Division problems with remainder: A study on strategies and interpretations with fourth grade Mexican students

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Abstract: The present research focuses on analyzing how fourth-grade elementary school students (ages 9 to 10) solve and interpret the result of non-routine problems, precisely division measurement and division-partition with remainder. The methodology is qualitative, with a descriptive and interpretative approach. The information was collected using a questionnaire consisting of three problems (two of quotitive division and one of partitive division) and a clinical interview. The results showed the importance of using the division, multiplication, and addition algorithms to give a realistic answer to the problems. In the same way, it was possible to demonstrate the graphic strategy combined with counting to give a realistic answer to the problem. However, students were found to use division correctly but without an interpretation of the remainder or quotient. Likewise, they struggled to choose the correct procedure to solve the problem. These data suggest including realistic problems in mathematics classrooms to make sense of mathematical concepts in real life or the student's context. Likewise, this study provides implications on the mathematical problems that the teacher proposes in the classroom, where not only the division algorithm should be taught mechanically, nor focus on posing routine problems that lead the student to use a single heuristic resolution strategy. Essentially, the teacher is required to include real-world problems, where the student can awaken creativity to represent in different ways the understanding of a problem and, therefore, different strategies to solve it. In addition, that the student has the ability to check the result of the problem, with the conditions, situations or circumstances imposed by reality or everyday life.

Keywords: Division problems with remainder, division-quotitive, division-partitive, resolution strategies, primary education.

INTRODUCTION

To talk about problem-solving is to consider the importance of schools as a suitable setting for learning. It is here where the student manages to develop mathematical strategies to face challenges where the solutions to these do not require an obvious answer. However, different researchers interested in the subject show that the problems brought to the mathematics classroom lack authenticity. That is, they leave aside the students' context and reality. These types of problems do not help students learn to apply mathematical procedures in situations in their daily lives (Chamoso et al., 2014; Jiménez & Ramos, 2011).

Frequently, problem-solving is only a processing of the numerical data that appear in the statement without understanding what is being sought (Eslava et al., 2021). For Vicente et al. (2008), the most significant difficulties in students occur when working with real-world problems. In this sense, one of the most studied non-routine realistic problems, and where difficulties arise, are division problems with remainder (DWR). That is, problems in which the dividend and the divisor are integers, and the division gives rise to a non-integer result. In this type of problem, the result must be interpreted by the real-world constraints that give meaning to the problem (Verschaffel et al., 2009). According to Jiménez (2008), "realistic problems are verbal problems that describe real-life situations and that the application of an arithmetic operation does not simply lead to the solution of the problem" (p. 38). For Dewolf et al. (2014), verbal problems are an important way to bring the real world into the mathematics classroom and to teach mathematical modeling and applied problem-solving. A specific feature of realistic verbal problems is that they often do not contain all the information required to obtain a correct solution (Krawitz et al., 2018). For Payadnya (2021), realistic problems play an important role when you want to learn mathematics. That is, on the one hand students are required not only to understand the concept but also to apply the concept to solve everyday problems. Therefore, in verbal problems, the student must use real-world knowledge to give meaning and coherence to his or her answers. In that sense, the real world is the starting point where the development of mathematical concepts takes on meaning and relevance (Agustina et al., 2021).

An example of a realistic problem found in the literature was: "An army bus holds 36 soldiers. If 1128 soldiers are being transported by bus to their training site. How many buses are needed?" (Carpenter et al.,1983, as cited in Verschaffel et al., 2009). Two situations may arise in this problem. The first is related to not having difficulty in identifying division as the correct solving operation, and the second is linked to the tendency to give an incorrect answer 31.3 buses because they do not emphasize the non-integer quotient, taking into account the situation of the problem (Lago et al., 2008).

The structure of the division problems with remainder allows us to see which element of the division (remainder or the non-integer quotient) the analysis is focused on. According to Fischbein et al. (1985). They propose two intuitive problem models of division: the quotitive and the partitive. The first could also be called division by measure, which seeks to determine how many times a given quantity is contained in a larger quantity. For this type of division it is required that the dividend must be greater than the divisor. That is, it is established how many times a given quantity is contained in a larger one (Lago et al., 2008). An example of this model is shown by Zorrilla et al. (2021) "9 fans want to travel to the stadium in another city. Each cab can carry four fans. How many cabs do they need?" (p. 1322).

In the case of partitive division model we rely on what is established by Fischbein et al. (1985): “an object or collection of objects is divided into a number of equal fragments or subcollections. The dividend must be larger than the divisor; the divisor (operator) must be a whole number; the quotient must be smaller than the dividend (p. 7). Likewise, this refers to the approximation of the partitioning. That is, a given number of equivalent groups is formed to define the number of each group (Lago et al., 2008). For Buforn (2017), split-partite problems refer to problems where the number of objects per group is unknown. An example of this model is also shown by Zorrilla et al. (2021) "A dance academy has distributed in a class eight tickets for a musical. The dancers in the class were three, and all received the same number of tickets. How many tickets did each dancer receive?" (p. 1322).

It should be emphasized that these two types of division models with remainder are the ones we intend to address in the present research. The analysis we want to develop also focuses on how students interpret both the remainder and the non-integer quotient.

