Empirical Research

The Role of Domain-General and Domain-Specific Skills in the Identification of Arithmetic Strategies

Joanne Eaves [1], Nina Attridge [2], Camilla Gilmore [3]

[1] *School of Psychology, University of Nottingham, University Park, Nottingham, United Kingdom.* **[2]** *Department of Psychology, Portsmouth University, Portsmouth, United Kingdom.* **[3]** *Centre for Mathematical Cognition, Loughborough University, Loughborough, United Kingdom.*

Corresponding Author: Joanne Eaves, School of Psychology, University of Nottingham, University Park, Nottingham, NG7 2RD, United Kingdom. E-mail: joanne.eaves2@nottingham.ac.uk

Abstract

Individuals solve arithmetic problems in different ways and the strategies they choose are indicators of advanced competencies such as adaptivity and flexibility, and predict mathematical achievement. Understanding the factors that encourage or hinder the selection of different strategies is therefore important for helping individuals to succeed in mathematics. Our research contributed to this goal by investigating the skills required for selecting the associativity shortcut-strategy, where problems such as '16 + 38 − 35' are solved by performing the subtraction (38 − 35 = 3) before the addition (3 + 16 = 19). In a well-powered, pre-registered study, adults completed two tasks that involved 'a + b − c' problems, and we recorded a) whether and b) when, they identified the shortcut. They also completed tasks that measured domain-specific skills (calculation skill and understanding of the order of operations) and domain-general skills (working memory, inhibition and switching). Of all the measures, inhibition was the most reliable predictor of whether individuals identified the shortcut, and we discuss the roles it may play in selecting efficient arithmetic strategies.

Keywords

associativity, domain-general, domain-specific, arithmetic strategies

Arithmetic Strategies

The way that individuals solve arithmetic problems reflects their knowledge of mathematical principles and procedures, potentially more so than the accuracy of their answer or time taken to reach it. Strategy use also predicts future expertise with mathematics, and education and employment success (Verschaffel et al., 2009). Consequently, the last forty years have seen increased research into the types of strategies that individuals use (Baroody et al., 1983). The majority of these studies focused on how individuals solved two-term problems (e.g., Lemaire & Lecacheur, 2011), fewer studies investigated strategy use on problems with more than one operation or two operands. However, measuring strategy use on more complex problems is important because those strategies are derived from higher-level principles, principles that are necessary for bridging the gap from basic to more advanced mathematics (Knuth et al., 2006).

Our research aimed to fill this gap by focusing on three-term addition and subtraction problems of the form 'a + b − c' and the strategies individuals use to solve them. More specifically, we were concerned with strategy selection, which

is one component of strategy use (Verschaffel et al., 2009) and refers to a conscious or unconscious process of choosing how to solve a problem. Studies into strategy selection typically measure *what* strategies individuals choose, with a minority investigating *when* individuals choose strategies for the first time, a process we refer to as 'identification' (Eaves et al., 2020). Understanding the process of identification is important because it forms the basis of models of strategy use (Siegler & Araya, 2005), and could help to develop advanced strategy competences such as flexibility and adaptivity. Flexibility refers to knowledge of multiple strategies and the ability to switch between those strategies, and adaptivity refers to the ability to select and execute the most efficient strategies taking into account the person, problem and context (Heinze et al., 2009; Star & Rittle-Johnson, 2008; Verschaffel et al., 2009). If an individual cannot *identify* valid strategies for solving problems, or takes substantial time to do so, they are unlikely to switch between different strategies across problems and solve a series of problems efficiently. In other words, if factors prevent individuals from identifying strategies in their strategy repertoire, their flexibility and adaptivity will be limited. Our research investigates factors that might be involved in identification.

The Associativity Shortcut Strategy

Arithmetic knowledge can be crudely divided into procedural and conceptual: conceptual knowledge refers to understanding the principles that underlie a domain, while procedural knowledge refers to knowledge of the mathematical symbol system and the rules for solving problems (Hiebert & Lefevre, 1986). One arithmetic principle is associativity, which refers to the fact that because some operations are related, some problems can be decomposed and recombined in different ways to produce a correct answer (Canobi et al., 1998). Thus, the principle allows some problems to be solved through different strategies, for example, 'a + b + c' can be solved as 'c + b + a'. The principle affords multiple strategies on multiplication (e.g., 'a × b × c = b × c × a'), multiplication and division (e.g., 'a × b ÷ c = b ÷ c × a') and addition and subtraction (e.g., 'a + b − c = b − c + a') problems.

We focused on 'a + b − c' problems, and two strategies that can be used to solve them, a) a left-to-right strategy and b) a right-to-left strategy. For example, '16 + 38 − 35' can be solved left-to-right if the addition (16 + 38) is performed before the subtraction (54 − 35), or right-to-left if the subtraction (38 − 35) is performed before the addition (3 + 16). For this problem, the right-to-left strategy is quicker and easier, and we refer to it as a shortcut. Compared to strategies derived from other arithmetic principles, the associativity shortcut is used infrequently (Robinson et al., 2018): only 15 − 25% of 6 − 10 year olds use associativity shortcuts (Robinson & Dubé, 2009), 30% of 11 − 13 year olds (Dubé, 2014), and 50 − 60% of adults (Dubé & Robinson, 2010). Individual differences in selecting efficient strategies are therefore substantial and prevail throughout life, and education practitioners have called for a greater emphasis on efficient and flexible arithmetic strategy use (National Mathematics Advisory Panel [NMP], 2008).

