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GeoGebra Assisted Multiple Representations for Enhancing Students' Representation Translation Abilities in Calculus

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Abstract

Multiple representations cultivate students' mathematical mindset. However, research results have reported that students do not benefit from these tools due to lack of representational fluency. This study was designed to determine the impact of GeoGebra assisted multiple representations approach on students' representation translation performance in calculus. Pretest - posttest quasi experimental design was implemented. Three intact groups of first year first semester of social science students in the 2019/2020 academic year of size 53, 57 and 54 at Jigjiga and Kebri-Dehar Universities in Ethiopia were considered. The groups were taught with GeoGebra supported multiple representations (MRT), multiple representations (MR) and comparison group (CG). Representation translation test was given before and after the treatment. Furthermore, students' translation errors were categorized as implementation, interpretation and preservation errors and analyzed using frequency and percentage. The ANCOVA result revealed that significant difference was obtained on the adjusted mean of RTF posttest (F (2,160) = 5.29, P = 0.006, Partial η^2 =0.062) in favor of the MRT. The interpretation error was the most frequently committed among the groups. Recommendations were forwarded that included the use of GeoGebra and the need to conduct further study with different participants to generalize to the entire population.

Keywords: Calculus, Errors, GeoGebra, Multiple representations, Representation translation, Source representation, Target representation, Translation.

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Contents

Contribution of this paper to the literature
The study provides a step forward on the implementation of GeoGebra to enhance the quality and quantity of using multiple representations for learning calculus. Furthermore, the study explains the plethora of multiple representations on representation translation abilities through detecting potential sources of errors to facilitate students' learning.

1. Introduction

Inherently, mathematics is endowed with multiple representations (MRs) used for problem solving, conceptual understanding, and disciplinary discourse. Mathematics stands as a discipline due to these representations; otherwise it would have been an imaginary idea. Upon the advent of versatile computer technologies, MRs becomes a conspicuous instructional approach in mathematics to create a cohesive and comprehensive conceptual understanding on students' side. There are plenty of computer tools that support dynamic links of MRs. GeoGebra is one of them. It is a fascinating mathematical software application used to experiment and explore mathematical contents via dynamically linked MRs (Dahal, 2019). The ability of MRs is paramount important to characterize students who excel in mathematics. Mathematical representation skills is one of the five mathematical abilities in the school mathematics standards, including (1) communication, (2) reasoning, (3) problem solving, (4) connection, and (5) representation. The most commonly used representation types in a typical calculus classroom instruction are numerical, graphical, algebraic, verbal, and their possible combinations. The notion of MRs is the delivery of a concept using at least two different representation types (Ainsworth, 2006; Arifah, 2020). The advantage of learning mathematics with MRs are explained from the perspectives of information processing theory (dual coding) (Clark & Paivio, 1991) , and pedagogical functions (complement , constrain and construct (Ainsworth, 2006) In the process of designing an appropriate multi-representational learning environment to support students' learning, the rhythm must be based on the pedagogical functions that MRs provides along with design parameters and cognitive tasks (Ainsworth, 2006).

The chalk and board approaches are no longer supportive of implementing MRs in mathematics classroom. However, the advent of multiple interface computer technologies has increased the quantity and quality of using MRs in mathematics instruction. The emergence of GeoGebra makes MRs more accessibly with more quantities and qualities in calculus learning (Arifah, 2020). To get the optimum benefits that MRs can offer , students need to develop multiple representation abilities: the ability of creating, interpreting, implementing and translating multiple representations for problem solving (Fonger, 2019). MRs ability is a land mark for meaning making in mathematics learning. These abilities are equally important for novices and experts (Pedersen, Bach, Gregersen, Højsted, & Jankvist, 2021). However, novices lack the required abilities for MRs (Nurrahmawati, Sa'dijah, Sudirman, & Muksa, 2019; Rahmawati, 2019). The ability with MRs enables seamlessly translate one representation to another and flexibly implement various representations in solving mathematical problems. These abilities are at the heart of successful mathematics learning (Adu-Gyamfi, Stiff, & Bossé, 2012; Fonger, 2019).

MRs as method of instruction is defined as the implementation of two or more than two different representations simultaneously in classroom instruction when a single representation is no longer adequate to provide a complete picture of a concept (Ainsworth, 2006; Fonger, 2019; Nurrahmawati et al., 2019). Representation translation refers ,in this study, to a process of constructing a mathematical representation (called ,target representation) from given representation (called , source representation) in solving mathematical problem (Adu-Gyamfi et al., 2012; Afriyani, Sa'dijah, & Muksar, 2018). The goal of representation translation is to preserve semantic congruence between two mathematical representations.

