Journal of Education and Learning; Vol. 11, No. 6; 2022
ISSN 1927-5250 E-ISSN 1927-5269
Published by Canadian Center of Science and Education

# Relation Between Mathematical Proof Problem Solving, Math Anxiety, Self-Efficacy, Learning Engagement, and Backward Reasoning

Yuno Shimizu[1]

[1] Center for Curriculum Redesign, Hyogo University of Teacher Education, Hyogo, Japan

Correspondence: Yuno Shimizu, School of Management, Center for Curriculum Redesign, Hyogo University of Teacher Education, 942-1, Shimokume, Kato, Hyogo, 673-1494, Japan.

Received: July 23, 2022 Accepted: September 10, 2022 Online Published: September 22, 2022

doi:10.5539/jel.v11n6p62 URL: https://doi.org/10.5539/jel.v11n6p62

**Abstract**

Providing adequate proof is one essential mathematical method, but it is a difficult task for many school-age students. While previous studies have revealed the cognitive factors of proving the proof, the function of affective factors and learning engagement has been largely unexplored. This study examined the effects and processes associated with math anxiety, self-efficacy, engagement, and backward reasoning on the provision of proof when solving math problems about geometric similarity. A survey was conducted on 160 junior high school students, from which it was found that: (a) self-efficacy was indirectly and directly positively associated with the provision of proof when solving math problems and was mediated by math anxiety and backward reasoning; (b) math anxiety was indirectly and negatively associated with the provision of proof when solving difficult proof problems and was mediated by backward reasoning; (c) the use of backward reasoning was positively associated with the provision of proof when solving proof problems; (d) cognitive engagement was indirectly and positively associated with the provision of proof when solving proof problems and was mediated by self-efficacy and backward reasoning. The results suggest that improving cognitive engagement, self-efficacy, and backward reasoning may be effective in the provision of proof when solving proof problems.

**Keywords:** mathematical proof problem solving, math anxiety, self-efficacy, engagement, backward reasoning

## 1. Introduction

STEM (science, technology, engineering, and mathematics) education has become increasingly important over the past decade, with mathematics being the foundation for fully understanding and expressing the various STEM phenomena. Mathematical knowledge and thinking are strong predictors of later academic success in school (Duncan et al., 2007; Siegler et al., 2012; Watts et al., 2014). In addition, attainment in mathematics predicts various adult outcomes, including future earnings (Murnane et al., 1995), STEM occupations (Wang et al., 2015), and risk comprehension and medical decision-making (Reyna et al., 2009). Hence, there have been a lot of fundamental and practical studies aimed at developing mathematical knowledge and thinking (e.g., Mayer, 1998; Hembree, 1992; Papadakis et al., 2017).

One essential mathematical method and activity is the provision of adequate proof, which has been at the core of mathematics since ancient Greece. Thus, the provision of proof is placed as one of the main curriculum components of mathematics learning (Ministry of Education, Culture, Sports, Science, and Technology in Japan, 2008, 2017; National Council of Teachers of Mathematics, 2000; Victorian Curriculum and Assessment Authority, n.d.).

Providing adequate proof is a difficult task for many school-age students (National Institute for Educational Policy Research, 2016, 2017; Recio & Godino, 2001; Senk, 1989; Weber, 2001). For example, the correct answer rate for the descriptive type proof problems in the national mathematical achievement survey of Japanese third-graders in junior high school was only 30% in 2016 and only 45% in 2017 (National Institute for Educational Policy Research, 2016, 2017). Senk (1989) conducted a study in which high school students were asked to prove four geometrical theorems; the average number of theorems the students were able to prove was 1.75, and 29% of the students were unable to prove a single theorem. Previous studies showed that students' difficulties in proving were due to cognitive factors, such as their lack of knowledge and strategies regarding proving (Makino, 2014; Moore, 1994; Weber, 2001). Weber (2001) investigated the provision of proof on the homomorphism for university students and graduate students, finding that their problem solving failed due to a lack of strategic knowledge about the specific techniques needed to provide proof and the importance and usefulness of theorems.

On the other hand, affective as well as cognitive factors may contribute strongly to proof problem solving. In general, previous studies showed that not only cognitive factors such as knowledge and heuristics but also affective factors such as confidence and anxiety determine mathematical problem solving (Hembree, 1992; Mayer, 1998; Schoenfeld, 1992). Furinghetti & Morselli (2004) criticized proof problem solving research for its exclusive focus on cognitive factors and examined the relationship between the provision of proof for problem solving and cognitive and affective factors. The results showed that not only cognitive factors such as lack of algebraic knowledge but also affective factors such as lack of confidence and internal causal attribution were associated with problem solving failure.

