Students' Difficulty with Problems Involving Absolute Value, How to Tackle this Using Number line and Box Method

Aradhana Kumari

Borough of Manhattan Community College of the City University of New York, USA,

akumari@bmcc.cuny.edu

Abstract: *Solving problems involving absolute value is one of the hardest topics for students learning in elementary algebra course. This is an important topic in student's mathematics life since absolute value functions are important example of a function which is continuous on the real line but not differentiable at the origin. A deep understanding of these topics is very important for students. Some students understand the definition of absolute value but are still unable to apply the definition to solve the problems. Others do not understand this concept. This article focuses on using the number line when introducing/explaining this topic. A visual technique called box method helps students to write each step with correct reasoning when solving problems involving absolute value. The number line and box method helps them to see the connection between the concept and procedure used to solve the problems involving absolute value.*

Introduction

Solving absolute value equations and inequalities requires students to memorize a series of steps and follow these steps blindly. This article focuses on emphasizing definitions to students rather than just introducing them to symbols in mathematics. The second thing this article focuses that it is very important is that we ask students to critique each step they wrote in their work. This process of asking students to critique helps us as instructors to understand what they know and where we need to focus. In this article, I present some of the students' work on solving equations and inequalities involving absolute value. For many semesters, I used to teach the absolute value concept as presented in Table 1. I would define $|x|$ as the distance of x from the origin. We read $|x|$ as "absolute value of x. I would explain $|x| = 5$ means, we are looking for those numbers whose distance from zero is five units. In the definition and explanation, I did not use the number line which is why I think students struggled to understand this concept. Usually, a book begins a lesson on how to solve inequalities involving absolute value as shown in Table 1, and I used the same technique as the book does to introduce the definition of absolute value equations and inequalities. The struggle was that students did not understand the absolute value concept. They try to memorize Table 1, and hence they made mistakes. At the end of the semester during the final exam period, none of the students wrote the correct solution for the problem involving absolute value inequality, which made me think to teach this concept differently for better student understanding. I started looking at some articles on absolute value and found many excellent resources (Mark W. Ellis and Janet L. Bryson, 2011). The key idea in these resources was that they all used number line in introducing/explaining the concept of absolute value to students. I also used the number line and a visual technique called "box method" (Kumari, A. 2021) which was helpful for students to understand this concept.

Let c be a positive real number.

Equations and inequalities with absolute value symbol	**Meaning without using absolute value symbol**
$\lvert x\rvert = c$	$x = c \ \ or \ \ x = -c$
$\lvert x\rvert < c$	$-c < x < c$
$\lvert x\rvert \le c$	$-c \le x \le c$
$\lvert x\rvert > c$	$x < -c \ \ or \ \ x > c$
$\lvert x\rvert \ge c$	$x \le -c \ \ or \ x \ge c$

Table 1.

Students' difficulty

Students do not understand when and why the procedure works (Givvin et al., 2011; Stigler et al. 2010.). Students try to memorize certain "rules" without integrating reasoning and sense-making (Goldrick-Rab, 2007: Hammerman & Goldberg, 2003) and hence sometimes they make mistakes. They used the rote/memorization technique to "understand" absolute value equations and inequalities. They try to memorize the above Table 1 without understanding and hence made mistakes in solving problems involving absolute values. Using the number line sweeps away the mystery of working with absolute values and empowers students to make connections between procedures and concepts (Mark W. Ellis and Janet L. Bryson, 2011).

Analysis of students' work

After I introduced the absolute value topic to students, I gave them an assignment. One of the problems of the assignment was solve for x where $|6x - 3| \geq 9$. Below is some of the student work shown for this problem. From the work of students, A, B, C and D, we see that there is a disconnection between conceptual understanding and procedural fluency. Students are unable to translate the inequality involving absolute value to an inequality without absolute value, and hence they are unable to solve the problem correctly. We see that all the students (listed below) made the mistake of not using the keyword "OR" in their procedure. Student A made the mistake at step 6x-3 ≥ -9 since they did not translate the absolute value inequality correctly and hence rest all the procedure is wrong. Student B made the same mistake as student A when he/she translated the absolute value inequality as $-9 \leq$ 6x-3, another mistake this student made was translating the absolute value inequality "greater than or equal to" as a combined inequality $-9 \leq$ 6x-3 $\geq$ 9. Student C was unable to apply the definition of an absolute value inequality. Student D wrote each step correctly except for the missing keyword "OR". Students E and F started with a similar procedure, but when I asked them to explain why they wrote 6x-3 $\geq$ 9, 6x-3 $\geq$ 0; -(6x-3) $\geq$ 9, 6x-3 $<$ 0, their explanation was that they found it online and they did not understand the steps.

