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Research Article

The effect of problem posing-based active learning activities on problem-solving and posing performance: The case of fractions

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This study aims to examine the effect of problem posing-based active learning activities on students' problem-posing skills and problem-solving achievement. To this aim, an experimental design with pre-test post-test control group was employed. The participants consisted of two groups of sixth graders, one experimental group (N=23) and one control group (N=25). Students in the experimental group were exposed to seven problem-based active learning activities over the course of six weeks. The study used problem-solving and problem-posing tests to collect data. The results revealed that eventhough the intervention was not statistically significant, the increase in the problem-solving mean score of the experimental groups was higher than that of the control group. Problem posing pre- and post-test scores of the experimental group differed statistically significantly with a high level of effect size ($\eta^2 = 0.80$). Finally, educational implications are discussed, and recommendations are made for future research.

Keywords: Mathematical problem-solving; Problem-posing; Active learning; Operations with fractions

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1. Introduction

Rather than achieving the result of the problem encountered, problem-solving involves determining the right way to accomplish the goal (Polya, 1957). Therefore, Polya (1957) defines problem-solving as using mental processing skills to solve a problem. Accordingly National Council of Teachers of Mathematics [NCTM] (2000) indicates that problem-solving develops students' mental processing skills, such as processing, systematic thinking, and organizing their thoughts. As a matter of fact, developing problem-solving skills has become one of the major objectives of the Mathematics Course Curriculum (Turkish Ministry of National Education [MoNE], 2018).

Problem-posing, on the other hand, typically involves problem-solving, develops as a continuation of problem-solving, and as such, needs to be addressed as a whole in conjunction with problem-solving. The problem-posing process is not a separate process from the problem-solving process, as Gonzales (1994) describes it as a continuation of the problem-solving process.

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Stoyanova and Ellerton (1996) define problem-posing as the creation of meaningful mathematical problems based on one's mathematical experience and evaluations developed for concrete situations. In this context, problem-posing is used by researchers (Abu-Elwan, 2002; English, 1997; Silver, 1994) as a tool for evaluation, discovery and mathematical thinking and plays an influential role in the studies carried out to improve school mathematics (Silver, 2013). As a consequence, to enhance the problem-posing skills of students in mathematics lessons, various activities should be undertaken in which students can actively participate (Lowrie, 2002; Nicolaou & Philippou, 2007).

Eventhough the ways in which students actively participate in a class differ, they are basically grouped under the name of active learning techniques (Bilgin & Acar, 2007). Kyriacou (1992) defines active learning as non-rule, open-ended teaching in which students control the learning activities applied to them, thereby shaping their own learning experiences. By combining problem-posing, which is an active learning method, with different active learning techniques, more effective and interesting activities can be prepared for students. According to existing studies, active learning activities improve academic achievement (Rotgans & Schmidt, 2011), persistence in learning (Bédard et al., 2012) interest and motivation towards mathematics (Angelini et al., 2018), as well as the students' mental abilities (Shroff et al., 2019). In active learning activities, students learn by doing while at the same time integrating the information they have learned into new situations. By using these techniques, it may be possible to turn problem-posing, which is perceived as difficult, into a fun activity for students to develop skills that can be obtained by problem-posing, and to ensure that the skills are retained. In the end, mathematics is a fun game that can be enhanced with active learning methods as well as serious work (Sarıer, 2020). This study examined students' problem-posing skills and their problem-solving achievement through a series of activities designed with active learning techniques.

Research on problem-posing in the literature has been conducted using various experimental designs (Cankoy, 2014; Cankoy & Darbaz, 2010; Chen et al., 2015; Kopparla et al., 2019; Özgen et al., 2019; Terzi, 2021). These studies examined the impact of a designed learning environment on students' problem-posing skills (Cankoy, 2014) while others examined the effect on both problem-posing and problem-solving skills (Chen et al., 2015; Kopparla et al., 2019; Özgen et al., 2019; Terzi, 2021). In most of these studies, the problems established after individual or group activities were analyzed by groups or classes.