However, two important questions remain unresolved in the research to date. First, to our knowledge, there are no quantitative empirical studies that simultaneously address cognitive and affective factors and examine their relationship to the provision of proof for problem solving. The simultaneous consideration of cognitive and affective factors makes it possible to identify factors and processes that explain the provision of proof for problem solving in a detailed and meaningful way. In addition, since Furinghetti & Morselli (2004) was a qualitative case study of one female undergraduate student majoring in mathematics, a quantitative empirical study needs to capture students' tendencies more broadly. Second, no previous studies on the provision of proof for problem solving in Japan have focused on geometric similarity. Previous studies on the provision of proof for problem solving in Japan have focused on character expression and geometric congruence problems (Kunimune, 2017). The Japanese junior high school curriculum guidelines place learning about the provision of proofs for geometric similarity in the third junior high school year (Ministry of Education, Culture, Sports, Science, and Technology in Japan, 2008, 2017), that is, the students are expected to complete mathematical reasoning and argumentation courses as part of their compulsory education. Therefore, clarifying the relationship between the cognitive and affective factors and the provision of proof for problem solving geometric similarity is important to ensure students can properly grasp the concepts.

With this in mind, this study examined the relationships between the provision of proof for problem solving and the cognitive and affective factors, and the impact of learning engagement on these variables. The following section describes the variables discussed in this study and the assumed associated processes.

### *1.1 Math Anxiety*

Math anxiety is defined as the feelings of tension or anxiety that interfere with arithmetical or mathematical problem solving in a wide range of situations, such as everyday life and academic environments (Richardson & Suinn, 1972). Math anxiety can result in avoiding math-related activities and situations and interfere with mathematical problem solving (Barroso et al., 2021; Hembree, 1990; Ma, 1999). For example, math anxiety was negatively correlated with the extent of high school math ($r$ = -.31, Hembree, 1990) and math achievement ($r$ = -.28, Barroso et al., 2021; $r$ = -.27, Ma, 1999). PISA survey showed that many 15-year-olds have math anxiety, with 59% of students worrying that mathematics classes would be difficult for them (OECD, 2013). PISA 2012 also showed that math anxiety explains about 14% of the variance in mathematical literacy (OECD, 2013). Therefore, as math anxiety is a major affective factor in the provision of proof for problem solving, it was considered a factor in this study.

The disruption account is the prevalent theoretical explanation for the negative association of math anxiety with problem solving (Ramirez et al., 2018). This account is that mathematical problem solving is inhibited because cognitive resources are allocated to processing the negative emotions generated by math anxiety. This is also consistent with attentional control theory (Eysenck et al., 2007), in which general anxiety reduces task performance by weakening the inhibitory executive functions. Math anxiety is therefore considered to have a negative impact on mathematical problem solving by preventing the use of cognitive problem solving strategies.

### *1.2 Self-Efficacy*

Self-efficacy is a judgment about one's ability to plan and perform a set of actions to achieve a specific performance (Bandura, 1986). In other words, self-efficacy is the self-belief that a particular action can be performed. In general, self-efficacy is a strong positive predictor of mathematical problem solving. PISA 2012 showed that 15-year-olds with higher self-efficacy were more mathematical literacy and that self-efficacy explained 29% of the variance in mathematical literacy (OECD, 2013). Furthermore, previous studies have shown that self-efficacy positively predicts mathematical problem solving, even when self-concept and prior academic performance are statistically controlled (Jiang et al., 2014; Pajares & Graham, 1999). Therefore, as self-efficacy could be a major affective factor in the provision of proof for problem solving, it was included as a factor in this study.

Self-efficacy is assumed to positively influence mathematical problem solving directly and indirectly through the mediation of math anxiety and problem solving strategies. In control value theory (Pekrun, 2006), self-efficacy is assumed to function as a frame of reference when cognitively evaluating emotions such as math anxiety. In other words, students cognitively process information from the external world based on their self-efficacy regarding a given mathematics task, which increases or decreases math anxiety. Empirical studies consistently show that self-efficacy and math anxiety is negatively correlated (OECD, 2013; Pajares & Kranzler, 1995; Pérez-Fuentes et al., 2020). Self-efficacy may therefore promote problem solving strategies by reducing math anxiety.

### *1.3 Backward Reasoning*

Problem solving strategy is a technique that does not guarantee a solution to a problem but helps students understand the problem and approach the solution (Gick, 1986). In mathematics education, problem solving strategies are referred to as heuristics. Pioneered by Polya (1945), problem solving strategies have received much attention as the dominant cognitive factor in mathematical problem solving. Previous studies showed that problem solving strategies were positively associated with mathematical problem solving (Hembree, 1992; Liljedahl et al., 2016; Schoenfeld, 1992). For example, Hembree's (1992) meta-analysis showed that problem solving strategies and mathematical problem solving were significantly positively correlated ($r$ = .31 for drawing a diagram and $r$ = .20 for using equations, Etc.).

Among the various problem solving strategies, backward reasoning is an effective strategy for the provision of proof for problem solving (Hasegawa & Miwa, 2004; Kirby & Williams, 1991; Polya, 1945, 1965). Backward reasoning is the examination of conditions and rules of reasoning by going back to the conclusion but not the assumption. Polya (1945, 1965) illustrated that backward reasoning is one of the effective problem solving strategies for the provision of proof for problem solving through concrete geometric proof problems. In addition, Hasegawa & Miwa (2004) instructed eighth-graders in backward reasoning and concept maps to determine the proof for figures. Therefore, this study focuses on backward reasoning within problem solving strategies.

