# Analysis of the Mathematics Function Chapter in a Malaysian Foundation Level Textbook Adopted by a Public University

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**Abstract:** *In order to enhance students learning environments, mathematics lecturers need to prepare suitable materials in their lessons. So improving the materials in textbooks based on learning theories is so important to enhance the ability of students in mathematics learning through problem solving activities. The purpose of this qualitative case study is to analysis the function chapter of a Malaysian foundation level textbook to prepare deeper understanding for lecturers about the quality of materials and to identify textbook elements which can improve to enhance the quality of learning. This study was conducted to investigate the quality of function chapter especially about problem solving in the Mathematics 1 textbook of a foundation center of a public university in Malaysia in 2019. The method of this research is content analysis. In this current study, three theories namely Bloom's taxonomy, behaviorism and constructivism were used to analyses the textbook's materials. The findings represented that exercise solving based on the behaviorism theory highlights this textbook instead of problem solving based on the constructivism theory. Meanwhile, the textbook's materials mostly are related to the first two levels of Bloom's taxonomy. Finally, some errors in the textbook are addressed in order to lecturers improve them for the next editing.*

**Keywords:** Mathematics textbook, Mathematics function, Mathematics problem, Mathematics exercise, Higher order thinking

## 1. Introduction

The materials considered in the textbook as a primary instrument in teaching and learning mathematics are so important to enhance the students' ability in mathematics problem solving (Berisha et al., 2013). Doing suitable activities by students help them to have better performance in the classes and improve their abilities in mathematics problem solving. Brandstrom (2005) explained that the textbook has a very central task in the classroom for educators and learners in mathematics teaching and learning process. The mathematics function concept is a central and practical but difficult topic in secondary school curricula (Akkus et al., 2008; Ponce, 2007). For instance, "the topics inverse function and composite function is more conceptual and challenging among educators to transfer to students" (Oehrtman, Carlson, & Thompson, 2008, p.39). Mathematics functions apply in human life to modeling the real-world problems (Michelsen, 2006). Therefore, students should learn mathematics function conceptually as a practical topic in human life

The quality of mathematics materials in the textbooks plays an important role in teaching method among lecturers. If the materials in textbooks contain suitable problems and activities, lecturers need to improve their content knowledge and pedagogical content knowledge to use problem solving approaches in their classes. Otherwise, lecturers, by using low quality textbook through traditional method, cannot improve the students' abilities in problem solving. Educators utilizing traditional method of teaching mostly emphasize on the importance of lecture and mathematics exercise solving among students (Khalid, 2017; Mon et al., 2016). In fact, in this lecturer-centered approach of teaching, many of students only memorize the theorems, formulas and methods of solutions and apply them in mathematics exams or homework problems (Gholami et al., 2019). In mathematics education, learning theories are used to explain how students learn the new concepts. Therefore, the learning theories help educators and mathematics experts to understand the complex process of learning among students. In educational studies, researchers discuss two main perspectives in learning theories namely Behaviourism and Constructivism. Traditional method in mathematics teaching is supported by the behaviorism learning theory in education. *Behaviorist* refers to the learning theories emphasizing changing behavior which is resulted from learners' associations of stimulus-response (Ormord, 1995). According to this theory, learning among learners is a change in their behavior because of their experience (Ormord, 1995). The problem solving approach is based on the constructivism learning theory. *Constructivism* is the theory that students construct their own understanding of the mathematics concepts through experiencing problem solving activities and reflecting on those experiences (Simmons, 1999). In fact, mathematics problem solving helps learners develop a wide domain of complex mathematics structures and obtain the ability of modeling variety of real-life problems (Tarmizi & Bayat, 2012).

In behaviorism learning theory, educators does not teach critical thinking, rather it excludes any form of cognition (Von Glasersfeld, 2008). Behaviorists think that mathematics knowledge and materials transfer from one person to another by means of reinforcements and conditioning. This kind of mathematics learning involves rote learning, repetition and external rewards to elicit behavior. This educational theory prepares an incomplete way to teaching according to memorization method. Some appropriate behaviorist strategies need to be performed in order to encourage and motivate participants to learn the basic knowledge of mathematics. For instance, when lecturers teach the concept of composite function, it is appropriate to consider some mathematics exercises as students' activity to help them learn the definition and concept of composite function to engage students with suitable problems based on their abilities.