Chen et al. (2015) conducted discussion classes and group meetings for Chinese fourth-year students as part of their training program. Following this instruction, students' problem-solving performance and attitudes toward problem-solving and posing improved in favor of the experimental group. In contrast to the control group, there was a greater improvement in the originality dimension of problem-posing in the experimental group. In another study, Kopparla et al. (2019) examined the development of students' problem-posing and problem-solving performance after completing problem-solving and posing intervention groups. It was found that students in both groups improved their problem-posing and solving performance, but the students in the problem-solving group improved their problem-posing performance more. Using Ellerton's (2013) active learning framework, Özgen et al. (2019) designed a training programme. Throughout the course of this designed training, it was determined that GeoGebra software was effective for developing students' problem-solving and posing skills. Cankoy and Darbaz (2010) designed an instructional model, *problem posing-based problem-solving instruction*, for their problem-posing activities. A three-step approach involves establishing a problem, solving the problem, changing the data in the problem, and establishing another problem. This method considers problem-posing as a continuation of Polya's (1957) problem-solving steps. During this cycle, problems are discussed in the classroom first, then erroneous data is corrected to create a new problem. A class evaluation is conducted in terms of the suitability of the established problems to grammatical rules, logicality, and solvability. In the process of evaluating, erroneous points are corrected and a new problem is created. The problem posing-based approach to problem-solving was found to be very effective in students' understanding of problems, in their permanent traced learning, and in their qualitative reasoning. In this research, the problem-posing based problem-solving model developed by Cankoy and Darbaz (2010) was applied in the first stage of the intervention. The reason for applying this model before problem posing-based active learning activities is to ensure that students gain awareness of their mistakes in the problems they have individually established through collective class discussion.

The activities used in this research were created for operations with fractions. According to van de Walle et al. (2013), operations with fractions are important because they are closely related to decimal numbers, percentages, ratios and proportions, measurement, algebra, and probability. It has been found, however, that students, prospective teachers, and even teachers have difficulties with fractions, particularly with operations with fractions (Brown & Quinn, 2006; Hansen et al., 2017; Şahin et al., 2016). Moreover, some studies revealed that student misconceptions in operation with fraction (e.g. Karaoglan Yilmaz et al., 2018), while others showed conceptual errors in posing problems for operations with fractions (e.g. Kar, 2015). To identify misconceptions and conceptual errors related to fractions and to develop conceptual comprehension skills for fractions, problem-posing activities were used in this investigation. It has been shown throughout the literature that the activities designed to develop students' problem-posing skills were quite limited (e.g. Işık & Kar, 2012; Işık & Kar, 2015; Örnek & Soylu, 2021). In this study, different from other studies in the literature, various activities based on active learning methods were provided. During this study, activities were designed to contribute to efforts to use problem-posing in mathematics lessons and to provide an alternative to traditional problem-posing studies. Applied activities also included a variety of problem-posing activity types (free, structured, or semi-structured). This study examined how the designed problem posing-based active learning activities affected the problem-posing skills of the students as well as their ability to solve problems.

The subject of operations with fractions learning outcomes were assessed by implementing problem posing-based active learning activities within two 6th grade classes, which were divided into experimental and control groups. The application was evaluated in terms of its effects on students' problem-solving achievement and problem-posing skills. In order to achieve this goal, the following research problems were identified:

RQ 1) Are there any differences in problem-solving achievement of students in the control group versus those in the experimental group?

RQ 2) Are there any differences in problem-posing skills of students in the control group versus those in the experimental group?

2. Method

2.1. Research Model

Current investigation employs an experimental design with pre-test, post-test control group. In experimental research designs, the effect of one variable on other in controlled settings is examined. In other words, an experimental design is the approach in which the cause and effect relationships between two variables are measured (Miller et al., 2020). This study included two groups with random assignment, an experimental group and a control group. Different from the control group, active learning activities based on problem-posing were implemented in the experimental group. While active learning activities based on problem-posing were the independent variables of the research, the dependent variables were the students' problem-solving achievement and problem-posing skills.

2.2. Participants

The study sample consists of 48 sixth grade students, 26 girls and 22 boys, who are enrolled in public middle schools in the 2020-2021 academic year. In order to identify the school where this research was conducted, a purposeful sampling method was employed. Using purposeful sampling is a technique widely used to identify and select information-rich cases to maximize limited resources (Duan et al., 2015). The school had five classes that were at sixth-grade level.