To encourage individuals to use efficient strategies we must first understand the skills that are required to identify those strategies. Difficulties with particular cognitive skills (e.g., working memory, inhibition, calculation skill) could make different aspects of mathematics challenging; for example, an individual might be able to perform the calculations in the associativity shortcut ('b − c', 'a + b') but struggle to identify the strategy because they have a difficulty with other cognitive skills. By understanding the factors involved in the process of identification can we help individuals to make efficient strategy selections.

Skills Required for Identifying Associativity Shortcuts

Here, we focus on two domain-specific skills (calculation skill and knowledge of the order of operations) and three domain-general skills (visuospatial working memory, inhibition and switching) that might help in identifying the associativity shortcut.

Domain-Specific Skills

Calculation skill refers to the accuracy and speed with which an individual solves arithmetic problems and is often used to measure procedural knowledge. While the precise relationship between procedural and conceptual knowledge remains debated (Rittle-Johnson et al., 2015; Rittle-Johnson et al., 2001; Star, 2005), most scholars agree that it is positive and iterative. Individuals who are better at calculating usually perform better on tasks that involve conceptually-derived

arithmetic strategies (Gilmore et al., 2018; Rasmussen et al., 2003). However, the mechanism through which calculation skill and conceptually-derived strategy selection relate is unclear. It may be that proficiency with calculation gives individuals time to solve problems through one strategy and then validate their answer using a different strategy, offering more opportunities to identify the shortcut. Alternatively, quick computation may free domain-general resources such as working memory, inhibition and switching skills (Watchorn et al., 2014), skills that might then enable identification.

A second domain-specific skill that might be important for identifying the associativity shortcut is knowledge of the order of operations, the convention that for problems with mixed operations (e.g., '2 + 4 × 5'), multiplication and division should be performed before addition and subtraction, but within multiplication and division and within addition and subtraction order does not alter the result. Individuals who understand the order of operations are more likely to be aware of different permissible ways that problems can be solved, which in turn could help them to identify the shortcut. For example, McMullen et al. (2017) found that children with a better understanding of operation order generated more arithmetic strategies, and more complex strategies than those with a poorer understanding. A good understanding of the order of operations may therefore help individuals to identify arithmetic strategies.

However, many individuals have a limited or misconceived understanding of operation order (Zazkis & Rouleau, 2018), particularly in the UK, USA, Canada and Australia where acronyms such as BODMAS ('Brackets, Orders, Division, Multiplication, Addition, Subtraction') are used to teach it. This is because the acronym is unclear in specifying that some operations have equal precedence. For example, some children and adults possess 'literal' interpretations of the acronym (Eaves et al., 2022), erroneously believing that division must be performed before multiplication, and addition before subtraction, because of the order in which the letters appear. This interpretation can lead to incorrect answers (e.g., answering '10 to 18 − 6 + 2') and in the case of 'a + b − c' associativity problems, hinder shortcut identification.

Domain-General Factors

Recent years have seen an increased recognition that executive functions (working memory, inhibition and switching) are important for mathematics achievement, and this might be particularly so for selecting efficient arithmetic strategies. While both verbal and visuospatial working memory are associated with mathematics performance (Peng et al., 2016), visuospatial working memory may be particularly important for manipulating visually presented mathematical expressions. Visuospatial working memory (VSWM) is the ability to simultaneously maintain and update visual and spatial information (Baddeley & Hitch, 1974) and therefore might help individuals to identify the shortcut on 'a + b − c' by building 'mental models' of the problem (Edwards, 2013). Mental models are visual images of problems that are spatially represented in memory, and in our case, may allow individuals to move their attention around the problem and locate the shortcut. After locating the shortcut individuals might then need working memory to evaluate the subexpression 'b − c' (i.e., to judge whether it is easy or difficult), navigate back to the left-hand side, compare it to 'a + b', and choose a strategy.

To date, three studies have investigated whether VSWM relates to the use of conceptually-derived arithmetic strategies. Rasmussen et al. (2003) found a positive correlation between VSWM and accuracy scores on 'a + b − b' inversion problems in 4-5 year olds, and Edwards (2013) found a positive correlation between VSWM and the frequency of self-reported shortcut use on 'a × b ÷ b' and 'a × b ÷ c' problems in young adults. However, Cragg et al. (2017) found no evidence of a relationship between VSWM and recognition of arithmetic strategies derived from a mixture of principles, with participants aged 9 – 14 years and young adults. Evidence of a link between VSWM and shortcut strategies is therefore positive, but not consistent. The inconsistency in the previous literature cannot be explained by differences in participant ages, but could be due to differences in measures of conceptual-strategy use.

A second executive function is inhibition, which refers to "the stopping or overriding of a mental process, in whole or in part, with or without instruction" (Macleod, 2007, p. 5). Several taxonomies of the structure and function of inhibition exist, which often decompose inhibition into a family of three functions. Friedman and Miyake's (2004) taxonomy refers to these functions as a) resisting distractor interference, b) resisting proactive interference, and c) resisting prepotent responses, each of which could play a role in strategy identification. For example, in 'a + b − c' problems, the digit 'a' could be a distractor as it is extra visual information that is not relevant to the first-step of the

shortcut strategy (distractor interference). Individuals might also need to overcome interference from previously learnt strategies (proactive interference) and to pause before selecting a strategy (resisting prepotent responses).

Indeed, two studies provide supporting evidence for the role of inhibition in strategy selection (Gilmore et al., 2015; Robinson & Dubé, 2013). Of these, Robinson and Dubé's study (2013) is most relevant because they used similar arithmetic problems to us. In their study, 8 – 10 year olds solved 'a + b – b' inversion problems and 'a + b – c' associativity problems, and self-reported how they solved them. They also completed a Go/No-Go task, a widely used paradigm for measuring inhibition. Children who reported using both the inversion shortcut (solving 'a + b – b' problems without calculating) and the associativity shortcut had significantly better inhibitory skills than those who used neither, and inhibition accounted for 6 – 11% of the variance in shortcut use. Thus, inhibition might help individuals to resist prepotent left-to-right responses.

