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Learning to Learn in Mathematics: Two Fulbright Distinguished Awards in Teaching Fellows' Narratives

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Abstract

Two middle school educators earned a Fulbright Distinguished Award in Teaching fellowship. A Fulbright Finland Foundation inter-country travel grant provided the grantees with a unique opportunity to connect and collaborate at the University of Helsinki. Within this research, they described their inquiry experiences. The research included examining authentic student-centered learning continuums and phenomenon-based learning in Finland and teachers' adaptability in relation to meeting the needs of linguistically and culturally diverse math classrooms in the Netherlands. This paper summarizes how cross-cultural dialogues, classroom observations, and informal interviews with educators, students, and thought leaders informed each grantee's discovery of how student-centered learning is structured, delivered, and valued in Finland and the Netherlands. This article (1) describes how communication empowers middle school mathematics students, (2) analyzes the learning-to-learn framework, and (3) provides insights into how to utilize language diversity in a mathematics classroom.

Keywords: student-centered mathematics, language diversity, learning-to-learn framework, phenomenon-based learning, teacher professional development, professional inquiry

Introduction

The European Commission (2018) stated that "everyone has the right to quality and inclusive education, training, and lifelong learning to maintain and acquire skills that allow full participation in society and successful transitions in the labor market" (p. 1). All students gain skills, such as problem solving, critical thinking, the ability to cooperate, creativity, computational thinking, and self-regulation. The European Commission's goal is to raise the level of achievement of basic skills, such as literacy, numeracy, and basic digital skills, that support the development of *learning to learn*. This competence serves as a foundation for lifelong learning and participation in society. The European Commission promotes cross-disciplinary learning and language awareness in schools. Likewise, the National Council of Teachers of Mathematics (2014) in the U.S. encourages teachers to facilitate meaningful mathematical discourse and develop connections among areas of mathematical study and between mathematics and the real world.

This article describes inquiries conducted by two Fulbright Distinguished Awards in Teaching fellows, S. Goldberg (Researcher 1 - hosted by the University of Helsinki in Finland) and J. Dean (Researcher 2 - hosted by Utrecht University in the Netherlands). Both the researchers served as early secondary or middle school field-based teachers and teacher-educators. During their fellowships, they focused on two aspects. Goldberg's study targeted authentic student-centered learning continuums and phenomenon-based learning. Dean's focused on language use in middle school mathematics to understand how Dutch and German mathematics teachers adapt student-centered practices to more linguistically and culturally diverse classrooms in the Netherlands and Germany. Both educators engaged in cross-cultural dialogue and examined mathematical practices in their host countries. Classroom observations and informal interviews with teachers, students, and thought leaders revealed the expertise of host educators.

Literature Review

Mathematics has long been considered a gatekeeper to higher-level learning rather than a key to unlocking authentic student-centered learning dimensions. However, this dichotomy has created many opportunities and challenges for mathematics educators who intend to develop best practices for middle school mathematics students. The phrase *authentic student-centered learning* describes the criteria for student-centered learning of real interest or genuine concern for students. Authentic tasks and assessments feature performance analyses that "require students to construct extended responses, perform on something, or produce a product" (Koh, 2017, p. 3). Essentially, *authentic* describes students' investigations or products based on their interests and actions.

O'Neill and McMahon (2005) determined that *student-centered learning* is used interchangeably with flexible learning, experiential learning, and self-directed learning. In "student-centered learning contexts, learning outcomes focus on what the student will be able to do rather than the content covered by teachers" (O'Neil & McMahon, 2005, pp. 30-34). This alternative approach aims to empower students to learn mathematics via student-centered methods and to provide mathematics activities that can help pupils contextualize the use of mathematics in the real world. Thus, student-centered learning validates how students conceptualize mathematics; how they think, feel, and experience mathematics (Goldberg, 2008).

Phenomenon-based learning involves the pursuit of a topic negotiated by students or suggested by teachers. Typically, phenomena are observable facts or trends that students may express an interest in studying in-depth, such as topics that pique their interest during their learning. Scientific inquiry topics, such as artificial intelligence, autonomous vehicles, energy, fingerprints, climate change, and black holes, may intrigue students. These scientific topics can promote mathematical literacy because mathematics is the language of science and is a unique, people-centered discipline.

Ricks (2009) found that students benefit from mathematical activities or tasks that challenge them to "do what mathematicians do when they do mathematics" and "mathematize" (p. 2). Also, conceptually rich mathematical tasks that feature novel or non-routine problem-solving challenges (e.g., project work) require students to apply knowledge and critical thinking skills.