Among these classes, two sixth-grade classes, a control group of 25 people and an experimental group of 23 people, were randomly assigned.

2.3. Data Collection and Analysis

A problem-solving test (PST) and a problem-posing test (PPT) developed by the researcher were used to collect data for the study.

2.3.1. Problem-solving test (PST)

Before the development of the PST, the sixth-grade textbooks approved by the Ministry of Education and the study guides used for sixth graders were examined. The PST draft was prepared and reviewed by an expert in mathematics education. The test then was administered to 62 students from three different seventh grade classes. Following the pilot test, item difficulty and item discrimination indices were calculated. Test items with a discrimination index of zero or negative were removed from the test, and items with a discrimination index between 0 and 0.30 were modified (Tekin, 2008). An item difficulty between 0 and .39 is considered difficult, while items between .40 and .59 are considered moderately difficult, and items between .60 and 1 are considered easy (De Boeck & Wilson, 2004). After revisions to the draft version of the PST, the number of items was determined to be 20, and the final version of the PST was administered. The KR-20 reliability coefficient of the PST was estimated to be .82, and it was deemed appropriate to measure the problem-solving achievement of the students.

The PST consists of 20 multiple-choice items, each worth five points. Answers were analyzed as either five or zero points, with five being the correct answer, and zero being the incorrect or blank answer. According to the normality tests conducted for PST, the pre-test ($p = .286 > .05$) and post-test ($p = .232 > .05$) data of the experimental group show a normal distribution. Therefore, the dependent group t-test was used to determine whether there was a significant difference between pre-test and post-test scores of the experimental group. Since the pre- ($p = .001 < .05$) and post-test ($p = .005 < .05$) scores of PST did not show normal distribution for the control group, a Mann-Whitney U test was performed. Similarly, the Mann-Whitney U test was performed to determine whether there was a significant difference between the post-test scores of the PST experimental group and the control group.

2.3.2. Problem-posing test (PPT)

To measure the problem-posing skills of the students, the problem-posing test (PPT) consisting of nine open-ended items was prepared for student performance in operations with fractions. During the developoment of the instrument, studies on problem-posing in operations with fractions (e.g. Kar, 2015; Toluk-Uçar, 2009), 6th grade textbooks of MoNE (2018), and expert opinions were taken into consideration. The final version of the PPT is presented in Appendix 1.

The PPT primarily contains items from free, semi-structured, and structured problem-posing situations created by Stoyanova and Ellerton (1996). In the first semester of the 2019-2020 academic year, the prepared PPT was applied to a 6th grade class of 19 students as a pilot study. A reliability coefficient was calculated between the scores given by the two evaluators after implementation. Using the reliability coefficient formula, a reliability coefficient of .73 was calculated, which indicates significant agreement between the calculated value and the scores achieved (Landis & Koch, 1977). It was therefore decided to use PPT to measure students' problem-solving skills.