Finally, switching, the ability to shift attention between different tasks (Cragg et al., 2017) might help to identify the associativity shortcut. In everyday life and in experimental studies, arithmetic problems vary and lend themselves to different strategies that individuals must shift between. In the context of 'a + b – c' problems, problems may differ in how much benefit is gained from using a right-to-left procedure compared with a left-to-right procedure, depending on the numbers in the problems. Problems that are more conducive to a shortcut provide greater benefit if the individual switches from a left-to-right and a right-to-left procedure. Three studies are particularly relevant here; Watchorn et al. (2014) found that 7 – 10 year olds' switching skills predicted the frequency with which the children used the inversion shortcut, however, Cragg et al. (2017), with participants aged 9 - 14 years and adults, and Gilmore et al. (2018), with children aged 8 – 10 years, found no evidence for a relationship between switching and individuals' recognition and application of strategies derived from different principles. Thus, evidence for the role of switching in selecting conceptually-derived arithmetic strategies is inconclusive. The inconsistencies in the previous literature may be due to differences in the measures of conceptual-strategy use. This highlights the importance of using sensitive and comprehensive tasks to assess strategy use.

The Present Research

Understanding why individuals fail to select efficient strategies is important if we are to improve current standards and outcomes in mathematics education (NMP, 2008). Our research addresses this by investigating the domain-specific and domain-general skills involved in identifying the associativity shortcut strategy on 'a + b – c' problems.

Method

The study was approved by Loughborough University's Ethics Approvals (Human Participants) Sub-Committee. Before data collection, the study hypotheses, design, sample size, exclusion criteria and analysis plan were pre-registered. The pre-registration protocol is provided in the Supplementary Materials.

Participants

120 adults aged 18 – 59 years (*M* = 22.43, *SD* = 8.84, 99 female, 21 male) participated. Participants were recruited via Loughborough University, and the majority were students and staff. Almost all participants had been educated in the UK. All participants were proficient in English and were not currently studying for, and had not historically studied for, a mathematics degree.

The sample size was determined by the minimum number of participants required to detect an odds ratio of 2.15 for a binary logistic regression (where 1.49 = small, 3.45 = medium and 9.00 = large) with 5 predictors, 90% power, and an alpha level of 0.05. The effect size was guided by previous studies (Eaves et al., 2019) that provide data on the number of adults who identify the shortcut (approximately 14 – 63%).

Design

Participants completed seven tasks. Two tasks measured the identification of the associativity shortcut, a shortcut self-report task and a trial-by-trial shortcut task. Two tasks measured domain-specific skills, a calculation skill task and an instrument that measures knowledge of the order of operations. Three tasks measured domain-general skills, an odd-one-out task (working memory), a Go/No-Go task (inhibition) and a letter-number categorisation task (switching).

Materials and Procedure

The odd-one-out, Go/No-Go, letter-number categorisation tasks and the trial-by-trial shortcut task were presented on a 15" laptop using PsychoPy (Peirce et al., 2019). All other tasks were presented on A4 paper. The materials for the tasks are provided as Supplementary Materials. The tasks were presented in the same order to all participants: the shortcut self-report task was presented first, followed by the trial-by-trial shortcut task, the letter-number categorisation task, the odd-one-out task, the Go/No-Go task, the calculation skill task and the order of operations instrument. This order minimised the influence of one task on another; for example, the odd-one-out and letter-number categorisation tasks require individuals to look at different parts of the screen, which might influence whether an individual identifies the shortcut. They were therefore presented after the tasks that involved associativity shortcuts.

Shortcut Identification Tasks

Materials for the shortcut identification tasks can be found in the Supplementary Materials.

Shortcut Self-Report Task — The shortcut self-report task measured early identification of the shortcut, i.e., identification on the first three-term problem that was presented in the study. Participants were presented with five arithmetic problems one at a time, and were asked to solve each problem mentally, write down their answer in an A4 booklet, and respond to the open-ended question "How did you solve the problem?" They were instructed to report how they were trying to solve the problem, even if they did not compute an answer. The third problem was '6 + 38 – 35', which contains an associativity shortcut, while the other four problems were two-term problems with no associativity shortcuts. We were interested in responses to the associativity problem, which measures spontaneous use of the shortcut on a single item. Each problem was presented for 15 seconds, and the problems were presented in the same order for all participants.

Participants were classed as an identifier of the shortcut if their self-report contained any indication of having performed the subtraction before the addition (e.g., "I did the right-hand side first"). If participants reported using both strategies (left-to-right and right-to-left), they were classed as an identifier because they had technically identified the shortcut.

Trial-by-Trial Shortcut Task — The trial-by-trial shortcut task (Eaves et al., 2020) measured a) whether and b) when individuals identified the shortcut while solving a series of three-term arithmetic problems. The task uses response time for solving 'a + b – c' problems to measure the trial number on which an individual first identifies the shortcut, which we refer to as their identification point (IP).

