MATHEMATICS TEACHING RESEARCH JOURNAL
Vol 13, no 2
SUMMER 2021

A Visual Way to Teach How to Find Square of an Algebraic Expression

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Abstract: *After teaching last 10 years and analyzing student work, I realize that in developmental and Elementary Algebra courses there is a disconnection between the students conceptual understanding in context in contrast to their procedural fluency. In some cases, students can perform the procedure, but do not understand when the procedure is valid. In other cases, students do not understand the definition/concept but were able to perform the correct procedure. In Elementary Algebra classes, I have found that students have difficulty in squaring an algebraic expression. They can square and simplify expressions like* $(x^3y^4)^2$*, but they make mistakes when they simplify expressions like* $(x^3 + y^4)^2$*. After analyzing student work, I have concluded that they did not understand the underlying concept that is squaring an algebraic expression. A visual approach using the "box" technique helps students to understand the concept of squaring an algebraic expression and write each step with correct reasoning.*

Introduction

Consider the following problems:

Problem (I): Simplify $(x^3y^2)^2$

Students showed their work for this problem as: $(x^3y^2)^2 = (x^3)^2 (y^2)^2 = x^6 y^4$

Problem (II): Simplify $(x^3+y^4)^2$

Students showed their work for this problem as: $(x^3+y^4)^2 = (x^3)^2 + (y^4)^2 = x^6+ y^8$

Analysis of Students work

With regards to Problem (I), when I asked students the reason they can write $(x^3y^2)^2 = (x^3)^2 (y^2)^2$, they responded that they can distribute the exponent 2 which is the correct reasoning. My next question asked them to explain why they were able to distribute the exponent 2. They were unable to explain their reasoning. Concerning Problem (II), I asked them to critique the equation $(x^3+y^4)^2 = (x^3)^2 + (y^4)^2$. Their response was the same; namely that they can distribute the exponent 2. In this case, distributing the exponent is not correct. I gave them a counterexample to overcome this so-called freshman dream; namely that $(a + b)^2 = a^2 + b^2$ is incorrect. For example, if we let a = 3 and b = 4. Then $(3+4)^2 = (7)^2 = 49$ while $(3)^2 + (4)^2 = 9 + 16 = 25$.

Students' difficulties

After analyzing the students work, I realized that students had the following difficulties:

- they did not understand when and why they can distribute the exponent
- they did not understand the meaning/definition of squaring an algebraic expression

How to overcome student difficulty

To overcome this difficulty, I used a visual approach using the "box" technique. In the box technique, we replace algebraic expression with a box. This box technique helps students to understand this basic concept of squaring an algebraic expression and it also helps them to know the correct reasoning as to when and why they can distribute the exponent 2.

Question: What is the definition of the square of x ?

Answer: x^2 means x times x.

In symbol $x^2 = x \times x$ (1)

If we replace x with a box □ then the equation given by (1) becomes

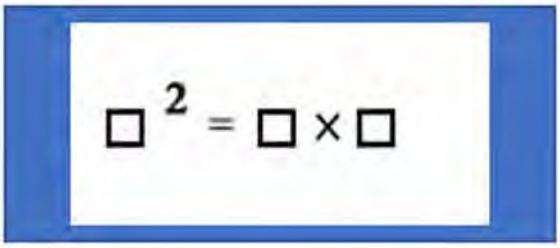

Figure 1

In the above equation (given in figure 1) if we replace the box □ with (x^3y^2) then we get

$(x^3y^2)^2 = (x^3y^2) \times (x^3y^2)$, we can simply further and write as

$(x^3y^2) \times (x^3y^2) = x^3 \times y^2 \times x^3 \times y^2 = x^3 \times x^3 \times y^2 \times y^2 = x^6\, y^4$

Students figured out the reason "why" they can distribute exponent 2 in the Problem (I) is because $(x^3y^2)^2$ means $(x^3y^2) \times (x^3y^2)$ and rearranging the terms will lead us to write

$(x^3y^2) \times (x^3y^2) = x^3 \times x^3 \times y^2 \times y^2 = (x^3)^2\, (y^2)^2$.

In the above equation (given in figure 1) if we replace the box □ with (x^3+y^4) then we get

$(x^3+y^4)^2 = (x^3+y^4) \times (x^3+y^4) = \{x^3 \times (x^3 + y^4)\} + \{y^4 \times (x^3 + y^4)\}$

$= (x^3 \times x^3) + (x^3 \times y^4) + (y^4 \times x^3) + (y^4 \times y^4)$

after combining like terms, we obtain

$(x^3+y^4)^2 = x^6 + 2\,x^3\,y^4 + y^8$

Outcomes of this study

- **Student Comments:** After presenting the box technique and presenting students how to write each step with the correct reason (as shown above) is when the aha moment happened, and students realized their mistake. Below are some student comments

  "Aha I got it", "Now I see it", "I see my mistake".

- While student understanding improved, I hope to follow up the study to quantify the level of understanding.
- Later in the semester where the student had to square algebraic expressions in order to solve problems like, solve for y where $\sqrt{y+15} = y + 3$, students were able to apply the box technique (as mentioned above) and were able to square both sides of the equation correctly, and they were able to successfully solve the problem.

Conclusions

By presenting the lesson on how to find square of an algebraic expression using the "box method" students realized their mistakes. They understand the meaning of squaring an algebraic expression and they can find square of more complicated algebraic expressions. They understand when and why the procedures work. They can derive the formula for $(x + y)^2$ and $(x - y)^2$ by themselves and hence they do not need to memorize it. It would be interesting to follow up on the qualitative aspects of the student experience in a follow-up study.

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