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Mathematics education, body and digital games: the perception of the body-proper opening up horizons of mathematical knowledge constitution

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Abstract: *This article investigates the perception of students in the first year of high school in relation to their own body language, while mathematically they conjecture a way to improve their performance in an electronic bowling game, which uses body sensors for own actions of the game. Interactions were performed by a group of four students from a public school in Brazil, in order to perform mathematical activities with the Sports game on Xbox One with Kinect. Mathematically, the students discussed issues related to angulation, velocity, position relative to an axis and correlated to their digital being (game avatar) controlled by their biological body. In this sense, we are drawing up on embodied cognition articulated with the conceptions of perception and body-proper arising from the phenomenological view discussed by Merleau-Ponty. We understand that students' perception is shown by the acts of being-with, thinking-with and knowing-doing-mathematically-with-Digital-Technologies. Nevertheless, the perception of the body- proper opens up horizons of knowledge constitution that are not commonly seen in the mathematics classroom.*

MOVING TOWARDS RESEARCH

This research began with an investigation about the constitution of mathematical knowledge with Digital Technologies (DT). In this perspective, video games are thought-provoking DT, when one wants to combine playful resources with the constitution of mathematical knowledge by students. Thus, in our research, we worked with Sports game, which is a game played with Xbox (which is a kind of video game) and with Kinect device (which is a resource that is responsible for recognizing bodily movements). With these resources, we launched us, philosophically, to the primacy of perception, which is an idea developed by Merleau-Ponty (1990). This can be affirmed, once these technological resources open a new horizon to the perception of the body and, consequently, of the cognition with it. Thereby, we understand the body, according to Merleau-Ponty (2006), as being body-proper, a unit, not fragmenting what is physiological from what is psychological (mind and matter), being a singular organism, which until then is not without the materiality of its body, or the world, where it meets the other. Also, Rosa and Bicudo (2019)

understand that, like the life-world, the body-proper is not taken as given in itself, in an objective way, but as the different ways that constitute this carnality and the different ways of being alive. For that, the living body is constituted in a double way, being physical, biological, which allows it to evaluate its qualities and feel sensations, launch itself into the world.

Particularly, there is the constitution of mathematical knowledge with DT, because there are sensory devices that constitute and reproduce the player's bodily movements (which happen through player's avatar) and create a world in which mathematics can be perceived with other nuances. It happens because,

[...] all perception takes place in a certain horizon and finally in the world [...]. The perceived world would be the background always presupposed by all rationality, all value and all existence. Such a conception destroys neither rationality nor the absolute. Seeks to get it down to earth. (Merleau-Ponty, 1990, p. 42).

Thus, by diving into the philosophical thought of the Frenchman Maurice Merleau-Ponty (1908 - 1961) about perception, developed under the bias of phenomenology, we obtained significant understandings about embodied cognition, which we propose to discuss in this article. We will do this, therefore, assuming the embodied cognition through the idea of perception phenomenologically understood, that is, as a non-tetic and pre-reflexive experience, we understand that we will have access to the truth (Merleau-Ponty, 2006). It is this significance that will guide our steps in the production of knowledge in this investigation. Therefore, our research question is: "how does the perception of students in the 1st year of high school show up in relation to their own body language, when they conjecture mathematically (in terms of knowledge constitution) a way to improve their performance in an electronic bowling game, which uses body sensors for the game's own actions?"

Therefore, investigating the students' perception enable us to understand how the constitution of mathematical knowledge will take place. In this sense, perception depends on the subject, who is already with the "world". For this reason, it is necessary to know the technological resources chosen, that is, to glimpse the portrait of the microworld constituted for the investigation. Thus, the experience with Sports-with-Xbox-One-with-kinect is not just the combination of the game and the sport. When the players choose to live this game experience, they can have a movement analysis experience different from the one they have when playing sports in a traditional way, or when playing another type of game, as if they were immersed in another world, besides of the one commonly perceived for that (mundane reality). They are in the game, represented by their digital bodies drawn on the television screen, and their movements have different responses from the ones they would have, if there were no videogame, although there are similarities. Notwithstanding, there are differences and similarities, which allow the mathematics of this analyzed background in

this research to project itself, with specific meanings, to the universe of the young subjects who were investigated.

