Commognitive Conflicts in Mathematics Teachers' Pedagogical Discourse in Lesson Study

Anita Tyskerud
University of Stavanger

Reidar Mosvold
University of Stavanger

Raymond Bjuland
University of Stavanger

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The study reported in this paper applies the commognitive theory as a lens to study teacher learning in lesson study to better understand how and why teachers learn from lesson study. Video recordings were made from reflection meetings in three lesson study cycles with a group of four Norwegian lower secondary school teachers, one person from the school administration and a participating observer. The research revealed the potential of commognitive conflicts to explain teachers' pedagogical discourse. The analyses identified and described patterns in the teachers' pedagogical discourse when focused on observations of student learning during the reflection meetings. The discourse aligned more with a pedagogical delivery discourse rather than an explorative pedagogical discourse that was the aim of lesson study participation. Three examples illuminate the mismatch in the teachers' discourse with the canonical pedagogical discourse.

Keywords · mathematics teacher education research · commognitive conflicts · pedagogical discourse · reflection meetings · lesson study cycles

Introduction

When describing the Japanese approach to lesson study, Stigler and Hiebert (1999) emphasised how teachers worked steadily to improve teaching through small, incremental changes over time. They considered this to diverge considerably from common efforts to rapidly improve teaching through reforms in the United States. Since the reform approach had proven not to work, Stigler and Hiebert (1999, p. 131) suggested that "something like lesson study" was necessary in order to improve teaching on a national level in the United States, and similar arguments could be made for many other countries. In the years following Stigler and Hieberts' seminal study, researchers and educators have explored the potential of lesson study as a tool for teacher learning and professional development (e.g., Cerbin & Kopp, 2006; Dudley, 2015; Fernandez, 2002). In their review of literature on teacher learning in lesson study, Xu and Pedder (2015) point at two particular weaknesses in these efforts. First, they observed that most studies were unclear about what theoretical frameworks applied to investigate teacher learning, and they called for more theoretical work to improve our understanding of how and why teachers learn in lesson study. Second, they noticed that most studies analysed what teachers say about their experiences with lesson study as descriptive evidence for claims about teacher learning rather than focusing on teachers' talk in the classroom as a mediator of learning and development. The aim of this study is to mitigate these two weaknesses by applying Sfard's (2008) commognitive theory in analysis of teachers' communication through several cycles of lesson study. The research examined teachers' pedagogical discourse from the reflection meetings convened during three lesson study cycles. Instead of searching for evidence of learning, the goal was to identify misalignment of lesson study outcomes with the kinds of discourse needed to better understand what might prevent teacher learning in lesson study.

Literature Review

Ever since lesson study became known outside of Japan (Stigler & Hiebert, 1999), its potential for teacher learning and professional development has been emphasised. Several studies suggested that lesson study supports the development of instruction (Lewis et al., 2006; Pang, 2016), of teachers' knowledge and skills (Hunter & Back, 2011; Lewis & Perry 2014), and of teachers' professional growth (Gee & Whaley, 2016). In order to better understand these efforts to study teacher learning in lesson study, we elaborate on some key issues. We start by pointing at some different interpretations of the lesson study cycle before we explore how studies have attempted to consider teacher learning in terms of change in knowledge or practice. We then conclude the literature review by elaborating on two critiques of research on teacher learning in lesson study, each of which have been important to the formation of our study.

Lewis and Hurd (2011, p. 2) defined lesson study as a four-step process that constitutes a lesson study cycle: study curriculum and formulate goals, plan, conduct research lesson, and reflect. However, it is worth noting that different definitions of a lesson study cycle appear in the literature due to the addition of re-teaching. For instance, Murata (2011) described the lesson study cycle as considering learning goals, careful planning of the lesson, observation and data collection, and reflection based on observations; re-teaching of the research lesson is voluntary. Others, like Larssen and Drew (2015), included re-teaching of the research lesson as an integral part of the lesson study cycle. Dudley (2013) described planning, teaching the research lesson, interviewing students, followed by reflection discussions as the rudimentary lesson study cycle, but he then added that the lesson study process—also referred to as a "cycle"—can consist of three or more such rudimentary cycles. In the present study, we adopt a definition of a lesson study cycle as including one repetition of the research lesson.

Repetition of the research lesson is one aspect of the lesson study cycle that varies across studies, another is the length of the planning phase. In a critical discussion of how lesson study has been implemented outside of Japan, Fujii (2014, p. 67) highlighted "the lengthy period of planning a lesson" as a key characteristic of Japanese lesson study and commented that the process of planning a research lesson can sometimes take more than half a year. When lesson study has been implemented in other countries, the planning period has often been shortened both in time and depth (Yoshida, 2012).

Many studies of teacher learning in lesson study included analyses of teachers' developing knowledge and experiences. For instance, Lewis and Perry (2014) assessed participants' developing knowledge of fractions by using a combination of multiple-choice items, open-response items, written reflections, videos, and artefacts. When comparing the tests of teachers in an experimental group with a control group, results indicated that the experimental group had a more significant reduction in errors in the post-test than the control group. In another study, Cajkler and colleagues (2014) investigated teacher learning in lesson study by analysing data from evaluation meetings and interviews. Whereas some studies analysed results from assessments or observations (e.g., Cajkler et al., 2014; Lewis & Perry, 2014), others, like Lewis and colleagues (2013), investigated teacher learning in lesson study by analysing the experiences of teachers throughout the lesson study process. Puchner and Taylor (2006) also investigated the experiences of participating teachers, but their analysis explored the potential of lesson study instead of focusing on evidence for teacher learning in and through lesson study.