Methods

Searching for Phenomenon-Based Learning and Learning How to Learn the Finnish Way

Phenomenon-based learning has become a critical aspect of the 2014 Finnish National Curriculum Framework (Halinen, 2015). This development prompted Researcher 1 to explore authentic student-centered inquiry and phenomenon-based learning in Finland. While investigating phenomenon-based learning, the learning-to-learn framework (Fredriksson, 2013; Hautamäki & Kupiainen, 2014; Hautamäki et al., 2002) was discovered by Researcher 1. The learning-to-learn framework has served as a foundation for the core Finnish education system since 1995 (Hautamäki & Kupiainen, 2014).

Classroom observations occurred in 12 geographically diverse schools in Finland included two teacher training sites. Informal conversations with field-based teachers, teacher-educators, administrators, students, and Finnish thought leaders provided the Researcher 1 with rich descriptive data and answers to the following questions: (1) How do Finnish teachers promote equitable access to deeper learning in mathematics via phenomenon-based learning? (2) How do teachers in Finland feel about phenomenon-based learning? (3) Do Finnish teachers promote equitable access to deeper learning in mathematics via phenomenon-based learning? (4) How do Finnish educators plan, implement, and assess phenomenon-based learning?

Findings

The school buildings in Finland are accessible, spacious, and quiet. Besides student artifacts on display in corridors, most of the schools' hallways feature portraits of former rectors (e.g., school administrators) and educators. Foosball and ping pong tables, vending machines, and modular seating arrangements dominate common areas for students. These symbols of popular culture reinforce the value of play in Finnish culture. In contrast, traditional landscape paintings and taxidermy of mounted mammals and birds in glass displays and in-classroom curio cabinets underscore the cultural history of Finland and its love of nature, wildlife, and hunting.

Finnish schools do not practice ability grouping. Teachers conduct classes in Finnish, one of the mother-tongue languages in Finland (the other two are Swedish and Sami). In Finnish-speaking schools, students learn in Finnish, and all students learn how to speak English. Students who immigrated to Finland from Russia, Estonia, Ethiopia, Somalia, Iran, and Syria but are not fluent in Finnish, Swedish, or English receive educational support services. Special education teachers instruct students who are recent refugees living in government-subsidized homes without their parents before being mainstreamed with their same-age peers. These practices reflect Finland's determination to support "equity and create uniformity" (Krzywacki, 2016, p. 13). Special education teachers also assist mathematics teachers and provide students with linguistic support during classes. Scaffolding students with special reading or writing needs is done in class unobtrusively. Teacher discretion allows students to remain seated with peers of the same age and to continue to work autonomously.

Mathematics classrooms are well equipped. Whiteboards, chalkboards, projectors, and charging cabinets for laptops are built in or rolled into classes, and Newline Document Cameras are fixtures in every class. Students in 7th–9th grade have access to calculators, mathematics texts (printed in paperback), graph paper, and computer laptops. Like students in New York, students wear sneakers, hoodies, wireless earbuds, or headphones and carry cell phones. Generally, there are 18–24 students per class. Students are seated randomly in conventional rows or arranged into seating clusters for collaborative inquiry.

Observations of Mathematics Classroom Practices in Finland

Most of the observed mathematics classes were conducted in English to accommodate the researcher. Classes generally commenced with a review of the previous day's topic. Homework was checked for completion only, and discussions about homework were student-led. Each of the teachers observed instructed from their desks or in the front of the room while using desktop document cameras to project problem-solving tasks and illustrate solutions. In addition, each assigned practice problems from mathematics texts, and students participated in solving these tasks orally, in writing, and digitally.

During practice routines, each teacher circulated the room and responded to student queries. The teachers either sat with the students at their desks or stationed themselves at the front of the room. Students approached the teacher independently or in pairs for assistance and were receptive to the researcher's queries. For example, in one 8th-grade geometry class, students were prompted to be interactive, and they took turns calling on each other to plot points on a digital coordinate plane. Another 8th-grade class worked on the same topic in pairs and used *GeoGebra Math* to plot coordinates.

At their own pace, the students worked on the tasks assigned in their textbooks. Incomplete tasks became homework. Although teachers rarely assigned homework, when they did assign *tasks* from students' mathematics texts, only 3-5 problems were assigned. Students who requested more challenges received a selection of challenging tasks to complete. Alternatively, teachers classified homework tasks by labeling problems that were more difficult.

