Children's Errors in Written Mathematics

Chin Ying Liew*, Siow Hoo Leong, Nor Hazizah Julaihi, Tze Wee Lai, Su Ung Ting, Chee Khium Chen, Anniza Hamdan

Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Cawangan Sarawak, Kampus Samarahan 2, 94300 Kota Samarahan, Sarawak, Malaysia

*Corresponding author: cyliew@uitm.edu.my

Abstract: *Studies of errors in mathematics are essential for mathematics educators to design and contextualize a whole new instruction accordingly. Nonetheless, less attention has been given to the mathematical writing errors when compared to the mathematical conceptual and procedural errors. It is mainly because the former mistakes usually do not affect the final answer or are seemingly irrelevant to students' mathematics knowledge. Most of the studies surveying the mathematical writing errors focus on the undergraduates, followed by middle school students. Little has been done towards the primary school pupils. This paper aims to identify and classify the types of mathematical writing errors committed by primary school pupils in three domains: number and operation, measurement, and statistics. A qualitative approach is employed to identify and classify the mathematical writing errors committed by 29 above-average students from three schools. The results show that six categories of mathematical writing errors are observed. Lack of term or phrase and misuse of mathematical symbols top the overall categories of errors. The close-up views of the solutions with written errors provide some insights to tackle the problem by the mathematics teachers starting from the tender age of the students.*

Keywords: Elementary education; Lack of term or phrase; Mathematics grammar; Misuse of mathematical symbols; Writing error

INTRODUCTION

Studies of errors in mathematics are pertinent in the field of mathematics education. Mathematics educators believe that a whole new instruction could be designed and contextualized accordingly based on the errors identified (Bray, 2013; Rushton, 2018). By realizing the errors committed, students tend to experience a more rewarding learning experience, promoting enjoyment and confidence (Huang & Lin, 2015) and leading to an increased level of positive attitude towards mathematics. These students are more likely to engage in active and constructive mathematical learning processes and hence perform well in mathematics. Students who have performed well in mathematics are reported to have a higher possibility to join the Science, Technology, Engineering and Mathematics (STEM)-related field career (Blotnicky et al., 2018).

Mathematical writing errors, conceptual errors, and procedural errors are the general types of errors committed by the students (Sumartini & Priatna, 2018). Many studies have been done in researching issues related to students' mathematical errors with many of them focusing on the conceptual and procedural aspects (Arhin & Hokor, 2021; Ekawati et al., 2022; Mutambara & Bansilal, 2021; Payadnya et al., 2021; Rushton, 2018). However, Usiskin (2015) has stipulated that familiarity with mathematics language, both spoken and written, is a prerequisite to the understanding of mathematics concepts and procedures. It is commonly associated with the use of pronunciations, symbols, notations, and operations (Khoshaim, 2018; Matthews et al., 2012). The same researcher commented that the negligence of it would cause severe distortion of the intended mathematical meaning and solution, and perpetual bad mathematical writing habits of which one's mathematical professionalism could be greatly impaired.

Seo (2015) considers mathematical writing as an intertwining of three aspects: words, symbols, and images. It is referred to as "a thematic condensation of symbols, terms, and images to convey mathematical knowledge and meaning" (p.135). It implies mathematical writing as an endeavor to communicate mathematics between the students and their audience where the writing should be understandable by the audience. Surprisingly, relatively less attention has been given to the mathematical writing error specifically, when compared to the mathematical conceptual and procedural errors. This is especially true when such mistakes do not affect the final answer or are seemingly irrelevant to the students' mathematics knowledge (Freeman et al., 2016; Guce, 2017; Khoshaim, 2018). Researchers commented that the students who are offered a place in the university generally do not know how to write mathematics (Gunns et al., 2020). Although the students enrolled in STEM-related programs at the varsities show satisfactory performance in the technical aspects of mathematics, their skills in showing and explaining the technical mathematical results draw concerns among the researchers (Arévalo et al., 2021). These students used to self-perceived that they are "good at math" but "poor at writing" mathematics (Li et al., 2019). This may be due to the fact that written mathematics is still yet a common practice in the schools where most of the mathematics classes relied on mathematical skills building and conceptual understanding activities (Urquhart, 2009). Researchers generally regard mathematical writing as an underutilized method of instruction, be it in college or school (Heavner & Devers, 2020b; Urquhart, 2009).

In a study analyzing the mathematical writing errors of students enrolled in a calculus course, Khoshaim (2018) reported that missing a symbol like the equal sign or a notation were the most common inattentive errors observed, followed by the misuse of the equal sign and other symbols. In another study conducted by Guce (2017) that involved students from an advanced calculus course, it revealed that the most committed mathematical writing errors were incorrect grammar and misuse of mathematical symbols. A study on the types of mathematics error among the students attending a ring theory course found that the students had problems in using standard mathematical symbols to express mathematical definitions correctly, and no proper reasons were given for the final answers they obtained (Fatmiyati et al., 2020). Heavner and Devers (2020a, 2020b) claimed that undergraduates should be furnished with mathematical writing skills in order to communicate the mathematical concepts competently, which will eventually contribute to increased understanding of college mathematics. Studies conducted on middle and elementary school students reveal that the majority of students had difficulties communicating mathematical ideas and using the correct mathematical terms or notations (Adnan et al., 2021; Ng, 2020). The primary school pupils' weaknesses in translating the word problems into mathematical terms or notations were noticed (Rosli et al., 2020). In a worrying situation, pre-service teachers were reported to show impediments in understanding the mathematical notations and terminologies (Makonye & Ramatlapana, 2017).