In that sense, interpretation plays a vital role in these division problems with remainder. That is, not only is it required that the student uses the mathematical algorithm correctly, but also that the answer makes sense with the real situation of the problem. This implies two situations: in the first one, it must be kept in mind that the existence of the remainder forces the student to recognize the value of the quotient plus one unit as a result. The remainder is not contemplated in the second, but the non-decimal quotient is based on the partition's context (Buforn, 2017; Lago et al., 2008; Parra & Rojas, 2010; Verschaffel et al., 2009).

On the other hand, further research has focused on studying the use of strategies in division problems with remainder in elementary school students (Downton, 2009; Ivars & Fernández, 2016; Sanjuán, 2021; Zorrilla et al., 2021). In that sense, Downton (2009), in his study with third-grade students (ages 8 to 9 years), evidenced the use of modeling, multiplicative thinking, repeated addition/subtraction, and counting that were employed as strategies in both division models.

For their part, Ivars and Fernández (2016) in their research with students from 6 to 12 years old, evidenced modeling and counting strategies in both division models and number fact strategies and multiplication of equal addends in division-measurement problems. In Sanjuán's study (2021) with students aged 6 to 12, strategies such as direct modeling, repeated addition, grouping, combination, and the multiplicative strategy were found in division-measure problems.

As for division-partition problems, non-anticipative, additive, and multiplicative coordination strategies were used. Finally, Zorrilla et al. (2021), in their study, coincide with the strategies observed by Ivars and Fernandez (2016), highlighting a strategy modeling with counting (graphical strategy), additive and subtractive strategies (use of successive addition-subtraction), and known number facts and those derived from addition and multiplication.

In this research, we analyzed how fourth-grade elementary school students solve division problems with quotient and remainder considering the division-quotitive and division-partitive models. With this objective, we seek to answer the following questions:

How do students interpret the quotient and the remainder in non-integer division problems? What strategies do students employ in solving division problems with remainder?

METHOD

The present research is qualitative, descriptive, and interpretative, according to Hernández (2014), since it attempts to make sense of the phenomena in terms of the meanings people give them. A questionnaire validated by a group of researchers was first applied. The participants were 50 fourth-grade elementary school students from a private school in Puebla, Mexico, whose ages ranged between 9 and 10 years old, selected by convenience. This questionnaire is made up of three non-routine problems of multiplicative structure, two division-measurement problems, and one division-partite problem (see Table 1). Verschaffel et al. (2009) propose three situations for these types of problems. The first requires the quotient to be rounded up, the second consists of rounding down, and the last suggests keeping the result of a division with the remainder as a decimal.

In this sense, Zorrilla et al. (2021) classified these situations into three types: the first implies that the presence of the remainder forces to recognize as a solution the value of the quotient plus one unit (type 1). The remainder is not considered in the second but the non-decimal quotient (type 2), and the third is the quotient plus the fractional part of the remainder (type 3). It should be noted that we will focus on type 1 and type 2 situations (see Table 1).

Questionnaire problems	Problem Characteristics
A candle vendor in the Emiliano Zapata market in Puebla, Mexico, packs 30 candles in a box. How many boxes will the vendor need if he has to pack 122 candles?	Quotitive Division. The remainder forces to recognize, as a result, the value of the quotient plus one unit Zorrilla et al. (2021) (Type 1).
A museum has given away 75 tickets to an art exhibition to 14 schools. The schools have received the same number of tickets to be distributed to their best students. How many tickets does each school receive?	Partitive Division. The remainder is not considered. The solution is the non-decimal quotient Zorrilla et al. (2021) (Type 2).
Twenty-two players of the Puebla soccer team want to travel by cab to the training sports venue. Each vehicle can carry four players. How many cabs do they need?	Quotitive Division. The remainder forces to recognize, as a result, the value of the quotient plus one-unit Zorrilla et al. (2021) (Type 1).

Table 1: Measurement and partitive division problems

Secondly, an interview was conducted with the seven selected students to understand how children construct their worlds, think, and work cognitive processes (Ginsburg, 2009). That is, to know how they solve problems and interpret the quotient and the remainder in the division-measure and division-partite problems. It should be noted that the interview was applied to seven students, which were audio-recorded and immediately transcribed for subsequent analysis and triangulation of the information. In addition, the students were assigned codes S1, S2, S3, S4, S5, S6 and S7 for the interviews' excerpts. The letter R stands for the researcher.

RESULTS AND ANALYSIS

The results presented in this section are organized into two sections. The first section presents the analysis of the responses to the division problems with residue, distinguishing their realistic and unrealistic character. The responses were classified into realistic responses with the application of division, realistic responses without the application of division, unrealistic responses applying division, and other responses. The second section presents the strategies used by the students when solving problems 1 and 3 of type 1 of the quotitive division model. The strategies evidenced are realistic responses.

Realistic responses with the application of division.

In these answers, the individual uses division appropriately and considers the realistic aspects of the problem. The solver interprets the remainder or the quotient to give a realistic answer. This response is evidenced in problem 3 of type 1 of the quotitive division model, as shown in Figure 1.