1.1. Representation Translation

The key competency for meaningful learning of mathematics is representational fluency (RF), which refers to the ability to create, interpret, translate between, and connect MRs (Fonger, 2019). Representation translation fluency (RTF) is one component of RF that is considered to be a foundation for building up of conceptual understanding and mathematical thinking. Students most often fail to translate one representation to other representation. Dealing with the representations without connecting and translating resulted in a fragmented knowledge of the concept. Many researchers reported that students perennially demonstrate difficulty in correctly accomplish the translation tasks (Adu-Gyamfi et al., 2012; Afriyani et al., 2018; Bossé, Adu-Gyamfi, & Cheetham, 2011; Rahmawati, Hidayanto, & Anwar, 2017). Numerous factors interact to make some translations more difficult than others. A widespread of research results with the courses ranging from algebra to calculus indicate that there is a lost in translation when attempting to go from one representation of a mathematical situation or relationship to another (Adu-Gyamfi et al., 2012; Duval, 2006). The degree of difficulty is categorized into student-centered factors and content centered factors. Student-centered factors are including: the translation action, dual translation (through intermediate translation) and classroom experience. The factors related to the representation type include: different representation types required different interpretation techniques (like local versus global, or syntactic versus semantic), some translation are inherently more complex, and some require greater number of steps to accomplish (Rahmawati et al., 2017). The number of fact gaps associated with either the source or target representation involved in a translation may speak to the difficulty of the translation (Adu-Gyamfi et al., 2012; Bossé et al., 2011).

Any conventional mathematical representation is identified by its characters, configurations, syntactic and semantic rules in order to be communicable in the disciplinary discourse. In the process of translation, two types of representations are involved, source and target representations. In this process, semantic congruence must be preserved without loss of information while mapping the source to the target representation. To do so, learners must unpack the densely packed micro-concepts embedded in the source representation to recombine in constructing the target representation (Duval, 2006). This is in contrast to the black box, which concerns with the source and target representations ignoring the process.

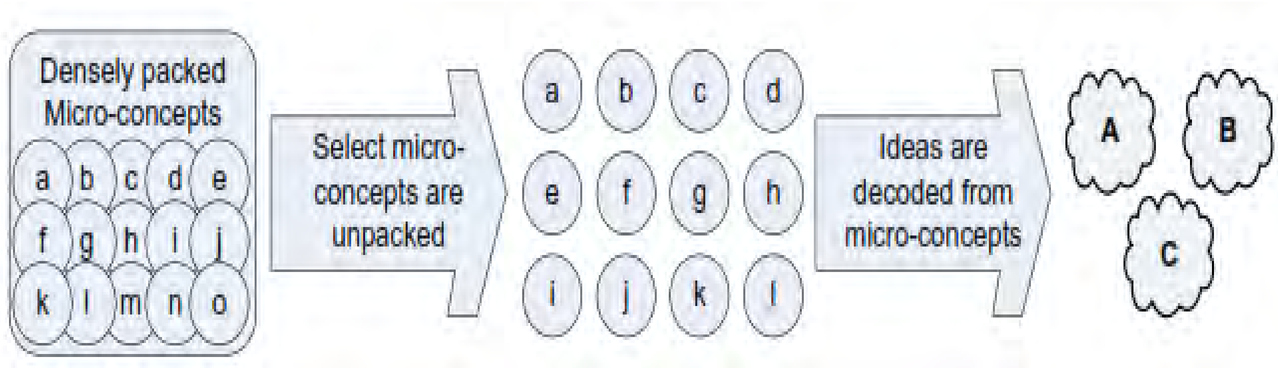

Figure 1. Unpacking the source representation (Bossé, Adu-Gyamfi, & Chandler, 2014).

All of the micro-concepts packed in the source representation are not helpful to construct the target representation. Some basic information or concepts from the source representation are required that guide to construct congruent target representation. Figure 1, illustrates that the densely packed micro-concepts (a-o) in the source representation are unpacked and some (m-o) may immediately be discarded as unnecessary and others may be overlooked or not fully recognized. Through investigating and interacting with the micro concepts of a representation, the learner decodes mathematical ideas (A, B, and C) which were captured in the source representation (Bossé et al., 2014). For instance, the basic concepts required for sketching a perfect graph representation of a function from its algebraic representation, more or less, are: domain, intercepts, symmetricity, asymptotes, intervals of monotonicity, extreme points, intervals of concavity, inflection points, behavior of the function at infinity. Adu-Gyamfi et al. (2012) identified three types of translation errors:

- Implementation error- steps in an algorithm are incorrectly executed.
- Interpretation error- incorrectly characterized the properties of either the source or the target representation.
- Preservation error – attributes of the source representation are not properly coded in the target representation.