METHODOLOGICAL MOVEMENT

We chose the electronic game Sports, which features different sports, including the simulation of bowling because there are many mathematical elements associated with this game. When each player chose to "live" the gaming experience, they reported to have a different movement analysis experience from the one they have while playing sports in the traditional way or when playing another type of game, because they were immersed in the mundane reality and their horizon were extended to the cyber world. Therefore, the four students were invited to analyze their individual and group plays in order to improve their performances in the game. They used the perception of their own body, the perception of their colleague's body and the perception of their body on the TV screen when performing each bowling ball throwing motion during the game. In addition, they were oriented and asked, for example, as follows: a) by making similar throws or trying to vary the speed in which the ball is thrown. Were the results very different? b) would it be possible that the ball movies so slowly that it does not reach the end of the lane? Justify your answer. c) how does the team ensure that the throws have different speeds, when compared one to another? If not, repeat them carefully. d) how does the game define the speed in which the ball will move? Justify your answer.

The game rounds experience was filmed, transcribed and analyzed. For that, four three-hour long meetings were necessary, in which the students got sufficiently acquainted with the game, created their avatars, tested the ways of throwing the ball and sought through the orientations to improve their individual performances as well as their performance as players.

THEORETICAL MOVEMENT

Research on body language defines the term "gesture", as it is used in a broad sense: a physical movement of a body part (e.g., hands, arms) (McNeill, 1992) in communicative situations. However, it is not only a pure movement of a body as a visible object, but the actions performed through it (Rosa & Caldeira, 2018), because,

Once the detriment of sensation is removed, a face, a signature, a conduct are no longer mere "visual data" that we would need to look for in our inner experience for psychological significance, and the other's psyche becomes an immediate object as ensemble impregnated with immanent significance. More generally, it is the very notion of the immediate that is transformed: henceforth, the immediate is no longer the impression, the object that is one and the same as the subject, but the sense, the structure, the spontaneous arrangement of the parts. (Merleau-Ponty, 2006, p. 91).

Thus, the meaning given by the gesture is more than mere a physical classification. There are aspects of "intentionality", for instance, the manner of motion (how the motion was carried out) and the path of the motion (the directionality of the motion) that also play a role. In the process of throwing the ball, the students' body posture may reveal plausible aspects of the student's own mathematical thinking. At the moment in which they perceive their gestures, senses, structure in their actions, this same perception takes place and it allows us to

[...] link the "physiological" and the "psychic" to each other [which makes it possible] [...] the fact that, reintegrated into existence, they no longer distinguish themselves as the order of the self and the of themselves, and that they are both oriented to an intentional pole or to a world. (Merleau-Ponty, 2006, p.129).

They are both (physiological and psychic) connected to the world and, thus, our body-proper moves and:

Each determined movement occurs in a medium, on a background that is determined by the movement itself (...). We perform our movements in a space that is not 'empty' and unrelated to them, but which, on the contrary, is in a very determined relationship with them: movement and background are really only artificially separate moments from a single whole. (Merleau-Ponty, 2006, p.192-193).

This artificiality causes theorists to separate the body into parts and pursue a classic, objective, objectifying categorization. As an example, McNeill (1992, p. 11) used the term "gesture" to mean "arm and hand movements ... closely synchronized with the flow of speech". Kendon (1980) refers to the spontaneous hand movements produced while speaking of "gesticulation". In similar lines, Sfard (2009, p. 194) defines "gesture" as a "body movement that fulfills the communicational function". In fact, it is a clear way to categorize and to try understanding these movements and express them from parts of the whole (Farsani, 2015, 2016). However, for us, not treating the background in terms of the surrounding world makes limited actions in terms of what phenomenology advocates. Therefore, we define "gestures" as movements of hands or arms that are articulated while the gesturer (student or teacher) intends to attribute cognitive or educational meaning to what they speak in the context of the world, its setting, with what surrounds it. Thus,

The gesture is before me as a matter of fact, it points me to certain sensitive points of the world, invites me to meet him there. Communication takes place when my conduct finds its own way on this path. There is confirmation of the other for me and of me for the other. (Merleau-Ponty, 2006, p. 251-252).

The starting point of perception based on Merleau-Ponty's phenomenology is showing how the classical analysis of perception, especially which carried out by empiricism and intellectualism, let the phenomenon of perception escape. Characterized by the relentless pursuit of structuring

knowledge, they started from different assumptions: the empiricism of experience and the intellectualism of thought as the ultimate basis for absolute knowledge (Silva, 1994).