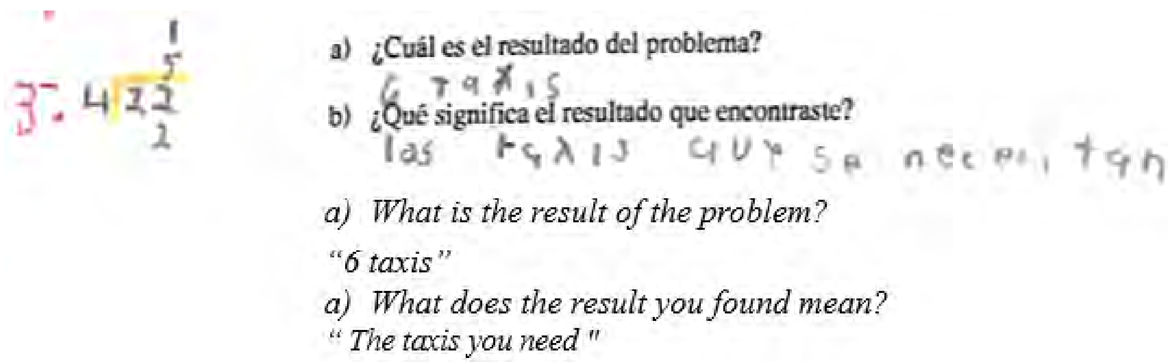


Figure 1. Realistic response to the division operation.
Source: student's response (4th grade)

In the interview, the student was asked the following questions:

R: What is the exercise asking you?

S1: How many cabs do the soccer team players need to go to the sports venue?

R: What procedure or operation did you perform there?

S1: A division.

R: You tell me that they are...

S1: 6

R: Why do you arrive at six cabs? Could you explain it to me?

S1: Because I divided: 22 by 4, it gives me 5, 2 are left, and the other two players leave in another cab. That would be six cabs. Five full cabs and another one with two players (4th-grade student, Interview excerpt, May 13, 2022).

Here the student uses division as an adequate procedure, expressing that to reach the problem result, he had to divide 22 by 4. In addition, he considers the real facts of the problem when he emphasizes that six cabs are needed to transport the 22 players to the sports venue. This is because the student understood the problem and interpreted the remainder of the division (2 players), assigning to this remainder an extra cab. He emphasizes that the division gives five as the quotient, and two are left over, expressing that there are five cabs full of players and another with two players for a total of six cabs.

Another realistic response result applying the division algorithm was evidenced in problem 2 of type 2 of the partitive division model, as shown in Figure 2.

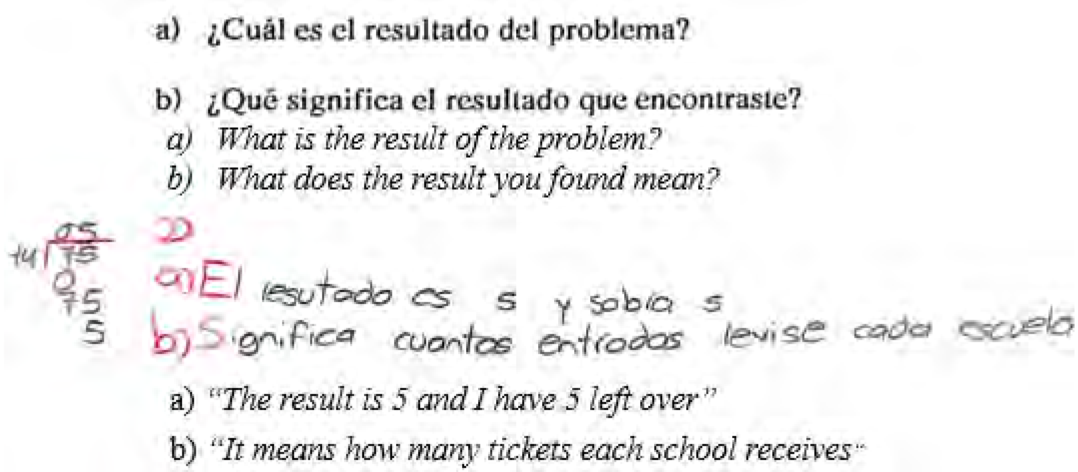


Figure 2: Realistic response to the division operation.
Source: student's response (4th grade)

In the interview, the student was asked the following questions:

R: What is the problem asking of us?

S2: A museum has given away 75 tickets to 14 schools. Each one is going to send its best students. So I divided to find out how many tickets each school gets.

R: Do you think you can do another procedure here?

S2: A sum could be

R: Can you explain how you would use addition in this problem?

S2: Adding until I get a result, but not more than the number of tickets we were given, and putting the remainder as a remainder.

R: Ok, so how many tickets does each school get?

S2: Five tickets

R: Ok, what does this residue mean?

S2: The leftover tickets

R: Would it be convenient to distribute those five tickets among the fourteen schools?

S2: No because it would be unfair

R: What would be the most convenient thing to do with those five tickets?

S2: Give them as a gift to someone else. (4th-grade student, Interview excerpt, May 13, 2022).