Students could choose whether to collaborate with their classmates while problem solving or work independently. They sought help and clarification from a teacher one-on-one or in small groups, or exercised autonomy. For instance, some students who were not on task or looked idle claimed that they had completed the work. Alternatively, one student stated that he would work with friends after school in an online *chat room* to *check answers*.

Students in the 8th grade used computer laptops to access the *GeoGebra Math* and *ViLLE* problem-solving mathematics programs developed at the University of Turku. Students in grades 7-9 also accessed *Khan Academy* videos and problem-solving tasks on their laptops. In addition, students used their personal cell phones to access *Photomath* and *Wolfram Alpha*. The *Photomath* application allows a user to photograph a math problem and then reveal a solution to it step-by-step. *Wolfram Alpha* is an application and calculation engine. The teachers provided the names of these two applications and endorsed their use.

In every class observed, 3 or 4 students played video games, listened to music during class, or engaged in texting without ridicule or reprisal from a teacher. One teacher explained this phenomenon: "They are learning to learn." Another teacher stated, "They will be assessed based on task completion and test scores. They will figure it out." One teacher simply stated, "They know

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what they need to do." A fourth teacher cited research by Mona Moisala, a neuroscientist. Moisala's research supports mobile phone use in a classroom (Lonka et al., 2015; Moisala et al., 2016). According to Lonka et al. (2015), "Instead of being denied technological tools, the pupils should learn how to use them in socially and pedagogically acceptable ways. They need to learn how to regulate their use of mobile devices inside and outside of school" (p. 11).

Some teachers collected cell phones from students as they entered a classroom. The students were visibly annoyed by this turn of events. The disgruntled expressions on their faces and pointed stares at the researcher indicated that they associated their loss of cell phone usage with the researcher's presence. Some teachers had either mixed feelings or divergent viewpoints regarding learning to learn and cell phone usage during mathematics classes or in the researcher's presence.

These observations are significant because Hautamäki and Kupiainen (2014) defined learning-to-learn competencies as "the ability and willingness to adapt to novel tasks, or the adaptive and voluntary mastery of learning action" (p. 181). Based on this premise, transversal competencies are considered "pedagogical goals" and viewed as "educational outcomes" (p. 179).

Teachers' remarks about students' *learning to learn* and comments regarding motivation and self-regulation helped the first author identify the learning-to-learn framework and its relationship to transversal competencies. This discovery led Researcher 1 to re-evaluate behaviors observed during classroom observations, which debunked the researcher's assumptions about textbook-driven and traditional teacher-centered practices. As a result, a deeper understanding of authentic, student-centered learning continuums began to evolve.

Teachers modeled content expertise and were acutely aware of students' behaviors, yet they never raised their voices or asked a student to behave differently. Instead, teachers "noticed and were responsive" (Campbell, 2018, p. 24) to students' individual needs and questions. As a result, the tempo and rhythm of the students' inquiries varied. Some of the educators waited for students to approach them, while others did not. Nevertheless, student agency and empowerment were evident as they constructed meaning, deeper understanding, and ownership of mathematical content and procedures.

Teachers provided specific information about math tasks, steps in the solution process of tasks, or advice on how to self-regulate while problem solving (Voerman et al., 2012). These instances showcased "students' active action, task performance, and perspective of hope and personal goal attainment" (Hautamäki & Kupiainen, 2014, p. 186). Teachers also helped pupils see how they could learn from errors made during a learning process (Hautamäki & Kupiainen, 2014, p. 195). Finally, students assessed their progress by utilizing various tools, such as math menus, which individual teachers digitally designed. Alternatively, students accessed a digital-learning management system in which they evaluated themselves topic by topic and task by task. Altogether, the quality of communication, particularly inter-communication between students and teachers, stood out.

Observations of Phenomenon-Based Learning in Finland

Drake and Burns (2004) noted three approaches to an integrated curriculum: interdisciplinary, multidisciplinary, and transdisciplinary. An interdisciplinary project is thematic. Multidisciplinary projects combine two or more disciplines. Transdisciplinary projects correspond to transversal competencies and are based on students' questions, concerns, or negotiated curriculum (Eronen et al., 2019). Examples of transdisciplinary project work in Finnish middle school mathematics classes included the following: A 7th-grade project challenged students to pick a topic or theme to study in depth with one caveat: they had to integrate a strand of mathematics. Working in small groups of 4–6 peers, students provided *PowerPoint* presentations on Sonya Kovalevskaja, the first Russian woman to earn a doctorate in applied mathematics; Pi competitions and Daniel Tammet, an autistic savant and world memory champion; Pythagoras, the Pythagorean theorem and number theorist; Archimedes and buoyancy theory; and Katherine Jackson, a human-computer and African American woman made famous by the movie Hidden Figures. After each presentation, the students informally evaluated themselves, each group, and each group member.