Responses to the items in PPT were analyzed using a grading rubric developed by Katrancı (2014, p.357) in terms of four dimensisions (problem text criterion, compatibility with mathematical principles, type and structure of the problem, and solvability of the problem) each of them has different weight (see Appendix 2). Accordingly, the *problem text criterion* (language and expression) received 7 points, the criterion of *compatibility with mathematical principles* received 8 points, the criterion of *type and structure of the problem* received 6 points, and the criterion of *solvability of the problem* received 8 points. Additionally, the problem text and the type and structure of the problem criteria are composed of four-point scales, while the other dimensions are composed of three-point scales. Therefore, the highest score for each item in the PPT is 100, while the lowest score is 0. In other words, the highest possible score from the PPT is 900, while the lowest is 0. Based on the consistency coefficients for different raters and the same raters, consensus percentage was .720, and the Kappa value was .700. As example of the scoring applied according to the specified rubric, the student with code D4 from the experimental group posed the following problem for the second item of the PPT pre-test: "What is the result of the $\frac{1}{6}+\frac{5}{18}$ operation ?". The same student posed the following problem in the post-test: "If Veli climbs $\frac{1}{6}$ of the stairs, and then $\frac{5}{18}$ of the stairs, how many stairs did Veli climb in total? ". Due to the linguistic problems that disrupt the flow of the problem text, 2 points were scored in the *problem text* criterion. In addition, 3 points were scored for the criterion *compatibility with mathematical principles*. The problem created is a simple word problem. For this reason, in terms of *type and structure of the problem* criterion, it was scored 3 points. Due to the lack of deficiencies and nonconformities in the data, 3 points were scored for *solvability of the problem*. In this case, the total score of the D4 was 80 for the second item of the PPT post-test. Similarly, the problem posed by D1 for the first item in the pre-test was "Order the fractions $\frac{3}{2}, \frac{2}{8}, \frac{4}{2}, \frac{5}{8}$ from least to greatest." Same student posed the following problem for the same item: "Ahmet bought $\frac{1}{8}$ of a cable, Selma bought $\frac{3}{16}$ of the cable, and Hatice bought $\frac{8}{36}$ of the cable. In that case, who bought the greatest and least cable?" Due to the simple structure of the first item of the PPT pre-test, D1 scored 1 for *type and structure of the problem* criterion. Furthermore, due to the simple level of the text, 1 point was given for the *problem text* and its *compatibility with the mathematical principles* criteria. Finally, the problem was solvable, but the data used for the problem text remained simple, so 2 points were given for *solvability*. This resulted in a 37 point score for the first item of the PPT pre-test posed by the D1-coded student. When examining the problem posed by the same student in the PPT post-test, it is seen that a verbal problem is created instead of the simple exercise type. Accordingly, the problem was given 3 points for *type and structure of problem* and *problem text*. In addition, when the posed problem was evaluated according to the criterion of *compatibility with mathematical principles*, the error of not being able to consider the whole-part relationship, one of the types of errors made in posing fractional problems, was not observed. Therefore, 3 points were scored for the aforementioned criterion. As the data used in the established problem provide evidence of its solvability, 3 points were scored on the *solvability of the problem* criterion. A total score of 87 was calculated for the first item of the PPT post-test for D1.

The normality test results for experimental group PPT pre-test ($p = .00 < .05$) and post-test ($p = .028 < .05$) showed normal distribution. For this reason, Wilcoxon signed rank test was performed to determine whether a significant difference exists between the mean of the pre-test and post-test scores. Since PPT pre-test ($p = .004 < .05$) and post-test ($p = .004 < .05$) data did not show normal distribution for the control group, Mann-Whitney U test was employed. Similarly, Mann-Whitney U test was performed to determine whether a significant difference between the experimental and control groups PPT post-test scores exists. To calculate the magnitude of the difference emerging from the paired samples t-test applied to the experimental group, the effect size (η^2) was calculated. The effect size (partial eta squared) ranges from 0 to 1. An efect size of .09 was considered as small, .25 as medium, and values greater than .25 were considered as large (Büyüköztürk, 2013).

2.4. Process

The implementation was carried out by the researcher in a middle school in 2020-2021. In the first week of implementation, pre-tests were administered to the experimental and control groups. Following the pre-tests, experimental procedures were conducted. The experimental group participated in seven different problem-based active learning activities over the course of six weeks. Three class hours were applied to the activity practices in the experimental group each week. Last week, the last activity took place outside of school hours for three hours. In the control group, the curriculum was applied according to the objectives. Following the implementation process, the groups were tested post-implementation. Table 1 summarizes the weekly content of the curriculum implementation process that was conducted according to the acquisitions in the control group.

Table 1

Implementation Process carried out in the Control Group

Week	*Acquisition for the Curriculum Conducted*	*Duration*
1	Compares fractions, orders them, and shows them on a number line.	3 class hours
2	Performs addition and subtraction operations with fractions.	3 class hours
3	Multiplies a natural number by a fraction and makes sense of it.	3 class hours
4	Makes sense of the multiplication of two fractions.	3 class hours
5	Divides a natural number by a fraction and a fraction by a natural number, making sense of this process.	3 class hours
6	Makes the division of two fractions and makes sense of them.	3 class hours
	Predicts the results of operations with fractions.	3 class hours

Lessons related to operations with fractions were processed with plain expressions and question-answer techniques in the control group. Then, problems related to the topic were solved together with students. Alternatively, Table 2 displays the activities and implementation process applied in the experimental group.