Khoshaim (2018) categorized the mathematical writing errors into three: missing symbol or notation, misuse of symbol or notation, and miss-order of operations, which are errors that occurred due to a lack of attention or inattentive errors. These categories can be refined into a more detailed classification of mathematical writing errors by Guce (2017) with nine categories: misuse of mathematical terms; misuse of mathematical symbols; incorrect notation; incorrect grammar; incorrect capitalization; no or incorrect punctuation; vague term; incorrect term or phrase; and lack of term or phrase. The classification has been adopted in the analysis of written error in the, for example, trigonometry topic (Setiawan, 2021) and online mathematics courses (Heavner & Devers, 2020b). Most of the studies scrutinizing the errors in written mathematics are on the college or university students, followed by middle school students. Studies about types of mathematical writing errors among elementary or primary school pupils are scarce. Findings from studies by Hornburg et al. (2022), Fyfe et al. (2020), and Vincent et al. (2015) imply that formal understanding of mathematical knowledge at the elementary level advances the positive mathematical development of the children at the higher level. This includes inculcating proper mathematics language, focusing on the use of words and symbols in relating the mathematical concepts learned and mathematical procedures performed among the children. Consequently, this study aims to identify the mathematical writing errors committed by primary school pupils when executing their procedural explanations and classify the types of these errors based on the classification of mathematical writing errors by Guce (2017), with quantification of the error's occurrences. The mathematical writing studied here focuses mainly on symbolic representation and the use of words. Knowing the importance of possessing mathematical writing skills, it is thus imperative to nurture primary school pupils at the beginning level of their mathematics education.

The organization of this paper is as follows: Section One gives the introduction and background of this study; Section Two provides the method and material used; Section Three presents the results obtained from this study and its corresponding discussions, and the last section concludes the study together with some recommendations for future research.

METHOD AND MATERIAL

A total of 29 sixth graders participated in this study. They were from three primary schools located among the communities of similar family socioeconomic backgrounds. These schools were referred to as Schools X, Y and Z in this study, with a total of 24, 43 and 41 sixth graders respectively. The students were purposively selected based on their score of at least 65% in an instrument adapted from a set of sample questions released by the national examinations board. The format and scope of this instrument are equivalent to the primary school national public examination of Malaysia. The instrument consists of sixteen questions covering three domains: number and operation (Question 1 and 3 are under the topic of number and operation, while Question 2, 13, 15 and 16 are under the topic of fraction and percentage), measurement (Question 4 and 5 are under the topic of weight, whereas Question 6 and 7 are under the topic of volume of water), and statistics (Question 8, 9 and 10 are under the topic of mean, while Question 11, 12 and 14 are under the topic of pictograph), and the total score is 100%. The students who scored at least 65% were expected to possess adequate mathematical conceptual and procedural skills (Guce, 2017). Therefore, they were selected for the analysis of mathematical writing errors.

A qualitative approach was employed in this study to identify and classify the mathematical writing errors. The mathematical writing error classifications by Guce (2017) was adopted given its conclusive and adequate consideration of all main distinctive errors in written mathematics across all education levels although the findings presented are on undergraduate students. By adopting the same mathematical writing error classifications, the results obtained from this study on primary school pupils add value to the research community where comparisons among different levels of education could be made. In this study, the solutions with the correct final answers from all the students were analyzed for the mathematical writing grammar used. Only the solutions with correct final answers were scrutinized for the written errors because these solutions were assumed to have possessed a satisfactory level of mathematical procedural and conceptual knowledge.

After all occurrences of mathematical writing errors committed by the students were identified and classified accordingly, quantification of the errors was carried out to provide richer information. The number of occurrences of the errors was summarized in total and percentage. Each occurrence of an error category was recorded using its corresponding coding and was counted as one error. The process of identifying and classifying the errors was repeated at least twice for confirmation. It was carried out by two researchers separately and their findings were compared. If the categories of an error classified by the two researchers were different, a discussion was carried out to reach an agreement. Should a dispute arise, and no agreement could be reached, a third researcher was called to finalize the error category. In this paper, the categories of all errors were agreed upon by two researchers. Table 1 provides the codes and brief descriptions of the nine categories of mathematical writing errors used.

Category	Code	Brief description
Misuse of mathematical terms	MMT	Incorrect use of mathematical terms such as mathematical equations and expression.
Misuse of mathematical symbols	MMS	Incorrect use of mathematical symbols such as the equal sign "=" and upper-case and lower-case symbols.
Incorrect notations	IN	Incorrect use of mathematical notations, the symbolic expressions with their established meaning such as (Adnan et al.) and a_i, and $\frac{dy}{dx}$ and $\frac{\partial y}{\partial x}$.
Incorrect grammar	IG	Incorrect use of grammar and spelling when writing mathematics in sentences where standard grammatical rules of writing are violated.
Incorrect capitalization	IC	Incorrect use of capitalization where words either are capitalised when they should not be or are not capitalised when they should be.
No or incorrect punctuation	NIP	Incorrect use or absence of punctuation when writing mathematics in complete sentences.
Vague term or phrase	VT	A vague term refers to a term that is unclear of its meaning in which the distinctness of the mathematical statement becomes ambiguous or vague.
Incorrect term or phrase	IT	An incorrect term or phrase refers to the inappropriate use of a term or phrase in a mathematical statement which causes the inaccurate idea of the statement.
Lack of term or phrase	LTP	Lack of term or phrase refers to the presentation of a mathematical solution for a mathematics problem without any or adequate explanation such as defining a variable used in the solution, causing possible misinformation to the readers.