Another transdisciplinary project involved three teachers (e.g., language, art, and computer science) who teamed up to focus on the parliamentary elections. First, students used two political-survey applications on their cell phones to determine which candidate represents their political views. Then, they argued opposing viewpoints. Finally, students discovered what images and events influenced their political views and integrated virtual reality to craft a political infomercial/iMovie. Both transdisciplinary projects occurred after a marking period over a 3–4-day period.

In contrast, at the Lahti LUMA (mathematics, science, and technology) Solutions Science Center, interdisciplinary, multidisciplinary, and transdisciplinary projects were featured. For example, 2nd graders asked, *why does a fingerprint unlock an iPad or iPhone?* This project featured an inquiry into fingerprint patterns and phenotypes. Another presentation featured two middle school students who presented a hands-on workshop titled *Chemistry of Fragrances*, in which they crafted perfumes with natural essences that reflect their knowledge of chemistry and the olfactory system. A third team of 9th-grade students built a robot that mechanically casts an image of a rose through a prism. According to the students who presented this light show, their motivation was to communicate that *scientific inquiry can create beauty and hope*.

In the following section, Researcher 2 describes classroom observations in the Netherlands. Informal conversations with field-based teachers, teacher-educators, administrators, students, and Dutch thought leaders provided Researcher 2 with rich descriptive data and answers to the following questions: (1) How do teachers in the Netherlands adapt their instructional practices to meet the needs of linguistically and culturally diverse students in math classrooms in the Netherlands? (2) How do teachers in the Netherlands navigate linguistic diversity, promote inclusion and equitable access to deeper learning in mathematics?

Moving Toward a Language-Positive Math Class

Most mathematics classrooms worldwide exhibit language diversity. In some cases, teachers and students speak different languages and must find ways to communicate in a common tongue. In other arenas, teachers and students may share a common *school language* while home languages and informal ways of expressing observations differ. In all classrooms, students and teachers must navigate every day, school and technical register in one or more languages (Prediger & Wessel, 2011). As language diversity has increased in the Netherlands, so have calls for increased verbal engagement and sense-making on the part of students. This calls for more meaningful participation

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points to the importance of language-socialization events and that the tension of navigating across languages has also increased (Barwell, 2020).

It is a delicate undertaking to invite students to participate fully in a community that requires them to express ideas in the language(s) they are learning. Deploying one's breath to express an idea not fully understood requires a vulnerability that many students are unfamiliar with displaying. Especially in compulsory education, tension is inherent. Students do not choose to be there, yet most educators want them to participate once they are. Moreover, when teachers demand that students talk before, they have something to say, they cannot bring themselves fully to the learning experience. They might comply, but their participation will remain peripheral and superficial. Negotiating the tension between inviting and shaping participation is essential to supporting students in participating fully in their learning.

While participation need not mean speaking in front of a class, language production is essential because we need words to think about mathematical ideas, even if the dialogue is internal to the learner. Hence, explicit attention to a language used in math classes is essential. Barwell (2020) used the term *language socialization* to describe ongoing interactions that shape the uses of language in a group of people. Barwell sees "learning mathematics as socialization into mathematical discourses" (p. 9); that is, learning math is the same as learning to communicate about mathematics in a community of learners. Barwell's recent work (personal communication with the second author of this current article) in Canadian multilingual math classrooms highlights how the most important moments occur during the tension between speaking and listening; understanding and confusion; and silence and verbal participation.

Language-Positive Practices Observed in Dutch and German Schools

Barwell (2020) drew on Sfard's (2008) work on discourse to frame learning math as the process of being apprenticed into full participation in discourse communities. In Barwell's theory, the act of learning is not about acquiring information; instead, it is about increasing students' capacity to identify and use mathematical ideas and tools for making sense of mathematics for themselves. A classroom thus becomes a place of novices and experts. Often, a teacher or tutor is an expert, but students can fulfill this role for their peers. The role of an expert is not to *fix* students' thinking but to be curious about students' thinking and interact in ways that help them make sense of mathematics for themselves.