Table 2

Implementation Process carried out in the Experimental Group

Activity No	*Implementation*	*Active Learning Technique*
1	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to pose structured problems (2 class hours)	Four corners
2	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to establish semi-structured problems (2 class hours)	Tournament
3	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to establish open problems (2 class hours)	Chain note
4	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to pose semi-structured problems (2 class hours)	Marketplace
5	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to pose open-ended problems (2 class hours)	Speed sharing
6	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to pose structured problems (2 class hours)	Station
7	Problem posing-based problem-solving instruction (1 class hour) Active learning activity to pose structured problems (2 class hours)	Card pairing

As shown in Table 2, before each problem-posing-based active learning activity, problem-posing activities were implemented based on Cankoy and Darbaz's (2010) modular approach of *problem posing-based problem-solving instruction*. Three steps are involved in this modular approach: First, (1) the students pose a problem, then (2) the posed problem is solved, and finally, (3) the data in the posed problem is changed so that the students can pose a different problem. Every time a problem is posed in this cycle, a class discussion is held in terms of compliance, logic, and solvability according to the grammar rules, and then the problems are solved. In the end, a new problem is created. Following the above steps, worksheets were distributed to students in the first lesson hour of each activity, and problem posing-based problem-solving activities were carried out. The purpose of applying the problem posing-based problem-solving approach in the first phase of each activity was to highlight possible errors that may arise in class discussions.

Literature supports the practice of introducing posed problems for discussion in class (Christou et al., 2005; Ellerton, 2013; English, 1997; Lavy, 2015). Two course hours were dedicated to active problem-posing activities in the other phase of the implementation. Active learning methods and techniques were examined while designing active learning activities based on problem-posing (Açkgöz, 2014), and teaching was provided using different active learning techniques. The following section explains how active learning activities based on problem-posing are processed.

2.4.1. Problem-posing activity carried out using corners technique

For the first activity, the four corners technique, four students were chosen as group chairs and randomly assigned to four corners of the class to form four groups, while the remaining students were not yet grouped. During each group meeting, the chairmen were given cork boards with a problem for comparing fractions, ordering them, and showing them on a number line. Using the structured problem-posing technique, they were asked to pose a problem similar to the given problem within five minutes. Each student took turns visiting the corners and examining the problems at the end of the period. After the review, students were asked to form groups and choose the corner with the problem they liked. The group members were then asked to pose a new problem together. During the process of posing a new problems to the group members, the researchers visited each group and provided directions about each problem.

2.4.2. Problem-posing activity carried out using tournament technique

As part of the second activity, the students were divided into three groups for the tournament. The tournament technique was used to acquire the second acquisition (see Table 1). As course material, three activity papers with a bell competition and questions about symbolic operations were used. An activity sheet and a bell were given to the students in the first row of each group. The student in the first row then posed a problem for the initial operation on the paper and rang the bell. The researcher examined the problem posed by the student who rang the bell and provided ideas about it. Students were then asked to write a process related to addition or subtraction with fractions and give it to their group mate in the back row under the problem they had posed. The practice continued in this manner for the next student.

2.4.3. Problem-posing activity carried out using chain notes technique

In the chain notes technique that is used as the third activity, the activities are arranged according to the third acquisition of the subject, *multiplies a natural number by a fraction and makes sense of it*. Four different colored envelopes and A4 paper were used in this activity. Four groups were formed by equally dividing the students, and the envelopes were distributed with the paper. After that, each member of the group was asked to pose a problem that involved multiplication by fractions. Once the problem-posing was completed, the group selected a problem with a common decision and sent it to any desired group. At this point, researchers referred to the postman as '-I have a letter to the postman!' and instructed the postman to leave the letter at the desired address (another group). Consequently, the researcher selected as the postman handed over the preferred problem to another group. At the addresses where the letters arrived, the students examined the problems arising from the opened envelopes and sent them through the postman back to the original addresses.

2.4.4. Problem-posing activity carried out using marketplace technique

In the fourth activity, the marketplace technique was used as a semi-structured problem-posing activity aimed at acquiring *makes sense of the multiplication of two fractions*. Materials required for this technique included cardboard, a glass jar, and activity papers. The papers containing the multiplication of two fractions were placed in the glass jar. The class was then divided into two groups, marketers and customers. Students were randomly assigned a problem-posing question in the form of multiplication of two fractions in the glass bells they had. Students were asked to set up a problem with the cardboard cards given for the process they were pulling from the glass jar.