2. Statement of the Problem

In the availability of MRs, it is a fact that there is an urge to prefer , translate, implement and interpret representations (Ainsworth, 2006). However , students often fail to do so due to lack of RF (Rahmawati, 2019). Students also lack to relate and translate the representation with the underline mathematical concept (Samsuddin & Retnawati, 2018). Fonger, Davis, and Rohwer (2018) in turn, reported that instructional support enabled student to enhance RF. Students have a huge gap in flexible implementation of MRs during problem solving and bridging them through translation (Adu-Gyamfi et al., 2012). Van Meter, List, Lombardi, and Kendeou (2020) indicate that novice (students) fundamentally lack RF which enable them to understand the underline concepts. These representations can be mutually translated to each other without any loss of formal properties (Dey, 2019). The more complete the translation ability of the students have, the stronger the understanding of mathematical ideas they have. Many researchers argue that some mathematical translations are more difficult than others (Adu-Gyamfi et al., 2012). This study was framed to investigate the effect of GeoGebra assisted multiple representations approach on students' representation translation fluency in learning calculus.

3. Research Questions

1. What is the effect of GeoGebra assisted multiple representations approach on students' performance in representation translation of calculus problems?
2. Which representation error is most committed across groups in calculus learning?
3. Which representation type is difficult for students to translate in terms of the source and target representations?

4. Methods

The study implemented a multi-treatment pretest- post-test non-equivalent group quasi-experimental research design on purposefully selected groups of students that belongs to two distinct universities in Ethiopia. The research areas include Jigjiga University (JJU) and Kebri-Dehar University (KDU). The study was intended to compare the effects of three differentiated approaches: GeoGebra supported multiple representations approach (labeled as MRT), multiple representations approach (labeled as MR) and the control group (labeled as CG) on students' performance on representation translation on differential calculus concepts. Furthermore, this study investigated the prevalence of translation error committed by the students. In addition to that, students' difficulty in representation type was identified in terms of the source and target representation.

4.1. Participants

The study was conducted in 2019/20 academic year on first year first semester students of the social science stream who enrolled for the course mathematics for social science at Jigjiga University (JJU) and Kebri-Dehar University (KDU). According to the new Ethiopian higher education roadmap for undergraduate program, which was indorsed in the 2019/20 academic year, the course mathematics for social sciences incorporates calculus in a separate unit. According to this roadmap, first year students at JJU and KDU, who admitted from all corners of the country in 2019/20, had the natural and social sciences streams. The first-year social sciences students at JJU and KDU were assigned into their section (labeled as A, B, C,etc.) based on alphabetical order. Each section contained, on average, 58 students. In the JJU and KDU, there were 23 and 12 sections of social science students in the 2019/20 academic years, respectively. One section from JJU was assigned into the MRT (n = 53) and two sections from KDU were assigned into the MR (n = 57) and CG (n = 54). The learning contents of the course that were selected for the interventions cover limits, continuity, derivatives and application of derivatives.

The multiple representation approach (MR) group was received multiple representations based instruction, with special focus on the verbal, numerical, graphical, algebraic representations and their possible combinations. This group was not supported by any technology. Chalk and board as well as paper and pen were used for illustrating a content using different representation types and translating one representation type to other representation type. In the GeoGebra supported multiple representations treatment group (MRT), the multiple representations was supported by GeoGebra software. For this group, some classroom arrangement and classroom shifting was implemented during the intervention. Even if all of the classrooms in JJU have access of electricity infrastructures, there was erratic power supply. These classrooms are not sufficient to use mass technology, like liquid crystal display (LCD) to display mathematical representations on GeoGebra interfaces, since they are designed for chalk and board teaching approach and they have stretched rectangular shape. In the MRT classroom had two phases. First, GeoGebra was used side to side with the chalk and board/ pen and paper in the hall using laptop and LCD display. Second, a computer lap was used to practice the students on the GeoGebra worksheets and to construct their own at fly. The time allocation for these sessions of the group was based on the 3 credit hours and 2 tutorial hours of the course. The students were not constrained to use the software only in the class time, but many of the students downloaded and installed in their private electronic devices. They were using it in their dormitory for practicing, experimenting and learning mathematics contents with their own peace.

Due to the fact that the course instructor and the students were novices for the technology, most of the time , online sources were using for the classroom presentation and demonstration as well as for the computer lab practicing and experimentations. In the MRT group, by means of GeoGebra, the teaching and learning of calculus was shifted into more active, where students explored calculus concepts with linked multi-representations, which is often difficult using chalk and board. The CG was taught based on the conventional approach, which was more dominantly algebraic representation. The course instructor in the CG class might use different representations separately. But, he did not receive any training to use multiple representations intentionally with recommended model. However, the course instructors in the MRT and MR groups, the DeFT (Design, Function, Task) model was adopted. The three groups were taught by different instructors who had equivalent academic status and teaching experience. The intervention was lasted for about six weeks.