Here the student applies the division algorithm as a suitable procedure to answer the problem. In addition, he expresses that the repetitive addition would be another procedure that could be applied as long as it did not exceed the total number of entries and that the leftover entries would be the remainder. She also states that she interpreted the remainder as the five entries that were left over and the quotient as the number of entries that should be distributed to the fourteen schools equally (five entries for each school). In other words, the student interpreted the numerical answers correctly (context of the distribution) and successfully solved the problem.

Realistic responses without the application of division.

These are responses where the subject uses arithmetic operations other than division and manages to interpret the real situation of the problem. That is, the solver uses addition and multiplication as an adequate procedure. Likewise, they interpret the reality or situation described in the problem to solve it. This situation was evidenced in problem 3 of type 1 of the quotitive division model, as shown in Figure 3.

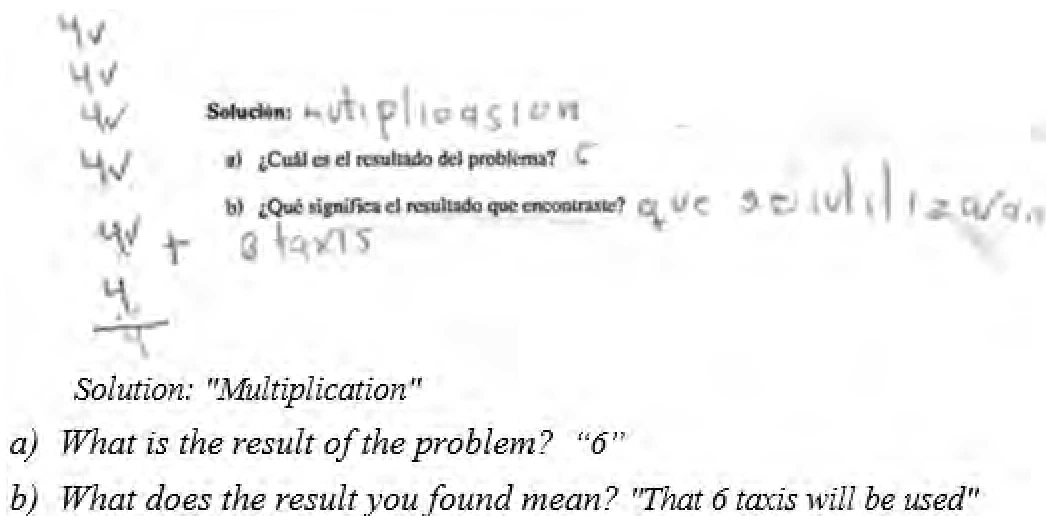


Figure 3: Realistic responses without the application of division.
Source: student's response (4th grade)

In the interview, the student was asked the following questions:

R: What is this problem asking you?

S3: I am being asked if 22 players of the soccer team want to travel in cabs to the training sports venue. Each vehicle can only carry four players. How many cabs are needed? I did multiplication.

R: Using multiplication, how did you arrive at the answer of 6 cabs?

S3: If each cab can carry 4, because 4 times... no I had to divide 22 by 4 and I got six.

R: Explain to me why you got the answer six cabs?

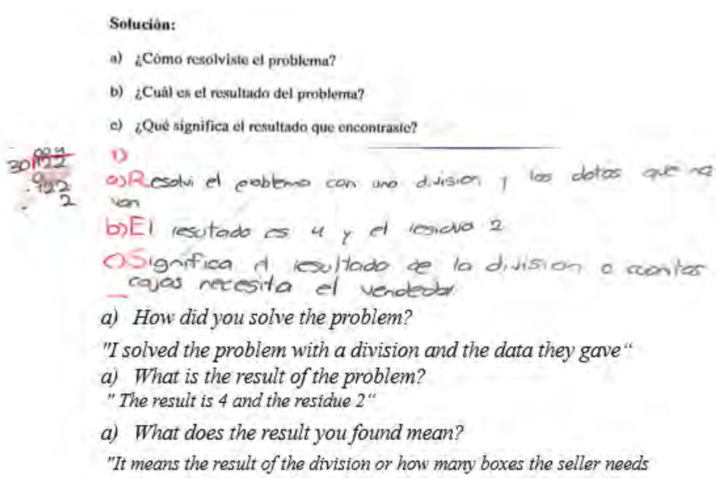
S3: The four players that fit in the cab I added them until I got 22, I also multiplied 4 x 4=16, 4 x 5=20, 4 x 6=24, now 4 x 5=20, the players are complete in the cab, and then I have two players left over.

R: What happens to those two players left over?

S3: They can go in a sixth cab, then they are 4 x 6= 24, but it happens, I know I have only 22 players, but the two missing players can go in a cab that fits 4, and nothing happens (4th-grade student, Interview excerpt, May 13, 2022).

Here the student used addition and repetitive multiplication as the appropriate procedure to answer the problem and gave it the correct interpretation. However, at one point in the questioning, he manages to reflect by stating that he has divided. However, when he begins to explain what he has done, he seems to have used addition and multiplication. That is, the student expresses that to arrive at the answer of 6 cabs, he added four by four until he got 22 players. In explaining how he performed the multiplication, he emphasizes that the product of 4×5=20 represents the players that can travel in the five cabs with four people. He also considered that the two surplus players could travel in a sixth cab, considering the multiplication of 4×6=24, although it exceeds the total of 22 players that must be transported.