Whereas some studies of teacher learning in lesson study emphasised teachers' knowledge or experience, other studies focused more on change in practice or student participation. For instance, Robinson and Leikin (2012), studied how students' behaviour and engagement changed after three main adjustments were made between two iterations of a research lesson: the way the teacher asked questions, the IRE pattern (initiations, responses, and evaluations) of classroom communication, and lesson settings (how students were divided into groups). They concluded that lesson study supports teacher learning with respect to these aspects of practice. Other studies claimed that participation in lesson study stimulates teachers' attention to students' discourse, students' thinking and questioning strategies, or fosters collaboration within a professional learning community (e.g., Gee & Whaley, 2016; Hunter & Back, 2011). Yet others exposed the benefits of structured observation (e.g., Lee & Tan, 2020) and facilitating reflection (e.g., Karlsen & Helgevold, 2019; Warwick et al., 2016).

Although research has reported on numerous "success stories" that provide evidence of teacher learning in lesson study, recent reviews have pointed out several shortcomings in many of the studies, for instance in terms of short time span and lack of theoretical foundation (Huang & Shimizu, 2016; Xu & Pedder, 2015). Whereas Japanese lesson study traditionally emphasised development through small and incremental changes over time (e.g., Stigler & Hiebert, 1999), studies that apply lesson study in the Western world tended to make claims about teacher learning after only one cycle (e.g., Lewis & Perry, 2014; Robinson & Leikin, 2012). Several researchers noticed this and called for studies that aimed at determining the nature of teachers' long-term learning and growth (Dotger, 2015; Gee & Whaley, 2016; Hunter & Back, 2011). Ono and Ferreira (2010, p. 71) suggested that, "any form of development would require a long-term gradual progression to change." Also, a crucial element of teachers' learning concerns how to facilitate lesson study (Murata, 2011) when initiated the first time. A suggestion is thus to follow the same group of teachers over several lesson study cycles in order to study teacher learning and development in lesson study over time.

In their review of teacher learning in lesson study, Xu and Pedder (2015) called for research that determines and explains learning with a clearer theoretical framing. A similar call was made by Larssen and colleagues (2018) in their review of lesson study in initial teacher education. Many studies have replied to such calls by applying sociocultural theories of learning (Huang & Shimizu, 2016). For instance, some studies used activity theory (Lee & Tan, 2020; Wake et al., 2016), whereas others used various approaches to discourse analysis (e.g., Pillay & Adler, 2015; Warwick et al., 2016). Some, like Karlsen and Helgevold (2019), combined theories—they combined theories of noticing (van Es, 2011) and interthinking (Littleton & Mercer, 2013)—to investigate teachers' learning processes in lesson study. Instead of adopting general education learning theories or theories developed in other fields (Karlsen & Helgevold, 2019), we decided to apply a theory of learning developed in the field of mathematics education and used in recent studies of teacher learning in other professional development contexts (e.g., Heyd-Metzuyanim & Shabtay, 2019; Heyd-Metzuyanim et al., 2018).

Theoretical Framework

Where cognitive theories consider learning as change in mental structures, the commognitive theory describes learning in terms of discourse (Sfard, 2008), and as a process of change in routines of participation in a certain community (Heyd-Metzuyanim et al., 2019). Sfard (2008) defined learning as a change in discourse, and she elaborated on how learning mathematics can be described as the process of becoming a participant in a mathematical discourse. From a commognitive perspective, teaching can be described as a communicational activity that aims to "bring the learners' discourse closer to a canonic discourse" (Tabach & Nachlieli, 2016, p. 299). Simply put, this can happen by the introduction of new words to the discourse, or by adjusting the rules that govern the discourse (e.g., Sfard, 2007). In other words, teaching relates to facilitating opportunities for students to participate in a certain mathematical discourse.

Professional development programmes for teachers often focus on "reform oriented" teaching practices trying to lead the teachers towards a practice of teaching that highlights discussion and exploration (Nachlieli & Heyd-Metzuyanim, 2021). Heyd-Metzuyanim and Shabtay (2019) distinguish between traditional teaching practice and reform-oriented teaching practice in terms of teachers' pedagogical discourses, where pedagogical discourses are described in the following way:

> Pedagogical Discourses shape and orient teachers towards *what* to teach students, *how* to teach them, *why* certain teaching actions are more effective than others and, often not talked about but still very important, *who* can learn (or not learn). (Heyd-Metzuyanim & Shabtay, 2019, p. 543)