Table 1: Codes and brief description of mathematical writing errors used

RESULTS AND DISCUSSION

This section presents the results of mathematical writing errors according to each domain for the three participating schools. The frequencies of writing errors from the three schools are first summarized as a whole, and then the frequencies breakdown from each school are tabulated and compared. Each school is analyzed separately to investigate if there is a different trend or pattern of errors across the schools, which could be due to the school environment such as the experience or training of their teachers. Besides, it also considers equal importance for each school as the numbers of students selected from the three schools for analysis of written errors are unbalance. The types of errors committed will thus not be dominated by the school with more students. Then, close-up views of the solutions are shown to explicate how a particular written error was made by the students.

Number and Operation

Table 2 shows the number of various written errors observed in the solutions with correct final answers for the domain of number and operation for the three schools. Five categories of errors were observed for this domain: MMS, IG, VT, IT, and LTP. However, out of the total of 68 errors observed, only one observation was noticed for each of the IG, VT, and IT errors. The majority of the errors recorded was LTP (73.5%), followed by MMS (22.1%). School Y recorded a very high percentage of LTP error (81.1%) when compared with School X (69.2%) and School Z (61.1%). Interestingly, more categories of errors (MMS, VT, IT and LTP) were observed in the solutions of students from School X, followed by three categories of errors (MMS, IG and LTP) were observed for School Y while only two categories of errors (MMS and LTP) were observed for School Z.

		School X (n_65% = 4)						School Y (n_65% = 19)						School Z (n_65% = 6)						Total (N_65% = 29)					
Domain	Q	✓	MMS	IG	VT	IT	LTP	✓	MMS	IG	VT	IT	LTP	✓	MMS	IG	VT	IT	LTP	✓	MMS	IG	VT	IT	LTP
Number and operation	1	4	1	0	1	0	0	17	0	0	0	0	0	6	0	0	0	0	0	27	1	0	1	0	0
	2	4	0	0	0	0	0	18	0	0	0	0	0	6	0	0	0	0	0	28	0	0	0	0	0
	3	4	1	0	0	1	3	14	2	0	0	0	12	6	4	0	0	0	4	24	7	0	0	1	19
	13	4	0	0	0	0	2	8	4	0	0	0	2	3	1	0	0	0	1	15	5	0	0	0	5
	15	4	0	0	0	0	4	19	0	1	0	0	16	6	2	0	0	0	5	29	2	1	0	0	25
	16	0	0	0	0	0	0	3	0	0	0	0	0	1	0	0	0	0	1	4	0	0	0	0	1
Total			2	0	1	1	9		6	1	0	0	30		7	0	0	0	11		15	1	1	1	50
Total Errors			13						37						18						68				
Percentage			15.3	0	7.7	7.7	69.2		16.2	2.7	0	0	81.1		38.9	0	0	0	61.1		22.1	1.5	1.5	1.5	73.5

Note: The domain is listed in the first column of the tables. The second column ‘Q’ refers to the question number in the instrument. The number of students who had answered each question correctly is given in the column with the column header ‘✓’. This is followed by columns showing the categories of writing errors classified and the number of errors identified for each category. The last column of the tables, with column header labelled ‘Total’ presents the overall findings of this study for the domain. The notation ‘n_65%’ refers to the number of students who scored at least 65% for each school; whereas ‘N_65%’ refers to the total number of students in this study that had scored at least 65%.

Table 2: Incidences of errors observed for the domain of number and operation

Figure 1(a) shows an example of an MMS error that involves the equal sign (“=”), where the subject of the equal sign is missing in the solution of “= 260/2 = 130”. This is the most common mathematical writing error observed for MMS in the domain of number and operation. Another common misuse of the equal sign is that the students exploited it as a way to refer to the final answer of their solutions. There was one particular example of MMS that is presented in Figure

1(b), where a student misused the equal sign and wrote “7 = 700 000” to express the final answer of “7” as the digit that carried the place value of hundred thousands in the given number of “2 736 981”. Figure 1(c) presents another serious misuse of mathematical symbols involving the equal sign. Although the vertical calculation shown on the left is correct, the horizontal procedure on the right is incorrect because “13 259 + 461 × 2” was not equal to “27 440”. This error suggested a partial insight error, a type of bit error due to the carelessness or lack of conceptual understanding (Sumartini & Priatna, 2018), on the priority of executing the addition and multiplication operations in this example.

