Teaching and Learning Processes for Prospective Mathematics Teachers: The Case of Absolute Value Equations

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Abstract: *Solving absolute value equations is one of the topics within the course of Selected Topics in School Mathematics (STSM) for prospective mathematics teachers. This research aims to investigate the implementation of the learning and teaching processes of the STSM course for strengthening conceptual understanding and procedural fluency of prospective mathematics teachers for the case of absolute value equations. This qualitative study was carried out through observations on online learning and teaching processes involved 47 mathematics education students, as prospective mathematics teachers, during the Covid-19 pandemic situation. The results of this study revealed that the learning and teaching processes are mainly implemented by using a deductive approach which aided with the use of the GeoGebra software as a tool for helping in the equation solving process and as an environment for developing mathematical concepts. The written student work from the assessment showed various students' solution methods and difficulties in dealing with absolute value equations. We conclude that the learning and teaching processes of the STSM course need to be improved so as to develop prospective mathematics teachers' conceptual understanding and procedural fluency in a balanced manner.*

INTRODUCTION

Solving absolute value equations is one of the algebra topics within school mathematics that is considered difficult either to learn or to teach (Almog & Ilany, 2012; Çiltaş & Tatar, 2011; Stupel, 2012; Stupel & Ben-Chaim, 2014; Wade, 2012). The difficulty in this topic is not only encountered by secondary school students, but also by mathematics education students as prospective mathematics teachers all over the world (e.g., Serhan & Almeqdadi, 2018; Stupel & Ben-Chaim, 2014), including in Indonesia (Aziz, Supiat, & Soenarto, 2019; Nisa, Lukito, & Masriyah, 2021). Difficulties in solving absolute value equations include, inter alia, determining intervals that make algebraic expressions within absolute value signs positive or negative and applying absolute value properties during an equation solving process (Aziz et al., 2019; Stupel & Ben-Chaim, 2014). For future careers of mathematics education students, as prospective teachers, these difficulties should be overcome. An endeavor to do so is by strengthening their conceptual understanding and procedural fluency in solving absolute value equations.

One of the courses for prospective mathematics teachers in Indonesia that strengthens conceptual understanding and procedural fluency in school mathematics is so-called the Selected Topics in School Mathematics (STSM) course. Solving absolute value equations is one of the topics within this course. In this course, each topic is addressed by emphasizing conceptual understanding and procedural fluency in a balanced manner. Concerning this STSM course, due to Covid-19 pandemic situation, we wonder how the learning and teaching processes are implemented online so as to strengthen conceptual understanding and procedural fluency of the prospective teachers, particularly for the case of solving absolute value equations.

To investigate online learning and teaching processes of the STSM course, we carried a qualitative study in the form of classroom observations for the case of solving absolute value equations which aided with the use of the GeoGebra software. Previous studies have shown that this type of investigative research, particularly with the use of digital tools in the learning and teaching processes, in Indonesian context, to certain extent, is rarely conducted (Jupri & Sispiyati, 2020; Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2016). Taking this into consideration, the current study aims to investigate the implementation of the learning and teaching processes of the STSM course and its effect toward prospective mathematics teachers' ability in solving absolute value equations.

THEORETICAL FRAMEWORK

Theoretical frameworks used in this study include types of learning and teaching approaches, didactical functions of technology in mathematics education, and algebraic proficiency. In general, we distinguish types of learning and teaching approaches into two, including inductive and deductive approaches (Prince & Felder, 2006). The deductive approach is implemented by applying deductive thinking in the learning and teaching processes, i.e., teaching mathematical concepts and principles from general to more specific cases (Jupri, Usdiyana, & Sispiyati, 2021; Ndemo, Zindi, & Mtetwa, 2017; Prince & Felder, 2006). As a consequence, the learning and teaching process is carried out consecutively from explaining concepts, definitions, and principles to using these concepts, definitions and principles in solving exemplified problems; to providing exercises and classroom discussion for students; and to conducting an individual written test.

