealth be distributed?” These questions can be qualified as philosophical ones, in that they explore political and ethical dimensions of our experience. Such questions are typically “big questions”—common to most people, central to our living together, and contestable, as they do not have one simple answer (Splitter&Sharp, 1995; Wiggins & McTighe, 2005). They are also questions that best lend themselves to communal, collaborative deliberation in a classroom setting, known among educators as a “community of inquiry” (Lipman, 2003, Kennedy & Kennedy, 2011)—a setting in which participants are engaged in putting forward arguments, and evaluating and making judgments about the arguments of others. “Inquiry dialogue,” as Walton (1998) calls it, is truth-directed and aims at collectively arriving at a conclusion or judgment on a common, central and contestable issue that is deemed the most reasonable and acceptable by the community. The criteria for reasonableness and acceptability of a judgment are as follows:

- ✓ The judgment must rely on sound argument and good reasons;
- ✓ The judgment must be clear;
- ✓ It must be well informed; and
- ✓ It must reflect multiple and diverse perspectives (Gregory, 2006, Lipman, 2003).

In other words, viable arguments and judgments are ones that can withstand the evaluation and critique of the community. In a community of inquiry, teachers invite students to propose, evaluate, and build on each other’s arguments, to agree and disagree in a spirit of ongoing, collaborative search for truth. They invite students to make certain logical and dialogical moves, which they

model in the course of the discussion: to ask questions, to agree or disagree giving reasons, to offer a hypothesis, explanation, or make a statement, to offer an example or a counterexample, make comparisons, classify/categorize, identify an assumption, offer a definition, make a distinction, self-correct, among others (Kennedy, 2013). They also facilitate the sequencing of student moves by inviting students to make interventions, by encouraging them to connect and respond to what has been said, by making or asking for clarifications or restatements, by offering or asking for summarization, and by managing turn taking. The teacher acts to orchestrate the inquiry with the goal of focusing and moving the inquiry further through scaffolding the inquiry process with questions, counterexamples, restatements and summarizations. In cases in which the community might align with only one side of an argument, the facilitator may offer a different perspective, another possible argument, or counterexample. For example, she might say “And what would you say if someone said.....”, or, “ What about thinking about this issue in a different way, for example...” Teachers typically choose the initial stimulus—in the case presented here a mathematical problem--and sometimes offer questions or ask students to generate them, one of which is typically chosen to become the focus of the dialogue. The discussion begins with a big question that is contestable, central to the initial stimulus and the main theme raised by it, and relevant to the students’ interest. Ideally, it is student-generated, and related to the math activity that preceded the communal construction of the agenda, for example “Do feelings come in quantities?” How good is math in describing emotions?, or “Is math a universal language?”

In themes related to social justice, the questions that might emerge following math problems about wealth distribution are typically centered around ethics--“Is it ethical to pay illegal immigrants less?”--or normative issues: “What should we do to correct this injustice?” Such “big” questions prompt students to examine and define complex concepts like “fair,” “equitable,” “just”. As participants in a genuine pursuit of meaning about questions that they have taken up by their own volition, students come to develop personally meaningful judgments. Such judgments are of another order from “accepted truth” typically passed on to them by the teacher. Instead, they are developed and pursued through engaged participation in collective, deliberative, reflective thinking about current social structures, and a result of careful evaluation of the ideas and assumptions aired by the group in response to an important question. Such a project draws heavily on Dewey’s idea that inquiry should begin with a particular experience—in this case an experience that involves a mathematical activity. The mathematical activity makes for a shared group event, and the problematization that follows in the discussion can motivate students to inquire further in a search for answers to related, relevant contestable questions. In other words the mathematical activity acts to structure the inquiry that follows it, or to use Dewey’s term, to “occasion” it. What is seen and felt as problematic and perplexing must reflect the experiences of the group of students-

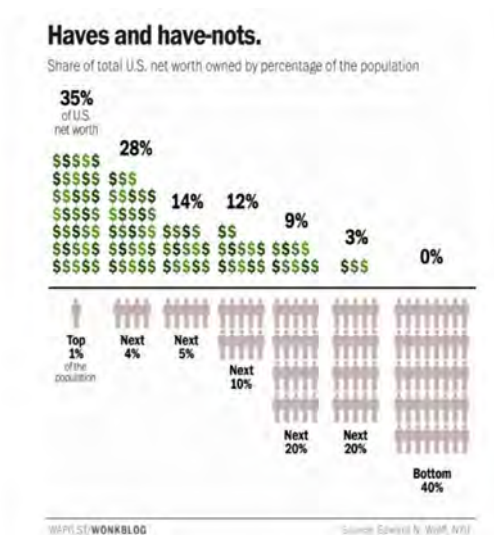
-not only those experiences related to the mathematical activity, but previous personal school and out-of-school experiences as well. Above all, the goal of the inquiry with contested questions related to mathematics is to help students reflect on and challenge deep-seated assumptions, critically explore values and social practices, and discuss social alternatives.