Initially, Merleau-Ponty (2006), brings the understanding of perception through empiricism. Perceiving an object, according to this view, means "[...] to have representations, 'sensory data' or 'ideas' of it caused by the way it acts on the sensory organs of the perceiver" (Matthews, 2010, p. 36). Thus, empiricists defined perception as sensation, that which is entrusted to the sense organs as a pure quality or impression.

For Merleau-Ponty (2006, p. 25), this analysis distorts the idea of perception. If we take as perception the quality of the object given by sensory organs, just as empiricism did, we would be assuming the object as full and determined, that is, "[...] quality is not an element of consciousness, it is a property of the object". This error stems, according to the author, from the loss of the world, from a world taken in itself, in which everything is determined. When our eyes experience the image shown in Figure 1, it may be possible to see how much we are "stuck" in the world of defined objects, trying to define exactly what we are seeing.

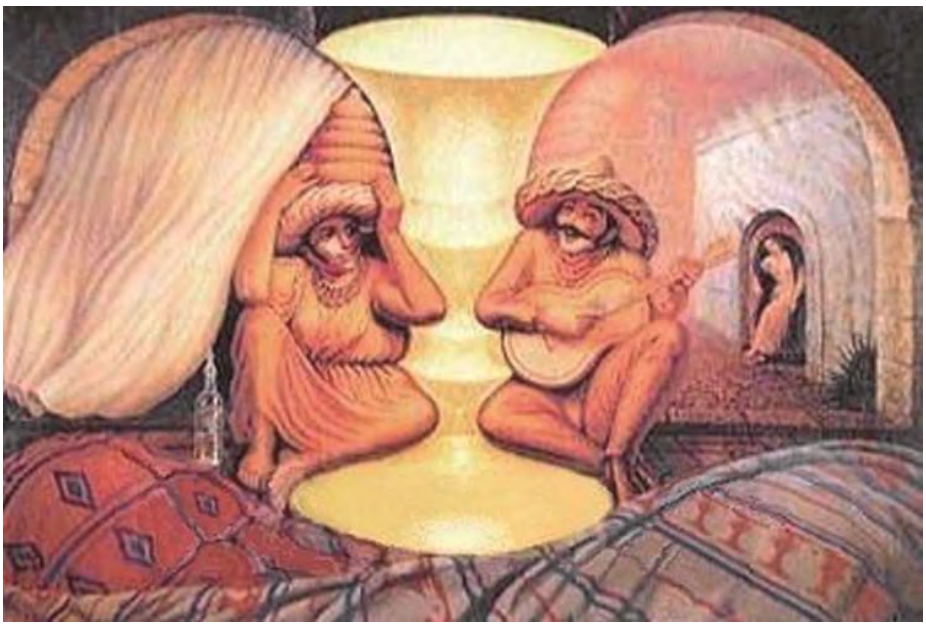

Figure 1 – Old Couple or Musician, Salvador Dalí (1930).

It seems to us that our vision strives to seek alternatives to determine what, perhaps, presents itself as a confused spectacle at the exact moment that this image is experienced. There are "[...] an

infinity of manifestations that refuse its full formalization in the name of the richness of the experience" (Basbaum, 2005, p. 27), having to retake it at every moment to define it again. In Figure 1, at first glance, two young men are observed by a young girl while one of them, who has a mustache, plays a serenade. Or perhaps we can catch a glimpse of an old couple who are smiling at each other. Who knows there is a golden cup right in the center of the figure? Thus, we no longer notice the serenade boy, his friend and the young girl, because the image has been transformed, being partially or totally replaced by a current one. This set allows knowledge to escape with determined, pure and absolute qualities that empiricists admitted (Merleau-Ponty, 2006), as it is not possible to arrive at a single precise and finished truth.

In this way, there is an inexhaustible flow of perspectives that can be experienced, because, at every moment, the totality of the perceptually field, which is intentionally constituted, opens to new possible experiences, without ceasing. Thus, the perceived image announces more than an isolable quality and immediately experienced with the sense organs, it announces "[...] a whole, which is the way we perceive an object that is always affected by its relationship with other [objects]" (Matthews, 2010, p. 40). This means that our experiences in the world are a whole (an unlimited background) in which the perceived is highlighted by detaching itself from this background in the plasticity of the movements of the structure named by Merleau-Ponty (1990) as "figure-and-background" or "figure on a background".