As mentioned in 1.1 and 1.2, it is assumed that math anxiety has a negative impact and self-efficacy has a positive impact on problem solving strategies.

### *1.4 Learning Engagement*

Learning engagement is defined as active involvement in learning activities and is considered a variable that captures the quality of the learning commitment (Christenson et al., 2012; Skinner et al., 2009). Learning engagement has three aspects (Fredricks et al., 2004); behavioural engagement, emotional engagement, and cognitive engagement. Behavioural engagement is a construct that indicates attention, effort, and involvement in learning. Emotional engagement is a construct that indicates positive emotional responses such as interest and learning enjoyment. Cognitive engagement is a construct that indicates cognitive participation, such as planning, monitoring learning, and using learning strategies. This study included behavioural, emotional, and cognitive engagement positioning engagement for the learning of geometric similarity.

Learning engagement is assumed to be a predictor of self-efficacy, math anxiety, backward reasoning, and the provision of proof for problem solving. Previous studies showed that learning engagement consistently positively predicts math performance (Fung et al., 2018; Shimizu, 2020; Putwain et al., 2018; Reeve & Tseng, 2011). Learning engagement is an essential variable that understands, predicts, and promotes not only academic performance but also affective factors such as psychological need and self-efficacy (Reeve & Lee, 2014; Umemoto & Ito, 2016). In other words, learning engagement positively may influence self-efficacy and negatively influences math anxiety. Furthermore, because learning engagement contributes to improved cognitive abilities such as creativity and metacognitive awareness (Sun et al., 2021), it is thought to affect backward reasoning, a type of problem solving strategy, positively.

### *1.5 Framework and Purpose of This Study*

Based on the previous discussion, this study examined the effects and processes associated with math anxiety, self-efficacy, engagement, and backward reasoning on the provision of proof when solving math problems about similar figures. The following hypotheses were formulated based on previous studies.

(Hypothesis 1) Math anxiety is negatively related to the use of backward reasoning and the provision of proof for problem solving.

(Hypothesis 2) Self-efficacy is negatively related to math anxiety and positively related to backward reasoning and the provision of proof for problem solving.

(Hypothesis 3) The use of backward reasoning is positively related to the provision of proof for problem solving.

(Hypothesis 4) Engagement is negatively related to math anxiety and positively related to self-efficacy, backward reasoning, and the provision of proof for problem solving.

This study tested a hypothetical model (Figure 1), which was constructed based on the above hypotheses. The hypothetical model (Figure 1) assumed a direct path from self-efficacy and math anxiety to the provision of proof for problem solving, as self-efficacy and math anxiety may influence the provision of proof for problem solving through factors other than backward reasoning.

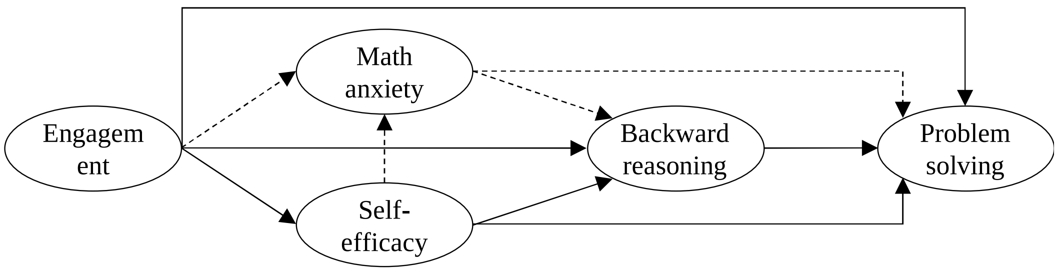


Figure 1. Hypothetical model of this study

*Note*. Solid lines represent positive relations; dashed lines represent negative relations.

## 2. Method

### *2.1 Participants and Procedures*

The participants in this study were 160 eighth-grade students enrolled in private junior high school A in the Tokyo area. All participants were ethnic Japanese. The participants had already learned geometric similarity by the time of the survey but had not yet learned the properties of circumferential angles or the Pythagorean Theorem.

Before the survey, the purpose of the research was explained to the math teacher at school A, and the teacher asked to cooperate with the research. After obtaining the teacher's consent for the survey, the survey was conducted during self-study time in math class. As part of the survey, the following were clearly stated: (i) the responses to the survey were voluntary and not related to class math grade; (ii) the participant's privacy is protected because the investigation content is statistically processed; (iii) the results from the provision of proof for a problem solving section are feedback to the participant through the teacher in charge; (iv) the researcher is responsible for disposing of the questionnaires other than the provision of proof for problem solving. The author also explained the subject orally. Informed consent was obtained from all participants.

The survey was administered in mid-February 2020 in a self-study time in math class at school A. The survey measured math anxiety, self-efficacy, and engagement, the provision of proof for problem solving, and backward reasoning. The response time was about 10 minutes for the math anxiety, self-efficacy, and engagement sections, about 40 minutes for the provision of proof for the problem solving section, and about 3 minutes for the backward reasoning section. The survey was administered anonymously.