Table 1. Participants' background information.

		Gender			
		Female		Male	
Group	**N**	**f**	**%**	**f**	**%**
MRT	53	16	30.2	37	69.8
MR	57	21	36.8	36	63.2
CG	54	18	33.3	36	66.7

In the three groups, female participants were fewer than their male counterparts. Based on the results reported in Table 1, the MRT group consisted of 16 female and 37 male students. These numbers represent 30% females and 70% males of the group, respectively. Similarly, the MR group consisted of 21 (37%) female and 36 (63%) male students. The CG encompasses 18 (33%) female and 36 (67%) male students. From this table, it can be observed that the proportion of female students across the three groups was approximately equal with very slight variation. But, the proportion of female students was much less than the proportion of male students across each group.

Table 2. Representation translation posttest items, with respect to source and target representations.

Item	Source representation type(s)	Target representation type(s) intended to use for solving the problem
1	Algebraic	To be able to sketch graph of the function using the applications of derivative, using central difference determine derivative of the function at a specific number, use the formula to compare the values of a derivative and write a sentences that describes the behavior of the derivative of the function on a given interval
2	Combinations of algebraic and graphical	To be able to generate numerical table for the area of the region enclosed by the graph using the formula as well as the graph and draw a scatter plot for the data
3(a)-(e)	Combination of verbal and graphical	To be able to label dimensions of the figures, give algebraic expression for the volume of the box, construct a table of values for the volume function, sketch a graph for the volume function , and explain the behavior of the volume function as x approaches a number
4	Numerical	To be able to discuss the slope of the tangent line at a point P to the graph of V using graph, algebra, numerical value and verbal explanation

The students in the three groups were given representation translation test on concepts of differential calculus. The test had four questions, in which question 3 contained five questions (a-e). Table 2 illustrates that the type of representation used to express the question, considered as source representation, and the intended representation that the students elucidate in their answers, called target representation. The students were required to translate mathematical information from the source representations to the target representation preserving semantic congruence. Students result was scored using rubrics.

4.2. Data Collection Instruments and Procedures

The data collection instrument for this study was representation translation fluency test (RTFT) (pretest and posttest). Tasks that involve representation translation and representation implementation could lend themselves to rubric assessment and to other assessment types suitable for open-ended activities. Hence, the students' score on the representation interpretation problem was quantified using rubric assessment technique.

The problem is presented in one or the combined form of representation(s). The students' were asked to solve the problem in the other forms of representation(s) keeping in mind that the different representations are equivalent. In the process of representation translations, the students' errors were scored and categorized as implementation error, interpretation error and preservation error (Adu-Gyamfi et al., 2012). The three groups were compared based on these error types using frequency and percentage.

4.3. Reliability and Validity of Instruments

To obtain obtaining reliable and valid information from the data collection instruments, several efforts had been made. The main types of validity that were tried to establish through different mechanisms included: face validity, content validity, construct validity and criterion validity. In order to establish validity of the representation interpretation problem, the supervisors' comments were used and the items were modified accordingly. In addition to the supervisors' comments, the opinions of mathematics experts, who are member of the academic staff in mathematics department at JJU, were consulted for checking the validity of concept and appearance from the aspects that it aimed to measure. Regarding to the face validity, the assessors evaluated the appearance of the items in each of the constructs in terms of feasibility, readability, consistency of style and formatting, and the clarity of the language used to the level of the participants experience. Panel of experts were also involved to evaluate content validity of the constructs in the way to ensure each of the constructs incorporates all the items that were essential in which they eliminated irrelevant items in any of the constructs. In addition to the panel of experts, the literature review was used to establish content validities of the constructs.

To establish the reliability of the instruments of each construct, a pilot test was conducted on second year mathematics department students. Thirty students (15 students for the pretest and 15 students for post-test) were participated in the pilot test. The students' solutions on the constructs were assessed through rubrics. Two iterators were involved in assessing the students' work using the predetermined rubrics for scoring students' solution of the items in each of the constructs. As a result of this, student's solution were analyzed separately by two mathematics department academic staff members at JJU and the calculation of the reliability were computed manually using the formula "Consensus/ (Consensus + Dissensus) X 100" recommended by Miles & Huberman (1994), cited in Rahmawati et al. (2017). Based on this, the reliability value of the RTF pretest and posttest were obtained to be 0.78 and 0.77, respectively. These results are in the acceptable range of reliability.

4.4. Data Analysis

To compare the three groups with regards to their performance on the representation translation in solving calculus problems, appropriate inferential statistic was used depending on the underlined statistical assumptions about the collected data. One way Analysis of Variance (ANOVA) was employed to detect variations of the three groups (if any) on their RTF pretests. One way Analysis of Covariance (ANCOVA) was implemented to compare the three groups based on their score on the RTF posttests using the RTF pretest as covariate. Percentage and frequency were used for describing students' status on the translation error. A narrative analysis was used for the qualitative data to determine students' difficulty of representation type in the process of translation.