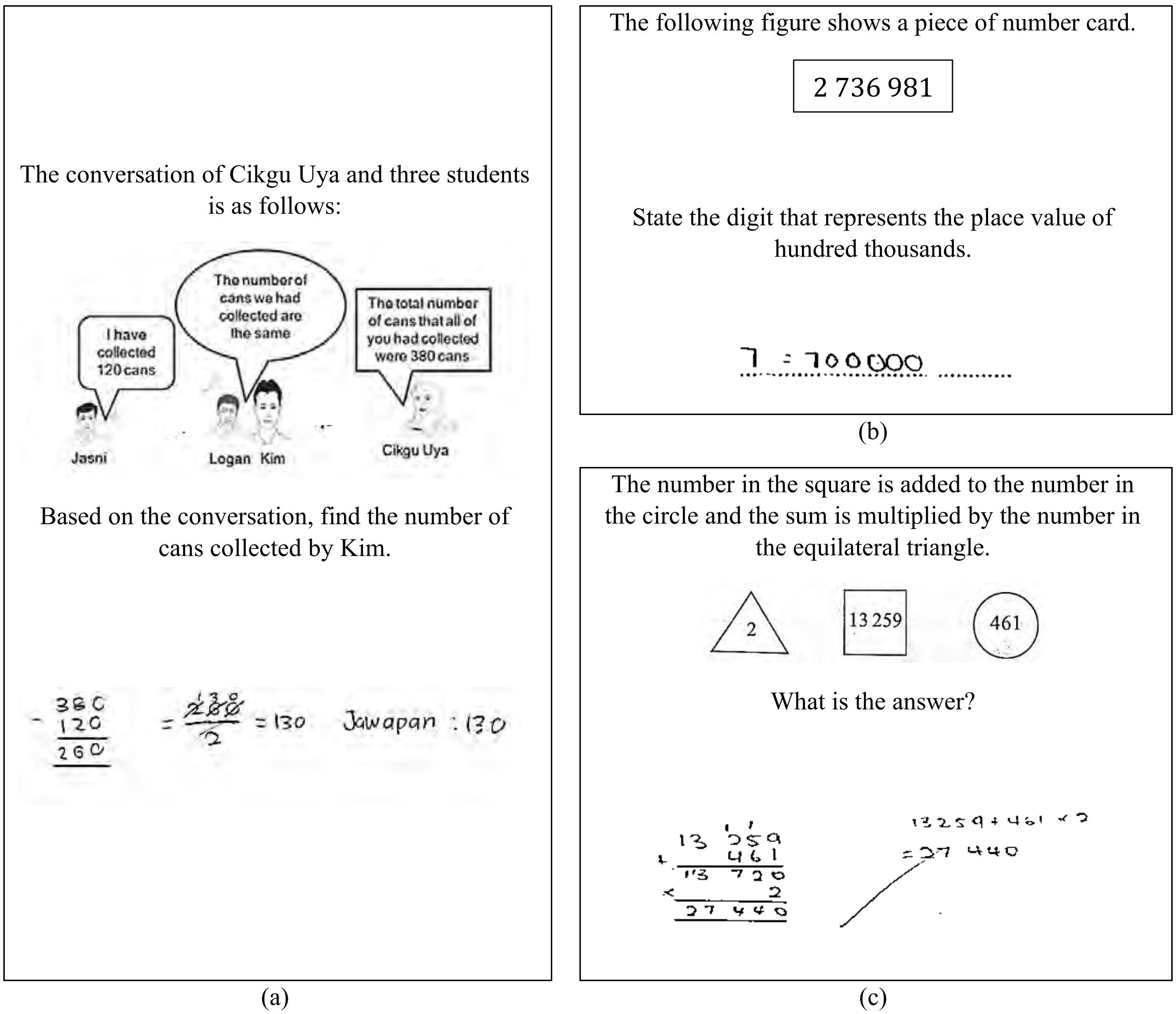
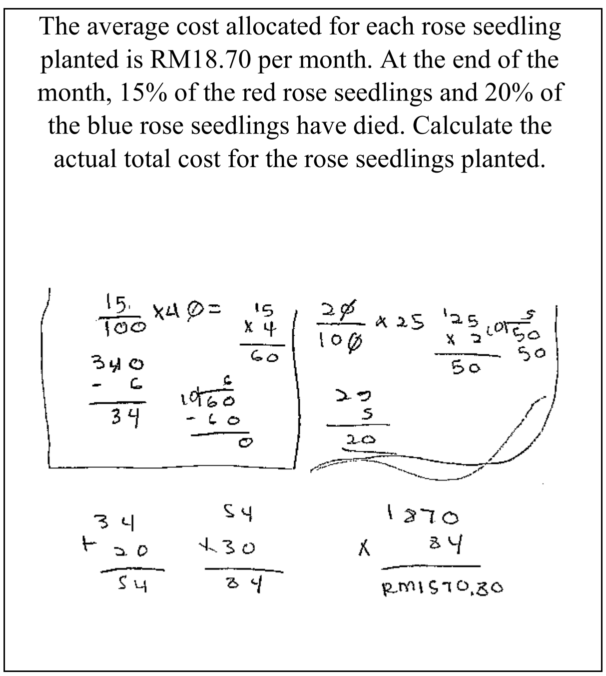
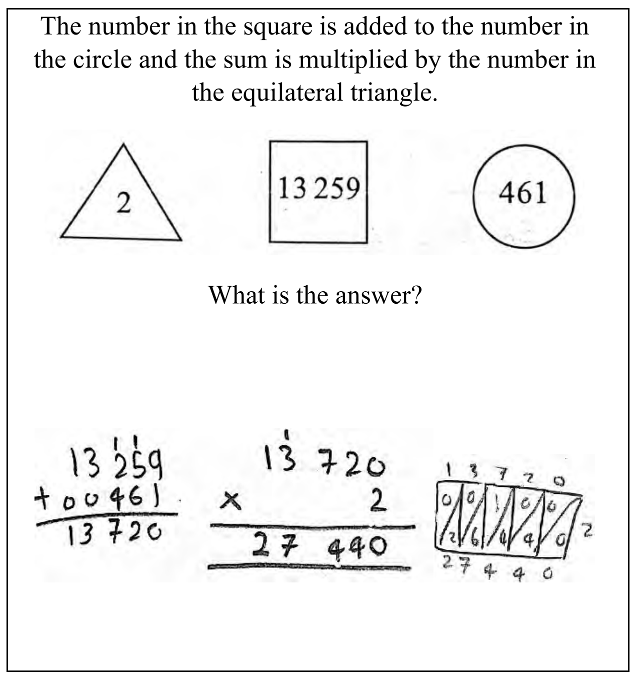