Unrealistic responses applying division. Although using division as an appropriate arithmetic operation, the answers given by the subject did not take into account the real part of the problem. The solver does not interpret the division's elements (the quotient and the remainder). An example of this unrealistic response to problem 1 of type 1 is shown in Figure 4.


Solución:

a) ¿Cómo resolviste el problema?

b) ¿Cuál es el resultado del problema?

c) ¿Qué significa el resultado que encontraste?

1) a) Resolvi el problema con una division y los datos que me dan

b) El resultado es 4 y el residuo 2

c) Significa el resultado de la division o cuantas cajas necesita el vendedor

a) How did you solve the problem?

"I solved the problem with a division and the data they gave"

a) What is the result of the problem?

" The result is 4 and the residue 2"

a) What does the result you found mean?

"It means the result of the division or how many boxes the seller needs

Figure 4: Unrealistic responses applying division
Source: student's response (4th grade)

Here the student divides 122 candles among the four boxes, obtaining; as a result, four boxes in the quotient and two candles as the remainder. Taking into account the above in the interview, he was asked the following questions:

R: In the first problem: What was I asking you to do?

S4: Divide how many boxes the salesman will need to put the candles in, and I already had to divide and get the result for the questions you were asking me.

R: So for this problem, you divided, right?

S4: Yes

R: What's the answer to this question? How many boxes will the salesperson need?

S4: Five boxes, right?

R: Five boxes? Why?

S4: Ah no. Four boxes.

R: And those two that are loose, what happens to them?

S4: They can't go because the boxes are already full. Yes, they could fit, but they would not be well arranged and out of the box.

R: What other option would you give?

S4: Buy another box

R: Apart from the division, do you think you could apply another procedure?

S4: No.

R: Then how many boxes would you need to ship the candles in total?

S4: 5 (4th-grade student, Interview excerpt, May 13, 2022).

In this interview excerpt, the student expresses that she used division to solve the problem. Furthermore, although she correctly applied the division procedure, she could not interpret the remainder (the two leftover candles). That is, the student did not consider the problem's real part. When answering, the student forgets the problem's text and takes the one found with the division algorithm as the correct answer, thus generating an incorrect result. It is worth noting that at one point in the interview, the student reflected that in order to pack the two leftover candles, an additional box was needed. That is, concluding that five boxes were needed.

Other responses.

The solver's responses in this situation are influenced by an inadequate procedure or calculation of the arithmetic operation. In addition, there is no interpretation of the result. An example of these responses was evidenced in problem 1 of type 1 in the quotitive division model, as shown in Figure 5.

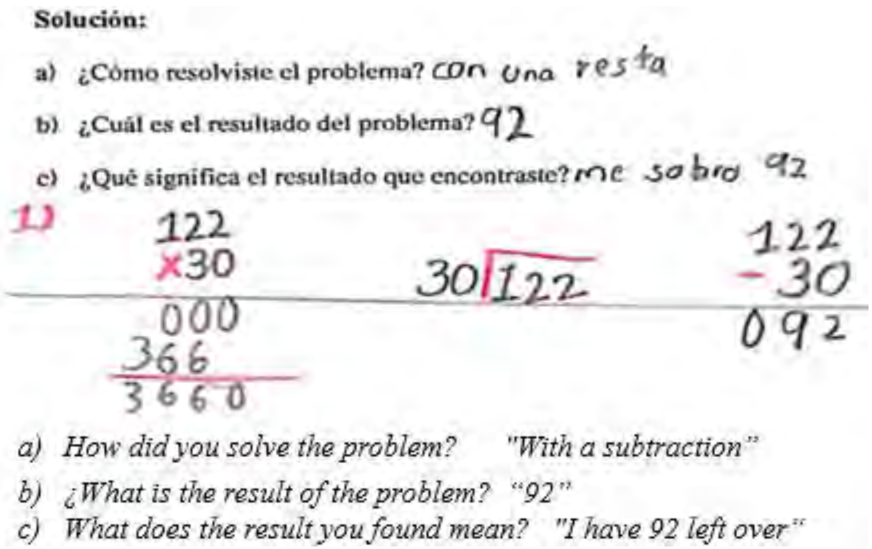


Figure 5: Other responses
Source: student's response (4th grade)

Here the student applies different arithmetic operations without arriving at a reasonable answer to the problem in question. The student performed arithmetic operations: from the application of subtraction and multiplication to division. However, there was no success in solving the problem.

In the interview, the student was asked the following questions:

R: In the first problem, what is he telling you, or what data is he giving you?

S5: In the Emiliano Zapata market in Puebla, Mexico, a candle vendor packs 30 candles in a box. How many boxes will the vendor need if he has to pack 122 candles?

R: What is the problem asking you?

S5: How many boxes will the seller need?

R: What procedure do you think should be done for this problem?

S5: A...

R: Here you said a subtraction. What should be subtracted?

S5: One hundred twenty-two minus thirty

R: Minus 30, which is what you did here.