5. Results

5.1. Pre-Intervention Results

The pretest results were the base line test scores taken from the three groups just before the intervention had been begun. The data were obtained from RTF pretest. From the RTF pretest, translation errors were identified and classified into three categories – implementation , interpretation and preservation (Adu-Gyamfi, Stiff, & Bossé, 2012). Hence, the test results involved both continuous and categorical data. For the continuous data, appropriate inferential statistics were used and for the categorical data, frequency and percentage were applied to describe the status of students before the intervention.

Table 3. Means and standard deviations of the representation translation fluency pretest for groups.

		Representation translation fluency pretest	
Group	**N**	**M**	**SD**
MRT	53	66.94	14.84
MR	57	57.96	13.09
CG	54	64.44	12.17

Mean and standard deviation were performed to summarize and describe the data set obtained from the continuous components of RTF pretest. The results reported in Table 3 shows that slight variations of mean scores were detected among the three groups with greater score for the MRT (M = 66.94, SD = 14.84) than the MR (M = 57.96 , SD = 13.09) and the CG (M = 64.44, SD = 12.17). However, these results do not enable to generalize with certain level of confidence and significance. Hence, generalizing the results was detained until the appropriate inferential statistics was performed.

Table 4. One-way analysis of variance results for the representation translation fluency pretests.

Variable	Source	SS	df	MS	F	P
RTF Pretest	Between groups	2381.91	2	1190.95	6.64	0.002
	Within Groups	28896.09	161	179.48		
	Total	31278	163			

The between subjects one way analysis of variance (ANOVA) was conducted to compare the three groups on the representation translation fluency (RTF) pretests. The stringent statistical assumptions for the ANOVA model had been checked before it was used for comparing the groups. These assumptions were met. The result reported in Table 4 shows that the three groups had statistically significant mean differences on the RTF pretest (F (2,161) =

6.64, P = 0.002) before the intervention had been begun. This result manifested that students in the three groups had various experiences on the RTF in learning on the pre-calculus concepts before the intervention. The post hoc comparison using the Tukey HSD test verified that mean score of the multiple representations group (MR) (M = 57.96, SD = 13.09) was significantly less than the GeoGebra supported multiple representations (MRT) (M = 66.94, SD = 14.84) and the comparison group (CG) (M = 64.44, SD = 12.17) on the RTF pretest. The MRT did not differ significantly from the CG on the RTF pretest (P = 0.60). The base line variations of the groups on the RTF before the intervention were considered as a covariate for the posttest and controlled statistically using analysis of covariance (ANCOVA).

Table 5. Translation pretest error – group cross tabulation.

Group	Error Type							
	Implementation		Interpretation		Preservation		Combination	
	f	%	f	%	f	%	f	%
MRT	16	30	32	60	2	4	1	2
MR	13	23	38	67	4	7	-	-
CG	15	29	36	67	2	4	-	-

The types of error that can be committed in the process of representation translations are categorized into implementation error, interpretation error, preservation error and combination of any of these errors. Frequency and percentages were used to estimate the most committed error type in the pretest. Based on the results reported in Table 5, interpretation error was the most frequently committed with slight variations among groups (60% of the GeoGebra supported multiple representation group (MRT), 67% of the multiple representations group (MR) & 67% of the CG) next to implementation error (30% of the MRT, 23% of the MR & 29% of the comparison group (CG)).

5.2. Post Intervention Results

Once the researchers become sure that there were no silly errors in the data set collected from the post administrated instruments (for instance, avoiding out of range scores in any of the instruments), a preliminary data analysis was done on the construct of RTF to check the stringent statistical assumptions for the underlined statistical methods. These preliminary activities paved the way to choose correct and appropriate statistical tools and techniques to address the formulated research questions of this study. As this research design was a non-equivalent groups quasi-experimental research design, most of the statistical tools that were chosen focused on estimating the differences between groups on data scores from the post administrated instruments as a result of the differentiated treatment type. In choosing the right statistic, a number of decisive factors were taken into great considerations.

Table 6. Means and standard deviations of the representation translation fluency posttest.

Group	N	Representation translation fluency Posttest	
		M	***SD***
MRT	53	22.79	3.09
MR	57	21.12	3.55
CG	54	20.94	3.02

Based on the descriptive statistics results reported in Table 6, slight variations were detected among the groups on the indicated variable. It would be a hasty generalization ahead of implementing the appropriate inferential statistic, however to provide conclusive information based on the results of the descriptive statistics.