Constructivist methods in mathematics teaching are student-centered and provide a suitable environment for students to learn the concept of mathematics deeply in a group. In fact, this learning theory emphasizes mathematics problem solving among learners individually and teamwork. According to constructivists, successful learning is the skill of the learner to explain procedures that would best interpret the environment (Ogwel, 2004). Bettencourt (1993) described constructivism as a theory that "involves a conception of the knower, a conception of the known, and a conception of the relation of knower-known" (p.39). Ogwel (2004) explained that "more still, to the constructivists, emphasis should be on the process of learning and not the product of learning" (p.2).

## 2. Theoretical Framework

Problem solving is so important in the process of mathematics learning. The teaching methods focusing on problem-solving has been a hot topic in the field of mathematics education among researchers and educators during last two decades (Hu et al., 2018). Therefore, including the suitable mathematics problems in the textbooks prepare vast opportunity for students to learn mathematical concepts deeply. According to the National Council of Teachers of Mathematics (NCTM) (2000), if students engage with a challenging task for the first time then this task is known as mathematics problem. On the other hand, if students follow some steps to solve a routine task then this task is called mathematics exercise. Therefore, problem solving points to engaging in a mathematics question that learners have not learned how to solve it before. Based on the study by Gholami et al. (2019) "the distinction between what is considered a problem and an exercise depends on many factors, including the grade level, mathematics competence, learning materials, the way it was taught, and the time given to complete the task" (p. 292). Asami-Johansson (2015) explained that open-ended problems and the level of problems depend on the students' mathematical ability. For instance, the following mathematics problem, after discussion in class, becomes a mathematics exercise.

Problem 1: If $\tan(x + y) = \frac{2}{3}$ and $\tan(x - y) = \frac{1}{4}$ then find the value of $\mathrm{A} = \tan 2y - \cot 2x$.

If lecturers consider a slightly change in this mathematics exercise, students will engage with another mathematics problem such as:

Problem 2: If $\tan(x + y) = \frac{2}{3}$ and $\tan(x - y) = \frac{1}{4}$ then find the value of
$\mathrm{B} = \tan(3y + x) + 2\tan(4x)$.

Furthermore, in this research, every mathematics problem related to the student's everyday life and other subjects such as physics, chemistry and biology are considered as practical problem. For example, the following task is a practical problem.

A farmer is growing winter wheat. The amount of wheat he will get per hectare depends on, among other things, the amount of nitrogen fertilizer that he uses. For his particular farm, the amount of wheat depends on the nitrogen in the following way:

$$Y = 7000 + 32N - 0.1N^2$$

Where $Y$ the amount of wheat is produced, in $kg$ per hectare, and $N$ is the amount of nitrogen added, in $kg$ per hectare.

i. How much wheat would he have if he uses $200\ kg$ of nitrogen per hectare?
ii. Could you have better suggestion for him to use the amount of nitrogen to produce more amount of wheat?

Polya (1945) suggested four phases for mathematics problem solving, namely, understanding the problem, planning a strategy, performing the plan, and confirming the answer. Since 1945 a lot of models for problem solving with different steps and phases have been introduced by educators and researchers but all models have described that students should understand the problem, choose a strategy, solve the problem and confirm the answer. It is important that mathematics textbook should be able to encourage and engage students with suitable problems according to their abilities and skills. The discovery of the use of mathematics problems and encouraging learners to describe the techniques and strategies they engage when solving problems are more pedagogically challenge among mathematics educators (Johnson & Cupitt, 2004; McDonald, 2009). So educators can improve the students' abilities and skills in problem solving and higher order thinking by engaging them with appropriate mathematics activities in the textbooks.

Educators use Bloom's taxonomy in mathematics assessments in order to ensure that learners assess on a variety of skills in problem solving. The revised Bloom's taxonomy (2001) describes the levels of learning in six categories, namely, remembering, understanding, applying, analyzing, evaluating, and creating (Anderson et al., 2001). There is no higher order thinking skills without lower order thinking skills. Therefore, there is a strong relationship between lower order thinking and higher order thinking (Mitana et al., 2018). When students engage with difficult mathematics problems, they need to use some definitions, theorems and methods that they have memorized or understood before. For instance, in the problem "Find the value of $A = \sin 22.5^{\circ} + \cos 22.5^{\circ}$." students can solve this problem in variety solutions methods such as

$$A = \sin 22.5^{\circ} + \cos 22.5^{\circ} \Rightarrow A^2 = (\sin 22.5^{\circ} + \cos 22.5^{\circ})^2$$

$$A^2 = sin^2 22.5^{\circ} + cos^2 22.5^{\circ} + 2\sin 22.5^{\circ} \cos 22.5^{\circ}$$

$$A^2 = 1 + \sin 45^{\circ} = 1 + \frac{\sqrt{2}}{2} = \frac{2+\sqrt{2}}{2} \Rightarrow A = \sqrt{\frac{2+\sqrt{2}}{2}}.$$