When the problem-posing phase was complete, the marketeers took their seats. In this stage, the goal was to create a marketplace atmosphere in the classroom. Thus, marketers tried to persuade customers to choose the problem at hand. Customers wandered around, examined the marketeers' problems, and matched. Customer and marketer pairs then evaluated each other's problems.

2.4.5. Problem-posing activity carried out using speed sharing technique

The fifth activity, the speed sharing technique, was prepared as a free problem-posing activity. This technique was used to acquire the fifth acquisition (see Table 1). Speed sharing involves placing students in two concentric circles. The students in the outer circle sat outwardly in the first stage of implementation, whereas those in the inner circle sat inwardly. Students were then asked to pose problems individually. Meanwhile, the researcher examined the problems of the students. The researcher blew the whistle after the students had completed their problem-posing task. Upon hearing the first whistle, students were asked to turn towards their friends and read their problems and evaluate each other's. With the second whistle blown, those sitting in the inner circle rotated and had a new partner. Every time a whistle sounded, the same process was repeated, allowing each student to see and evaluate the problems posed by their peers.

2.4.6. Problem-posing activity carried out using speed sharing technique

An active learning technique, the station technique, was used for the sixth activity. This technique was used in the learning of making the division of two fractions and making sense of it. Under the name of stations, six table layouts were set up for students in this activity. Each station was titled Station 1, Station 2, Station 3, Station 4, Station 5, Station 6, and their names were hung on the tables. The students were divided into six stations, with four students at each station and three at the final station. Station chiefs were chosen from the students distributed to each station. Station chiefs were responsible for explaining to the changing groups how each new group should proceed and what needs to be done each time the whistle blows. With the sound of the first whistle, each group posed a problem related to the division of two fractions at their station. After the second whistle blew, all but the station chiefs moved to the next station and attempted to solve the previous group's problem. As each whistle sounded, station chiefs summarized what the previous group had done to the newcomers. After problem-posing at the station was completed, the points that were deemed errors were evaluated. The next step involved posing a new problem, and the last step involved solving the problem. As soon as the third whistle sounds, the groups that changed stations again continue from where the previous group left off. Next, the errors in the posed problem were corrected, and a new one was posed.

2.4.7. Problem-posing activity carried out using card pairing

One of the active learning techniques, card pairing, was designed as an educational game as a final activity. We adapted the card pairing educational game to predict the results of fractional operations for the final acquisition of operations. Prior to implementation, the class was divided into three groups and seated one after another. Materials used included three competition bells, three cork boards, colored envelopes, and colored pins. Each envelope contained a paper with a problem of predicting with eight fractions. Cork boards were used to display the solution stages to the prediction problems. Using the board, the student in the first row tried to solve the problem by opening any envelope. With a pin, the student who found the solution fixed the solution on top of the problem. After ringing the bell, he or she went to the back. As a result, the game continued until all envelopes had been opened.

3. Findings

Research findings are presented in this section on the effect of problem posing-based active learning activities on students' problem-posing skills and problem-solving achievement. The results of the Mann Whitney-U test of the PST pre-test scores of the experimental and control groups are presented in Table 3 for the first problem of the study.

Table 3

Findings on PST pre-test scores of experimental and control group

Group	*N*	$\bar{X}$	*Mean Rank*	*Sum of Ranks*	*U*	*p*
Experimental Group	23	32.61	25.91	596	255.01	0.499
Control Group	25	33.00	23.20	580		

As shown in Table 3, no statistically significant difference between students' pre-test scores in experimental and control groups was found [$U = 255.01\ p > .05$]. The experimental and control groups are equivalent when it comes to their pre-implementation problem-solving achievement. Table 4 presents the results of the dependent samples t-test applied to compare pre-test and post-test scores of the experimental group.

