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Appropriation mediates between social and individual aspects of mathematics education,

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Abstract: *This research aimed to answer the following question: Why does dialogue deepen mathematics learning? In order to answer this question, the study author attempted to define the characteristics of appropriation, which connect the deepening of individual mathematics learning and that of social mathematics learning, by using the sociocultural approach. In addition, the study author proposed an extended model of sign appropriation and use by incorporating the learning community and the other individual learners as dialogue partners into it based on social constructivism in mathematics education. Finally, this paper explored the following research question: In order to deepen the learning of mathematics, it is necessary to have appropriation to compare and transform the original concept of the learner and that of the speaker, and the suitability of this appropriation is secured by dialogue. The author also identifies appropriation and its two characteristics—dynamic composition and mutual composition.*

Keywords: Appropriation, Sociocultural Approach, Dialogue, Dynamic Composition, Mutual Composition, Suitability, Creativity

INTRODUCTION

Many studies in the mathematics education field have empirically and theoretically referred to deepening mathematics learning through dialogue (e.g., Alrø & Skovsmose, 2004). Why does interacting with others in the learning community deepen mathematics learning? How can individual learning be connected to the learning community through dialogue and thus deepen mathematics learning? Furthermore, how can this connection be explained theoretically? Few previous studies have dealt with these questions. Ernest (1998) referred to the deepening of dialogue and mathematics learning from the viewpoint of social constructivism based on the sociocultural approach, but structurally, this study showed the deepening of mathematics learning within an individual. The explicit connection between individual and learning community was not specified. In addition, Sfard (2008), who pioneered the unique concept of commognition based on psychology and philosophy, stated that communication refers to thinking itself, and it emphasizes

the connection between individual thinking and the learning community. However, the structure between them was not shown in that research. The clarification of this structure prompts the consideration of the following questions regarding the deepening of mathematics learning and dialogue. This question is why does dialogue deepen mathematics learning. This question depends on the definition of the term dialogue in this context. In this paper, the author defines dialogue simply as the construction of new knowledge through the interaction of spoken language with someone who works together to solve a problem of interest. Considering this question by using the definition of this dialogue will indicate the necessity for—and quality of—the dialogue in mathematics learning, and this could prove very useful in future mathematics education practice.

PURPOSE AND METHOD OF THIS RESEARCH

This research aimed to clarify the dialogue structure in mathematics learning in order to answer the question: Why does dialogue deepen mathematics learning? An important concept to answer this question is appropriation in clarifying this dialogue structure. Therefore, this paper had a second purpose—that is, it presents the characteristics of dialogue related to this concept of appropriation and then answers why dialogue deepens mathematics learning. This research mainly involves literature research. In order to achieve these dual purposes, this study defined appropriation using the sociocultural approach in psychology and the sociocultural approach in mathematics education; furthermore, it pointed out characteristics of appropriation. Then, it connected the individual learners and their learning community through appropriation and clarified the structure of dialogue. Finally, based on this clarified dialogue structure, the study considers the other characteristics of appropriation.

THE CONCEPT OF APPROPRIATION AND ITS CHARACTERRISTICS

Ernest (1998) argued for deepening individual mathematics learning through dialogue, and Sfard (2008) argued for the importance of communication between dialogue partner's thinking. Both of these claims could assert the importance of the connection between the individual thinking and the learning community thinking through dialogue. This connection is related to the concept of appropriation. Furthermore, it enables the structuring of appropriation. This section provides a definition for the concept of appropriation and its characteristics.

As a concept, appropriation emerged from Vygotsky's psychology and Bakhtin's philosophy of language. Rather than "appropriation," Vygotsky used the term "internalization" (Vygotsky, 1978). In Bakhtin's philosophy of language, the appropriation concept appears in his explanation for the polyphonic nature of spoken language. Bakhtin's (1981) argument was as follows:

> The word in language is half someone else's. It becomes ''one's own'' only when the speaker populates it with his own intention, his own accent, when he appropriates the word, adapting it to his own semantic and expressive intention. (Bakhtin, 1981, p. 293)

This cited text outlines the concept of appropriation. This term refers to the process through which an individual discusses a certain concept after first borrowing it from another individual and then gradually forms their own concept based on the borrowed concept through the concept of self-concept formation. Forming one's own concept after borrowing another's concept implies that metacognition works to compare the self-produced concept and the concept produced by the other.