### *2.2 Instruments*

#### 2.2.1 Self-Efficacy

Two items were used to measure self-efficacy. Based on Pajares & Miller (1994), the instrument asked students to indicate their confidence in providing proof for problems (problems 1 and 2). Students rated the items on a 6-point Likert scale ranging from "not confident at all" (1) to "completely confident" (6) for each proof problem.

#### 2.2.2 Math Anxiety

Math anxiety was measured using the five-item measure from the PISA 2012 study (OECD, 2013). An example of an item was "I get very nervous doing mathematics problems." Students rated the items on a 6-point Likert scale ranging from "completely disagree" (1) to "completely agree" (6).

#### 2.2.3 Backward Reasoning

As far as could be determined, there is no scale to measure backward reasoning. Therefore, the procedure in Seo

(2005) was referred to, which was a scale that asks about the tendency to use “strategies to check failures in problem solving,” which is one of the problem solving strategies in mathematics. A scale was developed that specifically asked about the tendency to use backward reasoning when seeking proof for a problem. The scale was developed in two steps. First, a draft of the questionnaire items was developed by consulting the junior high school mathematics textbooks that mention backward reasoning (Suken Shuppan, 2015, 2016; Tokyo Shoseki, 2012). Secondly, two mathematics teachers from junior high and high schools judged whether the questionnaire item draft was appropriate for junior high school students and corrected the difficult expressions to understand. After following the above procedure, a questionnaire with four items was developed. The students rated the items on a 6-point Likert scale ranging from “completely disagree” (1) to “completely agree” (6) for each proof problem.

### 2.2.4 Learning Engagement

Twelve items were used to measure learning engagement. These items were adopted from Shimizu’s (2020) learning engagement scale, which was based on Skinner et al.’s (2009) scale for behavioural and emotional engagement and Reeve & Tseng’s (2011) scale for cognitive engagement. In addition, Shimizu’s (2020) learning engagement scale has adequate convergent and discriminant validity and reliability. The scale included the following three subscales: behavioural engagement ($n$ = 4; e.g. ‘I work as hard as I can on tasks in geometric similarity’), emotional engagement ($n$ = 4; e.g. ‘I enjoy learning geometric similarity’), and cognitive engagement ($n$ = 4; e.g. ‘I try to devise a learning method in geometric similarity). The students rated the items on a 6-point Likert scale ranging from “completely disagree” (1) to “completely agree” (6).

### 2.2.5 Proof Problems

In this study, two proof problems were developed based on basic examples in the junior high school mathematics textbook, one of which was developed from an applied end-of-chapter problem. One of the problems in this study was not included in the analysis of this study because the correct answer rate was 92%, and most of the subjects answered it correctly. The following describes the problems used in the analysis for this study.

(1) Problem 1: Figure 2 shows problem 1. This problem was developed based on basic examples in textbooks (Suken Shuppan, 2015; Tokyo Shoseki, 2012). The condition for similar triangles, the “AAA (Angle-Angle-Angle) rule,” is used to show that △ADE and △FBD are similar. Of the two sets of angles, ∠DAE and ∠BFD are 90°, which is easy to find because it is shown in the problem statement and figure. The corresponding angle ∠ADE and ∠FBD are equal to the other angles because the DE and BC sides are parallel.

In the figure, ABC is a right-angled triangle with ∠A = 90° . In the given △ABC, points D and E are parallel between segment DE and segment BC on segment AB and segment AC. In addition, point F is the intersection point between the perpendicular line from point D to segment BC. Prove that △ADE ∽△FBD.

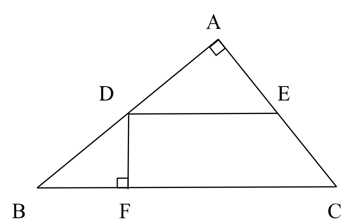


Figure 2. Problem 1

(2) Problem 2: Figure 3 shows problem 2. This problem was developed based on an applied end-of-chapter problem in the textbook (Suken Shuppan, 2016). Since $EB^2 = AB \times FB$ and EB: AB = FB: EB is equivalent, students need to show that △EBF and △ABE are similar. Students need to use the AAA rule to show that △EBF and △ABE are similar. Of the two sets of angles, ∠EBF and ∠ABE are common, so it is easy to find. The other set of angles shows that ∠BEF and ∠BAE are equal. To show this, students need to use the fact that AE is the bisector of ∠BAC and that △ABC and △ADE are similar, meaning that △ADF and △EBF are similar.

Problem 2 was more difficult than problem 1 because of the complexity in finding and proving that the figure of interest indicates that $EB^2 = AB \times FB$.

In the figure, △ABC ∽△ADE and ∠BAC = 80° . If segment AE is the bisector of ∠BAC and point F is the intersection point between segment AB and segment DE, then prove that $BE^2 = AB \times FB$.