Thirty 'a + b – c' arithmetic problems were presented one at a time on the screen. Half of the problems were 'conducive' to a shortcut strategy (e.g., '15 + 48 – 44') and half were non-conducive (e.g., '36 + 27 – 44'), see Eaves et al. (2020) for more details. Conducive problems were those where greater benefit was gained from using a right-to-left shortcut strategy, for example because b – c resulted in a small positive number. Participants were asked to solve each problem in their head and then say their answer out loud, and to respond as quickly and as accurately as possible. On each trial, response time (RT) was recorded. On the non-conducive trials, rolling means, standard deviations and 99.9% confidence intervals of the non-conducive RT data were calculated. For the first conducive trial, absolute RT was recorded. For the second, the mean RT was calculated and recorded. For the third conducive trial onwards, the median RT of the three most recent conducive trials was calculated and recorded. When the conducive median (or mean on the second stimulus) fell below the lower-endpoint of the confidence interval of the non-conducive mean, a 'trigger' was

generated. Three consecutive triggers defined the IP. With 30 trials, the earliest possible IP was the second conducive stimulus and the latest possible IP was the 13th conducive stimulus.

At the end of the task, participants were interviewed about the strategies that they had used to solve the problems (recorded using a voice recorder). Their responses were used to categorise them as an identifier of the shortcut or a non-identifier of the shortcut on the task in the same way as the shortcut self-report task, and most people could be easily categorised. Performance on the trial-by-trial shortcut task was therefore measured by two variables, 1) self-reported use of the shortcut after all of the trials had been presented (categorical), and 2) the percent of trials that the shortcut was used on (continuous), calculated from the IP.

Domain-Specific Tasks

Materials for the domain-specific tasks can be found in the Supplementary Materials.

Calculation Skill — This was a paper and pencil task where participants solved as many two-term addition and subtraction problems as possible in 90 seconds without a calculator. There were 64 problems in total, which varied by problem size (small: contained one double-digit and one single-digit; large: contained two double-digits), operation (addition or subtraction) and the presence of carries or borrows in the units (with or without a carry/borrow). The problems were presented in the same order to all participants, which was pseudorandomised prior to the study with the constraint that the correct answer was not the same on consecutive problems. Performance was measured by the total number of problems correctly solved in the time-limit.

Order of Operations Instrument — This instrument consists of 14 multiple-choice items that measures individuals' understanding of the order of operations (Eaves et al., 2022). Each item was a four-term problem, e.g., '46 − 39 + 14 + 22' with four response options, e.g., a) 46 − 39, b) 46 + 39 or 46 − 14, c) 39 + 14, and d) 46 − 39 or 14 + 22 and participants were asked to circle the option that contained the part(s) of the problem that could be solved first. They were told that there was one best answer (for this example '46 − 39 or 14 + 22') and that there was no set time-limit but all participants completed it in 10 minutes.

The instrument contained three types of items (brackets, precedence, misconception); bracket and precedence items had one correct response, which was the option that contained the bracket and the operation with precedence, respectively. For the misconception items, the response options distinguished between different types of understanding and misconceptions of BODMAS. For example, in the item above ('46 − 39 + 14 + 22'), the response options '39 + 14' and '46 − 39' are consistent with literal and left-to-right misconceptions of BODMAS respectively. '46 − 39 or 14 + 22' is the correct answer, and '46 + 39 or 46 − 14' intends to capture guessing behaviour. The instrument contained 3 bracket problems, 3 order problems and 8 diagnostic problems. The items and response options were presented in the same pseudorandomised order to all participants and performance was measured by the percent of literal responses on the misconception items. See Supplementary Materials for more information about the instrument.

Domain-General Tasks

Our domain-general tasks measured working memory (Cragg et al., 2017), inhibition (Kiehl et al., 2000; Maltby et al., 2005) and switching skills (Miyake et al., 2000; Rogers & Monsell, 1995). Where possible, digits were not used as stimuli to prevent inflating any correlation between the EF and arithmetic tasks.

Odd-One-Out Task — The odd-one-out task measured visuospatial working memory (VSWM; Cragg et al., 2017). In this task, sets of 3 to 7 items were presented. For each item, participants identified which of four stimuli was different (the odd-one-out). Then at the end of the set of items they were asked to recall the location of the odd stimuli for each item in the order that they were presented in the set. Performance was measured by the total number of items recalled in the correct order across all set lengths.

Each trial began with a blank screen (500 ms) followed by the presentation of a 3 x 3 grid of nine black squares. Four of the squares contained pictures of objects (e.g., circles, arrows, stars), three of which were identical, and one was different (e.g., an empty circle). Images remained on the screen until the participant identified the odd-one-out by

clicking on one of the squares. Once all items in the set had been presented, "?" was displayed at the centre of the screen and participants recalled the locations of the odd stimuli in order by clicking on an empty 3 x 3 grid. Each set length was presented three times, and if the participant recalled two of the sets correctly, the length increased by one. If they did not, the task ended. The stimuli and their location were pseudorandomised before the study and were the same for each participant.

Go/No-Go Task — A Go/No-Go task (Kiehl et al., 2000; Maltby et al., 2005) measured inhibition. On each trial, participants were presented with a letter at the centre of the screen; participants were asked to press the spacebar with their dominant hand each time they saw the letter 'X' (a 'Go' trial) and to withhold responding if they saw any other letter (a 'No-Go' trial), which in this case was always 'K'. There were two blocks of 80 trials, 80% of which were Go trials and 20% were No-Go trials: The proportion of Go to No-Go trials (4:1) was relatively high in order to increase the inhibitory demands of the task (Cragg & Nation, 2008). All trials began with a blank screen (250 ms) followed by a central fixation cross (250 ms), and a letter stimulus (300 ms) and participants had 1000 ms from the onset of the letter to respond. The total trial duration (approximately 1500 ms) and timing of the stimuli was therefore fixed across trials to encourage a regular pattern of responding (Cragg & Nation, 2008). The trials were presented in the same pseudorandomised order for all participants, and no more than two No-Go trials were presented consecutively. Performance was measured by the percent of false alarms (responses made on No-Go trials). Cronbach's alpha for performance on the No-Go trials was 0.82.