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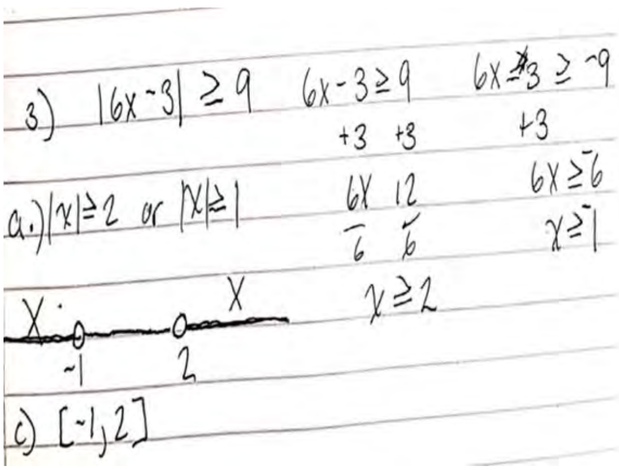
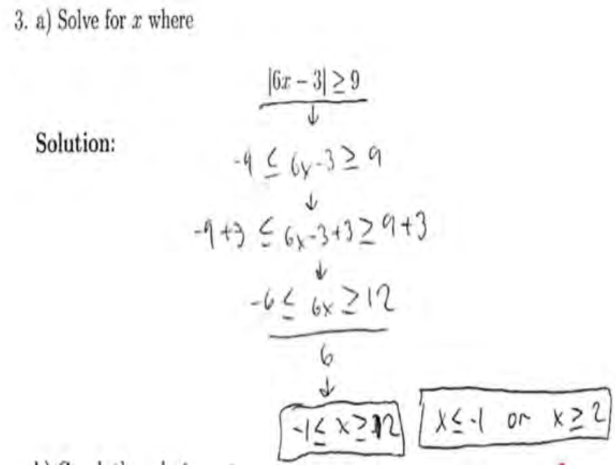
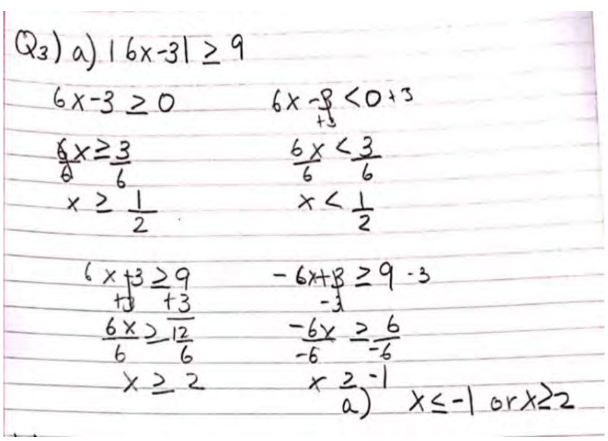