Barwell (2016) employed Bahktin's (1981) notion of centripetal and centrifugal language forces. These forces homogenize language for a given purpose, contrasting with those that introduce novel expressions and turns of phrase. The interplay between centripetal and centrifugal forces creates moments of tension. These moments are socialization events that can be powerful places and spaces to invite participation in a community of mathematicians. Barwell (2020, p. 150) called socialization practices that invite participation in mathematics *language positive* and those that do not *language neutral*. Since there is no such thing as an entirely language-positive classroom, language socialization occurs in the constant negotiation between people, and it lives in the space between us. Barwell (2020) named the following seven categories that help us see potentially fruitful moments of tension: (1) student use of a home language, (2) the occurrence of nonstandard accents, pronunciation, or orthography, (3) explicit attention to discourse, (4) encounters with genres, (5) the use of gestures, (6) explaining math thinking, and (7) moments of reduced participation.

Students' Use of Home Languages: Invite Them

In schools, there is often tension between the use of a home language and a language of instruction, which is usually positioned in a place of greater importance by both teachers and students; this increases the tension between languages (Barwell, 2016, 2018). Also, many school settings add to the tension between home and school languages by requiring students to speak only the language of instruction.

Teachers claim that students should not use their home language because it slows down their acquisition of the language of instruction. They also claim a need to understand each to *reinforce that students should use the language of instruction*. For example, one teacher in Utrecht stated, *If I let them speak Arabic, I do not know what they are saying and what they say may not be allowed*. Teachers also attend to students' relationships with each other. For example, a teacher of second-generation immigrant students in Amsterdam worried that Turkish students and those speaking Arabic would be in constant conflict unless he required them to speak Dutch to each other to work out their differences.

These places of tension—between home and school languages and between diverse home languages—provide socialization points, and what happens in these moments is essential for inviting participation. For example, a vocational high school hosts a program for newcomers that navigates the tension between home languages and the language of instruction by allowing and inviting students to use their home languages. The program's goal is for students to learn as much German as possible while making progress in mathematics to proceed to a mainstream setting. A middle school teacher and post-doc at the university, has developed materials to support tutors' work with students. Students work intensively with tutors who get to know them quite well as learners of math and language. Researcher 2 saw tutors who did not share home languages with their students productively use those languages to invite participation while moving students toward fluency in German.

In one group, a tutor allowed moments of meaning-making in a home language while being available for students to check their understanding. The session began with the tutor using German to introduce the ideas the students were to learn. Two girls, whose home language is Spanish, nodded as he spoke, and then they worked a bit. While doing so, they spoke to each other in Spanish, clarifying their understanding. Consistently throughout the session, the tutor listened to their Spanish without interrupting. They talked about how they already knew how to solve a problem. It was just the German that was new. The tutor's continued engagement via their intervals of using Spanish invited them to bridge from their mother tongue to the language of instruction. When they paused, he checked in with them in German. They worked together to translate their understanding for the tutor and for another group member whose home language is Arabic. The math concepts were brand new for this student, and the translation into German allowed him more access to math and a reason to further develop his German skills via his conversations with them. Because the tutor accepted the students' use of Spanish (even though he did not understand what they were saying), they could use all of their language resources. Each of the three students in the

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group spoke more German and had more access to math than they would have if the girls had not used their home language.

In the above-described scenario, the language of instruction remained central, and the students used all their collective language resources to solve problems together. The tutors invited continued participation by allowing moments of meaning-making in home languages and being available for students to check their understanding. It was also significant that students were engaged in problem-solving tasks rich in mathematical thinking and structured to support language development. Students knew that these learning expectations included learning math and language and that each is interrelated.

The Occurrence of Nonstandard Accents, Pronunciation, or Orthography: Redefining 'Help'

A classroom in Dortmund offered a glimpse of what it can look like to offer productive help with communication across language barriers. In that program, tutor-and-student interaction is about mutual understanding and using all students' language resources to learn math and German. While tutors in this program have developed unique ways of inviting students to participate, one group has developed nonverbal cues that allow students to be in charge of when they receive help.