Although this solution shows the students' higher order thinking skills, some facts related to the lower order thinking skills are used in this solution such as $(a+b)^2 = a^2 + b^2 + 2ab$, $\sin 2\theta = 2\sin\theta\cos\theta$ and $\sin 45 = \frac{\sqrt{2}}{2}$. In this study, based on the definition of lower and higher order thinking skills by Malaysian Ministry of Education (Ministry of Education, 2014), the first two levels of revised Blooms' taxonomy, remembering and understanding are considered as the skills of lower order thinking and four levels applying, analyzing, evaluating and creating are considered as the skills of higher order thinking. Berry, Maull, Johnson and Monaghan (1999) introduced the routine mathematics questions as lower order thinking skills that students can solve them easily using some steps and procedures. Non-routine questions need the application of mathematics materials such as definitions, theorems and methods in a new situation and critical thinking to find creative solutions. They further added one question may be considered routine for a student but non-routine for another. In other words, the categorization of mathematics tasks based on revised Blooms taxonomy depends on the students' abilities in problem solving. Therefore, mathematics education experts can do the classification of mathematics tasks in different levels of the revised Blooms taxonomy according to the students' problem-solving skills. For instance, Dartington (2013) categorized the following pre-university level questions in the levels of higher order thinking because in these questions, students engage with challenging mathematics problems and they require to think critically.

Example 1: A curve's equation is $y = f(x)$, where $f(x) = \frac{3x+1}{(x+2)(x-3)}$. Express this in partial fractions.

Example 2: The matrix $A$ is $A = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geq 1$,

$$A^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}.$$

The importance of using suitable materials in the textbooks in the process of mathematics teaching is supported by many educational theories and hence, preparing mathematics material in the course books based on the learning theories enable acquisition of problem solving skills among students such as making predictions and judgments, intuitive thinking, abstract thinking and extracting mathematical formulas to model the real world problems (Koparan, 2017). It seems the analysis of mathematical materials based on the theories of Bloom's taxonomy, behaviorism and constructivism not only provide golden opportunities for the lecturers to understand the weaknesses of the materials according to the different levels of the Bloom's taxonomy but also help them to enhance the skills of higher order thinking among students. The purpose of this current study is to analysis the mathematical materials in the function chapter of the Mathematics 1 textbook as respect to the problem solving and higher order thinking to prepare deeper understanding for lecturers about the quality of materials and to identify textbook elements to enhance the quality of learning. As scope of this research, the following research question was aimed to be answered:

Research Question: Is the mathematical materials in the function chapter of the Mathematics 1 textbook, appropriate in terms of problem solving and higher order thinking?

## 3. Methodology

This current study was part of a larger research study that was conducted in a foundation center in a public university of Malaysia during the academic year 2018-2019. Foundation program is a kind of Malaysian pre-university programs that run by selected universities. Several other universities also offer foundation programs in one year (two semesters) and the learners are almost similar in their qualifying entrance grades. Students pursuing university level are selected based on their performance in foundation level. So, students should engage with suitable mathematics materials to improve their abilities in problem solving and prepare them for better performance in mathematics courses at the university level. In foundation center, students used two mathematics textbooks, namely, Mathematics 1 and Mathematics 2 in semesters one and two, respectively. It is worth mentioning that these textbooks are taught by lecturers during one year (two semesters). The principal of the Foundation Center and the Head of Mathematics Unit signed the permission letter. The Head of Mathematics Unit explained that these textbooks are designed by all mathematics lecturers in this center (each lecturer designed one chapter) and these textbooks contain different chapters related to the algebra, calculus, trigonometry, geometry, probability and statistics.