Letter-Number Categorisation Task — The letter-number task (adapted from Rogers & Monsell, 1995; Miyake et al., 2000) measured switching. On each trial, a letter-number stimulus (e.g., 'G7') was presented in one of four quadrants on the screen and participants were asked to categorise it either according to the letter or the number that it contained. When the stimulus appeared in either quadrant at the top of the screen, participants categorised it based on the number (odd or even) and when it appeared at the bottom they categorised it based on the letter (consonant or vowel). Participants pressed 'x' if the letter was a consonant and 'm' if it was a vowel, and 'x' if the number was odd and 'm' if it was even. Stimuli remained on the screen for 5000 ms or until the participant responded, whichever came first.

There were 65 trials; 31 were 'no-switch' trials and 33 were 'switch' trials (the first trial could not be classed as switch or no-switch because there was no prior stimulus to compare it to). On no-switch trials, the vertical location of the current stimulus (upper or lower) was the same as the previous stimulus. On switch trials, the vertical location of the current stimulus (e.g., upper), and therefore categorisation type, was different to the previous stimulus (e.g., lower). Stimuli appeared in the upper quadrants on half of the trials, in the lower quadrants on the other half and were presented in the same pseudorandom order for all participants. Performance was measured by subtracting the median reaction time of correct responses on no-switch trials from the median reaction time of correct responses on switch trials. Cronbach's alpha for reaction times on the switch trials was 0.87 and for reaction times on the no-switch trials was 0.76.

Results

Pre-Registered Analyses

Descriptive Statistics and Correlations

Table 1 displays descriptive statistics of performance on the domain-specific and domain-general tasks; the data can be found in the Supplementary Materials. On all the tasks there was a good range of performance, with no evidence of ceiling or floor effects. For most of the tasks the data were normally distributed, although the percent of literal responses on the order of operations questionnaire were positively skewed.

Relationships between performance on the shortcut identification tasks, domain-specific tasks and domain-general tasks were explored with zero-order point-biserial correlations for the shortcut self-report variables and linear correlations for the percent use variable on the trial-by-trial shortcut task (Table 2).

Table 1

Descriptive Statistics for Performance on the Domain-Specific and Domain-General Tasks

Task	M	SD	Min	Max
Calculation skill (no. problems correct)	19.69	7.18	7.00	40.00
Order of operations (% of literal responses)	24.82	36.53	0.00	100.00
VSWM (total item score)	44.54	13.32	16.00	80.00
Inhibition (% of false alarms)	36.56	18.12	0.00	81.25
Switching (RT difference between switch and no-switch trials, ms)	504.20	269.18	-274.81	1257.80

Table 2

Correlations Between Performance on the Shortcut Identification, Domain-Specific and Domain-General Tasks

Variable	Shortcut self-report task (identifier, non-identifier)[a, b]	Trial-by-trial shortcut task (identifier, non-identifier)[a, b]	Trial-by-trial shortcut task (% use)[c]	Calculation skill	Order of operations	VSWM	Inhibition
Calculation skill	0.155	0.225*	0.219*	—	—	—	—
Order of operations	-0.037	-0.111	-0.181	-0.113	—	—	—
VSWM	0.054	0.096	0.127	0.319**	0.034	—	—
Inhibition	-0.125	-0.263**	-0.240**	-0.297**	0.059	-0.119	—
Switching (RT difference between switch and no-switch trials)	-0.213*	-0.037	-0.021	-0.032	0.067	-0.055	0.162

[a]Identifiers were coded '1' and non-identifiers '0'. [b]point-biserial correlations. [c]linear correlations.

*$p < .05$. **$p < .01$.

Identification on the Shortcut Self-Report Task

There were 33 identifiers and 86 non-identifiers (one participant could not be categorised). We checked whether the data met the assumptions of a logistic regression including a) linearity between each predictor and the logit of the self-report variable (Box & Tidwell, 1962) and b) no collinearity between the predictors. Both assumptions were met.

A hierarchical logistic regression was performed to assess whether identification of the shortcut on the shortcut self-report task was associated with the domain-specific and domain-general measures. Two regression models were computed, one with the domain-specific skills (calculation skill and knowledge of the order of operations) as predictors, and a second with the domain-specific skills and domain-general skills (VSWM, inhibition and switching) as predictors. Table 3 displays the result. Switching was the only variable that significantly related to identification. However, since neither of the regression models were significant the amount of variance explained is very small and therefore it is not appropriate to interpret the significance of individual predictors.

Table 3

Logistic Regression for Shortcut Identification on the Shortcut Self-Report Task

| Predictor	Model 1			Model 2		
	Exp(β)	Wald	Significance	Exp(β)	Wald	Significance
Calculation skill	1.05	2.78	0.096	1.04	1.67	0.196
Order of operations	1.00	0.05	0.822	1.00	0.00	0.998
VSWM				0.99	0.13	0.714
Inhibition				0.99	0.21	0.649
Switching				0.12	4.45	0.035
Model statistics	$\chi^2 = 3.04$, $p = .219$			$\chi^2 = 8.53$, $p = .130$		

Identification on the Trial-by-Trial Shortcut Task

The overall rate of identification on the trial-by-trial shortcut task was 53%, with 64 identifiers and 56 non-identifiers. Comparing this rate with the rate of identification on the shortcut self-report task indicates that there were 33 participants who reported using the shortcut on the self-report task, 27 of whom were also identifiers on trial-by-trial shortcut task. Furthermore, 86 participants did not report using the shortcut on the shortcut self-report task, 49 of whom also did not report using the shortcut on the trial-by-trial shortcut task. The trial-by-trial shortcut task includes many more opportunities to identify the shortcut and therefore we would expect the rate of identification to be higher.