A group started a session by reading a problem aloud together. As native speakers of Arabic, they struggled with many of the meanings and pronunciations of both the math vocabulary (e.g., inverse proportionality) and the words to describe the context (e.g., filling a swimming pool with water). Instead of correcting them, their tutor listened and watched each reader. The students had developed the practice of quietly inviting help via eye contact. The tutor supported the students when they hesitated enough to look at her. Only then did she provide help or correction. In this group, the girls spoke to each other in German and switched to Arabic only when they got stuck. After each exchange in Arabic, they checked their understanding of German with their tutor. The tutor's patience and focus on understanding and the power she gave them over their learning kept them focused on using all their language resources. She trusted them to make meaning of the math and gave them space to use math learning to become better at speaking German.

Explicit Attention to Discourse: Novices and Experts Interacting

A program in a neighborhood in Rotterdam uses discourse to support the learning of first, second and third-generation immigrant students from all over the world. Three elementary schools have partnered with a non-profit organization to offer to tutor every student in two grade levels four days per week. Rather than showing students how to solve problems, tutors give students opportunities to problem-solve ideas, and then they ask questions about students' sense-making. The relationship between tutors and learners was collaborative; they interacted about students' thinking and problem solving. The program director spoke at length about the tutors' ongoing professional development and students' massive gains. The tutors support their young students via tension-filled moments involving the students searching for words to express their ideas.

In Barwell's (2020) work, explicit attention to discourse consisting of voicing affirmation of students' mathematical verbalizations corresponded to the author's observations of interactions between tutors and learners. Tutors made subtle moves to keep students engaged in conversations about mathematics without rescuing them. Besides intentional ways of verbally engaging students, tutors gave students their undivided attention, even with a visitor in the room. Every child in this program received the message that all their mathematical ideas were important and valued. *Tutors were utilized less to explain or help students but were used more as an audience for students to work aloud on their ideas*. When it was necessary for students to know whether their answers were right or wrong, they retrieved an answer key and made corrections. The students chose when it was time to do this to confirm that their problem solving was making sense.

Explaining Math Thinking: Students Taking the Lead

A Dutch special education setting in Gorinchem illustrated a way to productively use tension to support students in verbalizing what they do not yet know. This school serves about 150 students aged 4–14 years. Under the guidance of school administrators, the school has a carefully articulated math program designed to support students in making meaning for themselves, regardless of their challenges with learning, socialization, language, or mathematics. The school utilizes a model called the *Vertaalcerkal* or *The Translation Circle* (Borghouts, 2018). This model has students constantly connect stories, calculations, physical actions, models, manipulatives, and drawings. Through each of these ways of working with and expressing mathematics, students revisit ideas and verbalize them in new ways. The program repeatedly regrounds students in what they can see and touch until they can formalize their understanding. Even then, teachers strive to reconnect the symbolic with the concrete. As a result, students always have something to think about and discuss.

Students with diverse cognitive and social needs all contribute ideas to solve the problems their teacher presents to them. One lesson involved potatoes and measurements. The teacher showed the children a 2.5-kilogram bag of potatoes and started a discussion about the bag's weight. Then, she asked the students to make two estimates orally: How many potatoes are in the bag, and what is the weight of one potato? The students knew that they would be cooking later in the afternoon, following a recipe, so the weight mattered. They also formed small groups and weighed their potatoes to check their estimates. This task demanded that they act out the division and gather around scales. Had the teacher done this for them, the students would have had less to ponder. As the students gathered around scales, they checked the weight of each potato and excitedly compared it to their estimates.

Their recipe called for a particular weight in peeled potatoes. They needed to know how much of a potato in a bag would be affected by peeling. Again, the students made estimates. There was only one way to determine whether their estimates were accurate: peel the potatoes. The students enacted subtraction with their hands, carefully capturing all the peels in bowls. Once each potato was ready, the class weighed the peeled potatoes and peels. While the students were busy, de Vries recorded the weights and kilogram-gram equivalents and created a number line on the whiteboard to support students as they explained their thinking.

As the class worked with the numbers, something peculiar emerged. The original bag, sold at 2.5 kg or 2,500 grams, contained 2,140 grams of potatoes and 392 grams of peel. When the students added the numbers, they discovered that the bag was overweight by 32 grams. This discovery raised another question about how measuring potato units might not result in exactly a 2,500-gram bag. Although this lesson did not include every element of the *Vertaalcerkal* model, it illustrated how the students could explain the ideas they were learning by constantly connecting the storyline of a recipe to physical actions and modeling relationships with numbers and the number line.