The method of this qualitative case study is content analysis of the Mathematics 1 textbook (version 2018) of a foundation center in a Malaysian public university. In the Mathematics 1 textbook, there are fifteen chapters that the researchers chose the function chapter randomly. This study aims to analyses the materials of the function chapter in order to find the appropriateness of them as respect to the problem solving and higher order thinking. The five topics related to the function chapter are shown in Table 1. Pedagogical approaches to these topics consist of following three theories: revised Bloom's taxonomy, behaviorism and constructivism. Furthermore, this paper tried to identify textbook elements which can be improved in order to enhance the quality of teaching and learning in mathematics.

**Table 1: The Topic of Lessons**

| Number | Topic |
|---|---|
| 1 | Relation and function concepts |
| 2 | Domain and range of the functions and algebraic combination |
| 3 | Composite function, inverse function, odd and even functions |
| 4 | Trigonometric functions |
| 5 | Exponential and logarithmic functions |

In this study, the researchers, the Head of Mathematics Unit and a lecturer were divided all the tasks of the Mathematics 1 textbook into two categories, mathematics exercises and mathematics problems according to the definitions of mathematics problem and mathematics exercise (NCTM, 2000). *Mathematics exercises* are regarded as questions solved using similar tasks, while *mathematical problems* are considered as applying these tasks to more challenging problems. Also, the mathematics problems were categorized based on the revised Bloom's taxonomy. These categorizations and the analysis of textbook materials were improved and confirmed by three professors in the faculty of mathematics at the same university through content review. Meanwhile, some factors about errors in typing, definitions and students misunderstanding are explained and addressed that deserve full consideration by lecturers in the next edition. Meanwhile, this textbook was designed in 2018 and lecturers taught it for the first time. The new textbook design allowed for researchers to study its strengths and weaknesses to improve upon in subsequent editions.

## 4. Findings

The findings of this study about the functions in the Mathematics 1 textbook are discussed in two parts, namely, problem solving and higher order thinking, and critique of the content.

### 4.1. Problem Solving and Higher Order Thinking

The analysis of materials related to these five topics in the Mathematics 1 textbook were insubstantial related to the mathematics problems. There are a few mathematics problems in the textbook. Meanwhile, there are not any practical problems in this chapter. In fact, the textbook focuses on solving mathematics exercises which is related to the behaviorist learning theory. This method encourages students to memorize some methods, formulas, theorems and shortcuts in order to apply them in some mathematics exercises using the lecturers' methods and steps. These lessons cannot improve their abilities in problem solving based on constructivism learning theory. For example, in the subtopic of composite function there are 18 similar routine questions on page 157 such as "find $fog$ for the functions $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$" that only the function rule of tasks is different in these questions. The questions "solve the trigonometric equation $4sec^2\theta = 3\tan\theta + 5$" on page 159 and "prove that for any positive, real number $x$ we have $\ln(\frac{1}{x}) = -\ln x$" are examples of mathematics problem because they are challenging task for students. The result of analyzing the textbook materials is represented in Table 2.

**Table 2: The Number of Mathematics Problems in the Textbook**

| No. | Topic | Exercise | Problem | Practical Problem |
|---|---|---|---|---|
| 1 | Relation and function concepts | 16 | 1 | 0 |
| 2 | Domain and range of the functions and algebraic combination | 19 | 1 | 0 |
| 3 | Composite function, inverse function, odd and even functions | 26 | 2 | 0 |
| 4 | Trigonometric functions | 10 | 1 | 0 |
| 5 | Exponential and logarithmic functions | 18 | 1 | 0 |
| | Total | 89 | 6 | 0 |

Regarding Table 2, in the case of these five topics, about 6.3% of tasks are mathematics problems and 93.7% of tasks are mathematics exercises. Meanwhile, there are no practical problems in each lesson to encourage students in learning mathematics by seeing some application of mathematics in the real world.

The mathematical tasks categorized by lecturers and three professors from the mathematics faculty confirmed them. For instance, the task "find the range of the function $h(x) = \frac{1}{\sqrt{4-x^2}}$" were categorized into applying level of revised Bloom's taxonomy because this task is not a routine question and students should find its inverse to consider the domain of invers function as the range of the function $h$. Table 3 shows the categories of tasks according to the revised Bloom's taxonomy for all topics.