Figure 1 displays the raw scores on the domain-specific and domain-general tasks for the identifiers and non-identifiers.

Figure 1

Raw Scores on Variables
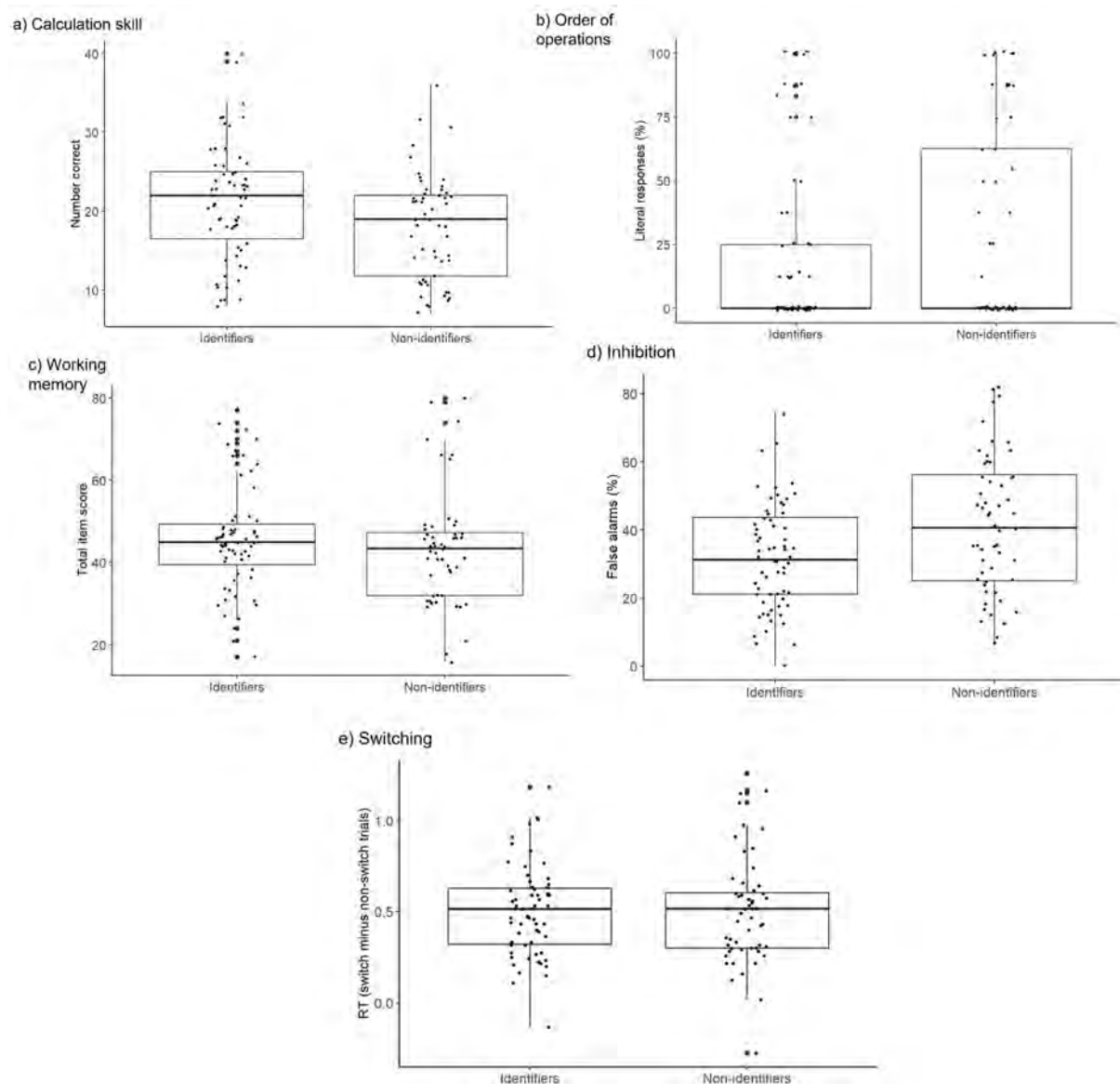
Note. Variables measuring a) calculation skill, b) literal misconceptions of order of operations, c) VSWM, d) inhibition and e) switching, for identifiers and non-identifiers on the trial-by-trial shortcut task.

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The data met the assumptions of a hierarchical logistic regression (above), and Table 4 displays the result. Model 1 was significant and calculation skill was a significant predictor, with an odds ratio to indicate that for one standardised unit increase in the number of problems solved correctly, the odds of being an identifier increased by 6.7%. Model 2 was also significant, where inhibition was a significant predictor with an odds ratio to indicate that for one standardised unit increase in the false alarms, the odds of being an identifier decreased by 2.5%. Hosmer-Lemeshow statistics for both models were not significant: Model 1, 2(8) = 12.35, p = .136 and Model 2, 2(8) = 4.53, p = .806, indicating that both were a good fit to the data. Statistics of the residuals were also explored via cooks distance, leverage, DF beta and standardised values, all of which were acceptable indicating that both models were a good fit.