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Encounters With Genres: Name Them and Explicitly Describe and Use Their Features for Communication

Genre, which usually applies to literary forms, can also describe communication practices in school, such as procedures, reports, and explanations (Smit, 2013). Some genres specific to math classes include graphs, argumentation, and proofs. Inviting students to participate in doing mathematics together necessitates inviting them to formulate ideas using socially recognizable genres. To an expert, the conventions of graphs are self-explanatory. The tension here is that reading and making graphs requires attention to the features of graphs that have become relatively automatic for experts.

Borrowing ideas from Boaler's (2018) Emoji graph, Researcher 2 worked with a teacher to make a graph about fruit (Stanford University, n.d.). This Emoji graph labeled the axes *cute* and *not cute* and *use* and *do not use*, while the fruits were graphed on continua from *tasty* to *not tasty* and *easy* to *not easy* to eat. Both the reading and creation of the graph permitted repeated language specific to understanding the genre of a coordinate graph.

Use of Gestures: Use and Notice Them

In face-to-face interactions, gestures go a long way toward augmenting communication. However, academic language is, by design, about conveying specific meanings without relying on the presence of a speaker with gestures that augment meaning. Tension lies in accepting everyday gesture-filled language while moving students toward a more precise expression of their ideas so that their meanings are apparent, even without gestures. In developing academic language, full-body, gesture-filled communication by both a teacher and students can play a significant role in inviting students' participation.

For example, in a secondary school in Amsterdam, many of the school's students while at home speak a language other than Dutch. As students entered the class, they were greeted with a handshake. The instructor began her lesson by inviting the students to offer equations to solve. Various students offered the following steps and answers as the instructor wrote on the board: In a warm manner, the instructor calmly recorded what the students offered, right or wrong, and let them catch their errors if they occurred. Here, she refrained from body language that could distract students.

When it was time to offer students something to think about on their own, the instructor invited them to describe a coordinate graph. A boy in the front of the room struggled with words and instead offered gestures. He spread his hands from side to side and moved them up and down. Instead of speaking for him, she mirrored his gestures and said, *I can see from your hands that you know what we mean by a coordinate graph*. Then, she drew axes on the graph on the board. Another student offered the labels x and y for the axes. By incorporating a student's gestures into the lesson, the instructor opened an additional way for her student to participate. Thus, while a gesture, such as a handshake, can communicate a warm greeting, gestures can also replace words to reinforce expectations more subtly and gently. Gestures accompanying mathematical meaning can help build a bridge between what students can think but cannot yet say.

Moments of Reduced Participation: Let Them Be

While Barwell (2020) defined reduced participation as moments in which students do not verbalize their understandings aloud, one needs to be careful when defining participation. Students can work hard on ideas and not appear to be producing language to prove this. Maybe they understand much more than they can say, yet their language creation is internal. Alternatively, they may be able to visualize ideas but not yet have the precise language to describe what they are imagining. Demanding that students verbalize before they are ready can create so much tension that they stop communicating altogether.

Conclusion

This article described the inquiries and experiences of two Fulbright Distinguished Awards in Teaching fellows in Finland and the Netherlands. Their findings promote cross-disciplinary learning and language-aware practices in schools. Richmond et al. (2019, p. 86) stated, "What is measured is a reflection of what is valued." In Finland, learning to learn is valued. Although students' beliefs and attitudes about learning to learn are measurable with questionnaires and surveys (Hautamäki & Kupiainen, 2014), more work is needed to emancipate mathematics teachers and their students. Given that a cell phone is a tool that can increase digital literacy (Van de Oudeweetering & Voogt, 2018) and support multi-literacy competencies for information and communication technology, students' cell phone use should be embraced as a tool for learning. Cell phones may optimize learning experiences for all students, "especially students and teachers who are not trained to use other digital tools and programs" (Lonka et al., 2015, p. 10).

Transversal competencies and transformational pedagogy are linked (Meriläinen & Piispanen, 2016). They empower teachers and students to critically examine their beliefs, values, and knowledge to develop a reflective base and appreciation for multiple perspectives. If teachers model collaboration, students will inculcate that collaboration is valued, and everyone's perception of mathematizing may expand.

As Meriläinen and Piispanen (2016, p. 204) surmised:

> Teachers' autonomy also enables them to close the door and go on as if nothing in the world has changed. This may result in great differences between schools and classes. Some teachers use technology, experiment with new assessment methods, and give up on desks. Others stick to the old ways.