**Table 3: The categorization of Tasks Based on the Revised Bloom's Taxonomy**

| Topic | Task | Remembering | Understanding | Applying | Analyzing | Evaluating | Creating |
|---|---|---|---|---|---|---|---|
| 1 | 17 | 5 | 11 | 1 | 0 | 0 | 0 |
| 2 | 20 | 7 | 12 | 1 | 0 | 0 | 0 |
| 3 | 28 | 14 | 13 | 1 | 1 | 0 | 0 |
| 4 | 11 | 4 | 5 | 0 | 1 | 0 | 0 |
| 5 | 19 | 6 | 12 | 1 | 0 | 0 | 0 |
| Total | 95 | 36 | 53 | 4 | 2 | 0 | 0 |

With respect to Table 3, about 94% of tasks are related to the lower order thinking and 6% of tasks are about higher order thinking. Therefore, the materials in the textbook are not appropriate for foundation level students to improve their higher order thinking skills.

### 4.2. Review of the Textbook's Materials

On page 143 of the textbook, a *relation* is defined as "the association or relationship between two sets of information or objects which is called a relation and every set contains some ordered pairs which is considered a relation". It seems that this definition is not appropriate for foundation level textbook. The professors who reviewed the content of the textbook suggested that it is better to define the relation by using Cartesian product ($A \times B = \{(x, y) | x \in A,\ y \in B\}$) because students can learn the concept of the function logically. On page 144, a *function* is defined as " a relation that assigns each input of $x$-values of the domain to exactly one output of $y$-values of the range.". For this case, considering a simple example like, "the height of an airplane in different time shows a function" in the textbook is useful to prepare the students mind for understanding the concept of the function conceptually. Because it is not possible that at the same time an airplane has two different heights, but it has the same height in two or more different times. Lecturers should consider an activity about this concept to provide some examples of function in daily life.

In the foundation level, the mathematics concepts should be transferred through logical and accurate materials. On page 148, we see the following definition "*rational function* is defined by $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials". In this definition, we need to consider $q(x) \neq 0$. Another definition provided is "*root function* is defined by $f(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$, where $n$ is a positive integer, and the domain of the root function is the set of real numbers if $n$ is odd, and the set of all positive real numbers if $n$ is even". Two points are important and should be considered in this definition. Firstly, in the function $f(x) = \sqrt[n]{x}$ the value of $n$ cannot be one and secondly, the domain of this function is non-negative real numbers.

Typographical errors create some challenges for students. In this chapter, there are some typographical errors. On page 158, there are two trigonometric formulas that are represented incorrectly $\sin x cos\ y = \frac{1}{2}(\sin(x+y) - \sin(x+y))$ and, $\cos x cos\ y = \frac{1}{2}(\cos(x+y) - \cos(x-y))$. Using these incorrect formulas confuses students. These formulas should be corrected as follows:

$\sin x cos\ y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$ and, $\cos x cos\ y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$.

Some mistakes in the mathematics concepts may lead to misunderstanding. On page 159, there is a sentence "for any $x, y \in (-\infty, +\infty)$, $\ln xy = \ln x + \ln y$" where the domain of the function $f(x) = \ln x$ is $(0, +\infty)$. In this case both variables $x$ and $y$ should be positive, real numbers.

Besides, there are some scientific problems such as the following definitions of even and odd functions: On page 156, the definition of even and odd functions are ambiguous for students and lead to misunderstanding. An *even function* is defined as follows:

"A function $f$ is said to be even if and only if $f(-x) = f(x)$ for all $x$.".

Also, an *odd function* is defined as:

"A function $f$ is said to be odd if and only if $f(-x) = -f(x)$ for all $x$.".

For example, students apply the above definitions for the question "the function $f(x) = x^2$ with $-3 \leq x \leq 2$ is even or not?" as this is an even function because this function satisfies the condition $f(-x) = (-x)^2 = x^2 = f(x)$. But this function is not even because the graph of this function is not symmetric with respect to the *y-axis*. For another example, some students by using the textbook definitions explain that the function $h(x) = \sqrt{2x} + \sqrt{-2x}$ is even because $h(-x) = \sqrt{-2x} + \sqrt{2x} = \sqrt{2x} + \sqrt{-2x} = h(x)$. But for this function $D_h = \{0\}$ therefore, $h = \{(0,0)\}$ it means that the function $h(x) = \sqrt{2x} + \sqrt{-2x}$ is both even and odd. Thus these definitions should be changed as follows:

A function $g$ is called an even function if the following two conditions are met.