Table 4

Findings on PST pre- amd and post-test scores of the experimental group

Experimental Group	*N*	$\bar{X}$	*SD*	*df*	*t*	*p*
Pre-test	23	32.61	12.32	22	-2.05	0.052
Post-test	23	40.00	19.24			

As presented in Table 4, compared to the pre-test scores, post-test scores of the students were found to be greater. This difference, however, was not statistically significant [$\mathrm{t}(22) = 2.05, p > 0.05$]. As a comparison of the mean PST post-test scores between the experimental and control groups, Table 5 shows the results of the Mann Whitney-U Test.

Table 5

Findings on PST post-test scores of experimental and control group

Group	*N*	$\bar{X}$	*Mean Rank*	*Sum of Ranks*	*U*	*p*
Experimental Group	23	40.00	28.24	649.50	201.50	0.075
Control Group	25	31.08	21.06	526.50		

Based on Table 5, the problem-solving achievement of students who participated in active learning activities based on problem-posing was higher than that of those who did not participate [$U = 201.50,\ p > .05$]. The findings of the Mann Whitney-U test of the PPT pre-test applied to the experimental and control groups are presented in Table 6.

Table 6

Findings on PPT pre-test scores of experimental group and control group

Group	*N*	$\bar{X}$	*Mean Rank*	*Sum of Ranks*	*U*	*p*
Experimental Group	23	138.78	21.28	489.50	213.50	0.127
Control Group	25	182.12	27.46	686.50		

It can be concluded from Table 6 that experimental and control groups have equal problem-posing skills prior to implementation. Furthermore, the experimental group's PPT pre-test and post-test scores were compared to examine whether active learning based on problem-posing improved their problem-posing skills. Table 7 presents the results of the Wilcoxon signed rank test applied in this case.

Table 7

Findings on PPT pre- and post-test scores of the experimental group

Post-test Pre-test	*N*	*Mean Rank*	*Sum of Ranks*	*z*	*p*
Negative Ranks	1	7.50	7.50		
Positive Ranks	21	11.69	245.50	−3.864	0.00
Equal	1				

Table 7 shows that 21 students have positive ranks and the average rank is 11.69, while one student has negative ranks and the average rank is 7.50. On the other hand, the mean pre-test score of the experimental group was 138.78 while the mean score of the post-test is 294.68. Wilcoxon signed rank test results indicated a significant difference in favor of the post-test [$z = -3.864$, $p < 0.05$]. According to the direction of the significant difference, one student of the experimental group's score decreases, 21 students' scores increase, and one student's score remains unchanged. The effect size of this difference was calculated to be =0.80, indicating a large level of effect.

As a final result, Table 8 presents the results of the Mann Whitney-U Test applied to the post-test data of the experimental group and the control group.

Table 8

Findings on PPT post-test scores of experimental group and control group

Group	*N*	$\bar{X}$	*Mean Rank*	*Sum of Ranks*	*U*	*p*
Experimental Group	23	294.68	27.78	639.00	212.00	0.119
Control Group	25	221.25	21.48	537.00		

According to Table 8, experimental and control groups had mean scores of 294.68 and 221.25, respectively. It was found, however, that there was no statistically significant difference between the PPT post-test scores of the groups [$U = 212.00, p > .05$].

4. Discussion and Conclusion

This section discusses the effects of problem-posing active learning activities on students' problem-solving achievements and problem-posing skills for the acquisitions of *operations with fractions*.

In the PST pre-test, students had low average scores, and the groups were equivalent in terms of problem-solving achievement. According to the post-test results of the PST, the experimental group had a higher average score than the control group based on PST pre-test results. Therefore, the problem posing-based active learning activities implemented in the experimental group increased students' problem-solving achievement. This increase, however, was not statistically significant. Similar results have been found in the literature (Turhan, 2011; Turhan & Güven, 2014). In a study by Turhan (2011), it was examined the effect of sixth grade students' problem-posing on their problem-solving skills in decimal fractions topic. The study concluded that the problem-solving post-tests of the experimental and control groups did not show any statistically significant differences. Furthermore, Turhan and Güven (2014) have found no evidence that teaching with a problem-posing approach is more effective than teaching from textbooks in terms of students' problem-solving skills.