Whereas traditional education literature has described it as a distinction between "traditional" and "progressive" education (Dewey, 1938), or as the difference between traditional and non-traditional lessons (Cazden, 2001). Heyd-Metzuyanim and Shabtay (2019) described it as the differences between a "delivery pedagogical discourse" and an "explorative pedagogical discourse." The delivery pedagogical discourse considers teaching as a process where teachers demonstrate and explain how to

carry out procedures and solve certain tasks—or as "delivering" the knowledge—whereas students "acquire" the knowledge by practicing and applying the skills that the teachers have demonstrated. A typical form of classroom communication within this discourse would be the initiation-response-evaluation (IRE) pattern of "traditional" lessons (e.g., Cazden, 2001). In contrast, an explorative pedagogical discourse can be described as a discourse that "values students' explorations, namely – agentive construction of their own mathematical narratives, externalization of thinking processes, group and classroom discussions, and students struggling with meaningful problems" (Nachlieli & Heyd-Metzuyanim, 2021, p. 348). The explorative pedagogical discourse thus aligns with what others refer to as progressive education or reform-oriented teaching. Within commognitive research, these terms have been used in several studies of teacher learning, where teacher learning is considered as a change in teachers' pedagogical discourse—from discourse aligned with delivery pedagogical discourse towards explorative pedagogical discourse (Heyd-Metzuyanim, 2019; Heyd-Metzuyanim et al. 2019; Nachlieli & Heyd-Metzuyanim, 2021). Heyd-Metzuyanim et al. (2018) show how there are misalignments in the discourse used in professional development programmes, and the discourse used by the teachers in their school classrooms (commognitive conflicts). Nachlieli and Heyd-Metzuyanim (2021) elaborated on how the learning mechanism towards explorative pedagogical discourse can be explained in terms of commognitive conflicts, and they identified four themes of conflict: 1) students' abilities, 2) who is in control of knowledge development, 3) limited view of the explorative pedagogical discourse, and 4) view of what constitutes an ordinary lesson.

Sfard (2008) initially coined the term "commognitive conflict" to describe a situation that arises in the learning process, "when communication occurs across incommensurable discourses" (p. 296). A typical example is when people use the same word, but with different meanings. Nachlieli and Heyd-Metzuyanim (2021, p. 351) extend the term commognitive conflict to teacher learning, with reference to situations "in which participants use the same words in different ways." In more general terms, a commognitive conflict can be regarded as misalignment between the discourse of participants who belong to different communities, or as misalignment between discourses. In our study, we consider commognitive conflicts as instances where there appears to be misalignment between participants' talk and the more canonical explorative pedagogical discourse.

Where Xu and Pedder (2015) called for more theoretical work that explores how and why teachers learn in lesson study, our study applies the commognitive theory to approach this indirectly by investigating commognitive conflicts in teachers' pedagogical discourse in lesson study to unpack some reasons why teachers may *not* learn in lesson study. We approach the following research question:

> *What types of commognitive conflicts may occur between teachers' discourses in lesson study and the canonical explorative pedagogical discourse?*

Methods

The study reported in this paper investigated data from a project that involved a lesson study group comprised of four Norwegian lower secondary school teachers and one person from the school administration. The teachers (one male and three female) had experience as mathematics teachers that ranged from two to twenty-three years. The first author was a participating observer and took the role of "knowledgeable other" in the lesson study group (Takahashi, 2014). The lesson study group carried out three lesson study cycles—a cycle typically involved planning, teaching, adjustment, re-teaching, and evaluation of a research lesson—over the course of one and a half years (each cycle within one school semester). Video recordings were made from planning meetings, research lessons (in one class at Grade 9 and one at Grade 10), and reflection meetings throughout the three lesson study cycles.

For the purpose of this study, we analysed the transcripts from the recordings of all the reflection meetings. The meetings were considered to be sites where teachers' pedagogical discourse would be most visible. In pursuit of identifying potential commognitive conflicts in the teachers' pedagogical discourse in the reflection meetings, we analysed the data in several iterations. The first iteration involved going through the transcripts from all reflection meetings by use of an open and inductive approach—often referred to as analytic induction (Cohen et al., 2007). The main focus in this iteration

was to identify and describe the nature of the discourse. Through this process of inductive coding, we identified four distinct themes in the discourse. The first theme involved the teachers' observation of student learning, where they shared what they observed from the students during the research lesson. A second theme involved talk about what to improve for the next research lesson. A third theme involved the teachers' reflections on their own learning, and, finally, a fourth theme included situations where teachers discussed how they could share their experiences with their colleagues.

In the next iteration of the data analysis, the data underwent reduction by focusing on the data related to the first theme, when the teachers' presented their observations of students' learning. Following an inductive process, the transcripts were divided into thematic episodes, each episode comprised of exchanges that had a primary emphasis on one of the three emerging foci: 1) talk that emphasised the *mathematical* content, 2) talk that emphasised *teaching* practice, and 3) talk that focused on *students*. In a further analysis of these episodes, we followed Nachlieli and Heyd-Metzuyanim's (2021) ideas about misalignments and commognitive conflicts to identify underlying assumptions of the teachers' discourse and considered how these assumptions might indicate misalignment between the delivery and exploratory discourses of teaching and learning.

Results

The main long-term goal in the lesson study participation was for the teachers to develop an explorative pedagogical discourse. The analysis identified three themes where there seemed to be misalignment between the teachers' pedagogical discourse and the canonical pedagogical discourse. Although such misalignments were often visible in more than one lesson study cycle, we used one example episode from each of the three lesson study cycles to illustrate the commognitive conflicts. In each of the episodes below, we present first the context and describe the episode, before commenting on the pedagogical discourse and identifying the underlying assumptions present that seem to indicate a commognitive conflict.

Episode 1: Discourse of Teaching

This episode is from the first lesson study cycle, where there was a strong emphasis on classroom organisation and use of time in the teachers' discourse. In the reflection meeting after the first research lesson, the teachers commented briefly on time management. They noted how they spent 10–15 minutes only to get started, but most of their discussions focused on how they were surprised that students struggled so much with the following mathematics problem:

> A sandbox has a volume of 500 litres. Can you build different sandboxes that would fit the same amount of sand? Make a drawing of these sandboxes and mark the sizes of the sides.