Table 7. Analysis of covariance result of the representation translation fluency posttests using the pretest as covariate.

DV	Source	Type III SS	df	MS	F	P	Partial η^2
RTF posttest	pretest	0.81	1	0.81	0.08	0.782	0.000
	Group	111.30	2	55.65	5.29	0.006	0.062
	Error	1684.88	160	10.53			

Note: DV: Dependent Variable.
R Squared = 0.063 (Adjusted R Squared = 0.045) for the RTF posttest.

Table 8. Pairwise comparisons on the representation translation fluency posttest.

DV	(I) Group	(J) Group	MD (I-J)	*P*
RTF posttest	MRT	MR	1.72	0.025
		CG	1.86	0.011
	MR	MRT	-1.72	0.025
		CG	0.14	1.000

Note: DV: Dependent Variable.

A one way analysis of covariance (ANCOVA) was carried out to compare the groups on the representation translation fluency (RTF) posttest while controlling the RTF pretest. Preliminary analyses were carried out to check the statistical assumption for the underlined statistical method and assumptions were met (Pallant, 2013). According to the one way ANCOVA result reported in Table 7 , there was statistically significant adjusted mean different among the groups on the RTF posttest after controlling the RTF pretest ($F(2,160) = 5.29$, $P = 0.006$, *Partial* η^2 =0.062). Regardless of the statistically significant difference, the mean differences between the groups were quite small. This was manifested in the effect size that 6.2% of the variance was explained by the treatment type. A *multiple comparison* results of the RFT posttest, reported in Table 8, shows that a significant difference was detected between the GeoGebra supported multiple representations (MRT) and the multiple representations group

(MR) (P = 0.025) and between the MRT and the comparison group (CG) (P = 0.011) in favor of the MRT. But, no significance difference between the MR and CG (P = 1.000). This result revealed that GeoGebra supported multiple representations approaches brought a better level of sophistication in RTF than the multiple representations approach only and the traditional approach.

Translation among mathematical representations has a powerful influence on the students' concept comprehension and problem solving (Afriyani et al., 2018). However, research results show that students often fail to construct target representation equivalent to the source representation (Adu-Gyamfi et al., 2012; Afriyani et al., 2018; Bossé et al., 2014; Bossé et al., 2011). As a result, the theme of the students' errors in translation is still the center of attention in today's mathematics education research (Adu-Gyamfi, Bossé, & Chandler, 2017; Afriyani et al., 2018; Rahmawati, 2019). Adu-Gyamfi et al. (2012) identified three distinct error types: Implementation Error, Interpretation Error and Preservation Error. These errors manifest at different stages of the translation process. Hence, this study was intended to determine the most committed translation error as a result of the treatment type.

Table 9. Percentage of translation error type of the posttest with group.

Group	Type of Error Committed		
	Implementation	Interpretation	Preservation
	%	%	%
MRT	21	66	13
MR	23	72	4
CG	9	80	11

Students were provided with different tasks of representation translating in the differential calculus content to describe the error type they committed in the process of representation translation. Percentage was used to compare the three groups on these translation error types. Based on the output displayed in Table 9, interpretation error was the most frequently committed across the groups (66% of the errors in the MRT, 73% of the errors in the MR & 80% of the errors in the CG). The implementation error was less frequently appeared across the groups (21% of the errors in the MRT, 23% of the errors in the MR & 9% of the errors in the CG) next to preservation error (13% of the errors in MRT, 4% of the errors in the MR & 11% of the errors in the CG). Despite the distribution of the error among the three groups appeared in a similar pattern, the prevalence of interpretation error was higher in the CG than the MRT and MR, and also higher in MR than in MRT. The students' translation errors in the three groups remained more concentrated in the interpretation error after the intervention. Hence, the treatment type did not bring a magnificent variation on the students' error type committed.

Table 10. Prevalence of translation error type with respect to source representation.

Group	N	Source Representation	Error Type					
			Implementation		Interpretation		Preservation	
			f	%	f	%	f	%
MRT	53	Algebraic	16	30.2	10	18.9	11	11.3
MR	57		12	21.1	12	21.1	11	19.3
CG	54		13	24.1	17	31.5	11	20.4
MRT	53	Graph	5	9.4	5	9.4	23	43.4
MR	57		10	17.5	7	12.3	3	5.3
CG	54		11	20.4	8	14.8	7	13.0
MRT	53	Verbal	9	17.0	6	11.3	7	13.2
MR	57		11	19.3	9	15.8	2	3.5
CG	54		18	33.3	8	14.8	1	1.9
MRT	53	Numerical	9	17	7	13.2	7	13.2
MR	57		13	22.8	10	17.5	3	5.3
CG	54		18	33.3	5	9.3	3	5.6

Based on the results reported in Table 10, in the process of translation from algebraic representation into the other form of representations considerable number of students from the MRT committed implementation error (30.2%). This number of students exceeds to the number of students who committed interpretation error (18.9%) and preservation error (11.3%) of that group. However, error type in the MR is nearly equally distributed among the implementation error (21.1%) , interpretation error (21.1%) and preservation error (19.3%). Some variation was observed in the CG in the error distribution with the greater share for interpretation (31.5%) that implementation error (24.1%) and preservation error (20.4%).