How does Vygotsky's concept of internalization differ from the appropriation concept? The sociocultural approach in psychology defines internalization as the reconstruction of knowledge within an individual; this process is brought "under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). This concept of internalization is triggered by interaction with friends and teachers in learning activities, which range from the whole process of reconstructing knowledge within an individual to the concept of reconstructing knowledge within a social process that occurs between individuals (Leont'ev, 1974). Cazden (2001) argues that internalization can be misleading and contrasts that internalization, appropriation, and constructivism. The misunderstanding in internalization involves the image of passive and static with regard to internalization, where learners easily copy and transmit the content of social interaction (Cazden, 2001). On the other hand, appropriation has active and dynamic implications. Cazden (2001) argued, "Internalization implies a unidirectional process: Only students are expected to internalize what they hear and see and read. Appropriation, by contrast, can be reciprocal" (p.76). In other words, internalization strongly envisions a one-way communication of learning content from the learning community to individual learners by means of social interaction. The appropriation concept emphasizes the mutual organization of the community's learning contents and the learner's individual learning contents through social interaction. This concept of internalization and appropriation is considered valid within the theoretical framework of sociocultural approaches (see Newman et al., 1989 and Lee & Smagorinsky, 2000).

Based on the above-mentioned claims about Vygotsky's internalization and the appropriation concept (Bakhtin, 1981 and Cazden, 2001), this paper defines appropriation as follows. Appropriation is a concept that has the characteristics of dynamic composition and mutual composition. Dynamic composition is a feature of the gradual formation of one's concept dynamically by contemplating a concept borrowed from another individual, and mutual composition also leads to the formation of a learning community in this process.

Thus, this concept definition shows that the concepts of individual learners and learning communities can develop interactively through appropriation. Up to this point, we have clarified the appropriation definition and its two characteristics. In the next subsection, this study will consider the appropriation structure in mathematics education.

RESEARCH ON APPROPRIATION IN MATHEMATICS EDUCATION

In a study that discussed appropriation with a sociocultural approach in mathematics education research, Moschkovich (2004) formulated the following questions regarding appropriation in

mathematics learning while reviewing the concept of appropriation: What does the learner appropriate? How does the learner appropriate? What are the central characteristics of appropriation? How does the learner's appropriation change further? In response to these questions, he identified three unique characteristics as follows. Appropriation forms the central concept of neo-Vygotskian psychology; it shows that learning is mediated by interactions and that it transforms meaning, behavior, and even goals. It is thus a dynamic concept. He then considered cases, but the concept of appropriation remained unclear, and he did not identify its process or structure (Moschkovich, 2004). Matsushima (2018a) also reviewed the characteristics of appropriation and tried to analyze the case study but, once again, did not mention its general process and structure. Oers (2000) and Radford (2000) conducted a detailed case analysis from the viewpoint of appropriation, but in both studies, they did not make any deep references to the appropriation concept itself.

A review of appropriation in mathematics education up until this point shows that the concept of appropriation is a new central concept in neo-Vygotskian psychology, and the meaning, behavior, and goals of learners and learning communities will be changed by dialogue. However, these features can be included in the appropriation definition that had two features—dynamic composition and mutual composition—which was mentioned in the previous section.

On the other hand, previous research that mentions the process and structure of appropriation in mathematics education includes a study by Ernest (1998, 2010). In the late 1990s, Ernest (1998) presented a structural model that created the meaning generation process related to social constructivism based on a sociocultural approach. Figure 1 shows this structural model, which was further modified and submitted as a model of sign appropriation and use (Ernest, 2010). Though Ernest (2010) does not define appropriation and its features, this model simply shows that the meaning of mathematics as conventionalized in the learning community, which is the public/collective domain indicated in the upper right of Figure 1, is appropriated to the individual use of signs. The use of signs here is based on Wittgenstein's philosophy (Wittgenstein, 1953) that the meaning of a word is determined by its use. Let us consider whether the model of sign appropriation and use in Figure 1 satisfies the two characteristics necessary for creating a valid appropriation definition in this paper.