During a discussion of how the students experienced this problem, the teachers frequently referred to the "weak" and "strong" students, and they were baffled that many of the supposedly more able students struggled. This struggle was described as an indication of failure, and the teachers were astonished to notice that the students struggled with content that had been covered in previous lessons. This observation sparked reflections about the connection between teaching and learning. One teacher asked, "We have taught this, but have they learned it?" She responded to her own question by adding, "At least we have told them." The teachers perceived lack of connection between what they had taught and what the students had learned and were able to use was a recurring theme.

In the next reflection meeting, the teacher who had taught the second research lesson in the first cycle reflected on her own use of time. She had struggled to decide how much time each group should be allowed to spend on the problem before moving on. The reason for her hesitation was that the students never seemed to provide the kind of explanations that the teachers had been aiming for in their lesson plan. Another teacher recalled that the learning goal for the lesson was to "calculate the

volume of different geometric shapes, and thereby use mathematics in everyday situations, develop critical thinking, and, finally, that they should be actively engaged in collaboration with other students." When engaging with the problem, however, the students only used trial-and-error without any reasoning. This observation caused astonishment and surprise among the teachers:

T3: No, I was a little bit surprised that so many only used trial and error.
T5: Yeah, because everybody did that, at least from what I could see.
T3: Yes.
T5: For the most part.
T1: Yes, and they even did this in the other class, and even the smartest ones!
T3: Yeah, but I also told them to select one of the sides, but they continued with trial and error. And then someone got 500.02, which was super close! (everyone laughs).
T1: In one of my groups, if we just had slightly less here and slightly less there, they would get it!
T3: Yes.
T5: Yeah, there is something about that as well.
T1: Trial-and-error, that is also a method.
T5: Yes.
T1: Right, those who don't ...
T5: Sometimes we write: "Show how you reached the solution by calculation, and then you can explain that this is what you want."
T1: Yes.
T3: Mhm.

Although the mathematics problem had the potential of inviting students into explorative participation with the aim of producing endorsed mathematical narratives (cf. Heyd-Metzuyanim et al., 2016), the students ended up in ritual rather than explorative participation. One example that illustrates this is when the teacher reflected on how the students used trial-and-error as a strategy in the solution process by randomly guessing three numbers where the product came close to 500. To put numbers into a formula in order to calculate the answer can be described as ritual participation, which is based on a procedure that has been conducted before. Even though one of the teachers (T3) had suggested that the students could select one side of the sandbox as a starting point, to stimulate a different approach, the students continued to use the trial-and-error strategy in their solution process. The episode above indicates that the teachers were both surprised and disappointed by the students' achievements, and the teachers experienced a gap between their own expectations and the students' performances. The teachers had hoped for the students to use equations to solve the problem, and we observe how T3 suggested that the students should start with one of the sides. Additionally, we underline that T5 has some exploratory assumptions for the students' participation, as he stated that teachers sometimes ask students to both show how they solve a problem and explain their reasoning.

In this episode, we identified three underlying assumptions that point towards misalignment with the explorative pedagogical discourse. The first assumption can be identified in the first reflection meeting, where one of the teachers commented that, "at least we have told them." This indicates an underlying assumption of teaching as telling, which aligns with a delivery pedagogical discourse. Furthermore, if the teacher explains a solution, the assumption is that students will learn by adapting the procedure and apply it to new contexts. This contrasts with the explorative pedagogical discourse where students actively engage in constructing mathematical narratives on their own. The underlying assumption that is present here communicates that knowledge is an object that can be delivered from teacher to students.

A second underlying assumption is that "strong" students are not supposed to struggle in mathematics, and that trying things out is not appropriate in mathematics. In the first reflection meeting, teachers were surprised that even the supposedly "strong" students struggled, and in the second meeting the teachers were astonished that most students only approached the problem by using trial-and-error. We noticed, for instance, how T1 commented that the students in another class—"even the smartest ones"—used trial-and-error. This is in conflict with the explorative pedagogical discourse, where the need for struggle is considered integral to mathematical learning, and where student struggle is not an indicator of failed teaching (Nachlieli & Heyd-Metzuyanim, 2021).

A third underlying assumption relates to the connection between teaching as exploration and the existence of a right way of approaching the problem. Although the teachers wanted the students to participate exploratively, their pedagogical discourse aligned with a canonical delivery pedagogical discourse that evidences the underlying assumption that the right way of solving this given problem is to use equations, when the teachers have already taught the students how to use equations to solve such problems. This points towards a discourse where knowledge is something that students adopt and apply, in contrast to a discourse about students constructing new and endorsed narratives as participants in a mathematics classroom.

Episode 2: Discourse of Learning Mathematics

In the first cycle, the teachers frequently reflected on the students' inability to apply previous knowledge of equations when trying to solve problems on volume. Throughout the reflection meetings, the teachers' observations had surprisingly little emphasis on the mathematical content. The episode that follows, taken from the second cycle, is illustrative of the few reflections that had some emphasis on the mathematical content, but, even here, the focus was more on how students struggled mathematically with the problem than on the content itself. One teacher did pay explicit attention to the mathematical content in the reflections, having noticed that the students said that their aim is to find the "square" and not the "square root." There is a strong emphasis on actions in the mathematical discourse—as it is presented in the teacher's reflections.