In the process of translation from graph to other form of representations, larger number of students from the MRT committed preservation error (43.4%) than the interpretation error (9.4%) and implementation error (9.4%). However, in the case of the MR, the prevalence of translation error ranges from implementation (17.5%) to interpretation (12.3%) and preservation (5.3%).Likewise, students in the CG who committed implementation error (20.4%) was greater than the interpretation error (14.8%) and preservation error (13.0%).

In the process of translation from verbal to other forms of representations, the number of students who committed were order as implementation error (17.0%), preservation error (13.2%) and interpretation error (11.3%).Similarly, the rate of distribution of translation error in the MR was ordered as implementation error (19.3%) , interpretation (15.8%) and preservation (3.5%). Likewise, greater number of students in the CG committed implementation error (33.3%) than the interpretation error (14.8%) and preservation error (1.9%).

In the process of translation from numerical representation into other form of representations, the number of students in the MRT who committed error was greater than implementation error (17%) than interpretation error (13.2%) and preservation error (13.2%). Similar order was reported in the MR ranged from implementation error (22.8%), interpretation error (17.5%) and preservation error (5.3%). Humongous students in the CG committed implementation error (33.3%) with less number of students who committed interpretation error (9.3%) and preservation error (5.6%).

6. Discussion

Over the past two decades, several efforts had been made to reform calculus curriculum. Multiple representations as method of instruction is highly demanded in the reformed calculus textbook (Chang, Cromley, & Tran, 2016). As a result, students' success in calculus learning is strongly associated with multiple representations abilities. One of these abilities is representational fluency (RF). RF refers to " the ability to create, interpret, translate between, and connect multiple representations " (Fonger, 2019). RF is key mathematical skill that enables to comprehend mathematical concepts. Representation translation fluency (RTF) is the main components of RF that characterize students who excel in mathematics. Due to the advent of multiple interface mathematical technologies, quantity and quality of MRs used in a classroom instruction is increased. GeoGebra is one of the dynamic software with multiple interfaces that support multiple representations. Hence, this study was mainly focused on GeoGebra assisted multiple representations approach on students' performance in RTF in learning calculus. Three intact groups from two Ethiopian universities participated in this study. Jigjig University (JJU) and Kebri-Dehar University (KDU) were purposely selected due to the parallel admitting of first year students in the 2019/20 academic year, the coinciding of time with the approval of proposal, similarity of the universities in terms of geographical and cultural context. One intact class from JJU and two intact classes from KDU were randomly selected as participants of this study. The intact class from JJU was assigned into the GeoGebra assisted multiple representations approach (labeled as MRT) and the two intact classes from KDU were assigned into multiple representations approach (labeled as MR) and comparison group (labeled as CG). The three groups were selected from first year social science stream students who were admitted based on the newly endorsed higher education educational roadmap and registered for the course mathematics for social science.

Pretest and posttest were used as means of data collection tools. The pretest was used to measure between group differences before exposure to the intervention. Any difference on the pretest was controlled using appropriate statistical tools so that the group variation on the posttest was accountable for the treatment type. The data obtained from the data collection tools were made ready for analysis and different statistical methods were used to analyze the quantitative data. In addition to the statistically significant values, the researcher used other indices like, effect size, frequency, percentage, categories and levels to interpret the clinical significance of findings.

The concepts of calculus covered in this study include: limits, continuity, derivative and application of derivative of function of single variable. A pretest was administrated to check the equivalence of the three groups on their performance on RTF just before the intervention had been begun. The contents of the pretest were compiled from function concepts, which are a pre-requisite for calculus. The treatment lasted for about six weeks. Upon the accomplishment of the treatment, a posttest was administrated immediately in the beginning of the seventh week of the treatment. The contents of the posttest were compiled from the contents covered during the intervention but with similar constructs with the pretest. Similarly, the types of translation error committed were categorized into implementation error, interpretation error and preservation error.