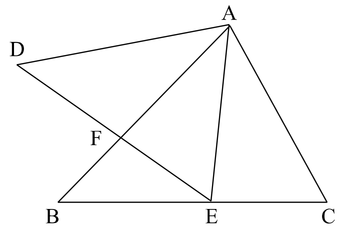


Figure 3. Problem 2

*2.3 Data Analysis*

There were three main steps in the data analysis. First, the evidence for the provision of proof for problems 1 and 2 (Tables 1 and 2) was extracted based on the National Institute for Educational Policy Research (2016, 2017) and Shimizu (2020). As in Shimizu (2020), when the evidence and the reason were correctly mentioned, it was coded 1, and when the evidence and reason were incorrectly mentioned or there was no answer given, it was coded 0. Then, the respective pass rates and IT correlations (Item-Total correlations) were calculated. In this study, IT correlations were used for item discrimination.

Table 1. Scoring criteria and correct answer example for problem 1

| Scoring criteria | Correct answer example |
|---|---|
| The following evidence (a) through (c) and their reasons shall be clearly stated. | In △ADE and △FBD |
| 【Evidence】 | ∠DAE = ∠BFD = 90°(Given) … (1) |
| (a) ∠DAE=∠BFD | Since DE // BC, ∠ADE = ∠FBD (Corresponding angle) … (2) |
| (b) ∠ADE=∠FBD | From (1) and (2), △ADE ∽△FBD (AAA rules) |
| (c) △ADE∽△FBD | |
| 【Evidence`s reason】 | |
| (a) Given | |
| (b) Corresponding angle | |
| (c) AAA rules | |

Table 2. Scoring criteria and correct answer example for problem 2

| Scoring criteria | Correct answer example |
|---|---|
| The following evidence (a) through (d) and their reasons shall be clearly stated. | In △ADF and △EBF |
| 【Evidence】 | ∠ADF = ∠EBF (△ABC ∽△ADE) … (1) |
| (a) ∠BAE=∠BEF | ∠AFD = ∠EFB (Vertically opposite angles) … (2) |
| (b) ∠ABE=∠EBF | From (1) and (2), △ADF ∽△EBF (AAA rules) |
| (c) △ABE∽△EBF | So, ∠DAF = ∠BEF … (3) |
| (d) $BE^2$=AB×FB | In △ABE and △EBF |
| 【Evidence`s reason】 | ∠ABE = ∠EBF (Common) … (4) |
| (a) ∠DAF = ∠BEF and ∠DAF = ∠DAE − ∠BAE = 40° | ∠BAE = 40°(Given) … (5) |
| (b) Common | ∠DAE = 80°(Given) … (6) |
| (c) AAA rules | From (5) and (6), ∠DAF = ∠DAE − ∠BAE = 40°… (7) |
| (d) △ABE ∽ △EBF | From (3) and (7), ∠BAE = ∠BEF = 40°… (8) |
| | Since (4) and (8), △ABE ∽ △EBF (AAA rules) |
| | So, AB: EB = BE: BF |
| | Hence, $BE^2 = AB \times FB$ |

Second, as the backward reasoning was measured on an original scale developed for the study, exploratory factor analysis (minimum residuals method and Oblimin rotation) was conducted to examine the scale's structure. Then, the ω coefficients were calculated to confirm the internal consistency of the scale.

Third, the arithmetic means of each item were adopted as the subscale scores. Then path analysis (maximum likelihood method) was conducted based on Figure 1 to examine the effects and processes associated with math anxiety, self-efficacy, engagement, and backward reasoning on the provision of proof when solving math problems. Positive covariance between behavioural, emotional and cognitive engagement was assumed in the path analysis because positive correlations were found between them (Reeve & Tseng, 2011; Shimizu, 2020).

The significance level of the path coefficients was set at 5%, based on convention.

To deal with the missing values, the path analysis was complemented by the full information maximum likelihood method, and the other analyses were pairwise processed. The data analysis was performed in R4.1.1, an open software environment.

## 3. Results

### *3.1 Results for the Provision of Proof for Problem solving*

The results for problem 1 are shown in Table 3. The pass rates for the items ranged from 65% to 94%, and the IT correlations ranged from.68 to.84, which indicated that there was a strong positive correlation between each item and the total number of correct items. Therefore, in this study, the total number of correct items was used as the scale score for problem 1 (0 to 6); the higher the score, the better the students were able to solve problem 1 correctly. The percentage of students who solved problem 1 correctly was 65%.

Table 3. Result of problem 1 ($N$ = 159)

| | pass rate | IT correlations |
|---|---|---|
| description of "∠DAE=∠BFD" | 94% | .71 |
| reason of "∠DAE=∠BFD" | 90% | .68 |
| description of "∠ADE=∠FBD" | 89% | .82 |
| reason of "∠ADE=∠FBD" | 70% | .83 |
| description of "△ADE∽△FBD" | 90% | .80 |
| reason of "△ADE∽△FBD" | 65% | .84 |

The results for problem 2 are shown in Table 4. The pass rates ranged from 19% to 28%, and the IT correlations ranged from.88 to.97, which indicated that there was a strong positive correlation between each item and the total number of correct items. Therefore, in this study, the total number of correct items was used as the scale score for problem 2 (0 to 8); the higher the score, the better the students were able to solve problem 2 correctly. Only 18% of students solved problem 2 correctly.