Perception becomes an 'interpretation' of the signs that our senses provide in accordance with the bodily *stimuli*, a 'hypothesis' that the mind evolves to 'explain its impressions to itself'. [...] In this way we are drawn away from reflection, and we construct perception instead of revealing its distinctive working (Merleau-Ponty, 2006, p. 61-62 - emphasis added).

Seen in this way, Merleau-Ponty (2006) understands that perception goes beyond the idea of sensation (empiricism) and of a simply logical act (intellectualism). The first, due to the fact that, for empiricism, the world is given in itself by the sensory organs, without a perceptual awareness; the second because this awareness is defined by the absolute exteriority of the parts as an absolutely true thought system.

Once the damages of the objective world are removed, the perception as Merleau-Ponty understands it, offers us "[...] a conscience that does not have the full determination of its objects" (Merleau-Ponty, 2006, p. 81), but which opens up about things through the experience lived. The perception opens up towards the truth through the relationship between me-other-world in a nascent state, the phenomenal field. This field, considered transcendental by phenomenology, awakens "[...] the thoughts that are constitutive of the other, of myself as an individual subject and of the world as the pole of my perception" (Merleau-Ponty, 2006, p. 94), overcoming the phenomenal field of the thinking "subject" as totally separate from any involvement with the world (Matthews, 2010) and having the body-proper as a general means of communication between the

subject and the world (Merleau-Ponty, 2006). Thus, bodily experience, as this means of connection, assumes an important role in perception.

MOVING US TO RESEARCH DATA

We present parts of a video extract that occurred on 06/19/2019. The activity proposed that the students prove that it is possible to throw the bowling ball in different speeds in the game and answer how the game identified this variation. To do this, the students measured the time it took for each throw to reach the first pin, considering different speeds. We present sequential images (Figure 2), that show the gestures of the students connected to the bowling game and its surroundings, in order to think mathematically about the presented question.

Figure 2: Sequence of Bowling ball throwing (side view)

The student was in the game, throwing himself into a digital body. The moving ball appears on the TV screen. The body-proper movements were perceived by the students as actions that directly interfered in the *position* and *speed* of the ball. They asked, for example: "Is this the reading of your hand in the game or is it your body's movement?". In this sense, the position of the hand or the movement of the body became agents of effectuation of the "ball effect" and would interfere, at first, in its speed. It was being-with-the-game that showed itself symbiotically. In dialogue, the answer emerges as: "I think it's body movement, because, with a closed hand [Figure 2A], you have more stability in your hand and, with an open hand [Figure 2C], you end up letting your hand go farther". This statement, for us, implies what Merleau-Ponty (2006) discusses, once there is the meaning that the hand is no longer just a hand, it becomes a ball in mundane reality and "hand + ball" with play, the sense it is no longer a mere visual datum of those we would seek in our inner experience for psychological significance. In this case, the *psyche* becomes an immediate object

as a whole impregnated with an immanent meaning, which is linked to the record of possible speed of the ball. This mathematical speed is perceived with the body itself. The obtainment of a faster speed of the ball in the game is determined by the velocity of the hand movement, which is executed, as well as, the angle of the player's body. Thus, in the movement of perceiving with the game screen, one of the students suggests "make this movement. Try to move with your leg [Figure 3A and Figure 3B], I think, it doesn't make a difference. For me it worked!", referring to the interference of the body in a move that allows the best direction of the ball in relation to the pins. He senses a know-how-with-the-game and, on the other hand, another student taking this situation as a base, says "angle, the angle with the lane, like, a… So, throwing the ball like this, then, more like it, it goes, it goes farther from the ground." in the sense that the course of the ball on the lane reaches a longer course, which suggests, in this case, a longer course on the same lane. The mathematical knowledge is constituted with the body, because the students perceive their movements and think-mathematically-with-the-body-with-DT. Although the concept of angle is initially remembered from a perspective of mathematics classroom, when the student said "angle, the angle with the lane, like, a…", this concept is constituted in the process of analyzing the body-proper through a situation with DT, with the game, with bowling lane. In addition, when the student said "angle, the angle with the clue, like, um…", this concept is constituted in the process of analysing the body-proper through the situation with DT, with the game, with bowling alley. That is, when stating "it goes farther from the ground" the student deals with the "ground" of the virtual environment. Also, when he stated "throwing the ball like this" the angle is projected, imagined, constituted with his body-proper. So, the mathematics is constituted through situated learning in a cyber world.