S5: Yes.

R: But here I see you were going to divide. Why didn't you do that?

S5: I couldn't, so I tried subtraction, giving me a result I thought it would be.

R: Why did you do multiplication?

S5: Because I was starting from multiplication to subtraction, then division, subtraction, multiplication, and addition [A confusing sentence, and the investigator did not go deeper].

R: And why did you stick with the subtraction result?

S5: Because I feel that they are the boxes that are going to accommodate 122 candles.

R: Ok, well, do you think that doing multiplication, division, or addition can also give me the correct result?

S5: No, because multiplication will be a more significant number, the division will be a bigger number, and the addition will be a bigger number.

R: Ok, then it's the subtraction. (4th-grade student, Interview excerpt, May 13, 2022).

In this interview excerpt, the student is convinced that the correct procedure is subtraction. In addition, he justifies that with this answer, the 122 candles can be accommodated and that multiplication, division, or addition would be a more significant number. In this problem, there is the belief that multiplication, addition, and division imply a more significant number and subtraction a minor number without considering what the problem is posing. In this case, the student shows an incorrect understanding of the problem and a lack of clarity when choosing the correct operation. The above could be why students tend to focus on superficial aspects of the problem statement and select inappropriate solution procedures

Strategies used in realistic responses

Graphic strategy. According to Zorrilla et al. (2021), this type of strategy occurs when the subjects offer a solution in which they use a drawing or diagram to solve the problem. For Matalliotaki (2012), drawings are one of how children express a complex phenomenon by facilitating the expression of the spatial relationships of objects. In that sense, the representation created by the students is helpful in interpreting the result, keeping in mind the real part of the problem. An example of this strategy is shown in Figure 6. The student drew the cabs needed to take the players, evidencing that the student is aware of the meaning of the rest.

Figure 6: Example of graphical strategy-counting in realistic responses.
Source: student's response (4th grade)

In the interview, the student was asked the following questions:

R: What does the picture you drew in this problem mean?

S6: The cab that takes the players

R: Well, explain to me what does this drawing consist of? Why do you place six cabs?

S6: Because there are 22 players

R: Ok, how many players are in each cab?

S6: Four players

R: Why do you put two players in the last cab?

S6: Because they are 1,2,3,4,5,6,7,7,8,910,11,12,13,14,15,16,17,18,18,19,20,21 and 22.

R: ok

S6: And here, fit two because they are all complete of 4.

R: Ok, they are complete of what?

S6: Of four

R: Does it matter that the last cab is of two players?
S6: It doesn't matter (4th-grade student, Interview excerpt, May 13, 2022).

Here the student employs the graphing and counting strategy to represent the solution to the problem. In the first instance, the graph made by the student illustrates the number of players with the total number of cabs that can be used to transport the 22 players. For this strategy, the student keeps in mind that the total number of players who can go in a cab is 4. The counting strategy is evidenced when he started counting from 1 to 22, concluding that a sixth cab was used. In addition, he considered that they completed five cabs with 20 players, and the remaining two could travel in a different cab.

Another realistic response where the graphing and counting strategy was used, as evidenced in problem 3 of type 1 of the quotitive division model, as shown in Figure 7.

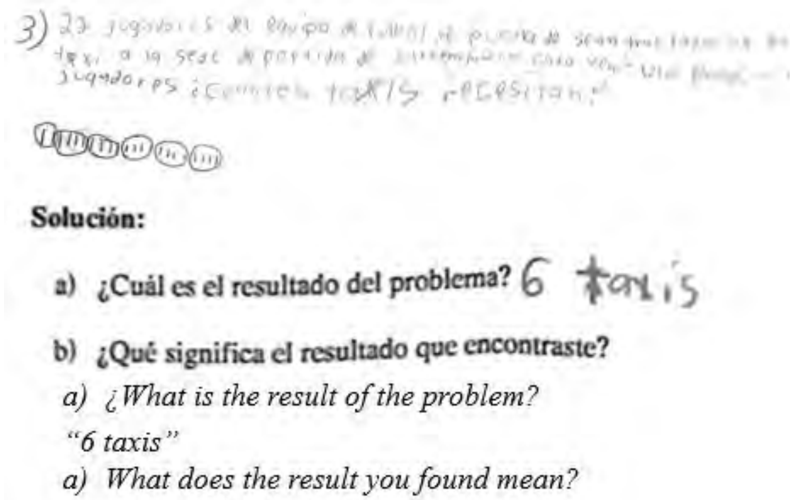


Figure 7: Example of graphical strategy-counting in realistic responses.
Source: student's response (4th grade)

In the interview, the student was asked the following questions:

R: In the third problem, what is the problem asking us, what data does it give us?

S7: 22 players of the Puebla soccer team want to travel by cab to the training sports venue. Each vehicle can carry 4 players. How many cabs do they need?

R: What is the problem asking you?

S7: How many cabs do you need to go to the stadium?

R: How many cabs do you need?

S7: Six cabs

R: How did you arrive at that answer?

S7: By drawing

R: What is the meaning of this drawing you made here?

S7: I put twenty-two lines and enclosed in fours.

R: Ok, you enclosed four by four, and in this last one how many were left?