Table 4. Result of problem 2 ($N$ = 159)

| | pass rate | IT correlations |
|---|---|---|
| description of "∠BAE=∠BEF" | 28% | .92 |
| reason of "∠BAE=∠BEF" | 25% | .88 |
| description of "∠ABE=∠EBF" | 26% | .95 |
| reason of "∠ABE=∠EBF" | 26% | .93 |
| description of "△ABE∽△EBF" | 23% | .97 |
| reason of "△ABE∽△EBF" | 19% | .92 |
| description of "$BE^2$=AB×FB" | 21% | .94 |
| reason of "$BE^2$=AB×FB" | 20% | .93 |

### *3.2 Exploratory Factor Analysis for Back-Ward Reasoning*

Exploratory factor analysis was conducted on the back- ward reasoning scale for each problem. Based on the Kaiser and MAP criteria, a one-factor solution was deemed appropriate for both problems, the analysis results for which are shown in Table 5. Because the factor contained all items and the factor loading was positive for both problems, this study named the factor "backward reasoning." The backward reasoning scale demonstrated high internal consistency (problem 1, ω = .80; problem 2, ω = .91) for both problems. Therefore, the average scores from the four-item scale were used to measure the backward reasoning for each problem.

Table 5. Results of exploratory factor analysis of backward reasoning (*N* = 159)

| | Problem1 | | Problem2 | |
|---|---|---|---|---|
| | Loadings | $h^2$ | Loadings | $h^2$ |
| I thought about what was necessary to conclude | 0.77 | 0.59 | 0.87 | 0.76 |
| I thought through the conclusion about what was good to be shown | 0.70 | 0.49 | 0.90 | 0.81 |
| I thought how to solve it before I solved the problem | 0.69 | 0.48 | 0.82 | 0.66 |
| I thought about what should be done to make the conclusion correct | 0.68 | 0.47 | 0.76 | 0.58 |
| Explained Variance | 0.51 | | 0.71 | |

### *3.3 Descriptive Statistics and Correlations*

The means, standard deviations, ω coefficients, and bivariate correlations between the study variables are shown in Table 6. Because the self-efficacy scale contained two items, it was not possible to calculate the omega-coefficient for the scale.

Table 6. Descriptive statistics and correlation matrix for each scale

| | *N* | *M* | *SD* | ω | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. behavioural engagement | 159 | 4.40 | 0.84 | .90 | − | | | | | | | |
| 2. emotional engagement | 153 | 3.96 | 1.02 | .92 | .63 | − | | | | | | |
| 3. cognitive engagement | 158 | 4.19 | 0.83 | .81 | .72 | .62 | − | | | | | |
| 4. self-efficacy | 157 | 3.76 | 1.00 | − | .41 | .46 | .53 | − | | | | |
| 5. math anxiety | 158 | 3.34 | 0.92 | .81 | -.37 | -.57 | -.41 | -.46 | − | | | |
| 6. backward reasoning in problem 1 | 159 | 4.75 | 0.81 | .80 | .48 | .37 | .36 | .36 | -.35 | − | | |
| 7. backward reasoning in problem 2 | 159 | 3.98 | 1.43 | .91 | .28 | .27 | .27 | .46 | -.38 | .32 | − | |
| 8. problem1 solving | 159 | 4.99 | 1.65 | .88 | .29 | .18 | .21 | .22 | -.17 | .41 | .29 | − |
| 9. problem 2 solving | 159 | 1.89 | 3.16 | .98 | .12 | .15 | .16 | .44 | -.25 | .18 | .53 | .24 |

### *3.4 Path Analysis*

The goodness of fit indices for the hypothetical model (Figure 1) were good (CFI = 1.00, TLI = .96, RMSEA = .06, SRMR = .02, AIC = 3708.02, and BIC = 3833.62). However, as there were some insignificant paths, the analysis was conducted while deleting these paths. The results for the re-analysis are shown in Figure 4. The goodness of fit indices for the model in Figure 4 were good (CFI = 1.00，TLI = 1.00, RMSEA = .00, SRMR = .05, AIC = 3686.08, and BIC = 3754.86). Because the model with the smallest AIC and BIC is selected in model selection (West et al., 2012), this study accepted the model in Figure 4.

The results in Figure 4 revealed that: self-efficacy was negatively related to math anxiety (β = -.26) and positively related to backward reasoning for both problems and the problem 2 solutions (respectively, β = .24, .34, and .25); math anxiety was negatively related to backward reasoning in problem 2 (β = -.21); backward reasoning for each problem was positively related to problem solving (respectively, β = .34, .41); behavioural engagement was positively related to backward reasoning in problem 1 (β = .37); emotional engagement was negatively related to math anxiety (β = -.44); and cognitive engagement was positively related to self-efficacy (β = .53). The model explained 27% of the variance in self-efficacy, 34% of the variance in math anxiety, 26% of the variance in backward reasoning in problem 1, 22% of the variance in backward reasoning in problem 2, 12% of the variance in the problem 1 solutions, and 32% of the variance in the problem 2 solutions.