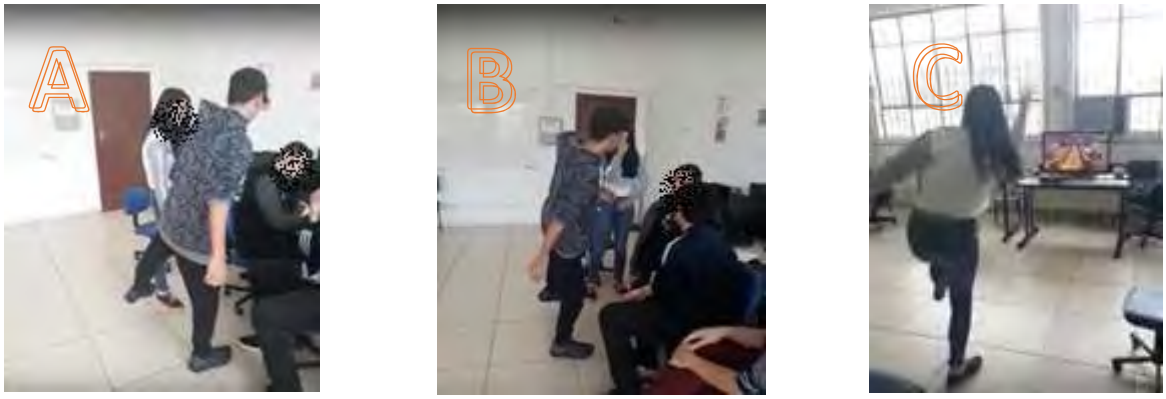

Figure 3: Leg movement as a way to increase the speed of the ball in terms of balance point

Reflection around the other's body perception continues and another participant states, "I think… Don't mine, I think it was the arm angle like when we made the normal move the arm is a little off-center, so to the side… So when she [Figure 3C] lowered her arm it was like this" which suggests to us, as Merleau-Ponty (2006), that the gesture is facing the student as a question to be

reflected, the gesture indicates certain sensitive points of the world, the centrality of the arm. It happens when the perception of movement finds meaning in the movement itself with the game. In addition, there is confirmation of your student's movement by the student and the avatar by herself. In this sense, there is the formal expression of the act of perceiving: “Okay, I put that: the more centered arm on a vertical line gives the ball more stability and precision. We take the acts of being-with, thinking-with and knowing-doing-with-DT (Rosa et al, 2018), , modes of students' perception of their body language, by mathematizing, in order to modify game actions, in order to understand the speed of the game. Each corporeal-with-the-game movement takes place in a mundane environment, on a digital background, because the game is already connected. This connection is determined by the very movement of throwing oneself into the digital, throwing the ball, reflecting mathematically on evolution strategy in/with the game. So, the mathematical knowledge constitution happened with the body-proper and this acquired a different sense from that occurs in classroom. The mathematics got meaning, because the students have wanted to improve their performance in the game.

In another situation, on 10/31/19, the group records themselves playing to analyse the movements that generated the effects of speed and curve, this second effect leaves a bluish trail on the ball, as shown in figure 4:

Figure 4: Blue effect

Two students are responsible for recording and analysing this blue effect. The question the pair wants to answer is “What is the relationship between hand movement and speed with the effects of the ball?” In this case, all four students understood that the effect was related to a different pitch, making a curve with the hand instead of just taking it forward. They considered that speed would have no influence in this case. After playing a game with the aim of unravelling this movement

pattern, recording themselves in videos, loading the videos on the computer and analysing them for a few minutes, the following scene occurs:

At the beginning of the scene one student starts answering the question about the similarity of the movements, but describes the difference between them more, saying first "He [the other student of the pair] opens [his arm] further back … And ... I [open my arm] [...] just in front [of my body] ... Yes, he [...] [takes his hand] back there and opens more. I go more closed here and open more in the front [of my body]". When the first student claims other opens further back [his arm], he refers to the beginning of the movement. Observing the initial frame of one and the other doing launching movements in figure 5, the difference calls the pair's attention.:

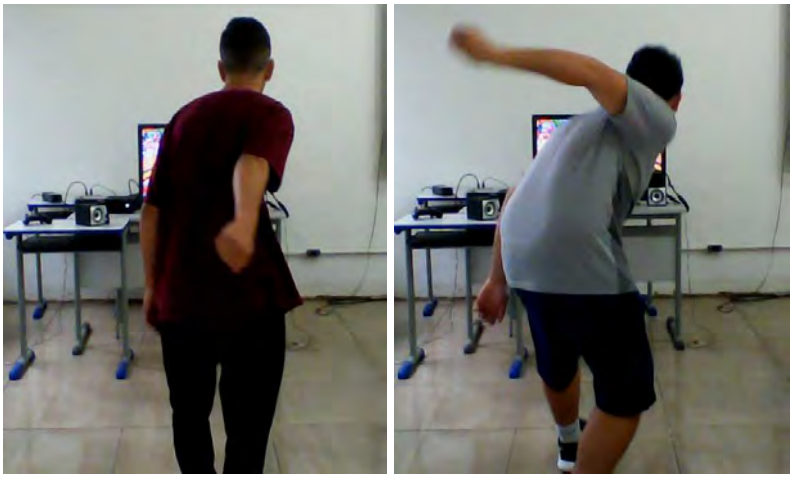

Figure 5: Lauching movements

Despite the discrepancy being so great, the pair had not identified this difference before analysing the videos. Perception connects being with the world and, even though two people who observe the same object may have different perceptions, even so, the perceived is neither created nor sealed, as Seidel and Rosa (2014) argue. Each player was launching himself in his movement, identifying movement patterns that resulted in the desired effect for analysis, without comparison with the other body. When they see each other in the video, they turn to differences, what stands out are the points of disagreement. At these different points, the pair finds movements that do not determine the effect of the event, from the other body moved in another way and still achieved the desired effect. Unlike the movements of the other body, each one identifies unnecessary movements of its own body. The perceived before was not wrong, it was the meeting of the intentional threads that connect the being in the world, but the perceived was his movement, and not his movement compared to another. Now, with the video, it is noticed that not all the pattern created in the movement of only one body defines the effect.

Despite understanding the similarity was the movement of the arm in front of the trunk and deciding to use the available images to measure the angle of the beginning of the movement of the arm in front of the trunk in relation to the trunk, this was not the angle that the first student begins to measure from the second student's body, only yours. When analysing himself in the video, he selects the angle he explained at the beginning of this process, outside this scene. The first student's movement was measured in a way, as he says "Yes, I measured it here (figure 5). However, [name of the second student]'s movements were not measured in the same way". The first student measures the angle that appears in initial movement of the second student, as shown in the figure 6:

Figure 6: Angle 1

Then, he measures his own movement, measuring the greatest angle that the arm makes with the torso at launch, as shown in figure 7 and he explains before this scene:

Figure 7: Angle 2

The first student understands what he wanted to do, but we observed that his perception on second student's movement was so different, in relation to his own movement, that it does not follow the pattern he defined. The first student intention of analyzing the second student's movement was the same as analyzing his movement, but when he sees the second student's movement on the screen, he launches himself from another angle. We understand that intentionality always directs the being towards the world (Merlau-Ponty, 2006), therefore, another angle, which is observed from the second student, is not interpreted by us as a slip. For the first student, it was more important to measure the angle of the beginning of the movement of the second student, to be able to quantify this, than to analyze similarities between the initial moments that he chose in the process. In a way, the need is to understand this movement, with quantification, because it is so different from his movement.

A reason why the first student's movement is so different from the second student' movement is suggested by the third student, who speaks "Look! He doesn't even play volleyball!". The others do not react to this sentence and the researcher asks, "How so?". The third student repeats the information "It doesn't even seem like he plays volleyball". This statement was not discussed in that moment. But, analyzing figure 6, we see that the initial position is similar to the initial position of a serves in volleyball. We have the expression of third student's perception, which connects the second student's movement with the volleyball serves movement. It is possible to observe the similarity between throwing the bowling ball and the movement of serves of the volleyball. We understood that the second student seems to have done this process, perceived by the third student.