The three groups were compared on the RTF posttests using RTF pretest as covariates. The ANCOVA result confirmed that there was statistically significant adjusted mean different among the groups on the RTF posttest after controlling for RTF pretest ($F(2,160) = 5.29$, $P = .006$, $Partial\ \eta^2 = .062$). Regardless of the statistically significant difference, the mean differences between the groups were quite small. This was manifested in the effect size that 6.2% of the variances on the RTF posttest was explained by the treatment type, which was quite small effect size. A *multiple comparison* result shows that the MRT score on the RTF posttest was significantly greater than the MR and CG. But, no difference was detected between the MR and CG. This result revealed that GeoGebra supported multiple representations approaches brought a better level of sophistication in RTF than the multiple representations approach only and the traditional approach. This result align with Bayazit and Aksoy (2010) idea that GeoGebra appears to be a powerful instructional tool that facilitates connection and translation of representations.

To investigate the type of translation error committed in the process of representation translation, the students were provided with different tasks of translating numerical, graphical, algebraic and verbal representations of calculus concepts to delineate the translation error they committed in the process of representation translation. Three distinct types of translation errors were identified that the students were committed and quantified as a frequency and percentage in the process of representation translation (Adu-Gyamfi et al., 2012). Students were provided with different tasks of representation translating from calculus concepts to describe the error type they committed in the process of representation translation. Percentage was used to compare the three groups on these translation error types. The obtained results revealed that interpretation error was the most frequently committed error across the groups (66% of the errors in the MRT, 73% of the errors in the MR & 80% of the errors in the CG). The implementation error was less frequently appeared across the groups (21% of the errors in the MRT, 23% of the errors in the MR & 9% of the errors in the CG) next to preservation error (13% of the errors in MRT, 4% of the errors in the MR & 11% of the errors in the CG). Despite the distribution of the error among the three groups appeared in a similar pattern, the prevalence of interpretation error was higher in the CG than the MRT and MR, and also higher in MR than in MRT. The students' translation errors in the three groups remained more concentrated in the interpretation error after the intervention. Hence, the treatment type did not bring a magnificent variation on the students' error type committed. These results revealed that all of the students in each group committed one or the other type of translation error. That is, no student perfectly constructed target representation from the source representation. Errors were detected in all the translation problems for each student. Altogether, these three error types encapsulated all possible errors that can occur during the translation process. These results conveyed the message that learning calculus concepts with multiple representations serves serious attention not to devastate the learning process of the students. Many contributing factors are involved for the students' difficulties in the process of representation translation. Adu-Gyamfi et al. (2012) confirmed that translation errors are often related to *attribute density*, "the amount of information inherently encoded in a given representation". Van Den Eynde, van Kampen, Van Dooren, and De Cock (2019) reported the influence of context, direction of translation, and function type on undergraduate students' ability to translate between graphical and symbolic representations of mathematical relations. The authors found that items starting from a graph are solved better than those starting from an equation.

To identify which source of representation contributes considerable translation errors across groups, a further analysis was done on each source representation. The analysis results obtained in this study revealed that the translation error types committed across groups vary among the source representation types. Based on the result, when the source representation is algebraic, the MRT committed more of implementation error (30.2%), but the MR and CG committed interpretation error (21.1%) and (31.5%), respectively. When the source representation is graphical, preservation was frequently committed (43.4%) in the MRT. However, the MR and CG committed implementation error (17.5%) and (20.4%), respectively. When the source representation was verbal, the most frequently committed translation error was implementation error, in the MRT (17%), in MR (19.3%) and in the CG (33.3%). Likewise, when the source representation was numerical, the implementation error was the most frequently committed in the MRT (17%), in the MR (22.8%) and in the CG (33.3%).

In relation to these results, Adu-Gyamfi et al. (2012) idea explains that a translation will be successful if elements or constructs expressed in the source representation are successfully articulated via structures available in the target representation. The goal of translation is to preserve *semantic congruence* between the source and target representation. These translation errors are considered as a sign of conceptual comprehension between the source and target representations (Duval, 2006).

7. Conclusion

The GeoGebra assisted multiple representations approach was more supportive for developing RTF in solving calculus problems than the multiple representations method of instruction and the conventional approach. Despite the variation across groups depending on the source representation, interpretation error was the most frequently committed in the process of representation translation across the groups. Hence, each representation type has its own usage for solving a particular problem, and the source representation contributes to the amount of committing translation errors.

8. Recommendations

Hence, the following bits of recommendation were forwarded.

- Mathematics instructors must emphasize multiple representations in calculus classroom beyond the algebraic representation alone.
- Academic officials, course instructors and students have to integrate GeoGebra into their calculus classroom instruction to implement the full "package" of multiple representations to develop cohesive conceptual understanding.
- Instructors and students must use GeoGebra to increase the quality and quantity of multiple representations in calculus learning.
- Further research needs to be conducted to determine the effectiveness of GeoGebra supported multiple representations in learning calculus.
- A further research needs to be conducted to identify the association between the students' representation translation fluency and problem solving skill in calculus learning.
- A further research is required to determine the interacting factors that can influence students' representation preference in solving a calculus problem.

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