It can be seen from figure 1 that the first requirement, dynamic composition, is satisfied. This is because, in the public/collective domain, it is clearly shown that the use of individual signs, which began to be appropriated through the use of signs such as statements and actions in learning communities, can be transformed through imitative use. However, the subject of the term "transformation" should be noted here. It is appropriated content. Since appropriation has dynamic characteristics, the content of the appropriation changes dynamically depending on the particular situation of the learner. In other words, it is important to regard that appropriation covers two domains, the lower half of which include the private/collective domain and the private/individual domain. It would be more rational to consider appropriation, including the transformation of appropriated concept. Therefore, in the model for Figure 1, it may be necessary to revise the term "transformation". When a dialogue begins, and a speaker speaks, a listener will first form thoughts using a concept by borrowing the speaker's concept; this gradually becomes a new concept that is

Ownership	*Social Location*		
	Individual		Collective
Public	Individual's public utilization of sign to express personal meanings	Conventionalisation →	Conventionalized and socially negotiated sign use (via critical response & acceptance)
	Publication ↑		↓ Appropriation
Private	Individual's development of personal meanings for sign and its use	← Transformation	Individual's own unreflective response to and imitative use of new sign utterance

Figure 1: Model of sign appropriation and use (Ernest, 2010, p. 44)

then integrated with their own original experience and knowledge. This entire process is the dynamic composition of appropriation. In this dynamic composition, it is important to note the uniqueness of the appropriation concept. The concept to be learned is conventionalized by the learning community in the public/collective domain; however, some differences emerge when each learner individually grasps the concept. This is because each learner has different experiences, knowledges, beliefs, and values. When the learner's original framework is different, the interpretation the concept will also be different—even if each learner watches and listens to information about the same learning target at the same time and in the same situation (Hanson, 1958).

It can be seen that the mutual composition requirements of the second feature are also satisfied by figure 1. Let us consider a case where the concept *x*, which is conventionalized in the public/collective domain of the learning community, is appropriated. When the appropriation begins, the concept *x*, which has been conventionalized in the learning community, is affected by the situation of individual learners and transforms into a concept *x'*, which is a bit different, in the private/collective domain. This concept *x'* is transformed into *x"*, which is even more different, in the private/individual domain based on the situation of the individual learner. It is then used and publicized as the concept *x"* in the public/private domain. Based on this publication, the new concept *y* will be conventionalized in the learning community. In other words, through the process of appropriation, both the concept of the individual learner and of the learning community are transformed. This is exactly the mutual composition itself. It can be said that the model in Figure 1 directly depicts the essence of the appropriation process and its structure, which includes both dynamic composition and mutual composition.

EVOLVING THOUGHT ON APPROPRIATION

Lerman continuously asserted the importance of sociocultural approaches in mathematics education. However, Lerman does not use the term "social constructivism" to describe the mutual composition of individuals and societies from the perspective of a sociocultural approach (Lerman, 2001). This is because the ultimate goal of the sociocultural approach is to consider social institutions, cultures, signs, and human meaning generation, whereas the term "construction" is tightly linked to human internal and universal developmental features (Lerman, 2001). This point is important for making the revised model. Certainly, the model in Figure 1 can also be regarded as a construction process model that is centered on the internal development of individual learners who develop their own concepts while interacting with the learning communities. When considering human meaning generation, social meaning generation is temporarily prioritized over individual meaning generation (Lerman, 2000). Therefore, further revision of the model in Figure 1 would be necessary in order to make a clear assertion that incorporating sociocultural perspectives is essential for this study. Ernest (2010) also pointed out that the model in Figure 1 provides a limited view of learning and knowledge generation, and it is necessary to construct a model that provides a viewpoint with a wider social structure and power structure.

The points made by Lerman (2000) and Ernest (2010) show that a revision of the model in Figure 1 would require a clarification of the sociocultural influence on human meaning generation. One method of making the revised model involves embedding the dialogue partner and the learning community in the model. Figure 1 tends to show an aspect from only one learner. Therefore, the model for the other learner is installed symmetrically as a dialogue partner. In addition, the public/collective domain can be regarded as a learning community, so two learners are connected at this point. This revised model is shown in Figure 2 as an extended model of sign appropriation and use. The model in Figure 2 is a revised model based on Matsushima (2018b), but its essential difference from Matsushima's model (2018b) is that this extended model is reconsidered using an appropriation perspective.

Consider the extended model. The learning community can be a small group of 2 to 4 people or even a whole class. In a learning community of any size, many people speak in dialogue, but only one person speaks each moment. For example, at the moment when learner A speaks, learner A speaks alone, while the others are listening to A's speech. After that, learner B begins speaking about A's utterance. The utterance of B is made in the learning community, but B's speech corresponds to that of A. That is, B's speech responds to A's speech.