The goal of the research lesson from the second cycle was to invite students to explore the Pythagorean theorem before being introduced to the formula. As reported in Tyskerud (2021, p. 163), the teacher who taught the research lesson had given the following verbal presentation of the problem:

> You get a sheet of paper with two triangles, and you are supposed to build squares on the sides of these triangles. When you have done that, you write in your own words what kinds of connections you see. If anyone manages to write this in mathematical language … I don't expect that you can, but you can try.

The episode from the reflection meeting begins with T2's utterance, emphasising how the students in her group only focused on the sides of the right-angled triangle when searching for the connection without considering the entire square:

> T2: My group was searching for the connection, but they didn't find it on their own. They only focused on the sides and not the entire square when searching for the connection. In the end, they managed, by using [a multiplication] table.
> T5: So, it served a purpose.
> T1: Yes, it did, … also with the group up front. They quickly found the connection when they got the table.
> T2: They found it immediately then, when they wrote down the numbers and knew what to be looking for; that they should use all the pieces and not just one of the sides. Then they struggled a bit more with the following task. They knew that they had to multiply this side with that side [to find the area], but they didn't know how to find the side when they knew the square. They tried to divide by two, and then by four, but they didn't understand why it wouldn't work. They tried multiple times. They then got a response, which they multiplied by itself. They managed to formulate it like this, but they didn't use the word "square root." Even though they explained that this is what they did. I think they used the word "square", "we have to find the square of this number." And then I think it was I who told them that they might be confused. That this is, this is the square, and that is the square root. So, when you explained it on the board, revisited and corrected this, they eventually figured it out. But they didn't use the right words. They didn't understand the square root or how to find it. Yet, they understood what to do, but they didn't have time to finish. Now I just remembered what they did! They found a multiplication table on the iPad and found the number that could be multiplied by itself to produce the given number. And then they found it. They knew that five times five is twenty-five, but they were insecure when the numbers got a little bit [larger].

We identified three underlying assumptions that indicate a conflict between the teachers' discourse and the canonical explorative pedagogical discourse in this episode. The first one can be seen in the dialogue between the three teachers, T2, T5, and T1. The teachers indicated that one particular representation—the table provided in the handout—was required to solve the problem. Following this, T2 confirmed that the students "knew that they had to multiply this side with that side." The teachers

do not elaborate on how and why this table would help students solve the problem, they only concluded that it was helpful for the students. By introducing a supporting tool like the table, the teachers simplify the problem and narrow students' opportunities to explore different ways of solving the problem. This way of simplifying the cognitive demands of problems is common and often deemed necessary within a delivery pedagogical discourse. Instead of letting the students truly explore the problem, an assisting tool like this guides the students towards ritual participation in the mathematical discourse.

A second underlying assumption in this episode is that mathematics is learned by a teacher explaining the content, followed by students' application. T2 said: "So when you explained it on the board, revisited and corrected this, they eventually figured it out." This is another example of an assumption that knowledge is something the teacher delivers to the students—if the teacher shows and explains how to solve the task, the students adapt and reconstruct their procedures.

A third underlying assumption is that using the right words is key to mathematical learning. T2 focused on the students' word use, and she was concerned that the students were not able to use the right word—as previously used by the teacher. Although word use is indeed part of the mathematical discourse, the emphasis is more on constructing endorsed mathematical narratives (Sfard, 2008). The focus on word use in combination with a lack of focus on how students use words thus becomes an indicator of a delivery pedagogical discourse. It seemed more important to the teachers that the students use the right words than considering how the students construct new narratives in their efforts to solve the given problem.

Episode 3: Discourse of Students

In the first cycle, the teachers frequently referred to "weak" and "strong" students, whereas their reflections in the second cycle focused more on what students did when working on mathematical problems. In the third cycle, talk about "weak" and "strong" students re-appeared. In this episode, one of the teachers twice described a group of students as a "strong group."

T5: T1, did you have an observation [you want to share]?
T1: Yes. Right then, I had a strong group, and they really started to generalise before they … they didn't use concrete materials a lot, at least not before they reached the last one. And I imagined that we perhaps should have developed a few additional tasks for them, some that were more advanced, because they solved this one immediately.
T2: The one with the table?
T1: Yes, the one with the table. And they were able to revert the formulas and … I had [name] and [name], and – who was that – [name] too.
T3: Yes.
T1: So, I had [another] strong group there. And they just looked at it, calculated with the formulas, inserted it…. And then they would sit and wonder what to do, so I gave them another one.

When describing the students, she considered to be a "strong group", T1 labelled them and used a language of static properties rather than actions. This indicated what Nachlieli and Heyd-Metzuyanim (2021) refer to as a "fixed mindset assumption." Such an assumption involves considering students' abilities as static, which is connected with a traditional view of learning as receiving knowledge from the teacher, instead of thinking of knowledge as something students are supposed to develop and construct by working together, by sharing thoughts, and by discussing strategies. In addition to this main issue of fixed mindset assumption, which is indicative of a delivery pedagogical discourse, this episode included three additional assumptions of what it means to be "strong" in mathematics. Each of these assumptions are misaligned with an explorative pedagogical discourse.