a. Domain $g$ is symmetric with respect to the point $(0,0)$
b. $\forall\, x \in D_g,\ g(-x) = g(x)$

A function $h$ is called an odd function if the following two conditions are met.

a. Domain $h$ is symmetric with respect to the point $(0,0)$
b. $\forall\, x \in D_h,\ h(-x) = -h(x)$

There are a limited number of mathematics problems in this section. Considering suitable problems in the textbook such as "how many functions both even and odd can we find?" can improve students' ability with problem solving. Lecturers can help students to solve this challenging problem as follows:

Since $f$ is even $f(-x) = f(x)$ also, $f$ is odd $f(-x) = -f(x)$ therefore,

$$f(x) = -f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0.$$

Based on this argument, there is only one function that is both even and odd. If we consider different domains for the function $f(x) = 0$ then we can find many different both even and odd functions. For example:

$$f = \{(-4,0),(-2,0),(0,0),(2,0),(4,0)\} \text{ or } g(x) = \begin{cases} [x] & if\ \ 0 \le x < 1 \\ 1 + [x] & if\ -1 < x < 0 \end{cases}.$$

On page 156, there is a mathematics problem related to even and odd functions as follows:

Problem: Show that

a. The sum and difference between even functions are even.
b. The sum and difference between odd functions are odd.
c. The sum and difference between even and odd functions are neither even nor odd.
d. The product between even functions is even.
e. The product between odd functions is even.
f. The product between even and odd functions is odd.

Although this is a good problem, all parts rejected considering a counter example $h(x) = 0$ and $k(x) = 0$. In the other words, all parts of this problem are incorrect. For instance, for part (a) if we consider $h(x) = 0$ and $k(x) = 0$ then $(h + k)(x) = h(x) + k(x) = 0 + 0 = 0$ thus the sum of two even functions is odd. Therefore, this problem should be corrected as follows:

Problem: Prove that

a. The sum and difference between non-zero even functions are even.
b. The sum and difference between non-zero odd functions are odd.
c. The sum and difference between non-zero even and non-zero odd functions are neither even nor odd.
d. The product between non-zero even functions is even.
e. The product between non-zero odd functions is even.
f. The product between non-zero even and non-zero odd functions are odd.

There are many similar mathematics exercises in each topic that consume a lot of time without improving the students' learning. For example, in topic 3, there are 18 exercises related to composite functions like the following exercise:

"If $f(x) = 1 - x$ and $g(x) = \frac{1}{x^2+1}$ find the function $fog$.".

For this example, lecturers can consider some problems to improve students' abilities and skills in problem solving such as:

Problem: If $f = \{(1,2), (3,5), (5,8), (4, -1)\}$ and $g = \{(2, -3), (3,1), (5,7), (-2,4)\}$ then find the function $fog + gof$.

Problem: If $f(x) = 2x - 5$ and $(gof)(x) = \frac{x-2}{x-4}$ find the function $g(x)$.

In the textbook, the domain of composite function discussed based on composite function rule that this method sometimes makes a misunderstanding for students. Thus, the logical definition $D_{fog} = \{x \in D_g | g(x) \in D_f\}$ is necessary to improve the ability of students in problem solving. For example, in the problem "If $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{1}{x-2}$ then find the domain of the function $fog$" according to the method of this textbook, students first find the function $fog(x)$ as:

$$fog(x) = f(g(x)) = f\left(\frac{1}{x-2}\right) = \frac{\frac{1}{x-2}+1}{\frac{1}{x-2}-1} = \frac{\frac{1+x-2}{x-2}}{\frac{1-x+2}{x-2}} = \frac{x-1}{3-x}.$$

Secondly, the domain of the function $fog(x) = \frac{x-1}{3-x}$ is $(-\infty, 3) \cup (3, +\infty)$. However, according to the logical definition of the domain of composite function, we have:

$D_{fog} = \{x \in D_g | g(x) \in D_f\} = \left\{x \in (-\infty, 2) \cup (2, +\infty) | \frac{1}{x-2} \neq 1\right\} = \{x \neq 2 | x - 2 \neq 1\} = (-\infty, 2) \cup (2, 3) \cup (3, +\infty)$.