When a certain speech is taken as the starting point of the dialogue in this way, the individual speakers may change one after another, but the nature of the dialogue is a person-to-person interaction. In the learning community, its dialogue is considered to be an accumulation of person-to-person interaction in the learning community field. Therefore, the structure of the social interaction related to dialogue in the learning community is considered to be basically represented in Figure 2. In the extended model, the two learners and the learning community are indicated using ellipses. If we consider the learning community as one virtual learner, the extended model could be considered as illustrating a tripartite dialogue model. Such a tripartite

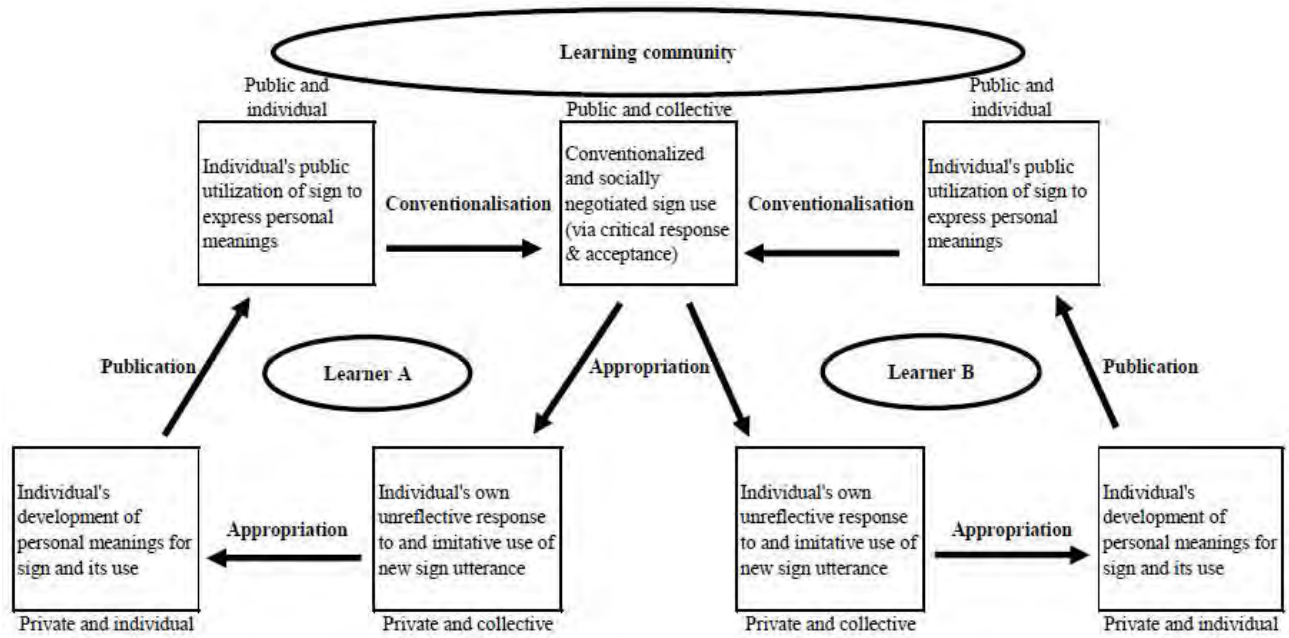

Figure 2: Extended model of sign appropriation and use

dialogue model has also attracted attention in the learning sciences (Paavola et al., 2004). When considering the meaning generation with regard to the learner's concept, elements of consideration should not be unnecessarily increased, but it is essential to include the learning community as an element of consideration when utilizing a sociocultural approach perspective. Moreover, by incorporating the learning community like a learner, the existence of appropriation may become more apparent. The distinction between the three learners may allow for comparisons between the conventionalized concepts in the learning community, speaker-appropriated concepts, and listener-appropriated concepts. This possibility will be useful for analyzing the deepening process of mathematics learning.

THE SUITABILITY AND CREATIVITY OF APPROPRIATION

Based on the extended model, appropriation can be repeated many times by continuing to publicize own sign use, and the individual learner's concept continues to change. However, it is empirically clear that some learners can deepen their learning without disclosing their ideas in the learning community. If appropriation is essential for deepening learning, isn't it necessary to publicize ideas in order to facilitate appropriation? Furthermore, does the direction of the concept, which is transformed by appropriation, guarantee that it will travel in the essential direction of mathematics? Are there any other characteristics that can result from the continuation of appropriation? In this chapter, the author considers the suitability and creativity of the appropriation related to these questions.