AN EXAMPLE

The excerpts below are taken from dialogues facilitated as part of a project with a group of nine homeschooled students, ranging from 11 to 17 years of age, in a small city in upstate New York, and their teacher, who agreed to co-facilitate four inquiry sessions with us of 1 hour each, using activities and prompts that we agreed on in advance. All discussions were audiotaped and transcribed.²¹

Have and Have-Nots: Representing US Wealth Distribution

For this session,²² we first offered the following diagram taken from the *Washington Post*, May 21, 2015:



We waited for students' exclamations to subside—they were clearly surprised to see the large disparities in wealth in the US. After a short discussion about the diagram, we introduced the mathematics activity below, which was designed to reflect this data:

²¹ The names of all students and the teacher have been changed. The author appears in the transcripts as Nadia.

²² Space does not allow us to include episodes of other dialogues conducted with the same group of students.

Activity 1: Wealth distribution in US²³.

Materials: 10 chairs; 10 students take part in the activity.

10 chairs are used to represent the total wealth of the USA in 2015. 10 students will represent the total population of US in 2015.

The wealth distribution in US in 2015 is as follows:

10% of the population owns 77% of the total wealth in the country; the next 10% of the population owns 12% of wealth, and the other 80% owns 12% of the wealth. Decide how to represent the data above given that 10 students represent the US total population and 10 chairs represent the total US wealth. Using the above data, how are you going to distribute the wealth?

Before the activity was introduced, the teacher and the nine students sat on chairs arranged in a circle. We asked whether the wealth in the room, represented by the chairs on which the students were sitting, was equally distributed. They all agreed that it was. The teacher and the nine students were to represent the US population. The students decided that the teacher would represent the “rich,” as they put it. They were asked to work as a group and decide how to distribute the ten chairs among the ten people—nine students and the teacher—so that the distribution reflects the given information.

First the students noted that the total percentages in the activity and in the diagram amount to 101%, and after a brief discussion they decided that this was a result of rounding and the choice to use pictorial representations in the diagram. They decided that they had to round in order to use chairs to represent 77% and 12% of the wealth. It didn’t take long for them to decide what the distribution would look like, and the activity did not require much facilitation. The group allocated eight chairs to the teacher, who represented the 10% who owned 77% of the wealth, one chair to Sam, who represented what they decided to call the “middle class,” and one chair to the other eight students, representing 80% of the population, who were immediately labeled, “the poor.” At the end of the activity, the teacher lay leisurely across his eight chairs, Sam sat on his chair, and the other eight students stood uncomfortably around the one remaining chair, which was impossible to share.

Students expected that after the mathematical activity everything would “go back” to the original setting—the chairs set in a circle and everyone comfortably sitting and facing everyone else, ready to continue the discussion. This did not happen. The teacher insisted that the group retain the “material context” for the follow-up discussion, which immediately produced the question among a few disgruntled chair-less students of “How is this fair?” Ignoring these protestations, he lay across “his” nine chairs while the chair-less students first stood, then sat on the floor. Although

²³ The idea of using chairs to represent the wealth distribution in Activity 1 is taken from “*Ten Chairs of Inequality*” by Kellogg (2006). It is also possible to use only the diagram or only the activity, and represent the information using chips or cookies instead of chairs.

they well understood the performative aspect of the activity, they were feeling uncomfortably “down.” Since the teacher “refused” to give away his chairs, all others except for one student sitting on “his chair” were obliged to continue the discussion sitting on the floor. There was a sense of shared dissatisfaction with the current chair distribution.

How Should Wealth be Distributed?

The teacher restated the students’ question “How is this fair?” as “Is this fair?” to which we added another two questions:

Should everyone have the same income?

If not, what constitutes equitable (fair) distribution of human goods?