S7: There are two

R: How many players are in this last cab?

S7: Two players

R: So how many cabs are needed?

S7: Six cabs (4th-grade student, Interview excerpt, May 13, 2022).

In this problem the E7 answers the question How did you solve the problem? giving as answer "6 taxis" and the second question What does the result you found mean? The student did not submit any response. First, the student used a graphic strategy or drawing to interpret and solve the problem. In addition, during the questioning, it was possible to identify that in addition to using the drawing, the student made a count by enclosing the lines representing the 22 players, four by four. In this problem, the student formed six groups representing the six cabs where the players should be transported, specifying that there are five groups with four players and one group with two players for a total of 22.

In summary, the results presented are a sample of how students interpret the remainder and the quotient in realistic problems. In that sense, in problems 1 and 2 (See Figures 1 and 2), students show a realistic response to having the division algorithm as a procedure. In these division models, students correctly interpret the result based on the reality of the problem. In the first instance, E1 interprets the remainder as adding one unit to the quotient. On the other hand, S2 interprets the context of division as a non-decimal result without emphasizing the remainder. These results disagree with those obtained by Zorrilla et al. (2021), who state that the algorithm's application does not result in a realistic response.

On the other hand, in some cases, when the student faces a realistic problem, it seems that he/she uses the arithmetic operations he/she handles correctly and uses it to solve the situation. In our study, the written production of S3 (See Figure 3), the procedure did not focus on the division algorithm but on multiplication and addition, thus generating a realistic response.

On the other hand, using division as an adequate procedure does not guarantee the student a correct interpretation of the result (See Figure 4). Rodríguez et al. (2009) pointed out that the choice of division as a solution procedure did not rule out inadequate numerical results. The division algorithm used in problem one by S4 was performed correctly; however, the difficulty was evidenced when associating the answer with the context of the problem. Galvão & Labres (2006) mention that children do not consider the remainder as a component of the division related to the other components. Another difficulty evidenced was considering the result of subtraction as appropriate without first interpreting problem 1 (quotient plus the unit) simply by considering that the result of multiplication, division, and addition is associated with a more significant number (See Figure 4). Jiménez et al. (2011) state that it is striking that, in order to solve problems with an apparently additive structure, many of the errors were produced by the application of subtraction, multiplication, division, or the combination of two arithmetic operations. That is to say, when solving a realistic problem; children tend to have difficulty selecting the arithmetic operation with which they intend to operate and making sense of the answer.

On the other hand, this study is a sample of the different resolution strategies that students use to give meaning and interpretation to the result of the problem. An example is a graphic and counting strategy, where S6 and S7 made a drawing to represent the situation of the problem. In problem three, the lines and dolls represent the players, and the circles and cars are the vehicles that will be used to move. In this representation, the student performs a count in rounds or consecutively. These strategies were also evidenced in research (e.g., Downton, 2009; Ivars & Fernández, 2016; Sanjuán, 2021; Zorrilla et al., 2021).

DISCUSSION

In the answers given by the students in problem 3 of type 1 as shown in Figures 1, 3, 6 and 7, the use of the algorithm of division, multiplication, and the fact of including drawings as a graphic strategy and repetitive counting are evident. These heuristic problem-solving strategies allowed students S1, S2, S6, and S7 to arrive at the correct solution and interpret the remainder or residue in terms of the actual situation or circumstance of the problem. In the case of using the division algorithm properly, Lago et al. (2008), pose that students' answers when applying correct resolution procedures are usually accompanied by correct interpretations. This is evident if we consider both the division model and the types of subtraction.

In the case where the student uses multiplication as a calculation to solve the problem, Downton (2008) asserts that young children can solve complex division problems when provided with a problem-solving learning environment that encourages them to draw on their intuitive thinking strategies and knowledge of multiplication. On the other hand, the fact of including graphic strategies such as drawing and performing successive counts, allowed the students to solve the problems. In this sense, Ivars y Fernández (2016), state that the student graphically represents the sharing process and counts the elements a group has. Likewise, using graphical strategies to represent the problem facilitates students to identify the structure underlying the problem (Santos, 1997, cited in Zorrilla et al., 2021).

In problem 2 of type 2 (See Figure 2) S2 interpreted the numerical answers correctly (context of the distribution) successfully achieving the solution of the problem. This result agrees with that obtained in Lago et al. (2008), where in this type of problem (non-decimal quotient), students always interpret the numerical answer with a high percentage of correct interpretations. However, it disagrees with the findings of Zorrilla et al. (2021), wherein in non-decimal quotient problems, realistic responses decrease from 81.4% to 62.5% between fourth and fifth grade. This decrease coincides with the increase of unrealistic responses in fifth grade, in which students solve the problems by giving the decimal quotient as the solution without considering the distribution context.