First, let us consider the extension model again and assess suitability. When a learner publicizes how to use a sign, the learning community determines whether the usage—that is, the learner's original concept—is provisionally accepted or partially or totally criticized. Next, more suitable concepts are created. Here, the suitability of the original concept itself is judged by the learning community. On the other hand, if appropriation occurs among learners who simply listen to the speaker's utterances without revealing their own concepts, it is assumed that this is caused by comparing their own concepts with the publicized ideas of other learners. In this case, the learner as the listener judges the suitability of the appropriation by oneself. In other words, the judgment regarding appropriation suitability differs between learners who publicize their ideas and those who only listen to other's utterances. This difference is related to the difference in the suitability of the speaker's and listener's appropriation. The sociocultural approach regards learning as a process that approaches more essential uses of artifacts through community dialogue. The sociocultural approach also considers that the deepening of mathematics learning itself is influenced by power relations and social situations. Thus, it is assumed that the deepening can be somewhat oppressively transformed into a socioculturally suitable direction through dialogue (Lerman, 1996). The social practice of dialogue in the learning community ensures the suitability of the direction selected for deepening mathematics learning. This assumption presents the possibility that proper appropriation may not have been carried out for the learners who do not publicize their own concept.

This paper's discussion of appropriation and its suitability up to this point answers the major research question of this paper: Why does dialogue deepen mathematics learning? Furthermore, an additional question is also provided to the author: Can mathematics learning be deepened without dialogue? The former question can be answered as follows: In order to deepen the learning of mathematics, it is necessary to utilize appropriation to compare and transform the original concept of the learner and of the speaker while the suitability of the appropriation is secured by the dialogue. The latter question can be answered as follows: There is nothing that cannot be deepened, but the suitability of the direction of the deepening is not guaranteed beyond the publication of one's concept. This is the essence of proper deepening and development of individual learners' mathematics learning through dialogues that publicize their own concepts. Dialogue is essentially related to the desired direction for concept formation. This also implies that it is important to incorporate dialogue in the lesson design in order to realize the deepening of mathematics learning.

Second, let us think about the relation between creativity and appropriation. The process of appropriation is described as follows: "Appropriation is a quite general process that can account for the emergent creativity of social interactions and the growth of flexible expertise in learners" (Newman et al., 1989, p. 143). The process of becoming a flexible expert can be explained by the occurrence of a continuous appropriation and its suitability. Here, we focus on appropriation and creativity. How is creativity related to the process of appropriation? It can be assumed that the extended model has two deviations and that these deviations are the source of the creativity.

First, we must acknowledge that there are gaps in the learning community. Even if learner A publicizes the concept x in the learning community, the concept x is perceived slightly differently

by each learner, and becomes *x'*, *x"*, and so on. This is because individual learners' own original experiences, knowledges, values, and cultural backgrounds differ. Therefore, it is virtually impossible to interpret concept *x* in exactly the same way as learner A. Secondly, we must also account for the deviation between the individual learners. Appropriation is the process of constructing new knowledge by comparing concept *x* of the speaker with the personal original knowledge and culture of the listener. Therefore, in such a situation, even if the concept *x* that is conventionalized in the learning community is the same, a deviation may occur each time through the learner's appropriation, and the deviation will then gradually increase. As a result, there will always be some deviation between learners.

Although these two types of discrepancies between the learning community and the individual learners can lead to errors in learning, this situation could also present the possibility of creating a new concept that was not included in the learner's original concept *x*. In other words, it can be said that the gap between the two points in the process of appropriation is the source of creativity that leads to new ideas that the learner may not have intended to create.

CONCLUSIONS

In this paper, the author attempted to use a sociocultural approach to answer the question, why dialogue deepens mathematics learning. In this process, the author also answered the following additional question: Can we deepen mathematics learning without dialogue? With regard to the former question, the author came to the following conclusion—that is, in order to deepen mathematics learning, it is necessary to have appropriation in order to compare and transform the original concept of the learner and that of the speaker, while the suitability of the appropriation is secured through the dialogue. With regard to the latter question, the author made the following finding—that is, there is nothing that cannot be deepened, but the suitability of the direction of the deepening is not guaranteed beyond the publication of one's concept. Thus, the answers to the two main research questions were derived by clarifying the concept of appropriation and its structuring. In addition, in this paper, the two features of appropriation, dynamic composition and mutual composition, were shown. Furthermore, the extended model of sign appropriation and use was proposed. This extended model could be a framework for facilitating qualitative analysis of dialogue with regard to mathematics learning.

However, the sociocultural approach emphasizes further scaled-up analysis that considers the background elements of learning, including rules and division of labor (Engeström, 1987). Future tasks in relation to this research could construct an analytical framework for dialogue in mathematics learning with greater emphasis on social and cultural aspects of learner's environment.

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