Episode 1

Rod: Rich people don’t just randomly get all this money, they worked really hard for it.

Voices: Not always.

Rod: You look at all of these rich people—CEOs of companies and all of that... It’s not easy to get huge sums of money.... so why should we take away something that someone worked really, really hard for.

Nadia: Rod is saying that we shouldn’t redistribute wealth as it’s “hard earned.” Do you agree with him? Does wealth need to be redistributed?

Teacher: Do I have to give you my chairs?

Zack: Did you fairly earn these chairs?

Teacher: I don’t know.

Zack: Let’s just say, you did.

Teacher: I worked really hard. These are my chairs. And I did it totally legally.

Zack: We’d like to have some of your chairs. You should give us some, but you don’t have to.

Teacher: Why are you saying this?

Zack: Because as far as I’m concerned, it’s the right thing to do to help poor people. But if the money is fairly earned.... it’s not unfair to keep all the money that you earned.

Jill: The person who has all those chairs fairly earned the chairs... and there is a bunch of other people who need chairs and want chairs, there is the consideration that they need to earn their own chairs, but they can’t...

Teacher: They can’t?

Jill: If they don’t have the resources to earn them...

Teacher: The resources are already used up. So, they can’t there are no opportunities for you to fairly earn your own chairs... And, therefore? What is your conclusion?

Jill: I mean...if the persons who has all these chairs takes into consideration what it is to not have chairs... and it’s up to what they decidebut right now what matters is who managed to get [earn] the chairs first. . . so it has already happened.

Teacher: So, does it make the situation not right? And it would be not right because...

Zack: There is nothing for us and you’re not giving us a chance to earn ...[chairs].... but

since you don't care and unfairly hold anything from us why don't we take it...why don't we steal it from you. If you're unfairly holding everything from us why shouldn't we not take it if necessary?

The dialogue then went on to explore several options for changing the situation and its respective consequences, such as the “haves” giving investment money and opportunity to the “have-nots” to earn a living, making donations, or even money being “taken away” from the 1%. At some point the group went back to the initial diagram in the mathematical activity, in order to evaluate the argument that Rich had put forward:

Episode 2

Rod: Actually, these chairs do not show properly how much has been distributed... how much is associated with each chair. It's really a lot...we're talking billions and billions....so one chair might equal a hundred million dollars, and distributed among 80% of the population might be still a good amount...

Nadia: So, you're saying that the one chair might be enough for the 80%. . .

Rod: Right.

Teacher: Everyone see that? Can someone else summarize?

Kye: What he is saying is that that chair can be worth a hundred billion dollars so ... but what I'm saying is that the rich people have so much for themselves ... they have so much that they don't need all of that. . . .

Rod: I just looked it up . . . each chair is approximately 1.8 trillion each . . . so really . . .this is lots of money.

Nadia: So, you're saying the 80% probably have enough.... you're also assuming that the wealth is equally distributed among these 80%, which is not the case... if you look at the first diagram that we looked at, the bottom 40% are shown to own nothing.

Teacher: Also, one chair might be 1.8 billion dollars, but you [the eight students] represent actually how many people?

Rod: One chair is worth 1.8 trillion,

Teacher: ... and this is how many people?

Rick: 80% of 360 mil, so ...uh ...288 mil

Teacher: So, 288 mil people get 1.8 trillion dollars. How much is that per person?

Jill. \$6250. That's not enough.

Next, a few new arguments were broached concerning people with investments who do not need to work very hard to make more money from investments, about rent, about hard earned money that should not be taken away, about the obligation that wealthy people donate to people in need, and about the feasibility of raising taxes so that everyone has at least a minimum living wage.

Episode 3:

Teacher: We have two issues on the table...the first situation is, if everyone has what they need but some have a lot more, the question is, is it fair? The second, Nadia brought the issue that not everyone has what they need anyway, and what should we do about it?

Zack: Have them have what they need.

Teacher: It seems that there is a lot of agreement that if people don't have what they need then there needs to be some sort of redistribution. Is it where everyone is?

Zack: Everybody, raise your hand if you agree with that?

Teacher: Everybody agrees but Eva.

Eva: I'm not completely sure...

.....

Teacher: So, an argument has been put out that perhaps there is enough money when redistributed, even though the wealthy guy like me has a tremendous amount of money. If that's the case, if people have what they need, and I have a tremendous amount is there still a moral question... should we still get rid of this or is it ok for me to have this?