Because it was assumed that engagement, self-efficacy, and math anxiety had indirect effects on problem solving, a mediation analysis (bootstrap method: number of resampling = 5000) was conducted, the results for which are shown in Table 7. Behavioural and cognitive engagement and self-efficacy were found to have positive indirect effects on the provision of proof for solving problem 1 (respectively, β = .13, .05, and .08); emotional and cognitive engagement and self-efficacy had positive indirect effects on the provision of proof for solving problem 2 (respectively, β = .04, .22, and .16), and math anxiety had a negative indirect effect on the provision of proof for solving problem 2 (β = -.09).

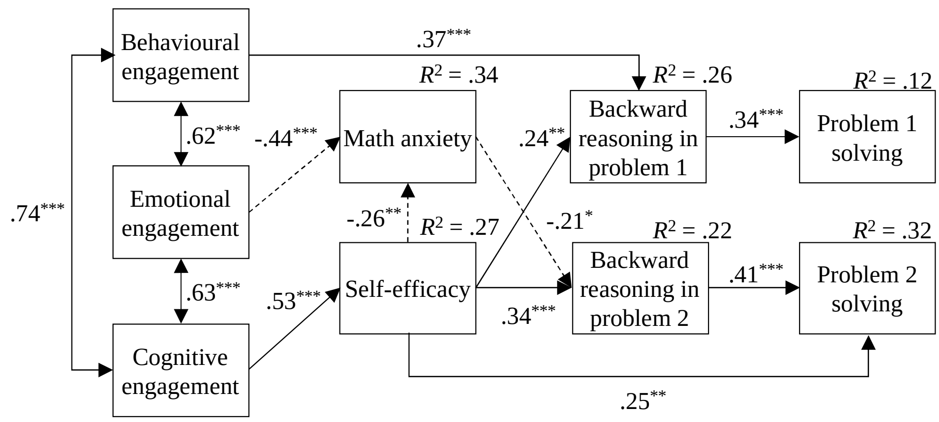


Figure 4. Result of path analysis ($N$ = 147)

*Note*. Solid lines represent positive relations; dashed lines represent negative relations. $^{***}$ $p < .001$, $^{**}$ $p < .01$, $^{*}$ $p < .05$

Table 7. Result of indirect effect ($N$ = 147)

| | $\beta$ |
|---|---|
| behavioural engagement → problem 1 solving | .13$^{**}$ |
| cognitive engagement → problem 1 solving | .05$^{*}$ |
| self-efficacy → problem 1 solving | .08$^{*}$ |
| emotional engagement → problem 2 solving | .04$^{*}$ |
| cognitive engagement → problem 2 solving | .22$^{***}$ |
| self-efficacy → problem 2 solving | .16$^{***}$ |
| math anxiety → problem 2 solving | -.09$^{*}$ |

*Note*. $^{***}$ $p < .001$, $^{**}$ $p < .01$, $^{*}$ $p < .05$

## 4. Discussion

This study examined the effects and processes of math anxiety, self-efficacy, engagement, and backward reasoning on the provision of proof for solving math problems. The following section interprets the results in this study in concert with findings in previous studies and outlines the possible implications for practice.

### *4.1 Math Anxiety and the Provision of Proof for Solving Problems*

This study found that math anxiety negatively affected problem solving through backward reasoning for problem 2 but not for problem 1, which partially supported hypothesis 1. This result was not anticipated; therefore, it was surmised that these results were probably because of the complexity of the problem. Ashcraft et al. (2007) pointed out that the higher the complexity of a problem, the stronger the association between math anxiety and problem solving. In addition, Hoffman (2010) showed that math anxiety was negatively correlated with hard mental multiplication but not with easy mental multiplication. Because problem 2 had lower pass rates than problem 1 in this study, it was therefore possible that the complexity in problem 2 was higher than in problem 1. Therefore, as math anxiety had a negative indirect problem solving effect only for problem 2, problem difficulty could be a moderator in the relationship between math anxiety and problem solving, as had also been found for calculation problems (Hoffman, 2010).

### *4.2 Self-Efficacy and the Pprovision of Proof for Solving Problems*

Self-efficacy was found to positively affect problem solving through math anxiety and backward reasoning, which supported hypothesis 2. This finding was in line with previous studies that found that self-efficacy was positively correlated with mathematical literacy and problem solving (OECD, 2013; Pajares & Graham, 1999; Williams & Williams, 2010). The standardized path coefficient values for problem solving were greater for self-efficacy than for math anxiety; therefore, as reported in Pajares & Graham (1999), these results suggested that self-efficacy was an affective factor that contributed more to mathematical problem solving than math anxiety. Put differently, teaching and learning that improves self-efficacy rather than promoting math anxiety could positively affect the provision of proof for problem solving.

A direct positive correlation was also found between self-efficacy and solving problem 2. The result indicates the potential presence of achievement emotions and problem solving strategies as mediator variables between self-efficacy and the provision of proof for solving problem 2. Therefore, a model that includes achievement emotions and problem solving strategies could better explain the provision of proof for solving problem 2. For example, enjoyment and ‘guess and test’ are positive predictors of mathematical problem solving (Bailey et al., 2014; Hembree, 1992).