Figure 1: MMS by (a) Z35 (no subject for “=”); (b) X7 (unequal value on both sides of “=”) and (c) Z32 (incorrect horizontal calculation)

On the other hand, the LTP errors recorded in this domain were due to the lack of term or phrase in the solution of the students. There was an absence of terms or phrases to connect and relate the concepts behind the mathematical expressions presented by the students in solving the questions. This is especially obvious when two or more different arithmetic operations are needed to obtain the final answer. In fact, many LTP errors committed by the students in this domain were because the students did not seem to bother to specify and inform the reader which solution was the final answer. Two examples are shown in Figure 2(a) and Figure 2(b). Moreover, the students did not indicate specifically the purpose of each steps written. These are typical examples of mathematical writing errors where "… *presenting only the mathematical solution without any explanation is like assuming that the solution will speak for itself* …" as purported by Guce (2017). An acceptable final answer without LTP error was shown in Figure 1(a), where Z35 had specified and informed that the final answer was 130 by indicating it using "Jawapan: 130" (in Malay), which means "Answer: 130".For the IG error, an example presented in Figure 3(a) showed that the student gave "15 bunga ros" (in Malay), which means "15 roses" as the final answer instead of the quantifier "rose seedlings" used in the question. This had caused incorrect grammar in the answer. For the VT error, the term "or" had been used in "7 or 700 000" for the solution of the question asking for the digit that is in the hundred thousands place value for the number of "2 736 981". This is presented in Figure 3(b), where "atau" (in Malay) means "or". So, by answering "7 or 700 000", the term "or" is vague in this context. As for the IT error, Figure 3(c) shows an example where the student wrote "130 bilangan" (in Malay), which means "130 number" instead of "130 cans" in the solution for a question asking about "number of cans collected". The term "bilangan" (in Malay) or "number" is incorrectly used.



(a)



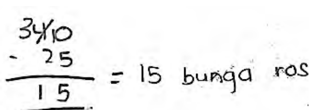
(b)

Figure 2: LTP by (a) Y32 and (b) Y36

Measurement

In the analysis of mathematical writing errors in the domain of measurement, the number of written errors observed in the solutions for Questions 4 and 5 from the topic of weight, and Questions 6 and 7 from the topic of the volume of water were grouped respectively in Table 3. The results show that MMS, IN, and LTP were three categories of error observed in the solutions of the questions under this domain. Out of the total of 48 errors observed, there was only one IN error, while MMS error topped the list (77.1%), and followed by LTP (20.8%). MMS was found to be the most common error, which is followed by LTP error across all the three schools in the domain of measurement. It is noticed that only one category of errors (MMS) was recorded from School X, whereas all the three categories of errors (MMS, IN, and LTP) were observed in the solutions from School Y and two categories of errors (MMS and LTP) were observed for School Z. This could be because School Y recorded the greatest number of students with the correct final answer while School X had the least number.

Calculate the difference between the largest number of rose seedlings and the least number of rose seedlings.

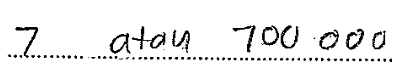


(a)

The following figure shows a piece of number card.

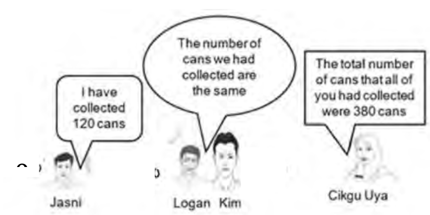
2 736 981

State the digit that represents the place value of hundred thousands.

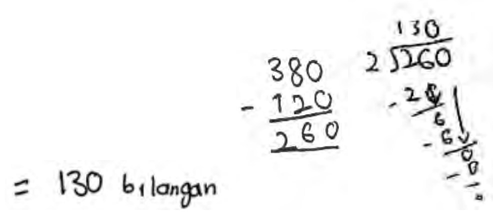


(b)

The conversation of Cikgu Uya and three students is as follows:



Based on the conversation, find the number of cans collected by Kim.