We understand from the precision that the analysis of the video allows that it is necessary to think about delimitations that were not necessary before, such as: when does the movement of launching begin? From the moment you want to launch, the movement for that starts, however, when is that? The group needs to define this beginning and, in this process, have the decision of one person, and the other person is subject to the decision. So, the process of constitution of knowledge does not run equally among the participants, because their subjectivities are presented.

The last scene, then, presents the measurement of the angulation of movements that are obtained as a game update a specific visual effect. However, when the first student makes the clippings of the video frames to measure the angle, he notices very different selected moments of the movement of one player in comparison with the movement of the other. We understand that this is justified by the fact that what drew attention in the comparison between movements was the difference in how it started. The first student makes the cutouts, defines the moment to measure the angle of his movement and, when looking at the second student, perceives such a big difference from his that he measures the angle of that moment specifically. He forgets the initial intention, which was look for similarities between the two movements, because the game's response to them was the same. He just sees the difference, because, intentionality is not subject to intention. It is shown in the acts of being of the body-proper. He needs to understand this difference and measure it. As he

measures, he observes that it looks like the volleyball's serve position and then finds an explanation for why the second student's body moves so differently his body.

Thereby, we understand that the constitution of the mathematical knowledge of students of the 1st year of High School when playing Sports Rivals (bowling) with-Xbox-One-Kinect is shown by the expression of the perception of the lived movement, while the students express/discuss their movements while playing and articulating mathematical thoughts related to the perceived; is shown by the expression of the perception of the movement of another body, when subjects are at the heart of their perception, in order to express perceptions during the movement of other colleagues playing, or discuss the perceptions at those moments, which show mathematical knowledge; and shown by the expression of the perception of movement recorded on video, which is revealed after students have recorded themselves playing and analyzed the videos produced, looking for patterns that the game would be identifying, in order to update their images in another technological resource with the same visual effect, expanding and improving your mathematical and player performance.

MOVING US TO FINAL CONSIDERATIONS

Understanding perception of students on their bodies when they think mathematically was the main topic of this article. Our interest was the investigation about embodied cognition under philosophical lens. As we illustrated it, students' perception in relation to their own body language, when they mathematically conjecture a way to improve their performance in an electronic bowling game, was not solely minimized to verbal messages as it has been traditionally studied. Students' perception and mathematical understanding can be manifested through movements, through the body-proper (Merleau-Ponty, 2006).

In this study, the constitution of students' mathematical knowledge happened with Digital Technologies, which uses body sensors for own actions of the game. But the knowledge has been constituted in a long process of perception about the students' bodies. The perception of movement finds meaning in the movement itself with the game. So, the corporeal-with-the-game movement takes place in a mundane environment, on a digital background, because the game is already connected to the students' bodies-proper. Phenomenologically, technology is a mode of revealing. Only when we allow our attention to rest on this fundamental characteristic does that which is new in modern technology show itself to us. Enframing means that way of revealing which holds sway in the essence of modern technology and which is itself nothing technological (Rosa, 2020). On the other hand, all those things that are so familiar to us and are standard parts of an assembly, belong to the technological. By considering the Enframing as an action beyond the use of frames, beyond dividing tools and resources, it is possible to propose a reflection on the relation with DT and consider it like being-with-DT, thinking-with-DT and know-how-to-do-with-DT. Our body-with-technologies is

no longer conceived as an object of the world, but as our means of communication with it, to the world no longer conceived as a collection of determinate objects, but as the horizon latent in all our experience and itself ever-present and anterior to every determining thought.(Merleau-Ponty, 2006, p.136-137).

So, the body-proper is not an object glued to the subject, but an implicit and confused unit (opposing the reflexive movement that separates the object from the subject and the subject from the object), because we are body, we perceive the world with our body.

Furthermore, as professional mathematics educators, we strongly believe that, regardless of a mathematics teacher's experience, it is always worth investigating the forms, styles and the quality of knowledge constitution that happens in mathematics classroom. For us, it is so important in professional teaching practice to pay attention in embodiment cognition, in perception of students' movements and in the proper movements of their bodies. We believe that optimizing of these very subtle and movements that are showed in classrooms can have a direct positive effect both, on teaching and learning process and also understanding students' perception of the mathematical task. One of the recommendations and practical implications in mathematics education is to perceive the body language in teacher education courses both for pre-service and in-service teachers in order to raise awareness to the understanding function of embodied cognition.

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