The inductive approach is implemented by applying inductive thinking in the learning and teaching processes, i.e., teaching mathematical concepts and principles from specific to more general cases (Jupri et al., 2021; Ndemo et al., 2017; Prince & Felder, 2006). Therefore, the learning and teaching process is carried out consecutively from posing specific problems for doing investigations; to constructing conjectures, principles, concepts, or formulas through solving the problems; to applying the concepts, principles, or formulas for solving problems; and to drawing general conclusions.

Regarding the use of technology in mathematics education, Drijvers, Boon and Van Reeuwijk (2010) identified three didactical functions of technology in mathematics education: as a tool for doing mathematics, as an environment for practicing skills, and as an environment for developing concepts. In the function of technology as a tool for doing mathematics, technology users do not need to know and to understand how the technology solves mathematical problems at hand. In other words, the process of obtaining results need not be visible to the users. In this case, technology only serves to help users use time efficiently. For example, when we draw a graph using the GeoGebra software, we need only the result and do not need to know the process of drawing the graph. In the function of technology as an environment for practicing skills, technology plays a role in strengthening users' skills in performing mathematical procedures. In this function, technology is usually used for solving routine problems. For example, the GeoGebra can be used for solving linear equations in one variable using CAS (Computer Algebra System) feature. In the function of technology as an environment for developing concepts, technology serves to help students in understanding a concept through, for instance, guided investigation process. For example, the GeoGebra can be used as an environment for investigating characteristics of quadratic function graphs.

Concerning algebraic proficiency, it can be interpreted as proficiency in symbolic representation (Brown & Quinn, 2007) which includes conceptual understanding and procedural fluency (Jupri, Sispiyati, & Chin, 2021; Van Stiphout, Drijvers, & Gravemeijer, 2013). Procedural fluency refers to skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; and conceptual understanding means a comprehension of mathematical concepts, operations, and relations (Kilpatrick, 2001). These two aspects of proficiency have to go hand in hand in encouraging proficiency in algebra and in developing algebraic expertise in particular. Algebraic expertise can be interpreted as an ability that ranges from basic skills such as procedural work with a local focus and algebraic manipulation to strategic work, which requires a global focus and algebraic reasoning and conceptual understanding (Bokhove & Drijvers, 2010; 2012; Drijvers, Goddijn, & Kindt, 2010).

METHODS

To investigate the implementation of online learning and teaching processes of the Selected Topics in School Mathematics (STSM) course, we conducted a qualitative study in the form of self-observations. The observations for the case of the topic of solving absolute value equations included two phases. In the first phase, we observed the learning and teaching processes (via Zoom platform) that are implemented in two meetings which lasted for 2 x 100 minutes, and involved 47 mathematics education students in one of the state universities in Bandung, Indonesia. In the second phase, we observed an individual formative written test on solving absolute value equations, which lasted for 30 minutes. After the test, each student should upload his or her answer sheet in an online learning management system of the university, which is called *SPOT* (Online Integrated Learning System). The first author did the teaching and self-observations on his teaching and learning processes, while the second author checked the observations based on learning and teaching data, including written student work, pictures of the learning and teaching processes, and power point slides with their corresponding additional notes.

Data that we collected in this qualitative study included field notes concerning steps of the learning and teaching processes, lecture notes in the form of power point presentation slides with their corresponding additional notes on the slides, and students' written work from the formative assessment.

In data analysis, data about online learning and teaching processes were analyzed using the frameworks of types of learning and teaching approaches and of the didactical functions of technology in mathematics education. Data on written student work were analyzed through the framework of algebraic proficiency. In analyzing written student work, we distinguished three methods of solving absolute value equations, i.e., definition, properties, and graph methods. By the definition method we mean the method of solving absolute value equations using the definition of absolute value. By the properties method we mean the method of solving absolute value equations using properties of absolute values. By the graph method we mean the method of solving absolute value equations using graphs, either produced using a mathematical software, such as GeoGebra, or produced manually by the students.