Theoretical Implications

The findings from Finland indicate that phenomenon-based learning relies heavily on communication and collaboration between students and teachers. After discussions on digital equipment and teachers' autonomy are validated, students can learn to learn and develop connections between mathematics and the real world. However, what if students do not collaborate, talk, debate, and share their ideas? The findings from the Netherlands could help us invite students to use their languages.

By paying careful attention to language, novices and experts can interact, which could provide students with access to academic language while engaging in mathematical thinking. In addition, the use of language-positive practices during moments of tension will help identify moments of

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hesitation and uncertainty as spaces for potentially significant learning. For example, suppose we can convince students to share their voices, which is an act of sharing their souls. In this case, we can use our power as teachers to explicitly draw their lively everyday language to more formal expressions of mathematical ideas.

Teachers can redefine *help* and support students' mathematical thinking even when they cannot come up with the academic language to say what they mean. Likewise, different genres in mathematics to students, name them, and explicitly describe and use their features for communication can be introduced. Teachers can use gestures and notice when students use their hands to augment communication. Teachers can let students take the lead and express their mathematical thinking and learn by thinking aloud to themselves in any language that makes sense to them. This approach may empower educators to see moments of reduced participation as signs of the tension inherent in communicating across languages and help them to see the effort behind silence. The most critical moments occur in the tension between speaking and listening, understanding and confusion, and silence and verbal participation when an educator's response to a student matter most.

Practical Implications

This research has practical implications for educator preparation and the importance of understanding academic language use (discourse). More importantly, this inquiry provides a range of ways to communicate and addresses the importance of cultural and other social phenomena that influence learning academic concepts in a new language, including for learners with special needs. Field-based teachers, professors, and novice teachers who have just entered the field of mathematics may benefit from critical aspects of the learning-to-learn framework. These research-based strategies promote student agency, collaboration, and social participation. Moreover, given the trend of increasingly diverse student populations, this research outlines ways of valuing and respecting students' language diversity and specifically students' home language while teaching content in a new language.

Overall, this research reveals that effective teaching practices address the most pressing issues in education: accessibility, diversity, equality, and inclusivity. For these reasons, the researchers are determined to share lessons learned in Finland and the Netherlands with longtime and newfound colleagues in host countries. Most importantly, the researchers are eager to collaborate with colleagues to implement language-aware practices in schools and learning-to-learn pedagogical approaches that will empower students and help them navigate the inherent tensions of learning mathematics, namely: listening, understanding, confusion, silence, and verbal participation. Supporting learning-to-learn learners feels promising to the researchers as mathematics educators and is practical for practitioners of disciplines beyond mathematics.

Limitations and Future Research

Critical limitations of the inquiry conducted in Finland and the Netherlands were the size and length of each investigation. Both researchers lived in their respective host countries for six months. Therefore, future research should examine a wider distribution of schools, over a longer extended period, include a larger sample of students from multiple grade levels and school policies and practices affecting the topics described in this research paper.

For example, one teacher's collection of cell phones in Helsinki might have occurred for various reasons. The teacher may have planned for students not to access their cell phones, and the decision to do so may have been unrelated to the researcher's presence. Future research could examine a more comprehensive array of schools and focus on teachers' attitudes and practices on using cell phones in the classroom and students' attitudes and practices regarding cell phone usage. An analysis of teachers' and students' responses might provide insights into technology's impact, specifical access to phone applications on the learning environment. Future research could examine a wide distribution of school policies and practices affecting cell phone usage in middle school mathematics classrooms in Finland. Future research could also target the impact of the learning-to-learn construct, transversal competencies, and phenomenon-based learning on a broader scale.

Likewise, culturally responsive pedagogy and positive-language practices reinforce the importance of the idea that immigrant learners experience access, equality, and inclusion in mathematics education. To foster accessibility, equality, and inclusivity, teachers must "consider the needs of students from other non-Indigenous, non-European settler cultures" (Golafshani, 2023, p. 114). Toward that purpose, future research could target policies and practices affecting the math education of refugee students (McBrien & Hayward, 2022) and other non-Dutch speaking immigrants in a larger sample of students from different grade levels over a more extended period. Moreover, exploring "peer relationships in enhancing language acquisition and structure of the learning environment on students' experiences" (Shahbazi et al., 2020, p. 44) is warranted. Students' perspectives on peer-to-peer relationships can enhance how math teachers in the Netherlands "support authentic learning and meaningful interactions" (p. 44) in language diverse math classes. Lastly, future research incorporating Barwell's (2020) language-positive practices could be essential to access and evaluate teacher attitudes and students' learning needs.

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