(c)

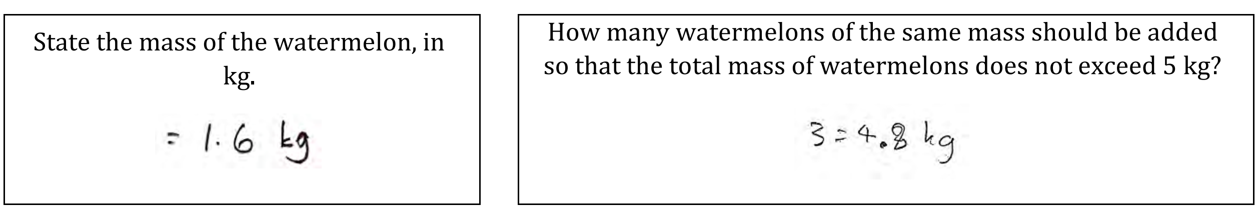
Figure 3: (a) IG by Y32, (b) VT by X15 and (c) IT by X20

		School X (n_65% = 4)				School Y (n_65% = 19)				School Z (n_65% = 6)				Total (N_65% = 29)			
Domain	Q	✓	MMS	IN	LTP	✓	MMS	IN	LTP	✓	MMS	IN	LTP	✓	MMS	IN	LTP
Measurement	4&5	3	1	0	0	18	17	0	1	5	4	0	1	26	22	0	2
	6&7	1	1	0	0	18	10	1	7	4	4	0	1	23	15	1	8
Total			2	0	0		27	1	8		8	0	2		37	1	10
Total Errors				2				36				10				48	
Percentage			100.0	0	0		75.0	2.8	22.2		80.0	0	20.0		77.1	2.1	20.8

Table 3: Incidences of errors observed for the domain of measurement (See Table 2 for Note)

The common errors observed under MMS were the misuse of the equal sign, especially when the subject was missing in the expressions of the solutions. An example is shown in Figure 4(a). Another instance of MMS error observed was when the students gave "3 = 4.8 kg" (Figure 4(b)) to express that the weight of three (3) watermelons was 4.8 kg. This was a typical MMS error where both sides of the equal signs were not equal.

On the other hand, the common LTP error observed under the measurement domain was similar to the number and operation domain, in which there were lacking terms or phrases to connect the steps of solutions. Two examples are presented in Figure 5(a) and Figure 5(b) where the former shows that the student did not specify which one was the final answer while the latter shows the lack of a phrase for the final answer such as "The mean volume of water is 0.18 liter" or "Answer: 0.18 liter". Besides that, the students always failed to provide a unit of measurement when it was needed in their solutions or missed out the correct arithmetic operation symbol as exemplified in Figure 5(b). As for the IN error, an example is depicted in Figure 5(c). The notations of a milliliter (mL) and liter (L) were erroneously used in the solution where milliliter was used in the question, but liter was given by the student, instead.



(a) (b)

Figure 4: MMS by (a) Z37 (no Subject for "=") and (b)Y42 (unequal value on both sides of "=")

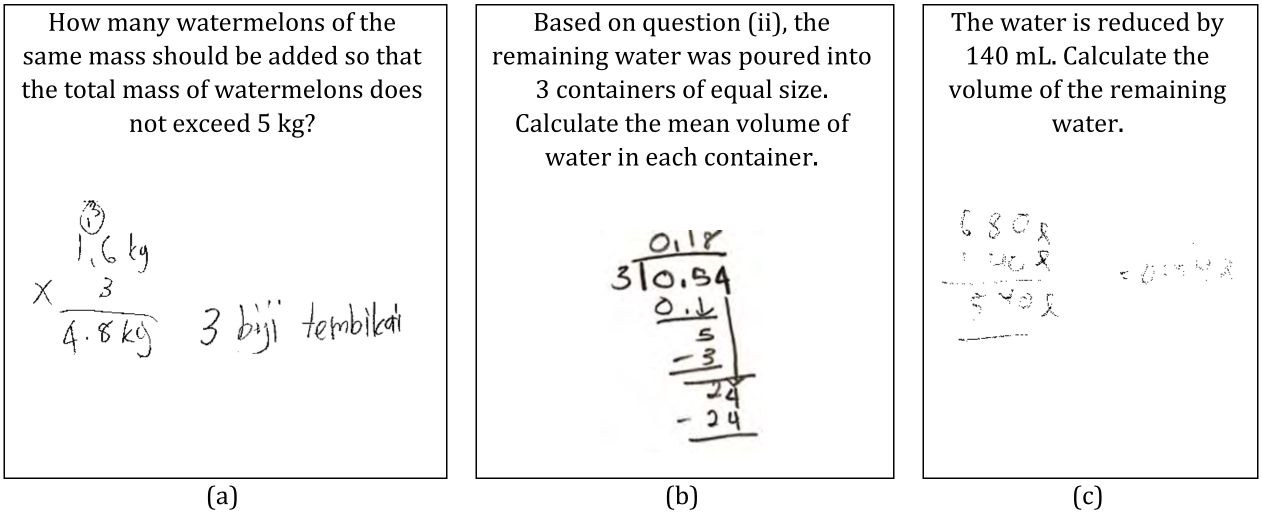


Figure 5: (a) LTP by X15, (b) LTP by Y35 and (c) IN by Y24

Statistics

The number of written errors observed in the solutions for Questions 9 and 10 from the topic mean, and Questions 11 and 12 from the topic pictograph were grouped together respectively in Table 4 for analysis. There were two categories of errors reported for the solutions of questions under the domain of statistics: MMS and LTP, which accounted for 23.9% and 76.1% of the errors respectively. The results reported for School X and School Y were similar, with more than 80% LTP error whereas the scenario was the opposite for School Z with almost 64% MMS error. No other category of errors was observed for this domain. This could be due to the nature of the questions that entail short solutions.