The first assumption is that a "strong" student is able to generalise without first using concrete materials. This can be identified in T1's first utterance about the "strong group", where she explained that the group engaged in generalisation rather than using concrete materials. This might indicate that the students engaged in explorative routines, but it might also indicate an assumption that use of concrete materials is an inferior route to mathematical exploration.

A second assumption seems to be that being successful in mathematics is related to students' ability to apply formulas. The second time T1 referred to the "strong group", she explained that the students had "calculated with the formulas", indicating that the students were able to apply the formula they had created by inserting different sets of numbers. This indicates that the group was identified as strong because they were able to carry out ritual routines.

Finally, a third assumption, which is embedded in the second, is that being "strong" in mathematics relates to students' ability to apply a formula to solve problems *quickly*. This is in conflict with an explorative pedagogical discourse, where productive struggle is considered to be an integrated part of learning mathematics by exploration.

These assumptions, along with the consideration of students' former achievement as a measure of their static ability—by labelling them as being "strong" or "weak" in mathematics—do not align with an explorative pedagogical discourse. Sfard (2008) referred to this as "subjectifying", and warned about describing students in that way, and suggested instead to focus on their actions and discourse.

Discussion

Above, we have used the commognitive theory (Nachlieli & Heyd-Metzuyanim, 2021; Sfrad, 2008) to analyse teachers' pedagogical discourse in their reflections about student learning in lesson study. We identified three themes where teachers' underlying assumptions seem to point towards potential commognitive conflicts, each of which indicated misalignment between the teachers' pedagogical discourse and the canonical explorative pedagogical discourse.

The first theme relates to the discourse on teaching. Although reforms tend to call for a shift from delivery to explorative pedagogical discourse, traditional teaching (delivery discourse) continues to prevail (Nachlieli & Tabach, 2019). In our analysis of teachers' pedagogical discourse in the lesson study group, we identified underlying assumptions about teaching as telling, and assumptions indicating that good teaching should help students know what to do and avoid struggle. In the first episode, we observed how the teachers were surprised that the students struggled with a problem that involved the use of algorithms that had already been covered in previous lessons. The teachers were also surprised that the students only used trial-and-error with no mathematical reasoning. Interestingly, the teachers presented the students with a list of steps to carry out (Tyskerud & Mosvold, 2018), and thereby invited the students to perform a ritual by using a formula (Heyd-Metzuyanim et al., 2016). We argue that this aligns with a delivery pedagogical discourse and a traditional way of thinking about teaching. Our findings here correspond with the findings of Nachlieli and Heyd-Metzuyanim (2021), who also identified a discourse of teaching as telling with delivery pedagogical discourse. These researchers also found a theme of commognitive conflict that involved assumptions that the teacher is responsible for constructing the mathematical knowledge in the classroom, and the assumptions identified in our study appear to point in a similar direction. The commognitive theory assisted in the discovery of the misalignment between the teachers' discourse and the canonical discourse.

These results of the research reported in this paper align with other research that studied the teachers' discourse when they presented tasks to the students. Findings show that the teachers presented the problem in a way that enabled participation in ritual routines, instead of presenting the problem in a way that required exploration (Tyskerud & Mosvold, 2018). Following Nachlieli and Tabach definitions of ritual and explorative routines (2019, p. 255), the teachers could have supported the students by prompting them to focus on a question of, "What do I want to achieve?" instead of a question of, "How do I proceed?" in order to develop an explorative pedagogical discourse. In close connection with the dominance of traditional teaching, the communication in mathematics classrooms tends to emphasise procedures and "doings". We observed this tendency in both of the first episodes.

A second theme identified in our study relates to learning mathematics, and we identified some underlying assumptions that learning mathematics is about adopting what teachers have explained. This corresponds with the findings of Nachlieli and Heyd-Metzuyanim (2021). In addition, we discovered some underlying assumptions about how support from the teacher—here in terms of providing a table as a supporting tool for exploration—is required to help students see the mathematical connections

expected. The analysis indicates that such actions and resources might simplify the demands of the problem and prevent students from engaging in true exploration. This is another example of a misalignment that may constitute a commognitive conflict between teachers' discourse and the canonical explorative pedagogical discourse.

Finally, a third theme—concerning students' abilities—corresponded with the first of four themes identified by Nachlieli and Heyd-Metzuyanim (2021). The data showed that the teachers frequently described students or groups of students as "weak" or "strong", thus considering their abilities as fixed.

Conclusion

Tyskerud (2021) reported that the teachers in this lesson study group aimed at giving their students opportunities to engage in explorative routines and produce endorsed mathematical narratives (Heyd-Metzuyanim et al., 2016). Results from the analysis in this paper, draw attention to commognitive conflicts, because the teachers' pedagogical discourse aligns more with a pedagogical delivery discourse rather than an explorative pedagogical discourse. Our current study illustrates how the commognitive theory can be useful for studying teacher learning. Xu and Pedder (2015) criticised studies for not focusing on talks as mediator for teachers' learning in lesson study, and this is precisely the focus of the commognitive theory (Sfard, 2008). The main goal for the teachers in the lesson study group was to develop an explorative pedagogical discourse. Our analysis identified several commognitive conflicts between their discourse and the canonical explorative discourse. The commognitive conflicts that we identified were in line with the commognitive conflicts identified by Nachlieli and Heyd-Metzuyanim (2021), and we agree with these researchers that identifying such conflicts can be challenging. Their study, however, provided clear indications of a gradual development in how the conflicts became explicit and were reflected upon. There was no clear evidence of explicit reflection in our study. This prompts us to suggest that an important role of the knowledgeable other in lesson study might be to identify potential commognitive conflicts between the teachers' pedagogical discourse and the canonical pedagogical discourse. It is important to make such conflicts visible to participants and challenge the underlying assumptions in order to promote development towards an explorative pedagogical discourse, and thereby enable the steady and incremental changes in practice that lesson study aims at. We suggest that this might be a productive application of the commognitive theory in lesson study, and we propose that future studies explore meaningful ways of applying the commognitive theory in the lesson study development process. Implicitly, this suggestion promotes a shift of emphasis from searching for *evidence* of teacher learning in lesson study to engaging in the *process* of changing practice through lesson study.