RESULTS AND DISCUSSION

This section presents the results and discussion of the two phases of observations, including the learning and teaching processes and written student work from the corresponding formative assessment for the case of solving absolute value equations.

Learning and teaching processes for the case of solving absolute value equations

The learning and teaching processes were started by the lecturer through introducing the definition of absolute value for a real number, that is, $|x| = x$ if $x \geq 0$ and $|x| = -x$ if $x < 0$. In addition, the absolute value of x, or $|x|$, is interpreted as the distance from x to 0. For example, $|3| = 3$ because the distance from 3 to 0 is 3. Similarly, $|-3| = -(-3) = 3$ because the distance from -3 to 0 is 3. Next, the lecturer used this definition to explain and to obtain some properties of absolute values for $x, y \in R$, including $|x| = \sqrt{x^2}$; $|x.y| = |x||y|$; and $|x|/|y| = |x/y|$ where $y \neq 0$. During the explanation, the lecturer posed relevant questions to students and would continue if the students provided relevant responses.

The lecturer then explained the use of the definition, interpretation, and properties of absolute values for solving absolute value equations. The lecturer gave two examples: (a) $|x - 1| = 1$ and (b) $|x + 1| = 2x$. For the first example, the lecturer explained how to solve $|x - 1| = 1$ using definition, interpretation, and properties of the absolute values. Using the definition, the equation can be written as $x - 1 = 1$ or $-(x - 1) = 1$, which lead to $x = 2$ or $x = 0$ as the solution of the equation. Using the interpretation, the equation $|x - 1| = 1$ is interpreted as finding numbers such that their distance to 1 is 1, which leads to $x = 2$ or $x = 0$ as the solution of the equation. In addition to this interpretation, the equation $|x - 1| = 1$ is interpreted as finding abscissas of the intersections of the graphs $y = |x - 1|$ and $y = 1$. To do this, the lecturer used the GeoGebra software to show the intersections of the two graphs (see Figure 1). From Figure 1 we could see that the abscissas of the intersections include $x = 0$ and $x = 2$, as the solution of the equation.