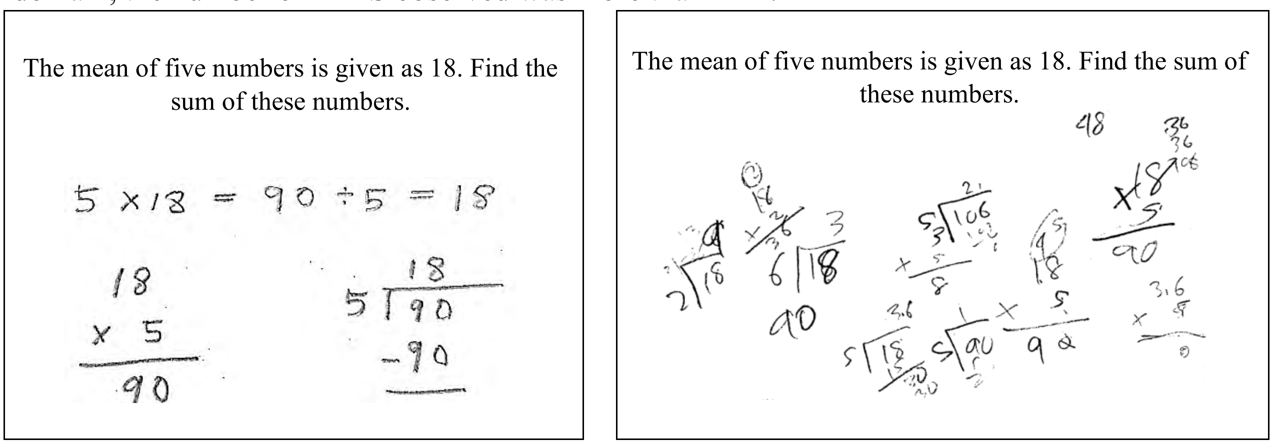
		School X (n_65% = 4)			School Y (n_65% = 19)			School Z (n_65% = 6)			Total (N_65% = 29)		
Domain	Q	✓	MMS	LTP	✓	MMS	LTP	✓	MMS	LTP	✓	MMS	LTP
Statistics	8	1	0	0	14	4	5	5	3	1	20	7	6
	9&10	2	1	1	14	2	21	1	3	0	17	6	22
	11&12	2	0	1	8	2	7	2	0	3	12	2	11
	14	3	0	3	15	0	9	5	1	0	23	1	12
Total			1	5		8	42		7	4		16	51
Total Errors		6			50			11			67		
Percentage			16.7	83.3		16.0	84.0		63.6	36.4		23.9	76.1

Table 4. Incidences of errors observed for the domain of statistics (see Table 2 for Note)

Two main common MMS mathematical writing errors committed in the domain of statistics were the use of equal signs throughout the steps of solution and the missing of a subject for the equal signs written in an expression of the solutions. An example is presented in Figure 6(a) where 5×18 was neither equal to $90 \div 5$ nor 18 although $90 \div 5$ did equal to 18. The solution shown in Figure 6(a) is a typical incidence as commented by Guce (2017) where "… *students tend to use equal signs to connect several lines of solution which are not actually equal*". An example of LTP error is given in Figure 6(b) where the student was "… *presenting only the mathematical solution without any explanation* …" which "… *is like assuming that the solution will speak for itself* …" (Guce, 2017).

Across Domain

Figure 7 presents the summary of overall errors identified and classified in this study across the three domains. Altogether, there were six categories of mathematical writing errors observed. LTP appeared to top the overall errors observed, followed by MMS. Most of the LTP was observed in the number and operation domain as compared to statistics and measurement domains. MMS was observed mostly in the measurement domain as compared to number and operation, and statistics domains. The number of LTP observed was more than MMS in the domains of number and operation, and statistics. However, in the case of the measurement domain, the number of MMS observed was more than LTP.