Following active learning activities based on problem-posing, the PPT post-test showed a significant increase in the average scores of the experimental group. In addition, a statistically significant difference was found between the average scores of the experimental group during the PPT pre- and post-test. Upon examining the effect level, a high level was determined. This suggests that problem-posing activities based on active learning improve students' problem-posing skills. The literature shows that activities with a problem-posing approach improve students' problem-posing skills (Cankoy & Darbaz, 2010; Chen et al., 2015; Karaaslan, 2018; Kopparla et al., 2019; Terzi, 2021). According to the results of the PPT post-test, the average problem-posing of the students in the control group also increased. In spite of the fact that the average scores of the experimental group students were higher than those of the control group students, there was no significant difference between the two groups. This situation can be attributed to the fact that students in the control group solved different types of problems during the instructional process. Nevertheless, the problem posing-based active learning activities applied to the experimental group might be better suited to developing the problem-posing skills of the students over the long run.

It is recommended to allocate more time to active learning activities based on problem-posing to increase the effectiveness of implementation. Moreover, the problem-posing active learning activities used in the study are limited to seven, and it is recommended that the activities be diversified with different active learning techniques. In this way, students' performance in problem-solving and posing and their motivation for problem-posing and mathematics lessons can be improved.

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Appendix 1. Problem Posing Test

No *Related Item*

1. Pose a problem in the space below that you can solve by comparing it to fractions.

2. Pose a problem for addition with the fractions given below.
$\frac{1}{6} + \frac{5}{18}$

3. Pose a problem with subtraction with the fractions given below.
$\frac{3}{4} - \frac{2}{24}$

4. Pose a problem by completing the problem given below so that it is for the product of a fraction by a natural number.
A total of 360 customers stay in a hotel. Of the clients,
………………………………………………………………………………

5. Pose a problem with multiplication with the fractions given below.
$\frac{3}{8} \times \frac{1}{2}$

6. Using the following data, pose a problem for dividing a natural number and a fraction.

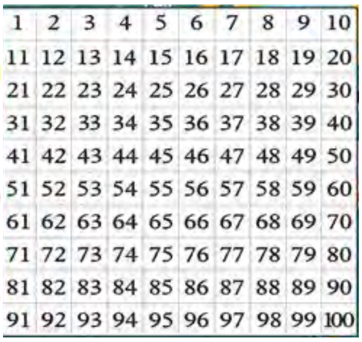
48, $1\frac{1}{3}$, Sack, Sachet, kg

7. Pose a problem with division with the fractions given below.
$2\frac{1}{5} : 1\frac{1}{10}$

8. Use one of the fractions in the table below to pose a problem for predicting the outcome of operations with fractions.

$\frac{32}{66}$ $\frac{20}{84}$ $\frac{120}{121}$ $2\frac{1}{39}$

9. Pose a problem that requires processing fractions by selecting data in the way you want from the tables given below.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$\frac{1}{2}$ $\frac{11}{7}$	$\frac{28}{49}$
$9\frac{2}{7}$	$\frac{40}{15}$
$2\frac{2}{3}$	$\frac{18}{35}$
$16\frac{2}{9}$	$\frac{9}{30}$

Appendix 2. Scoring rubric of PPT questions

Criterion / Indicators	Coefficient	Score
Problem text criterion	7	
The problem text is not clear and understandable. (1 point)		
Problem text is partially understandable. (2 points)		
The problem text is understandable, it is not clear what is asked. (3 points)		
The problem text is clear, concise and understandable. (4 points)		
Compatibility with mathematical principles	8	
The problem does not conform to the principles of mathematics (0 point)		
The problem partially complies with the principles of mathematics (such as the use of misconceptions). (1 point)		
The problem does not comply either mathematical or conceptual error, but the data used is not compatible with daily life (2 points)		
The problem complies with mathematical principles and real life. (3 points)		
Type and structure of the problem	6	
It is a simple exercise type. (1 point)		
It is of the exercise type. (2 points)		
It is of the simple word problem type. (3 points)		
It is of word problem type. (4 points)		
Solvability of the problem	8	
The data and information in the problem are not sufficient to solve the problem. (0 point)		
Although the given in the problem is sufficient, it cannot be solved because it is too complex. (1 point)		
The problem can be solved, but the data is either incorrect or missing. (2 points)		
The problem is solvable because the data and information in the problem are complete and appropriate. (3 points)		