Table 4

Logistic Regression for Shortcut Identification on the Trial-by-Trial Shortcut Task

	Model 1			Model 2		
Predictor	Exp(β)	Wald	Significance	Exp(β)	Wald	Significance
Calculation skill	1.07	5.33	0.021	1.05	2.55	0.110
Order of operations	1.00	0.92	0.336	1.00	0.95	0.330
VSWM				1.00	0.02	0.880
Inhibition				0.98	4.76	0.029
Switching				1.41	0.21	0.644
Model statistics	χ^2 = 7.19, p = .027			χ^2 = 12.28, p = .031		

Trial Number of Identification (Trial-by-Trial Shortcut Task)

For the self-reported identifiers who had an IP (N = 58), the mean percent of trials remaining after the IP was 39% (SD = 41%, range = 0 – 100%). A hierarchical multiple linear regression was performed to assess whether the percent of trials was predicted by the domain-specific and domain-general measures. As per the pre-registration, this analysis included identifiers' percent use scores, and non-identifiers who were assigned a score of 0%. The domain-specific variables were entered in the first model, and the domain-specific plus the domain-general variables in the second. Both models were tested to see if they met the assumptions of a multiple linear regression including a) linearity between the predictors and the outcome variable, b) no collinearity between the predictors, c) homoscedasticity and d) normally distributed residuals. Both models grossly violated all assumptions except collinearity, which we addressed by conducting a second multiple linear regression with non-identifiers (individuals with scores of 0%) excluded. The data still violated the assumptions of a linear relationship between the predictors and outcome, homoscedasticity and normally distributed residuals. Regression models are therefore not appropriate for the data and cannot be interpreted in a meaningful way. This analysis was therefore not progressed any further.

Exploratory Analyses

There was a significant correlation between calculation skill and inhibition, suggesting that the tasks may have partly measured the same construct. To investigate which variable drove the relationship with shortcut identification, Bayesian analyses were conducted to see if the Bayes factors were substantially larger for one variable (calculation skill or inhibition). Independent Bayesian *t*-tests found that the hypothesis of a difference between identifiers and non-identifiers (on the trial-by-trial shortcut task) in calculation skill was supported by a BF$_{10}$ of 3.090, and in inhibition was supported by a BF$_{10}$ of 9.297 (Bayes factors of 1-3 are 'anecdotal' and 3-10 are 'moderate' evidence for the alternative hypothesis; Jeffreys, 1961). The BF$_{10}$ for inhibition is approximately three times that of calculation skill, suggesting that inhibition is the more reliable predictor of identification.

Discussion

Strategy identification may be a key component of advanced strategy competencies such as adaptivity and flexibility, which in turn predict mathematical expertise (Verschaffel et al., 2009). The ability to identify valid strategies for solving problems is necessary if individuals are to select strategies that are the most efficient in particular contexts (adaptivity) and switch between those strategies on different problems (flexibility). Furthermore, shortcut strategies are particularly important as they are often derived from arithmetic principles that aid the understanding of algebra and ease the transition from elementary to advanced mathematics (Knuth et al., 2006). However, relatively few people select shortcut strategies when problem solving, with calls for researchers to investigate why (NMP, 2008).

Our research contributes to this goal by investigating whether domain-specific and domain-general skills are important for identifying shortcuts, and our findings suggest that of those skills, inhibition is the most important. In the next section we discuss the theoretical contribution of our findings to the literature on a) arithmetic strategy choice and b) arithmetic principles. We then discuss how the three functions of inhibition, a) resisting distractor interference, b) resisting proactive interference, and c) resisting prepotent responses, might help in the identification of the shortcut.

Theoretical Contribution

Executive Functions and Strategy Selection

A variety of computational models predict how and when individuals select among a repertoire of strategies and shift from using one strategy to another (e.g., Rieskamp & Otto, 2006; Siegler & Shipley, 1995). However, none of these models consider the role of executive functions in the selection process (although some do discuss attention). Given that strategy use is characterised by individual differences that do not consistently relate to age (Dubé, 2014) these models seem incomplete and need to be extended. Our findings imply that incorporating the role of inhibition is one way that this could be done.

Some studies have inferred a role of executive functions by comparing strategy selection of individuals of different ages, and individuals with and without cognitive difficulties (e.g., Sella et al., 2019). These studies used 'simple' problems (e.g., two-digit additions) with strategies that were prescribed to participants prior to problem solving. For example, in one study 8 – 12 year olds were informed of two estimation strategies, 'rounding up' and 'rounding down' for solving addition problems (e.g. '36 + 78') and the frequency with which they chose the 'best' strategy was recorded (Lemaire & Lecacheur, 2011). Inhibition and switching positively predicted the frequency with which participants chose the 'best' strategy, and mediated a relationship between age and performance. Thus, executive functions may enable individuals to select more efficient strategies, at least, for simple problems and in conditions where strategies are prescribed beforehand. To our knowledge, our study is the first to investigate the relationship between executive functions and strategy selection with more complex problems under conditions where strategies are not mentioned before problem solving.

In the arithmetic principle literature, studies have begun to explore the role of executive functions in children's use of arithmetic principles (e.g. shortcut use on 'a + b − b' inversion problems). For example, studies have investigated whether children's working memory and switching skills relate to solution accuracy and self-reported use of strategies derived from a mixture of arithmetic principles (e.g., Cragg et al., 2017; Edwards, 2013; Gilmore et al., 2018; Watchorn et al., 2014). These studies returned mixed results. The only executive function to consistently demonstrate a positive relationship is inhibition (Gilmore et al., 2015; Robinson & Dubé, 2013). Our findings support and extend this literature to associativity: in adults, we found no evidence for the role of visuospatial working memory or switching in the selection of a shortcut on 'a + b − c' associativity problems, but we did find evidence for a role of inhibition. Furthermore, by way of measuring a wide variety of domain-specific and domain-general skills, our study is the first to demonstrate that inhibition may be the most important factor.

It is possible that other factors, beyond those measured here, may also be involved in selection of an arithmetic shortcut. Regarding domain-general factors, we considered visuospatial working memory rather than verbal working memory, because our arithmetic expressions were presented visually and we were specifically interested in skills required to select an arithmetic shortcut rather than to calculate the shortcut. However, it is also possible that verbal

working memory may play a role. Verbal working memory is involved in arithmetic calculation (Cragg et al., 2017), and it may also be involved in strategy selection if participants use verbal strategies to reorder the parts of the expression.