References

Cajkler, W., Wood, P., Norton, J., & Pedder, D. (2014). Lesson study as a vehicle for collaborative teacher learning in a secondary school. *Professional Development in Education, 40*(4), 511–529. https://doi.org/10.1080/19415257.2013.866975

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning*. Heinemann.

Cerbin, W., & Kopp, B. (2006). Lesson study as a model for building pedagogical knowledge and improving teaching. *International Journal of Teaching and Learning in Higher Education, 18*(3), 250–257.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. Routledge.

Dewey, J. (1938). *Experience and education*. The Macmillan Company.

Dotger, S. (2015). Methodological understandings from elementary science lesson study facilitation and research. *Journal of Science Teacher Education, 26*(4), 349–369. https://doi.org/10.1007/s10972-015-9427-2

Dudley, P. (2013). Teacher learning in lesson study: What interaction-level discourse analysis revealed about how teachers utilised imagination, tacit knowledge of teaching and fresh evidence of pupils learning, to develop practice knowledge and so enhance their pupils' learning. *Teaching & Teacher Education, 34*, 107–121. https://doi.org/10.1016/j.tate.2013.04.006

Dudley, P. (Ed.). (2015). *Lesson study: Professional learning for our time*. Routledge. https://doi.org/10.4324/9780203795538

Fernandez, C. (2002). Learning from Japanese approaches to professional development: The case of lesson study. *Journal of Teacher Education, 53*(5), 393–405. https://doi.org/10.1177/002248702237394

Fujii, T. (2014). Implementing Japanese lesson study in foreign countries: Misconceptions revealed. *Mathematics Teacher Education and Development, 16*(1), 65–83. https://mted.merga.net.au/index.php/mted/article/view/206

Gee, D., & Whaley, J. (2016). Learning together: practice-centred professional development to enhance mathematics instruction. *Mathematics Teacher Education and Development, 18*(1), 87–99. https://mted.merga.net.au/index.php/mted/article/view/237

Heyd-Metzuyanim, E. (2019). Changing teaching practices towards explorative Instructions: The Interweaving of teacher Identity and pedagogical discourse. *Teaching and Teacher Education, 86,* Article 102862. https://doi.org/10.1016/j.tate.2019.06.016

Heyd-Metzuyanim, E., Munter, C., & Greeno, J. (2018). Conflicting frames: A case of misalignment between professional development efforts and a teacher's practice in a high school mathematics classroom. *Educational Studies in Mathematics, 97*(1), 21–37. http://dx.doi.org/10.1007/s10649-017-9777-0

Heyd-Metzuyanim, E., & Shabtay, G. (2019). Narratives of 'good' instruction: Teachers' identities as drawing on exploration vs. acquisition pedagogical discourses. *ZDM—Mathematics Education, 51*(3), 541–554. https://doi.org/10.1007/s11858-018-01019-3

Heyd-Metzuyanim, E., Smith, M., Bill, V., & Resnick, L. B. (2019). From ritual to explorative participation in discourse-rich instructional practices: A case study of teacher learning through professional development. *Educational Studies in Mathematics, 101*(2), 273–289. http://dx.doi.org/10.1007/s10649-018-9849-9

Heyd-Metzuyanim, E., Tabach, M., & Nachlieli, T. (2016). Opportunities for learning given to prospective mathematics teachers: Between ritual and explorative instruction. *Journal of Mathematics Teacher Education, 19*, 547–574. https://doi.org/10.1007/s10857-015-9311-1

Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: An international perspective. *ZDM—Mathematics Education, 48*(4), 393–409. https://doi.org/10.1007/s11858-016-0795-7

Hunter, J., & Back, J. (2011). Facilitating sustainable professional development through lesson study. *Mathematics Teacher Education and Development, 13*(1), 94–114. https://mted.merga.net.au/index.php/mted/article/view/48

Karlsen, A. M. F., & Helgevold, N. (2019). Lesson study: Analytic stance and depth of noticing in post-lesson discussions. *International Journal for Lesson and Learning Studies, 8*(4), 290–304. https://doi.org/10.1108/IJLLS-04-2019-0034

Larssen, D. L. S., Cajkler, W., Mosvold, R., Bjuland, R., Helgevold, N., Fauskanger, J., Wood, P., Baldry, F., Jakobsen, A., Bugge, H. E., Næsheim-Bjørkvik, G., & Norton, J. (2018). A literature review of lesson study in initial teacher education: Perspectives about learning and observation. *International Journal for Lesson and Learning Studies, 7*(1), 8–22. https://doi.org/10.1108/IJLLS-06-2017-0030