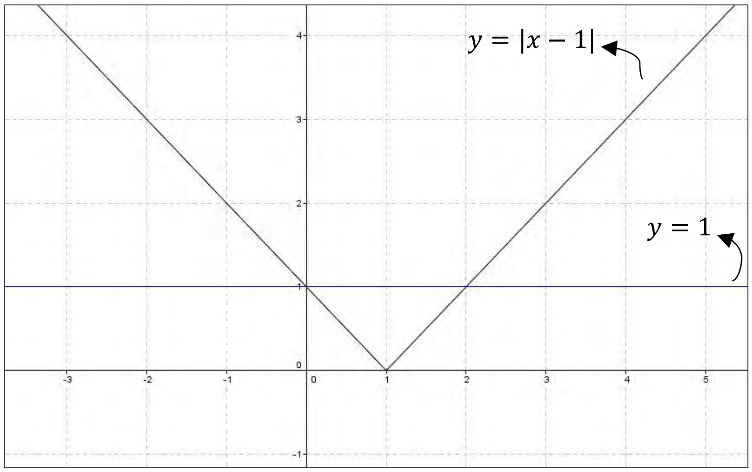


Figure 1: Graphs of $y = |x - 1|$ and $y = 1$

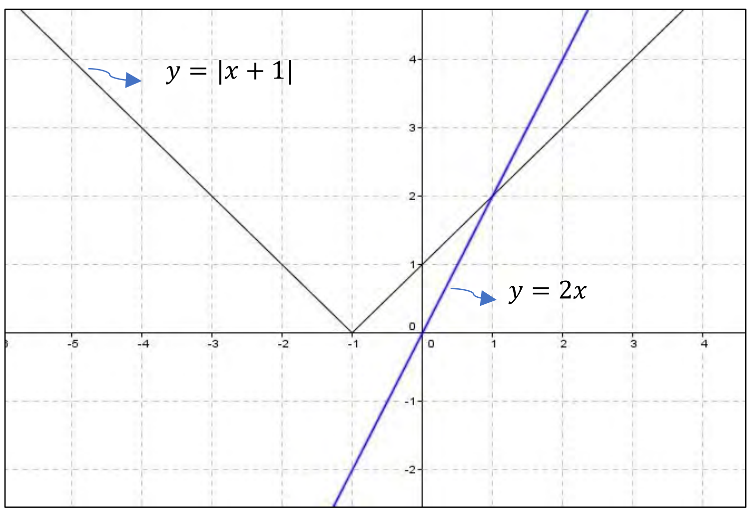


Figure 2: Graphs of $y = |x + 1|$ and $y = 2x$

For the second example, the lecturer explained how to solve $|x + 1| = 2x$ using the definition, interpretation, and properties of absolute values. Using the definition, the equation can be written as $x + 1 = 2x$ or $-(x + 1) = 2x$, which leads to $x = 1$ or $x = -1/3$. By an inspection, $x = 1$ is the only solution for the equation. Using the interpretation, solving the equation $|x + 1| = 2x$ is interpreted as finding abscissas of the intersections of the graphs $y = |x+1|$ and $y = 2x$. By using the GeoGebra software, the lecturer drew the two graphs (Figure 2), next he observed that $x = 1$ as the only abscissa of the intersection, and finally he concluded that $x = 1$ as the solution of the equation. Using properties of the absolute values, the equation $|x+1| = 2x$ can be written as $\sqrt{(x+1)^2} = 2x$, next both sides of the equation can be squared, be expanded, and be simplified to obtain $3x^2 - 2x - 1 = 0$. The solution for this last equation includes $x = 1$ or $x = -1/3$. By an inspection to the initial equation $|x+1| = 2x$, it can be concluded that $x = 1$ is the only solution for the equation.

After explaining the two examples above, the lecturer gave an exercise on solving absolute value equations. The tasks for the exercise—screenshot from the power-point presentation—is shown in Figure 3. The first three equations were addressed in the classroom discussion after the students were given sufficient time for solving them. The last four equations were then addressed in the next meeting.

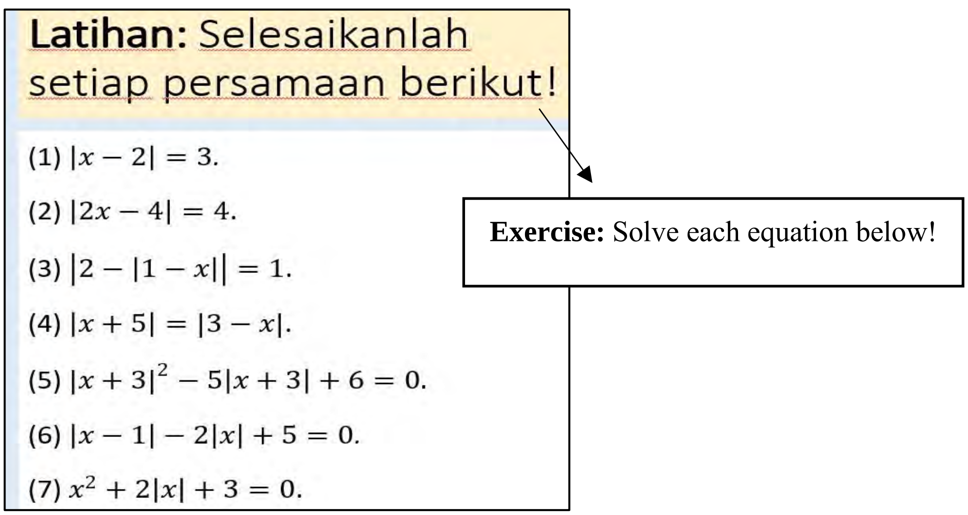
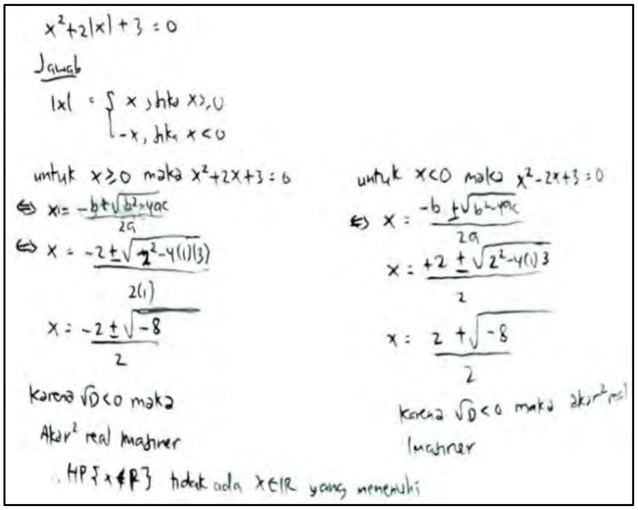