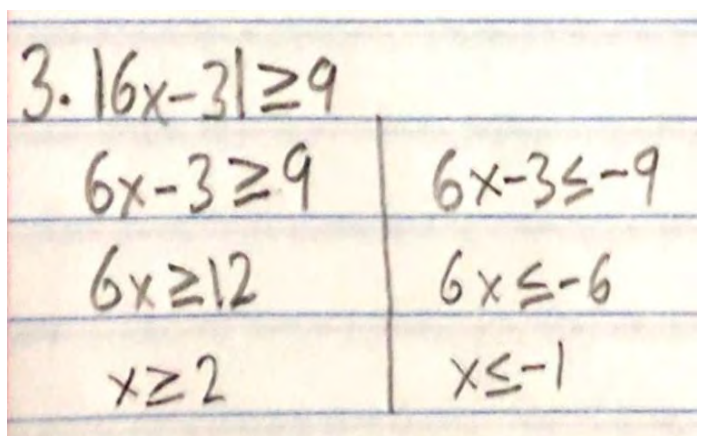
Student A



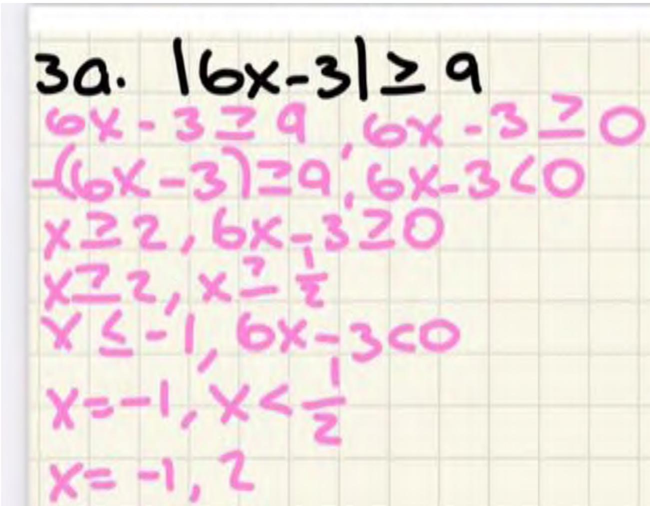
Student B



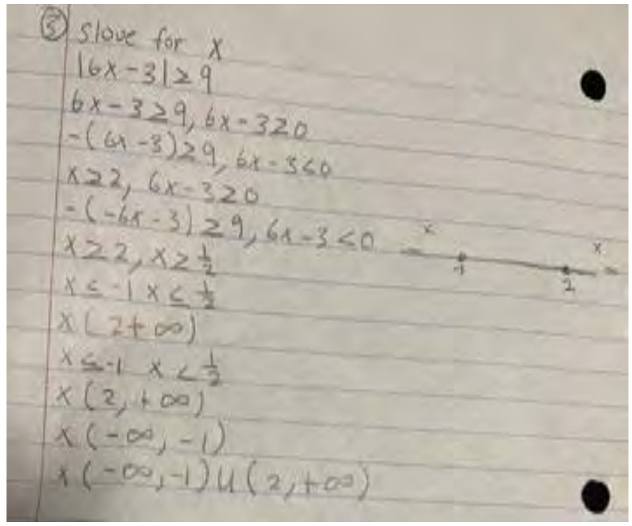
Student C



Student D



Student E



Student F

From the analysis of student work

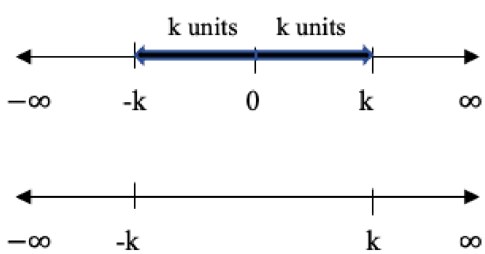
As shown from students works above, we see two kinds of difficulty students face to solve inequalities involving absolute value; firstly, some students do not understand the definition of an absolute value inequality as in the example of the work provided by students A, B, C, E, and F. On the other hand, some students do understand the definition of an absolute value inequality but are unable to correctly write the solution; for example, Student D's work missed the key word "OR".

How to help students understand absolute value concept using number line and box

I used number line and a visual technique called box method as explained in "A visual approach to solving equations and inequalities involving absolute value (Kumari. A. 2021). The summary is presented below:

1**.** Consider Table 1, where the main difficulty for students is that they do not understand how column 1 in the table is related to column 2. Hence, they try to memorize this table and consequently commit errors. We can overcome this difficulty by putting one more column in the middle (as shown in Figure 1) which uses the number line and the definition of absolute value. Figure 1 helps students understand the concept of absolute value and it also helps them to see the relationship between column 1 and column 2 in the Table 1.

2. We replace the algebraic expression inside the absolute value symbol with a box □. We transform the problem so that the right-hand side of the inequality is a non-negative real number k, we draw a number line and label k and -k on the number line (we draw k and -k because these are the two numbers whose distance from the origin is k units).



k and -k divide the number line into three pieces (as shown above). Reading from left to right, the first piece consists of numbers less than -k, the second piece correlates to numbers between -k and k and the third piece represents numbers greater than k. We find (using the problem) on which piece(s) we keep the box □, we write down the piece(s) using the mathematical inequality symbol and the box. Lastly, we replace the box □ by the algebraic expression we started with and then solve the inequality.

Absolute Value Equations and Inequalities Interpretation

Absolute Value Equations and Inequalities:	Interpretation using the number line and box:	Interpretation without using absolute value symbol:

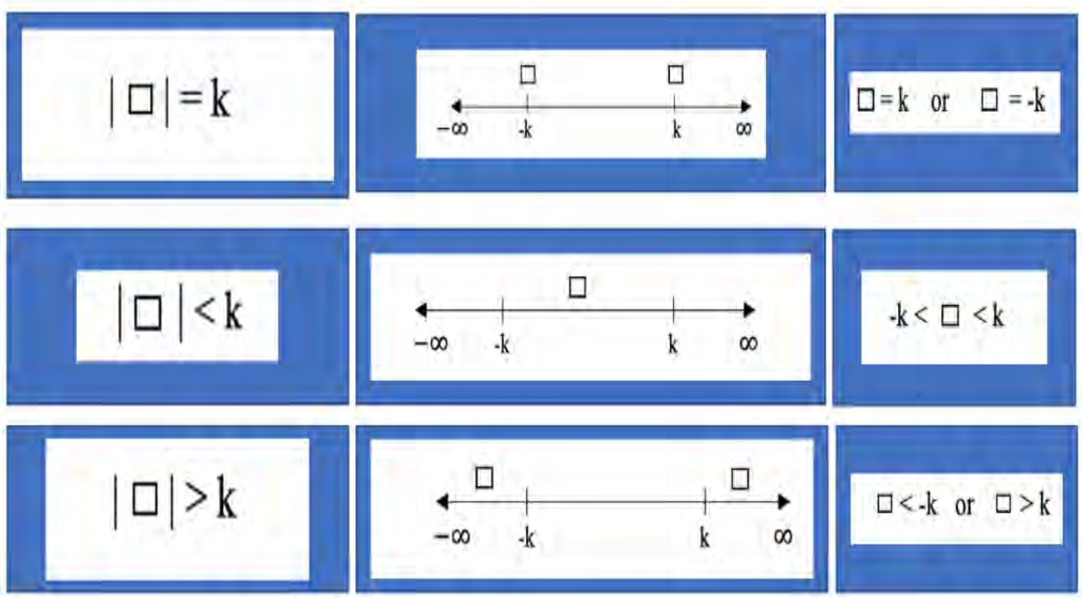
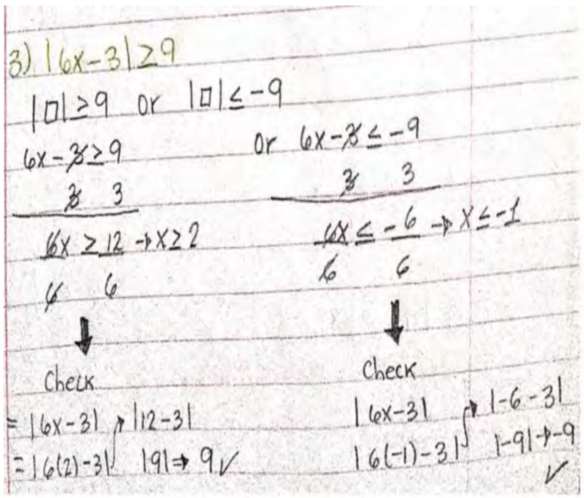


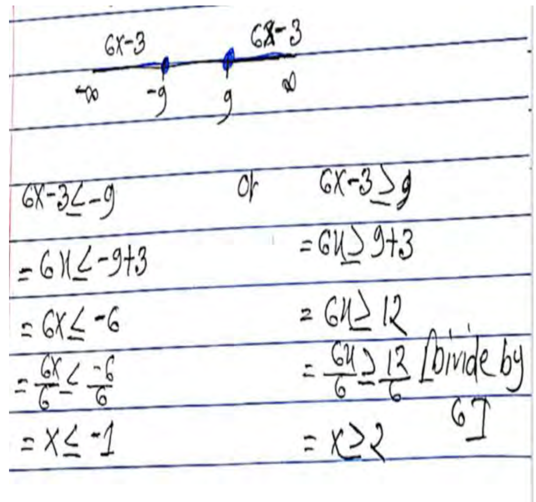
Figure 1. (Kumari, A. 2021)

Students work using number line and box method

Box method together with the number line helps students understand the concept of absolute value and it also helps them see the relationship between columns 1 and 2 in Table 1. Using the box method enabled students to write the correct procedure to solve problems involving absolute value equations and inequalities. Some student work is shown below. From the explanations of students G and H (shown below), they correctly translated the problem without using the absolute value symbol and wrote each step correctly. Finally, they solved the problem. For student G, the box method was helpful while student H used the number line to tackle this problem.



Student G



Student H

Conclusion

It is important that we ask students to critique each step they write in their explanation of a problem. This process of asking students to critique helps us understand what they learned and where we need to emphasize. It is very important for instructors to keep looking for the best method to teach as well as help students understand the concepts. Using the number line sweeps away the mystery of working with absolute values and empowers students to make connections between procedures and concepts (Mark W. Ellis and Janet L. Bryson, 2011). The box method helps students to solve problems involving absolute value equations and inequalities with the correct procedure and reasoning (Kumari, A. 2021).

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