Although, there are some weaknesses in this chapter there are some strengths such as covering different kind of functions that are applicable to other majors. Also, the authors organized and linked the topics appropriately. For instance, in topic 2 of the textbook, the authors considered a challenging problem related to the range of the function $f(x) = \frac{x+1}{x-2}$ that usually students cannot solve it with their prerequisite mathematical knowledge. According to the ideas of lecturers in this foundation center, most of students cannot find the range of this function despite spending a lot of time. When lecturers teach the concept of inverse function in the topic 3 and explain that for two functions $f$ and $f^{-1}$ we have $D_f = R_{f^{-1}}$ and $R_f = D_{f^{-1}}$ then students learn that one of the important applications of inverse function is to find the range of some functions. Therefore, they can find the inverse of $f(x) = \frac{x+1}{x-2}$ as $y = \frac{x+1}{x-2} \Rightarrow x = \frac{y+1}{y-2} \Rightarrow y = \frac{2x+1}{x-1} \Rightarrow f^{-1}(x) = \frac{2x+1}{x-1}$ and they easily see that $R_f = D_{f^{-1}} = (-\infty, 1) \cup (1, +\infty)$. For another example, two problems 7 and 8, on page 157 are about the composite function $fogoh$ it is a suitable example for students to generalize the definition of composite function. The exercise "identify the possible functions $f(x)$ and $g(x)$, given that $(gof)(x) = \ln(2x + 2)$" is an appropriate task for students to find different functions such as $f(x) = 2x + 2$ and $g(x) = \ln x$ or $f(x) = 2x$ and $g(x) = \ln(x + 2)$ or $f(x) = x + 1$ and $g(x) = \ln 2x$, recognizing that these functions are not unique.

## 5. Discussion and Conclusion

The results of this study confirmed that in this foundation center, the function chapter in the Mathematics 1 textbook emphasizes solving mathematics exercise based on behaviourism learning theory. A few of the tasks (6.3%) related to solving mathematics problem according to constructivism theory. Hence, problem solving is poor throughout the textbook. Furthermore, about 6% of tasks related to the higher order thinking skills. In Mathematics 1 textbook, each chapter is written individually by one of the lecturers. It seems that lecturers through collaborative work and using different educational theories can significantly increase the quality of this textbook. Thus, lecturers can collaboratively discuss the textbook material and decide how to improve their teaching. It is so important that mathematics lecturers need to have a strong foundation of learning theories and frameworks while planning to teach the materials in the textbook. They should be required to improve their knowledge about the learning theories such as the Bloom's taxonomy, behaviorism and constructivism in order to provide suitable materials in the textbook. For example, in Mathematics 1 textbook, there are 18 mathematics exercises about composite functions that emphasize drill-and-practice. There are similar examples based on behaviorism theory and the first two levels of the revised Bloom's taxonomy, remembering and understanding. But lecturers with suitable knowledge about learning theories and context can collaboratively improve the topic on composite functions. Based on the context, they should find how many mathematics exercises based on behaviorism theory about two levels remembering and understanding of revised Bloom's taxonomy and how many mathematics problems based on constructivism theory about the higher order thinking skills or four levels applying, analysing, evaluating and creating of revised Bloom's taxonomy should be considered in this lesson. Meanwhile, considering some practical problems could help students have better attitudes toward mathematics.

In this foundation center, the majority of mathematics lecturers taught precisely the same textbook materials, and the quality of the textbook about the mathematics function was deficient. Mathematics function is one of the essential topics used in many mathematics courses at the university level. Students in a foundation-level course need to have proper knowledge about the functions. So, they need to engage with practical problems in the textbook to learn the mathematics concepts meaningfully and experience its beauty. For example, suppose students learn even and odd functions conceptually. In that case, they can apply problem techniques and the properties of these functions to other related mathematics topics such as range of the functions and integration.

The Mathematics 1 textbook designed newly and taught for the first time in this foundation center thus it is natural to find some conceptual and typographical errors in different chapters including the function chapter. Therefore, this article could help lecturers improve the quality of the textbook according to the different learning theories. In fact, considering less lectures and more problem solving activities in regarding different mathematical topics through using constructivist learning theory has an important role in mathematics learning among students. Although this study focused on the function chapter, the results of this research still can be generalized for other chapters as well. For instance, the situation of mathematics problem solving can be improved throughout all chapters by lecturers in the new edition of textbook. For this case, collaboratively work among lecturers is so essential to share their knowledge, skills and experiences in order to produce a suitable textbook for foundation program students.

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