Figure 3: An exercise given in the learning and teaching process

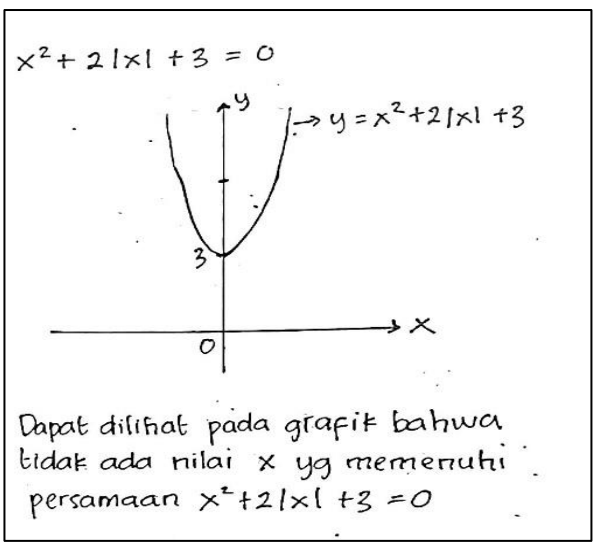
From the discussion of the last four equations, we observed two different solution methods for solving the equation (7), i.e., $x^2 + 2|x| + 3 = 0$, as shown in Figures 4 and 5. Figure 4 shows an example of written student work on solving the equation (7) using the definition method, and Figure 5 presents a written student work using the graph method. It seems that the use of the graph method is a direct consequence of the use of the GeoGebra as a tool for solving mathematics (Drijvers, Boon, & Van Reeuwijk, 2010). After discussing the last four equations, the lecturer then gave an individual written assessment on solving absolute value equations (addressed in the next subsection).



Since $\sqrt{D} < 0$, then the roots of the equation are imaginary.

Therefore, there is no $x \in R$ satisfying the equation.

Figure 4: A written student work using the definition method for solving $x^2 + 2|x| + 3 = 0$



It can be seen from the graph that there is no x satisfying the equation $x^2 + 2|x| + 3 = 0$.