Regarding domain-specific factors, we found that knowledge of operation order was not involved in selection of an arithmetic shortcut. This study was conducted in the UK, and the majority of previous research on associativity shortcuts has been conducted in Canada (e.g., Edwards, 2013; Robinson & Dubé, 2013) or the UK (Cragg et al., 2017; Gilmore et al., 2018) where acronyms such as BODMAS are used to teach operation order. It would be valuable to investigate whether knowledge of operation order had a stronger relationship with strategy selection in countries that have different approaches to teaching operation order.

We note that 6 participants who identified the shortcut on the self-report task then did not identify the shortcut on the trial-by-trial task. This could be because the practice trials for the trial-by-trial shortcut task were non-conducive, and could have caused self-doubt as to whether the shortcut was appropriate (i.e., because it conflicts with taught acronyms). Further domain-specific factors that were not measured here may also be involved in associativity shortcut selection. Identifying problems on which the associativity-based shortcut is advantageous requires estimation of the size of the quantities involved. Consequently, an additional domain-specific skill that may be relevant is estimating the magnitude of symbolic quantities. Tasks such as number line estimation or symbolic magnitude comparison would be worth considering as predictors in future research.

Functions of Inhibition

Go/No-Go tasks are established measures of the ability to a) pause before response selection and b) pause during response execution, both of which are relevant to identification. For example, on three-term inversion problems some individuals discover the shortcut before initiating any calculation, while others discover it part-way through executing a left-to-right procedure (Dubé, 2014; Robinson & Dubé, 2009). Similar routes to identification could exist on associativity problems, with some individuals identifying the shortcut before performing any computation, and others identifying it while performing 'a + b'; i.e., inhibition could encourage individuals to identify the associativity shortcut by helping them to resist prepotent responses, and this might initiate before or during the process of solving a problem. For those who identify the shortcut before problem solving, inhibition provides a moment of pause for them to consider different strategies and for those who identify the shortcut while executing a left-to-right strategy, inhibition enables them to interrupt an ongoing procedure.

Other functions of inhibition include a) resisting distractor interference and b) resisting proactive interference (Friedman & Miyake, 2004). Although our Go/No-Go task did not measure these functions, we do not rule out the possibility that they play a role in identification. On the contrary, there is good reason to think that both could be important. For example, resisting proactive interference might be particularly important for individuals in the UK, USA and Canada, where formal mathematics encourages left-to-right sequential approaches for solving arithmetic problems that can deter the use of alternative strategies (McNeil et al., 2010). Computational models echo this suggestion (Siegler & Araya, 2005). Future research could explore whether inhibition enables identification by resisting distractor and proactive interference by using a correlational design that uses a larger battery of inhibition measures such as flanker (Eriksen & Eriksen, 1974) and Stroop tasks (Stroop, 1935) that capture distractor and proactive interference respectively.

Practical Contribution

Our findings highlight the importance of inhibition in selecting efficient arithmetic strategies. Although our research was conducted with adults, it is likely that similar factors influence strategy selection in younger learners. These factors may be more important for younger children in whom executive function skills, such as inhibition, are still developing. Our findings can be used to raise teachers' awareness that the reason children sometimes choose suboptimal strategies is not because they lack an understanding of mathematics, but because they lack sufficient cognitive skills. As a result, teachers could devise techniques to lessen those cognitive demands, such as encouraging children to pause and think about how they are going to solve a problem before they begin. There is some evidence that "stop and think" interventions of this form can be beneficial (Wilkinson et al., 2020). Given that teachers are currently advised to encourage the selection and flexible use of efficient strategies (NMP, 2008), these findings are timely.

Conclusion

Identifying arithmetic strategies is necessary if individuals are to become adaptive and flexible in arithmetic strategy use. However, relatively few individuals select efficient 'shortcut' strategies on problems of the format 'a + b − c' and in a well-powered correlational study we investigated why. Of five domain-specific and domain-general skills, inhibition was the most reliable predictor of identifying the shortcut. We propose different functions that inhibition may serve, which could be communicated with educators to make them aware of the importance of cognitive resources in the selection of efficient strategies.

Data Availability: For this article, a data set is freely available (Eaves, Attridge, & Gilmore, 2022).

Supplementary Materials

The Supplementary Materials contain the following items (for access see Index of Supplementary Materials below):

- Pre-registration protocol
- Research data
- Materials for the tasks

Index of Supplementary Materials

Eaves, J., Attridge, N., & Gilmore, C. (2018). *Executive function skills and the identification of arithmetic shortcuts. (#14052)* [Pre-registration protocol]. AsPredicted. https://aspredicted.org/ZMO_NWK

Eaves, J., Attridge, N., & Gilmore, C. (2022). *The role of domain-general and domain-specific skills in the identification of arithmetic strategies - Data* [Research data]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.16438377

Eaves, J., Attridge, N., & Gilmore, C. (2022). *Supplementary materials to "The role of domain-general and domain-specific skills in the identification of arithmetic strategies"* [Materials for shortcut identification tasks]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.16438347

Eaves, J., Attridge, N., & Gilmore, C. (2022). *Supplementary materials to "The role of domain-general and domain-specific skills in the identification of arithmetic strategies"* [Materials for domain-specific tasks]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.16438356

Eaves, J., Attridge, N., & Gilmore, C. (2022). *Supplementary materials to "The role of domain-general and domain-specific skills in the identification of arithmetic strategies"* [Materials for domain-general tasks]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.16438329

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