(a) (b)

Figure 6: (a) MMS by Y14 (using "=" throughout) and (b) LTP by X15 (working all over the solution)

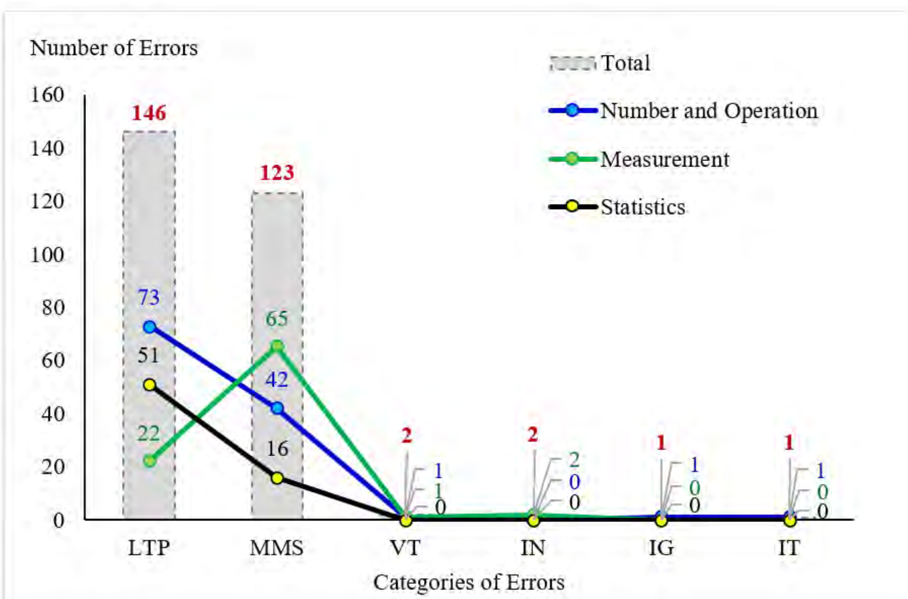


Figure 7: Cognitive process dimension

Observations of the other four categories of errors (IN, IG, VT, and IT) were very minimal and there were no incidents recorded under the category of MMT, IP, and NIP errors. This may be due to the nature of the instrument used in this study where most of the questions do not require lengthy solutions. As explained in Section 2, the instrument of this study consists of typical questions the sixth graders are expected to answer in the national primary school public examination. It is different from the expository-writing-nature instrument used by Guce (2017) where all categories of errors were observed. The findings of this study that involve primary school pupils are in accord with that of Khoshaim (2018) and Guce (2017), whose respondents were students in tertiary education institutes. Misusing mathematical symbols, especially the equal sign appears to be a writing error common to students across the levels of education.

The findings obtained in this study give rise to two implications worth highlighting: a) the correct use of equal signs; and b) the use of appropriate terms or phrases in presenting the mathematics solutions by the primary school pupils. It is vital for young children to possess a sound concept and interpretation of equal sign as this is closely related to mastering the algebraic skills and knowledge (Fyfe et al., 2020; Hornburg et al., 2022). Researchers have commented that algebra is a crucial step to not only mathematics studies at higher level but also higher education (Madej, 2022; Matthews et al., 2012). Henceforth, mathematics teachers are obliged to cultivate the good practice of using equal signs among their students and set a good example; use it only when the left-hand-side and the right-hand-side expressions of an equal sign do mean equality (Powell, 2012; Vincent et al., 2015). Both the relational and operational interpretation of equal sign among the students are important to holistic understanding of the meaning of equal sign and should be equally emphasized (Fyfe et al., 2020; Madej, 2022; Matthews et al., 2012).

At the same time, mathematics teachers ought to show and encourage the use of appropriate terms or phrases in the whole process of solving a mathematical problem, be it routine, partial-routine, or non-routine, particularly when it takes more than one step of solutions to come to the final answer of the problem. The mathematics teachers need to inspire the students to write and explain their solution and final answer. Using correct mathematical grammar is an important mode of thinking and learning for the students (Colonnese et al., 2018). The benefits are twofold: for the teacher to know both the depth and width of the conceptual and procedural understanding of their students.

Subsequently, the mathematics teachers must give equal emphasis to the grammar of the written mathematics alongside the intense attention given to the conceptual and procedural aspects of mathematics (Guce, 2017; Khoshaim, 2018), starting from the tender age of the students. As emphasized by the researchers, no toleration should be given to all possible writing errors. The power of correct mathematical writing should never be neglected in any mathematics instruction (Freeman et al., 2016). Although it may appear at a time that these mathematical writing errors are trivial to the students' conceptual understanding and procedural execution in the classroom, the consequences of continual carelessness and ignorant in mathematical writing are severe (Khoshaim, 2018).

CONCLUSIONS

This study looks into the mathematical writing errors of the sixth graders. It reveals the students' incompetency in written mathematics among the above-average performers. The findings should be taken seriously by educators as it is necessary to emphasize written mathematics besides stressing the importance of mathematical conceptual and procedural, starting from the tender age of the students (Ouyang et al., 2021; Valcan et al., 2020). As the students grow in their mathematics and English (or other language of instruction) language vocabulary at the elementary level, this could be made possible with teachers guiding the students in the class to practice mathematical writing alongside understanding the mathematics concepts and procedure in solving mathematics problems. In promoting good mathematics writing habits, teachers should pay attention to the mathematics writing errors committed in the students' homework and their formative and summative assessments. The students need to be informed clearly and explicitly of the errors and consciously be reminded that these errors are not acceptable. It is the responsibility of the teachers to be the role model and show appropriate mathematical writing in every mathematics class, when showing solutions in front of the class or to the students personally. Although highly identical patterns on the types of errors have been shown by the majority of the students from different schools of the communities with similar family socioeconomic backgrounds and located within the same geographical location, further studies in surveying any possible relationship between the demographic characteristics of students and the mathematical writing errors committed are recommended.

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