Figure 5: A written student work using the graph method for solving $x^2 + 2|x| + 3 = 0$

Based on the description above, we made the following three notes. First, the sequence of the learning and teaching processes for prospective mathematics teachers for the case of solving absolute value equations proceeds consecutively from explaining the definition, the interpretation of the definition, and the properties of absolute values that can be used for solving absolute value equations; explaining the application of the definition and properties of absolute values for solving equations through examples; doing classroom discussions; and conducting an individual written assessment. Considering these processes, which start from general ideas of the definition and properties of the absolute values to more specific ideas of applications, we view that the lecturer used a deductive learning and teaching approach (Bahri, Abrar, & Angriani, 2017; Prince & Felder, 2006). In these processes students were involved actively through a question-and-answer strategy. Therefore, even if the lecturer used the deductive approach, which is recognized as one of the teacher-centered approaches (Ramsden, 1987), the students were still encouraged to participate actively during the learning and teaching processes.

Second, we observed that the GeoGebra is used as a tool for drawing graphs which aids for solving absolute value equations in a more meaningful manner visually. This observation means that the use of technology in the learning and teaching processes includes two functions, namely as a tool for solving problems and as an environment for developing concepts (Drijvers, Boon, & Van Reeuwijk, 2010; Jupri et al., 2016).

Third, the use of the deductive learning and teaching approach aided with the use of the GeoGebra has influenced student thinking in solving equations. In the classroom discussion, we observed some students used the graph method for solving equations, and some other students consistently used definition and properties methods for equation solving processes. This observation is in line with other relevant studies where technology has influenced student mathematical thinking in the process of solving problems (e.g., Bokhove & Drijvers, 2010; 2012; Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2015).

With these three notes of observations, we obtain information about the learning and teaching processes for prospective mathematics teachers and its corresponding qualitative impact on their ability in solving absolute value equations. The ability in solving absolute value equations is further addressed in the next section based on written student work from the formative assessment.

Analysis of written work on solving absolute value equations

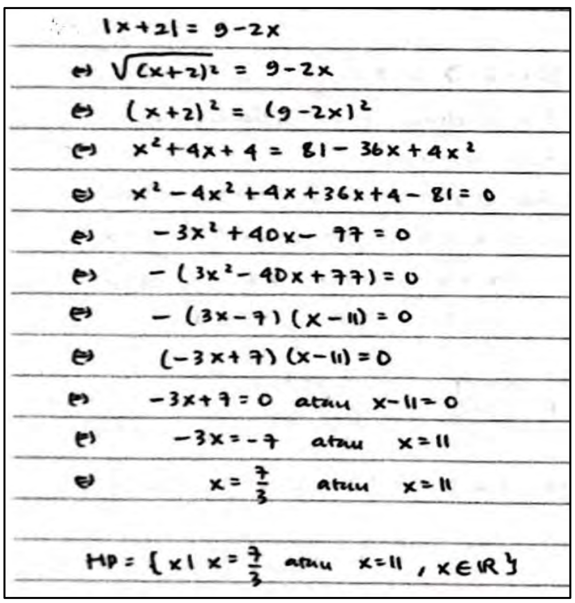
Table 1 presents findings of written student work on solving absolute value equations from the formative assessment. We view these findings as the effect of the deductive learning and teaching approach aided with the use of the GeoGebra toward students' algebraic proficiency. In general, the number of correct solutions for each task is more than about 75%, except for the Task 2 (44.6%). This indicates that the learning and teaching processes worked quite well and seem to have a positive effect to prospective teachers' ability in solving absolute value equations.

Tasks	#Correct solution (%)	Solution methods		
		#Definition method (%)	#Properties method (%)	#Graph method (%)
1. $\|3x - 2\| = 5.$	44 (93.6)	36 (76.6)	11 (23.4)	2 (4.2)
2. $\|x + 2\| = 9 - 2x.$	21 (44.6)	38 (80.8)	9 (19.1)	1 (2.1)
3. $\|1 - 2x\| = \|3x - 2\|.$	41 (87.2)	7 (14.9)	40 (85.1)	1 (2.1)
4. $6\|x - 3\|^2 - 19\|x - 3\| + 10 = 0.$	37 (78.7)	43 (91.5)	4 (8.5)	0 (0.0)
5. $\left\|\frac{2x-3}{3x+8}\right\| = \frac{1}{4}.$	35 (74.5)	37 (78.7)	10 (21.2)	0 (0.0)