Sam: Well if everyone has what they need ...they should work for more chairs ...but I feel that if you worked for this, you're fine.

Zack: If people have what they need and you have more, the only thing that can tell you to give away would be your conscience.

Teacher: Can someone come up with an argument against that? Sam and Zack have said, no you're fine. If people have what they need, it's ok, and if people want more they need to work. Is there an argument against that?

Lily: You don't really need that much money. And what's the point of having all this money if you don't need and use it. You can give it to other people so that they can have it and use it for what they need.

Teacher: Oh, but I can take really fancy vacations, and buy gold laced suits....

Rod: See...what it boils down to is ...if people have what they need to survive and therefore be happy, you're ok.

Teacher: But wait a second . . . if you have what you need to survive and no more and I have way more, I don't have to work, I can travel the world, can go hiking... you ...you have what you need but you can't go hiking or travelling, can't go to the swimming pools when you want. Does it change anything?

In these three episodes, taken from a one-hour session, the inquiry invokes several contestable concepts that emerge in the course of the dialogue, and a number of questions that could be pursued in the future, here presented in ascending levels of universality:

Is it fair to keep more than you need if others don't have what they need?

What is the relation between how hard you work and how much money that brings? Can it/should it be made into a formula that applies to every case?

Who should decide what is fair in matters of wealth?

Is wealth to be used as each individual wishes?

What does it mean to earn something fairly?

Is the use of violence an acceptable way to resolve unfair situations?

What is the relationship between time and money?

Is the legal always the ethical/moral?

What is conscience? Is conscience always right?

Who are we responsible for?

What does it mean to live in an ethical manner?

How does unequal wealth distribution might impact (advantage/disadvantage) society?

THE POTENTIAL ROLE OF PHILOSOPHICAL DIALOGUE IN THE CLASSROOM

As the above example suggests, philosophical dialogue provides space and “thinking tools” for questioning, challenging and critiquing implicit assumptions, misconceptions, and for reconstruction of concepts (Lipman, 2003). Philosophical dialogue in the classroom can offer the space for critical reflection on mathematics, and thereby facilitate acquiring an enriched overarching view of mathematics and its connections to the other school disciplines, society and self, and also potentially function as “under-labourer” -- to use Ernest’s term (2018a) – that is, to clear obstacles in learning mathematics such as misconceptions about mathematics or mathematical practice, beliefs about mathematics, or beliefs about oneself as a mathematics learner. It may aid in the opening of a “wider horizon of interpretations” that includes a critical dimension (Kennedy, 2018). Philosophical dialogue requires an “outsider” perspective and could furnish a more global view regarding mathematics, its nature, its instrumentarium, its aesthetic and ethical dimensions, and the cultural and political implications of its uses in our society. A widened mathematical perspective could also allow for exploring epistemological assumptions, in examining the role of mathematics in social reproduction, (e.g. in normalizing instrumental and calculative ways of thinking, Ernest, 2018b) and thus in organizing everyday experience. A widened perspective includes the examination of mathematics as a cultural product and an exploration of its aesthetic dimensions. In short, a widened view of mathematics promises to facilitate a deeper, more nuanced, and critical understanding of mathematics.

On the other hand, philosophical dialogue could play important role to counter undercurrents such as misconceived beliefs about doing mathematics and mathematics itself, negative attitudes towards mathematics, or received notions about who can do math and is good at it. Ernest’s use of the term of „under-labourer“ is particularly fitting to philosophical dialogue as a potential force in the reconstruction of students’ beliefs, attitudes, images of mathematics and thus could potentially work as a powerful mechanism in breaking the „failure cycles“ (negative attitudes — > reduced learning —> mathematical failure) that Ernest describes. Since philosophical inquiry is premised on the student’s questions and on their active engagement in the rethinking and the reconceptualizing of received views, philosophical dialogue typically involves a process of taking apart and putting together, weighing differences, reformulating and reconceptualizing. Such a collaborative engagement in developing students’ personal views promises to produce deeper engagement with the discipline and world-view of mathematics, and thus to influence student future mathematical experiences.

In closing, philosophical dialogue could play a role in developing an expanded and more critical

view of mathematics--one that offers more meaningful connections and interactions with students' personal experience and the broader culture. As such, dialogical inquiry represents a potentially transformative classroom practice in engaging, challenging and reconstructing students' beliefs, attitudes, and engagement with mathematics.

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