Larssen, D., & Drew, I. (2015). The influence of a lesson study cycle on a 2nd grade EFL picture book-based teaching practice lesson in Norway. *Nordic Journal of Language Teaching and Learning, 3*(2), 92–105. https://doi.org/10.46364/njmlm.v3i2.143

Lee, L. H. J. & Tan, S. C. (2020). Teacher learning in lesson study: Affordances, disturbances, contradictions, and implications. *Teaching and Teacher Education, 89,* Article 102986. https://doi.org/10.1016/j.tate.2019.102986

Lewis, J., Fischman, D., Riggs, I., & Wasserman, K. (2013). Teacher learning in lesson study. *The Mathematics Enthusiast, 10*(3), 583–620. https://doi.org/10.54870/1551-3440.1281

Lewis, C., & Perry, R. (2014). Lesson study with mathematical resources: A sustainable model for locally-led teacher professional learning. *Mathematics Teacher Education and Development, 16*(1), 22–42. https://mted.merga.net.au/index.php/mted/article/view/205

Lewis, C., Perry, R., & Murata, A. (2006). What is the role of the research in an emerging innovation? The case of lesson study. *Educational Researcher, 35*(3), 3–14. https://www.jstor.org/stable/3700102

Littleton, K., & Mercer, N. (2013). *Interthinking: Putting talk to work*. Routledge.

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. C. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 13–24). Springer. http://dx.doi.org/10.1007/978-90-481-9941-9_1

Nachlieli, T., & Heyd-Metzuyanim, E. (2021). Commognitive conflicts as a learning mechanism towards explorative pedagogical discourse. *Journal of Mathematics Teacher Education, 25*(3), 347–369. http://dx.doi.org/10.1007/s10857-021-09495-3

Nachlieli, T., & Tabach, M. (2019). Ritual-enabling opportunities-to-learn in mathematics classrooms. *Educational Studies in Mathematics, 101*, 253–271. https://doi.org/10.1007/s10649-018-9848-x

Ono, Y., & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education, 30*(1), 59–74. https://doi.org/10.15700/saje.v30n1a320

Pang, J. (2016). Improving mathematics instruction and supporting teacher learning in Korea through lesson study using five practices. *ZDM–Mathematics Education, 48*(4), 471–483. https://doi.org/10.1007/s11858-016-0768-x

Pillay, V., & Adler, J. (2015). Evaluation as key to describing the enacted object of learning. *International Journal for Lesson and Learning Studies, 4*(3), 224–244. https://doi.org/doi:10.1108/IJLLS-08-2014-0033

Puchner, L. D., & Taylor, A. R. (2006). Lesson study, collaboration and teacher efficacy: Stories from two school-based math lesson study groups. *Teaching and Teacher Education, 22*(7), 922–934. https://doi.org/10.1016/j.tate.2006.04.011

Robinson, N., & Leikin, R. (2012). One teacher, two lessons: The lesson study process. *International Journal of Science and Mathematics Education, 10*(1), 139–161. https://doi.org/10.1007/s10763-011-9282-3

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences, 16*(4), 565–613. https://doi.org/10.1080/10508400701525253

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511499944

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. The Free Press.

Tabach, M., & Nachlieli, T. (2016). Communicational perspectives on learning and teaching mathematics: Prologue. *Educational Studies in Mathematics, 91*, 299–306. https://doi.org/10.1007/s10649-015-9638-7

Takahashi, A. (2014). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development, 16*(1), 4–21. https://mted.merga.net.au/index.php/mted/article/view/204

Tyskerud, A. (2021). Utvikling av matematikkundervisning: En kommognitiv analyse av rutiner i klasserommet [Development of mathematics teaching: A commognitive analysis of classroom routines]. *Nordic Studies in Mathematics Education, 26*(3–4), 153–170.

Tyskerud, A., & Mosvold, R. (2018). Scrutinizing teacher-learner interactions on volume. *Nordic Studies in Mathematics Education, 23*(2), 49–67.

van Es, E. A. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing. Seeing through teachers' eyes* (pp. 134–151). Routledge. https://doi.org/10.4324/9780203832714

Wake, G., Swan, M. & Foster, C. (2016). Professional learning through the collaborative design of problem-solving lessons. *Journal of Mathematics Teacher Education, 19*(2–3), 243–260. https://doi.org/10.1007/s10857-015-9332-9

Warwick, P., Vrikki, M., Vermunt, J. D., Mercer, N., & van Halem, N. (2016). Connecting observations of student and teacher learning: an examination of dialogic processes in Lesson Study discussions in mathematics. *ZDM–Mathematics Education, 48*(4), 555–569. https://doi.org/10.1007/s11858-015-0750-z

Xu, H., & Pedder, D. (2015). Lesson study: An international review of research. In P. Dudley (Ed.), *Lesson study: Professional learning for our time* (pp. 29–58). Routledge.

Yoshida, M. (2012). Mathematics lesson study in the United States: Current status and ideas for conducting high quality and effective lesson study. *International Journal for Lesson and Learning Studies, 1*(2), 140–152. https://doi.org/10.1108/20468251211224181

Author contact details

Anita Tyskerud
University of Stavanger
anita.tyskerud@uis.no

Reidar Mosvold
University of Stavanger
reidar.mosvold@uis.no

Raymond Bjuland
University of Stavanger
raymond.bjuland@uis.no