Table 1: Results from data analysis of the written test (N = 47)

Concerning difficulties in solving absolute value equations, we found that the most common difficulties concern checking final results to an initial equation. For example, for the case of solving the equation $|x + 2| = 9 - 2x$, either using definition or properties method, when a student ends up at $x = 11$ or $x = 7/3$, she/he does not check whether each of this value satisfies the initial equation or not. Therefore, the student does not realize that $x = 11$ is not a solution. Figure 6 presents representative examples of written student work for the case of forgetting to check the final calculation to an initial equation of the Task 2. Other difficulties that we found, which are in line with other studies (e.g., Aziz et al., 2019), include difficulties in determining intervals for applying the definition of the absolute value and doing correct algebraic manipulations.

Concerning methods of solving absolute value equations, except for the Task 3, the definition method was used more frequent than the properties method or the graph method. The graph method was only used by some students along with the use of the properties method. This seems that the graph method is used only as a complementary method to ensure the use of the properties method correctly. Figure 7 presents a written student work showing the use of the properties and the graph methods. The more frequent use of the properties method than the definition method for the Task 3 seems to be caused by the fact that this task is easier to solve by applying properties of absolute values, i.e., by squaring both sides of the equation, expanding each term, and simplifying the whole equation into a quadratic equation. As the use of the definition method mainly depends on the definition of the absolute value, it can be considered that it tends to support the development of procedural fluency of prospective mathematics teachers in solving absolute value equations. As the use of the properties and graph methods needs a comprehensive understanding to equations before executing the solution process, therefore, this can be considered to support the development of conceptual understanding. In line with other relevant studies (e.g., Jupri & Sispiyati, 2020; Jupri, Sispiyati, & Chin, 2021), the findings of this study suggest that procedural fluency is more acquired than conceptual understanding. This might suggest that the algebraic proficiency of the prospective mathematics teachers needs further development to reach the balance between procedural fluency and conceptual understanding.



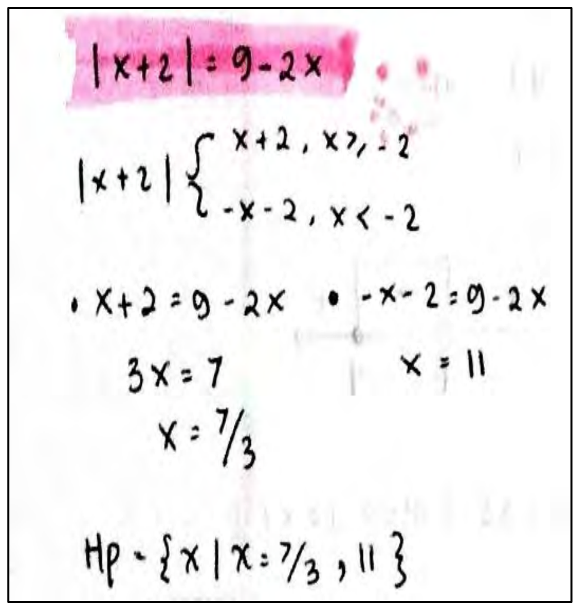


Figure 6: Representative examples of written student work of forgetting to check the final results to the initial equation