*4.3 Backward Reasoning and the Provision of Proof for Solving Problems*

This study found that backward reasoning positively affected the problem solving for both problems, which supported hypothesis 3 and was in line with previous findings that found that backward reasoning was a practical problem solving strategy (Hasegawa & Miwa, 2004; Kirby & Williams, 1991; Polya, 1945, 1965). The contribution of backward reasoning was more noticeable for problem 2 than for problem 1. In problem 2, the students needed to notice that $\triangle$EBF and $\triangle$ABE were similar as $EB^2 = AB \times FB$ and EB: AB = FB: EB were equivalent. Therefore, it was likely that the students who noticed that $\triangle$EBF and $\triangle$ABE were similar used backward reasoning to solve problem 2 correctly, which implies that backward reasoning could be instrumental in problems where the conclusion needs to be rephrased, such as problem 2. However, the contribution of backward reasoning may be less used in problems where it is not necessary to rephrase the conclusion, as in problem 1.

An association was found between math anxiety and backward reasoning for problem 2 but not for problem 1, which suggested that the association between math anxiety and backward reasoning was stronger for more difficult problems, that is, the more difficult the problem, the more significant the math anxiety, which inhibits the use of backward reasoning.

*4.4 Engagement and the Provision of Proof for Solving Problems*

There were three findings on the relationship between engagement and the provision of proof for problem solving. First, cognitive engagement was found to have a positive indirect effect on problem solving for both problems through self-efficacy, which supported hypothesis 4 for cognitive engagement and was consistent with previous studies (Fung et al., 2018; Shimizu, 2020). Fung et al. (2018) claimed that cognitive engagement is strongly related to achievement because it relates to various aspects of learning and is an indicator of learning motivation. In addition, Shimizu (2020) found that cognitive engagement directly predicted the provision of proof for problem solving. The findings in this study suggested that this process could be mediated by self-efficacy, math anxiety, and backward reasoning. Second, behavioural engagement positively affected the provision of proof for solving problem 1 through backward reasoning, which partially supported hypothesis 4. As problem 1 was based on a basic example from the textbook (Suken Shuppan, 2015; Tokyo Shoseki, 2012) the students were using, the students might have worked on similar problems. Therefore, behavioural engagement and solving problem 1 could be related. Third, emotional engagement had a positive effect on the provision of proof for solving problem 2 through math anxiety, which partially supported hypothesis 4. This result suggested that emotional engagement could improve the problem solving difficulties related to math anxiety. Taken together, to promote the provision of proof for solving problems, teaching and learning that increases cognitive engagement and other engagement aspects could be effective.

*4.5 Implications for Educational Practice*

The findings in this study have at least three implications for educators seeking to improve the provision of proof for problem solving. First, as encouraging backward reasoning is essential in the provision of proof for solving problems, rather than just explaining it as was done in Hasegawa and Miwa (2004), it may be more useful to incorporate this strategy into actual problem solving situations. Second, the findings suggested that increasing self-efficacy rather than math anxiety could improve backward reasoning and the provision of proof for problem solving. Usher & Pajares (2008) pointed out that mastery experience was the most crucial source of self-efficacy; therefore, educators should encourage their students to proactively use backward reasoning to provide the proof to solve problems. Third, cognitive engagement was found to be associated with self-efficacy, backward reasoning, and the provision of proof for solving problems. Yoshida & Murayama (2013) suggested that educators needed to explicitly teach effective learning strategies to students because students generally misperceive learning effectiveness. Therefore, explicitly training students to use learning strategies would be a useful practice when they are learning geometric similarity.

*4.6 Limitations*

This study had three limitations. First, a statement of clear causality was not possible. As this study was a correlation study at a one-time point, there is no certainty that the necessary causality condition that the independent variable precedes the dependent variable in time was satisfied. Therefore, more comprehensive longitudinal studies are needed to identify the causality more precisely. Second, there was a low value for the coefficient of de-termination for the provision of proof for problem solving (problem 1 was .12; problem 2 was .32). Therefore, it is necessary to examine the factors that might better explain the provision of proof for problem solving. For example, knowledge of figures, proofs, and strategies other than backward reasoning may better explain the provision of proof for problem solving. Third, there is still a need for further testing the validity of the backward reasoning scale developed in this study. Based on Messick's (1995) validity framework, this scale had content, structure, generalizability, and external aspect evidence; however, there was no evidence for the substantive and consequential aspects. Therefore, further work is needed to test the validity of this scale. For example, it would be desirable to conduct follow-up tests on other samples and examine the relationships with online scales, such as think-aloud protocols.

**Informed Consent Statement**

Informed consent was obtained from all students involved in the study.

**Compliance with Ethical Standards**

The study was conducted according to the guidelines of the Declaration of Helsinki. The author declares that data compiled for this study and its analysis met those ethical requirements.

**Conflicts of Interest**

The author declares no conflict of interest.

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