In the case of problem 1 type 1 as shown in Figures 4 and 5, on the one hand S4 applies procedure or algorithm of division properly, however it does not interpret the rest in terms of the actual situation of the problem. That is, the student forgets the text of the problem and gives as a correct answer the one found with the division algorithm, thus generating an incorrect result. For Verschaffel et al. (2009), students' weak performance in DWR problem solving is because they provide many mathematically correct but situationally inappropriate answers. Likewise, students present difficulties with problem situations about division. In addition, they require the activation of realistic considerations and sense-making to give an adequate interpretation of a non-integer quotient. Furthermore, Galvão & Labres (2006) posit that, children do not realize the meaning of the remainder in solution processes when it comes to divisions. In some cases, the remainder is seen as something superficial and not part of the problem's interpretation.

İncikabıa et al. (2020) showed that most students using the division algorithm successfully applied the operation steps but had difficulty interpreting the remainder. This result is interpreted from a clause of the didactic contract called formal delegation proposed by D'Amore and Martini (1997). According to the authors, solving a school problem coincides with finding the most appropriate operations, i.e., interpreting the text arithmetically and moving from natural language to arithmetic expression. The result of this operation is interpreted as the answer to the problem. At the end of this phase, the solver forgets the text and focuses on solving the operation.

Finally, in the S5 case at the time of solving problem 1 fails to understand the problem or identify the appropriate heuristic resolution strategy to solve it. That is, the fact of not understanding the problem leads the student to perform different arithmetic operations using trial and error but without reaching the correct solution. For Inoue (2005), students execute arithmetic operations without thinking, without evaluating their actions about common sense understanding of real-life practices. In that sense, Cooper & Harries (2005), when faced with a contextualized division with remainder, students fail to identify and use the appropriate operations. Likewise, S5 considers that multiplication, addition and division imply a larger number and for subtraction a smaller number without taking into account the understanding of the problem. For Jiménez and Ramos (2011), the incorrect beliefs generated by the didactic contract in the classroom seem to be responsible for the greater or lesser difficulty in solving the problems. In that sense, Parra and Rojas (2010) found that students perform any operation, following the rules learned in the solution of school arithmetic problems, in which the important thing seems to be to identify numerical data and use an arithmetic operation to give a numerical answer. In addition, As Rodríguez et al. (2009) state, students usually misinterpret problems from the beginning, and this initial error guides the entire solution process, including the interpretation stage. Also, students' difficulties in DWR problems did not seem to stem from the lack of interpretation of the correct numerical answer but the initial misunderstanding of the problems.

CONCLUSIONS

In elementary school, division problems with remainder (DWR) have been a source of conflicts or difficulties for students in finding the appropriate algorithm and making real sense of the problem. Likewise, the lack of contextualized problems in the classroom means that students are only prepared to solve routine problems. Lagos et al. (2008) argue that mathematical concepts must be perceived as useful in real life and that mathematics classes should favor understanding and reflection. For Inoue (2005), real-life knowledge plays an important role in mathematical thinking. In this sense, it is necessary for teachers' lesson plans to include this type of division problem with remainder and encourage students to use different resolution strategies.

In the problems of division with rest raised in the present research, it would be expected that the students of 4th grade of basic primary would solve these problems taking into account the following aspects: Read the problem carefully in order to understand it. That is, here the student must keep in mind the data or information provided by the problem and therefore the question that arises and the situation or circumstances that the problem presents. In addition, the student must create or design a path for resolution of the same, including heuristic resolution strategies. For example, mental calculations, trial and error, making a representation, outline or diagram, making a table and illustrations or drawings. Once this plan is executed and an answer to this problem is available, the student must relate the situation described in the problem with his reality. That is, the fact of relating it to your daily life will allow you to understand it better.

In that sense, teachers in their classrooms could present their students with different heuristic resolution strategies, making explicit the role played by the quotient and the rest in problems of this type. It is also suggested that students discuss among themselves and with the teacher the different meanings and interpretations that arise when solving this type of problems, specifically those of division with rest. The above could be useful to make it clear to the student that the realization of the algorithm in a strict way without making use of the interpretation of the result and reality is not always correct.

However, the analyzed results show that even though the student uses the division algorithm and correctly interprets the problem, it is still evident that some students use this algorithm without interpreting reality. It should also be noted that using arithmetic operations is not the only way the student uses to solve the problem. This is due to the use of strategies, such as the graphic strategy combined with skip counting or successive counting, where the student shows an answer coherent with the real part of the problem.

A possible limitation of our investigation would be the fact that we have not considered division problems in which, both in the dividend and in the divisor the figure of the units is zero. In the present work, problems related to the division with rest (DWR) were addressed. An example of this could be the following problem: "In a stationery store the employee needs to store 130 pencils in boxes with capacity for 20. How many boxes will you need?" One of the strategies that a student could have used would be to omit the zeros that appear in the quantities and therefore consider that the quotient and the rest would be respectively 6 and 1, showing with it the attachment to the algorithm (formal delegate) (D'Amore, 2006). However, the interpretation of the rest in realistic terms would be that the remaining 10 pencils would merit the use of an additional box, and therefore, the answer to the problem would not be 6 but 7 boxes.

Finally, it is suggested to present students with these types of problems to develop different strategies according to the context and the meaning given to the problem's solution. We consider that this would help them to give meaning to the procedures with arithmetic operations. For Ivars and Fernández (2016), giving students opportunities to solve problems using their strategies allows them to propitiate the confluence of their abilities with more formal approaches.

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