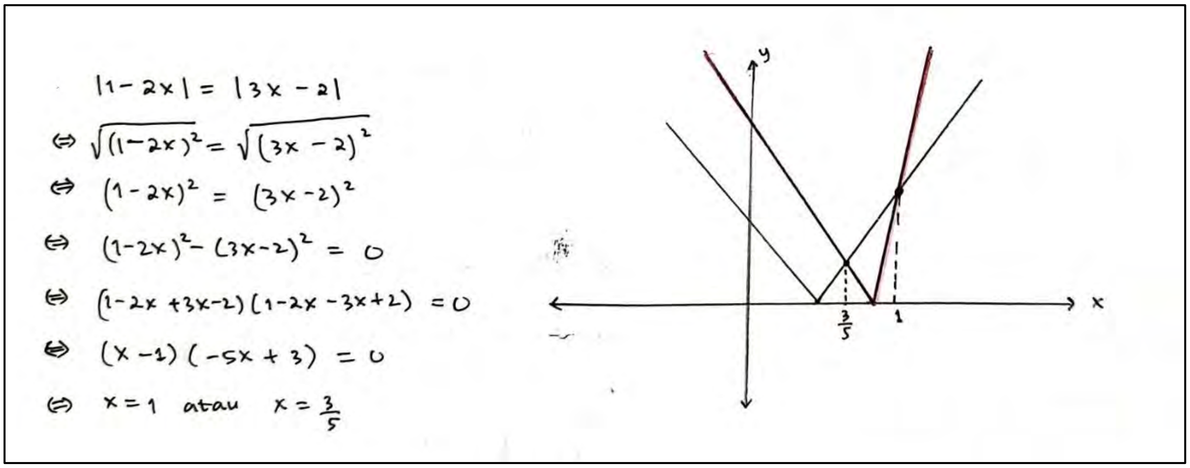


Figure 7: The use of properties and graph methods for solving an equation

CONCLUSIONS

From the description of the results and discussion above, we draw the following three conclusions. First, the observed learning and teaching processes of the Selected Topics for School Mathematics course for the case of solving absolute value equations mainly use the deductive learning and teaching approach aided with the use of GeoGebra and the question-and-answer strategy. In our view, this approach has a strong deductive character because the learning and teaching sequence proceeds from more general ideas, such as explaining definition and properties of absolute values, to more specific ideas of giving examples and explanations on solving absolute value equations. Even if the deductive learning and teaching approach seems work quite well in guiding prospective teachers' ability in solving equations, still an imbalance acquisition between procedural fluency and conceptual understanding is found. Considering this, we suggest to investigate the use of learning and teaching approaches that provide more opportunities to prospective mathematics teachers to think deeper in understanding of and in solving absolute value equations. This can be carried out, for instance, through providing activities of solving absolute value equations that explicitly request students to use various solution methods more independently. Therefore, the use of well-designed learning and teaching approaches that having inductive and explorative characters seem appropriate to be explored in future research.

Second, the more frequent use of the definition method than the properties method for solving absolute value equations shows that prospective mathematics teachers tend to be supported more on improving procedural fluency than on conceptual understanding. Therefore, for improvement of the learning and teaching processes, we suggest a balanced treatment regarding this, for instance, through putting more emphasize on applying the properties method (when appropriate) in solving absolute value equations.

Third, even if the learning and teaching processes seem work quite well, a number of students still encountered difficulties. These difficulties include making unnecessary mistakes in equation solving process, manipulating algebraic expressions correctly, understanding the meaning of absolute value equations before executing equation solving procedures, and forgetting to check the final calculations to the initial equations. For further investigation, in addition to use appropriate learning and teaching approaches, we suggest to put more emphasize on the use of technology, such as the GeoGebra, as a tool for solving problems and as an environment for developing mathematical concepts. In this way, we expect that the quality of prospective mathematics teachers, particularly in Indonesia, will improve in the future.

In spite of the conclusions above, we acknowledge that this study has some limitations. First, as this study depends on data of observations, field notes, teaching documents, and students' written work, the data triangulation needs to be enhanced through adding interview data. Through this way, we expect that more comprehensive results on prospective mathematics teachers' ability and difficulties in dealing with absolute value equations will be obtained. Second, as the observations in this study were carried out from only one cohort of students, we acknowledge that the findings could not be generalized. Therefore, for future research, we suggest to do more extensive observations, including more than one cohort of students, and use appropriate research methods to draw generalization from research findings.

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